

# Hydrodynamic load exerted on a moving bundle of hollow fibers using constraint dissipative hydrodynamics: Hydro-Rattle simulation

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# Outline

- 1** Introduction
  - Overview
  - Research Task
- 2** Theory and Simulations
  - Equivalent Sphere
  - Constraint Dynamics
  - Dissipative Hydrodynamics (DHD)
- 3** Results and Discussions
  - Hollow Fiber Specifications
  - Chain Dynamics Animation
  - Constraint Force on Short Chain
  - Constraint Force on Long Chain
- 4** Concluding Remarks
  - Conclusions
- 5** Acknowledgment
  - Thank You

## Submerged Membrane Bioreactor: Some Cases



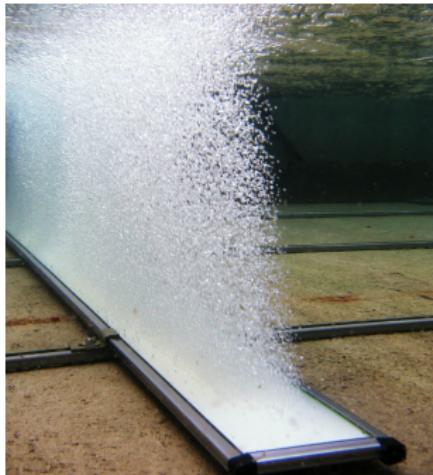
Hon. WWWTP, future MBR for greenhouse gas emissions (↓)



EnoQua Membrane Bioreactor



# How to Reduce HF Fouling: Blowing → SHAKING



AERO\*\*\*\*\*



Kolon, LENA

## ■ "Low Energy No Aeration" (LENA)

- 1 a new *counter-intuitive* membrane technology:
- 2 energy saving:  $\approx 25\%$
- 3 shaking consumes less energy than blowing.
- 4 video 1 & video 2

# Does Dynamics of Hollow Fiber Follow Wave Equation? NO!



(a) LENA

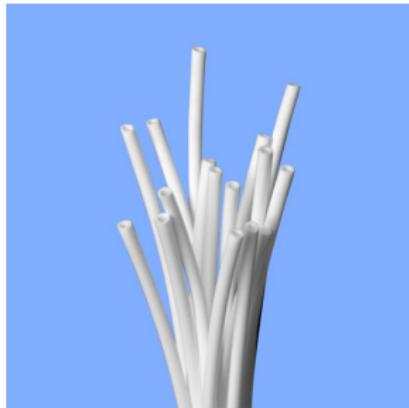
In (b), string image<sup>1</sup>



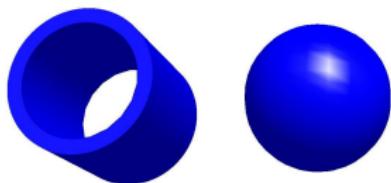
(b) Vibrating guitar strings

<sup>1</sup><https://www.sciencesource.com/archive/Vibrations-in-Guitar-Strings-SS2781967.html>

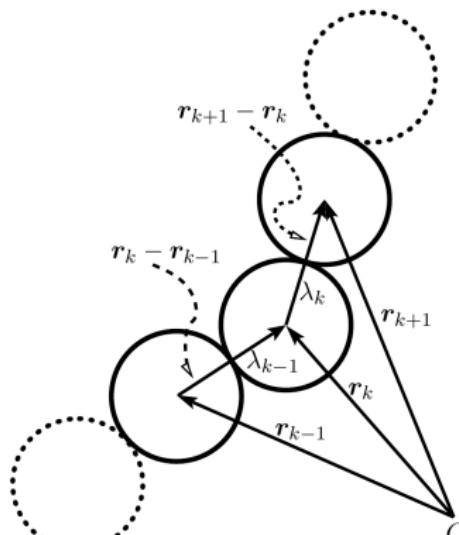
# Hollow Fiber Segment vs. Equivalent Sphere



(a) fiber bundle

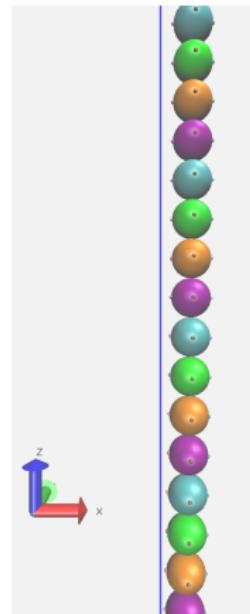
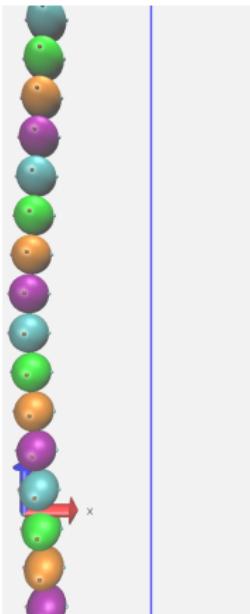


(b) fiber seg. & equiv. sphere



(c) fiber as linked spheres

## Hollow Fiber Segment vs. Equivalent Sphere (cont'd)



- Sphere colors are just for visualization
- Black dots (6 per sphere) is to see sphere rotation.

## Theoretical approaches



Holonomic constraints: center-to-center distance

$$\sigma_k = (\mathbf{r}_k - \mathbf{r}_{k+1})^2 - d_{k,k+1}^2 = 0 \quad (1)$$

Non-Holonomic constraints: position-velocity orthogonality

$$(\mathbf{r}_k - \mathbf{r}_{k+1}) \cdot (\mathbf{v}_k - \mathbf{v}_{k+1}) = 0 \quad (2)$$

Total Force = External + Internal

**1** External:

- 1** gravitational force (vector-wise)
- 2** hydrodynamic force/torque (tensor-wise)

**2** Internal

- 1** holonomic
- 2** non-holonomic constraint forces

## Constraint Forces



### Holonomic Constraint Force on sphere $k$

$$m_k \mathbf{a}_k^c = \lambda_{k-1} (\mathbf{r}_{k-1} - \mathbf{r}_k) - \lambda_k (\mathbf{r}_k - \mathbf{r}_{k+1}) \quad (3)$$

where  $\lambda$  is the Lagrange multiplier, to be instantaneously updated as responding to non-internal forces.

### Non-holonomic Constraint Force on sphere $k$

$$m_k \mathbf{a}_k^n = \kappa_{k-1} (\mathbf{r}_{k-1} - \mathbf{r}_k) - \kappa_k (\mathbf{r}_k - \mathbf{r}_{k+1}) \quad (4)$$

has the same mathematical form, but  $\kappa \neq \lambda$ .

### Constraint Dynamics Simulation

- How to efficiently calculate/update  $\lambda$  and  $\kappa$  for evol.

## Gravitational + Buoyant Forces

$$\mathbf{F}_j^G = \Delta m_j \mathbf{g} \quad (5)$$

where  $\Delta m_j = m_j - m_f$ , and  $m_f$  is the mass of fluid.

## Hydrodynamic Forces/Torques ← Stokesian Dynamics

$$\begin{bmatrix} \mathbf{U}^\infty - \mathbf{v}_j \\ \boldsymbol{\Omega}^\infty - \boldsymbol{\omega}_j \\ \mathbf{E}^\infty \end{bmatrix} = \sum_{k=1}^{N_p} \begin{bmatrix} \mathbf{a}_{jk} & \tilde{\mathbf{b}}_{jk} & \tilde{\mathbf{b}}_{jk} \\ \mathbf{b}_{jk} & \mathbf{c}_{jk} & \tilde{\mathbf{h}}_{jk} \\ \mathbf{g}_{jk} & \mathbf{h}_{jk} & \mathbf{m}_{jk} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{F}_k^H \\ \mathbf{T}_k^H \\ \mathbf{S}_k^H \end{bmatrix} \quad (6)$$

where  $[\mathbf{F}_k^H, \mathbf{T}_k^H, \mathbf{S}_k^H]^T$  is hydrodynamic force, torque, stresslet vectors, in an ambient flow field:

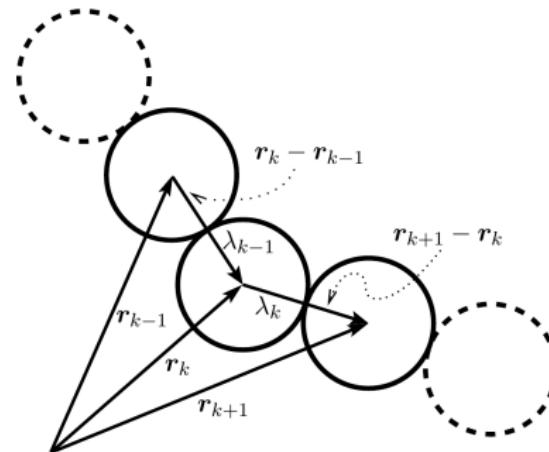
$$\mathbf{v}^D(\mathbf{r}_j) = \mathbf{U}^\infty + \boldsymbol{\Omega}^\infty \times \mathbf{x} + \mathbf{E}^\infty : \mathbf{x} \quad \text{for } \mathbf{x} \in S_j \quad (7)$$

# Grand Governing Equation for $N_p$ Connected Spheres

... requiring to solve a linear system of  $11N_p \times 11N_p$  matrix.

$$\begin{bmatrix} m_k & 0 \\ 0 & I_k \end{bmatrix} \begin{bmatrix} \mathbf{a}_k \\ \boldsymbol{\alpha}_k \end{bmatrix} = \begin{bmatrix} \mathbf{F}_k^H \\ \mathbf{T}_k^H \end{bmatrix} + \begin{bmatrix} \mathbf{F}_k^E \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{F}_k^C \\ 0 \end{bmatrix} \quad (8)$$

- If  $N_p = 1,000$ , the no. of element = 121,000,000
- Memory = element no. times 8 byte  $\rightarrow \approx 1.0$  GB.



## Simulation parameters: rack and fibers



	Variables	Value	Value
Rack	Fiber length [mm], $L_f$	1000	2000
	Number of spheres, $N_p$	500	1000
	Stack height [mm], $H = L_f / (1 + R_{st})$	998	1996
	Oscillation Amplitude [mm], $A$	6.0	12.0
	Oscillation frequency [Hz], $\omega$	0.46	0.46

Table: Simulation parameters of the rack and fibers, where the oscillation amplitude is calculated as  $A = 0.06 L_f$  and the stack ratio of  $R_{st} = 0.2\%$  is used. Based on the common frequency, the period of the rack oscillation is calculated as  $\tau = 2.1739$  s.

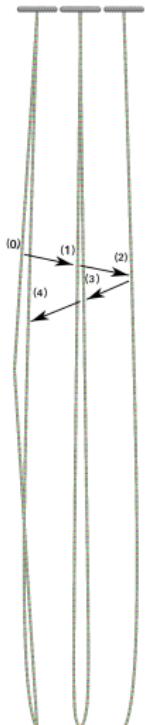
## Simulation parameters: rack and fibers



	Variables	PVDF	PET
Fiber	Solid density, $\rho_s$	1.780	1.380
	Outer diameter [mm], $d_o = 2a_f$	2.000	2.000
	Inner diameter [mm], $d_i = 2a'_f$	1.600	0.700
	Thickness [mm], $a_f - a'_f$	0.200	0.650
	Porosity [-], $\epsilon$	0.650	0.650
	Water-filled fiber material density, $\bar{\rho}_f$	1.273	1.133
	Mass of cylindrical segment [mg], $m_s$	6.901	7.017
Sphere	Volume [mm <sup>3</sup> ]	4.189	4.189
	Specific gravity [-], $s_g$	1.647	1.675

Table: Simulation parameters of the rack and fibers and equivalent spheres, made of polyvinylidene fluoride (PVDF) and polyethylene terephthalate (PET).

## Snapshots during 6 quarter periods



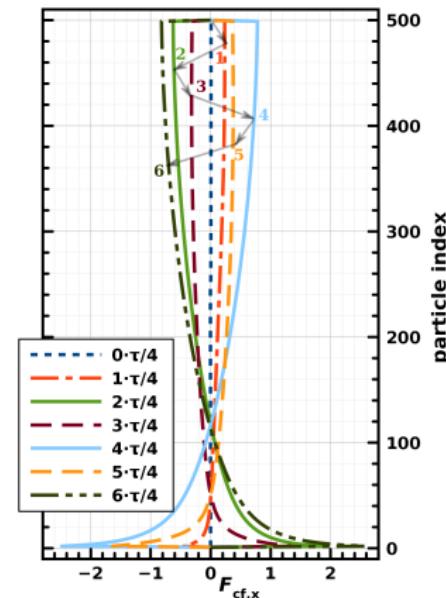
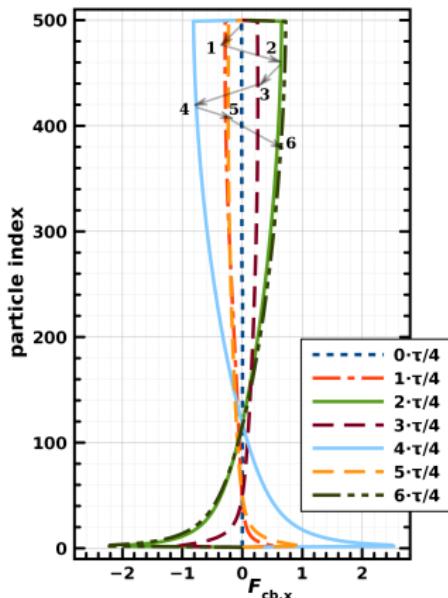
### Periodic motion

- 0: initial rightmost position
- 1:  $t = \tau_{1/4}$  after a **quarter period**
- 2:  $t = 2\tau_{1/4}$  after two quarter period
- 3:  $t = 3\tau_{1/4}$  after a half period
- 4:  $t = 4\tau_{1/4}$  after a full period
- 5:  $t = 5\tau_{1/4}$  after 1.25 period
- 6:  $t = 6\tau_{1/4}$  after 1.50 period

where  $\tau_{1/4} = \tau/4$  is a quarter period.

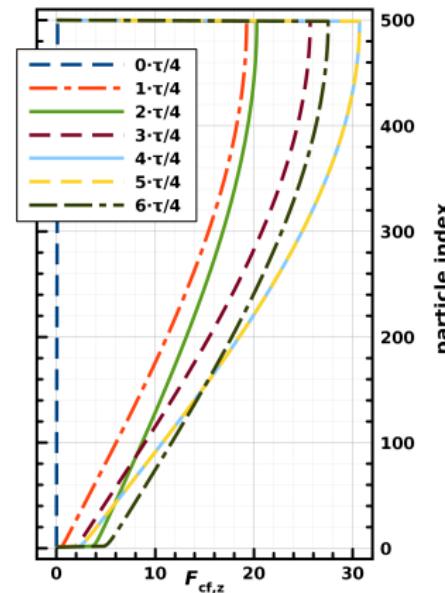
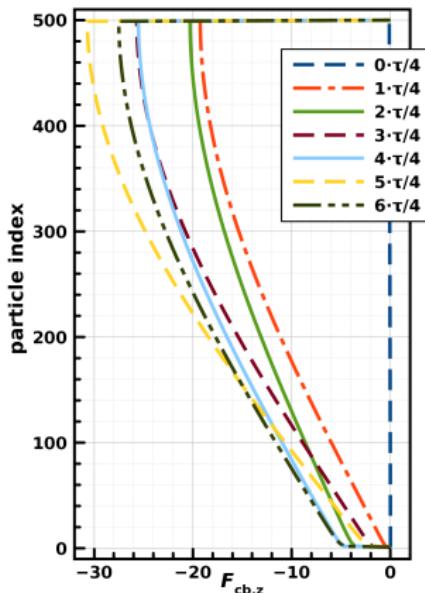
## [Short Chain] Constraint Forces in $x$ – Direction

- Backward & forward constraint forces: anti-symmetric.
- Near  $z = l_f/4$ , i.e., 25% height, almost  $F_x^c \rightarrow 0$  [nN].



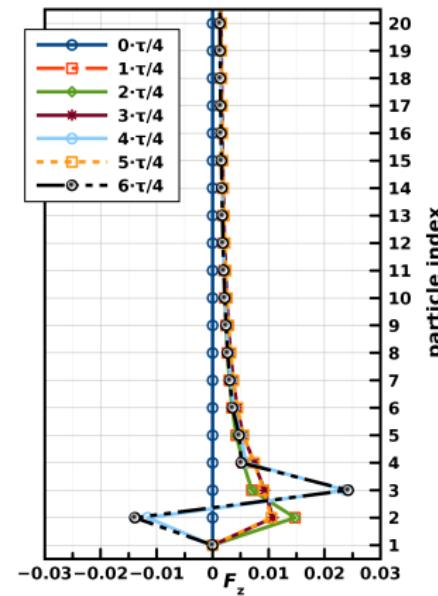
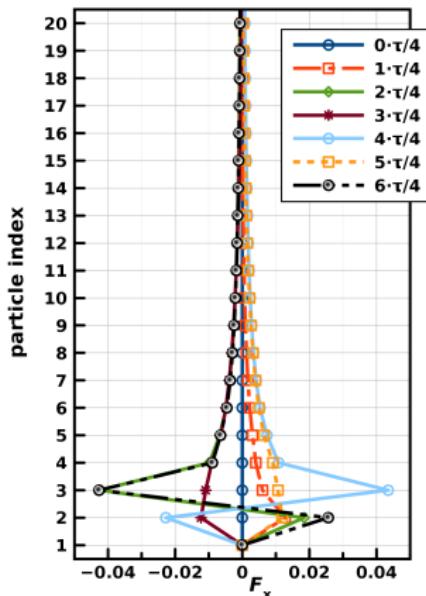
## [Short Chain] Constraint Forces in z – Direction

- $F_z^c$  is about 10 times stronger than  $F_x^c$  [nN].
- Anti-symmetry appears, but not perfect.



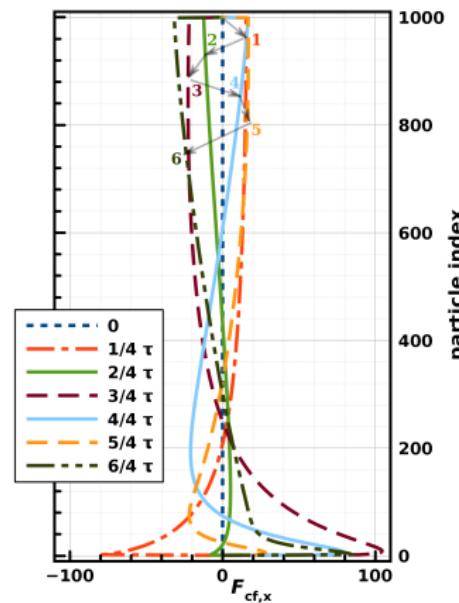
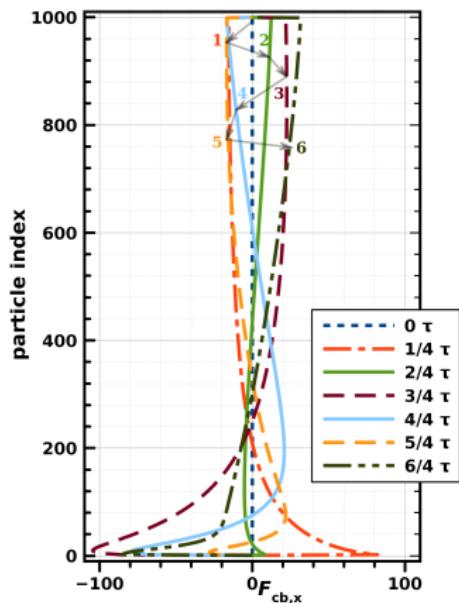
## [Short Chain] Total Force in $x$ - and $z$ - Direction

- The net total forces [ $\mu\text{N}$ ] exerted mostly near the bottom.
- Bottom and top segments can be mechanically reinforced.



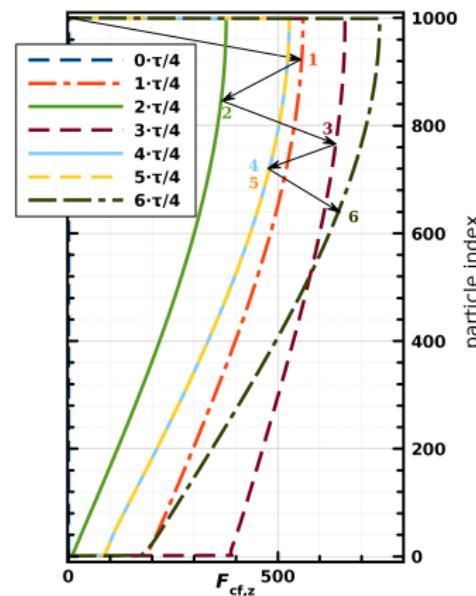
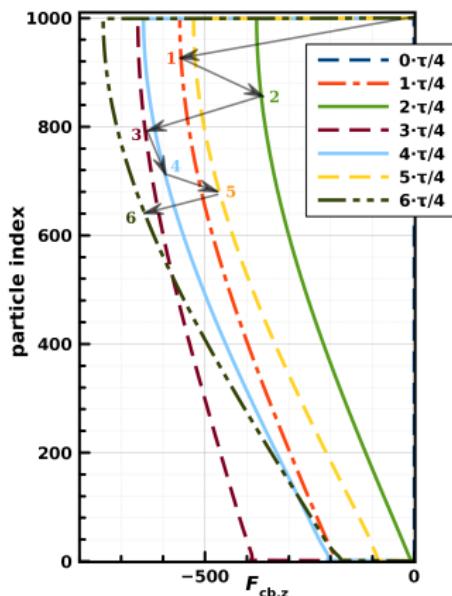
## [Long Chain] Constraint Forces in $x$ – Direction

- The zero-force [nN] zone becomes vague.



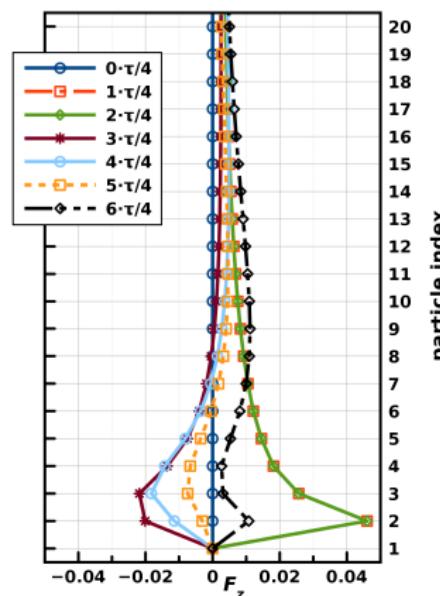
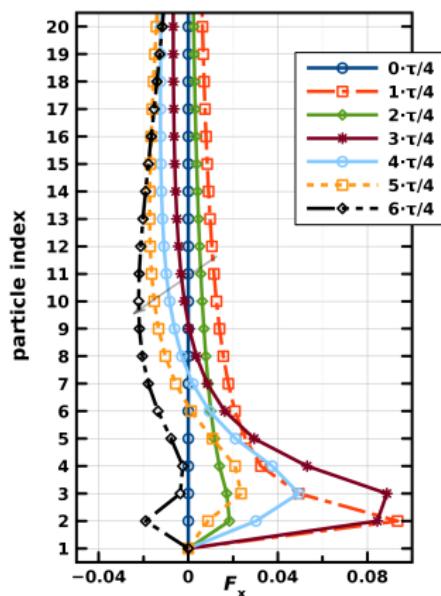
## [Long Chain] Constraint Forces in $z-$ Direction

- The periodicity and anti-symmetry become less apparent.
- Constraint force unit [nN]



## [Long Chain] Total Force in $x$ - and $z$ - Direction

- Unexpected similarity instead of periodicity in  $F_{tot}$  [ $\mu\text{N}$ ]



## Summary



- 1 A single hollow fiber is modeled as a sphere-connected chain using the equivalent sphere.
- 2 A short chain reciprocation is within intuitive expectation.
- 3 A long chain's geometrical configuration is less predictable. Because
  - 1 the return of the full fiber (i.e., the middle zone) is always after the returning of the rack.
  - 2 hollow fiber motion is energy-dissipative and entropy-increasing.
  - 3 periodicity is not fully conserved, generating random/chaotic displacements
  - 4 perhaps good for fouling reduction, but against the fiber durability.

## Acknowledgment



Kolon Industry, Inc.



KOrea + NyLON = KOLON (from 1957)

Korea Res. Inst. of Ships and Ocean Engineering (KRISO)

*For steady support and long-term collaborators ...*



KOREA RESEARCH INSTITUTE OF  
SHIPS & OCEAN ENGINEERING

## Contact Information



*Hydrodynamic load exerted on a moving bundle of hollow fibers using constraint dissipative hydrodynamics: Hydro-Rattle simulation*

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## Questions and Comments?

