Rendering polynomial curves

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Bezier curve

Charles Loop and *Jim Blinn* give a new way in rendering Bezier curve on GPU, Article on GPU Gems3

To render curve in high effecicy way without tessellation into many triangles, the key is to convert parametric curve$(x(t), y(t))$ to implicit function $f(x, y) = 0$. In their paper, Loop and Blinn use a very old result (Salmon 1852), It's only limited on Bezier curve, here i give a general way to show how to get the implicit equation for any general polynomial curve, so we can apply it on rendering other curves.

B(t) is defined by a set of control points, let P_i be the comtrol point, then B(t) is

$$\begin{aligned}\mathbf{B}(t) &= \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \mathbf{P}_i \\ &= (1-t)^n \mathbf{P}_0 + \binom{n}{1} (1-t)^{n-1} t \mathbf{P}_1 + \cdots \\ &\quad \cdots + \binom{n}{n-1} (1-t) t^{n-1} \mathbf{P}_{n-1} + t^n \mathbf{P}_n, \quad t \in [0,1],\end{aligned}$$

Let f_1= $\sum a_{1,i}t^i$, f_2= $\sum a_{2,i}t^i$, i=0, ... n

$deg(f_1) = deg(f_2) =$ n

Let $A(f_j, i) = a_{j,i}$

We define polynomial $k_2$and $f_3$as below:

f_1=f_2k_2+ f_3, $deg(k_2) = 0$, $deg(f_3) = n - 1$

k_2= $\frac{a_{1,n}}{a_{2,n}} = \frac{A(f_1,n)}{A(f_2,n)}$

again, we define $k_3$and $f_4$as below:

f_2=f_3k_3+ f_4, $deg(k_3) = 1$, $deg(f_4) = n - 2$

k_3= $\frac{A(f_2,n-1)}{A(f_3,n-1)}t$

generially, we have

f_j=f_{j-2}-$f_{j-1}k_{j-1}$

k_j= $\frac{A(f_{j-1},n-j+2)}{A(f_j,\ n-j+2)}t$

f_j=f_{j-2}- $f_{j-1}k_{j-1}$ $= \sum (a_{j-2,i} - a_{j-1,i-1}k_{j-1})\, t^i$, i=0...n-j+1

if f_{j-2}= $f_{j-1} = 0$, then f_j=0

Now let

f_1=$\sum a_{1,i}t^i$ - x

f_2=$\sum a_{2,i}t^i$ - y

Apply above steps on $f_1$and f_2, then we can get a $f_m, deg(f_m) = 0$, in each step, each factor of t^i will be a const or a linear function of x and y, so when we got f_m=0, we will get an equation of x and y.

Here we apply above steps on quadratice splines:

$f_1 = a_{1,2}t^2 + a_{1,1}t + a_{1,0} - x$

$f_2 = a_{2,2}t^2 + a_{2,1}t + a_{2,0} - y$
$f_3 = a_{3,1}t + a_{3,0}, \quad a_{3,0} = (a_{1,0} - x) - (a_{2,0} - y)\frac{a_{1,2}}{a_{2,2}}$
$f_4 = a_{4,0}$
$a_{4,0} = (a_{2,0} - y) - \frac{a_{2,2}}{a_{3,1}}a_{3,0}$

so finally we can get the implicit equation $f(w_1, w_2, w_3) = 0$, each $w_i$is linear combination of x and y. $w_i = c_{i,0}x + c_{i,1}y + c_{i,2}$

to be continued...