Introduction

The Exponentially Weighted Moving Average (EWMA) model is a widely used tool in financial risk management, known for its ability to capture the changing volatility patterns of financial time series data. The parameter λ, often referred to as the smoothing parameter, plays a crucial role in determining the responsiveness of the EWMA model to new information. Selecting an optimal λ is essential for accurately forecasting volatility and managing financial risk.

The RiskMetrics group, a pioneer in financial risk measurement, published a paper 'The RiskMetrics 2006 methodology', which recommends a λ value of 0.94 for monthly data for the following reasons:

- Empirical Testing: J.P. Morgan conducted extensive back-testing and simulations using historical market data. They tested various values of λ to determine which value provided the best balance between responsiveness to recent market movements and stability over time.
- Practical Considerations: The value of 0.94 was found to give a good compromise between being responsive to recent market changes and not being too volatile itself. This means it captures recent market volatility effectively without overreacting to short-term fluctuations.
- Long-Term Stability: A λ value of 0.94 ensures that the volatility estimate does not change too abruptly, which is important for risk management purposes. It provides a smoother and more stable estimate of volatility over time.

Overtime, the industry has adopted the use of 0.94 as the value of lambda. However, it's important to note that the optimal λ might vary for different assets and markets, reflecting the unique volatility dynamics and characteristics of each.

Motivation

Currently, there is limited literature exploring optimal lambda for various market. In this report, we investigate if there may be better lambda values other than the universal lambda that RiskMetrics has recommended. Our aim is to find if there could be a better lambda that could be utilized in Japan and Hong Kong equities by focusing our analysis on Nikkei 225 and Hang Seng Index. We do not aim to refute nor discredit the use of 0.94 as recommended by RiskMetrics.

We investigate the optimal λ of the EWMA model using historical data from the Nikkei 225 index. By analyzing the monthly returns of the Nikkei 225, we aim to determine the λ value that minimizes various error metrics between the realized volatility and the EWMA volatility estimates. This study not only highlights the importance of the smoothing parameter in the EWMA model but also provides insights into its practical application in the context of Japanese and Hong Kong financial markets.

Our approach involves calculating historical volatility, estimating EWMA volatility with varying λ values, and evaluating the performance of these estimates using error metrics such as RMSE and MAE. Through this process, we identify the λ value that offers the best fit for the EWMA model, ensuring accurate volatility forecasts and effective risk management strategies.

Literature review

Subsequent research has explored the applicability and effectiveness of the EWMA model across various asset classes and financial markets. For instance, Alexander (2008) in "Market Risk Analysis" discusses the impact of different λ values on the accuracy of volatility forecasts and highlights that the optimal λ can vary significantly depending on the characteristics of the underlying asset and the frequency of the data. Similarly, Poon and Granger (2003) in their comprehensive review "Forecasting Volatility in Financial Markets" emphasize the importance of selecting an appropriate λ to capture the unique volatility dynamics of different markets.

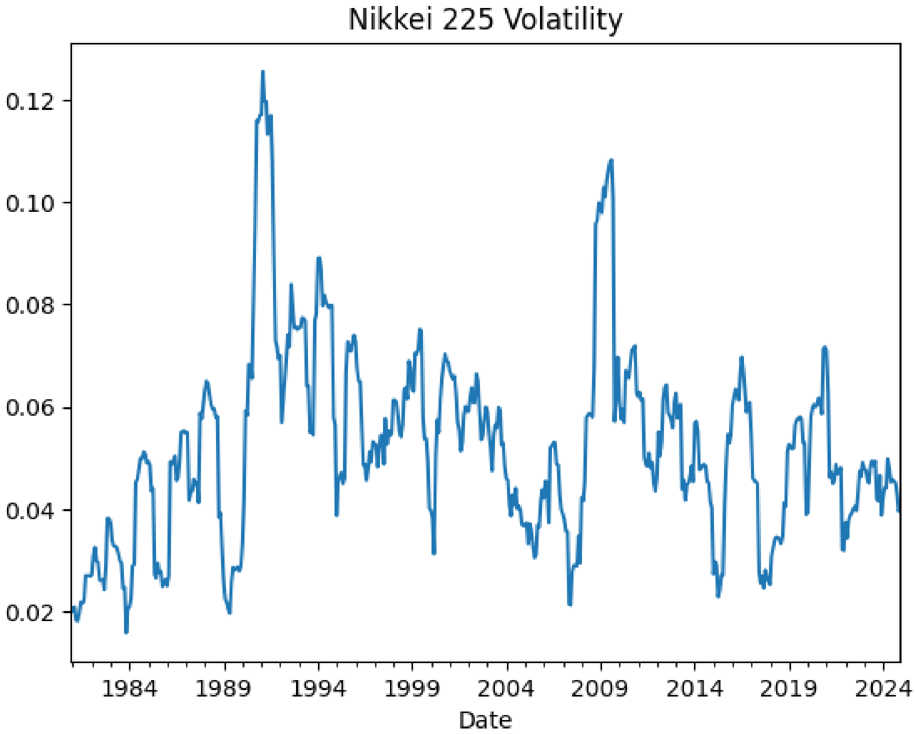
Moreover, Bollen (2014) in "What should the value of lambda be in the exponentially weighted moving average volatility model?" empirically investigates the optimal λ value for monthly data and finds that the commonly recommended value of 0.97 is not optimal. The study suggests that the optimal λ should be based on recent historical data and may vary across different assets and markets. Additionally, Araneda (2021) in "Asset Volatility Forecasting: The Optimal Decay Parameter in the EWMA Model" explores the optimal decay parameter for different forecasting horizons using a large time-series dataset of historical returns from top US large-cap companies. The study confirms that the optimal λ varies with the forecasting horizon and highlights the importance of short-term memory for lower forecasting horizons. This paper further evaluates the performance of EWMA using a time-varying decay parameter, which shows improved accuracy over a fixed parameter approach.

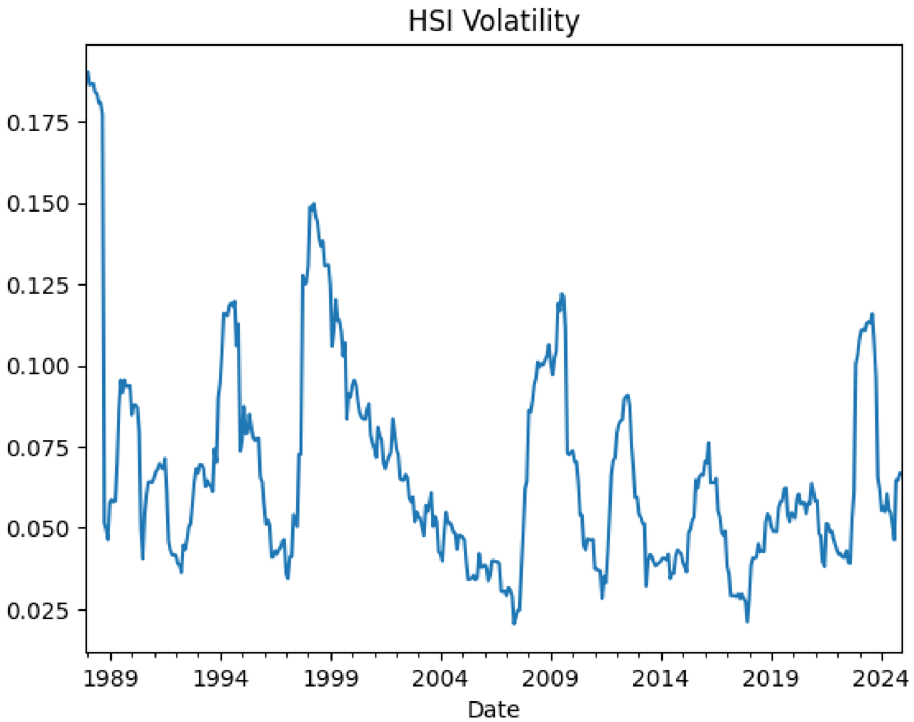
Data

The analysis is conducted using both daily and monthly data on the Nikkei 225 Index from January 1980 to December 2024 (680 trading months). For Hang Seng Index, the data is collect from January 1987 to December 2024. The data was sourced from the Yahoo Finance. To be more specific, the data is pulled out from the yfinance API via Python.

To calculate the monthly returns from daily data, we aggregated the daily returns monthly. The monthly volatility is then calculated using the volatility of monthly returns.

The figure below shows the monthly realized volatility of the Nikkei 225 between trading January 1980 to December 2024. We note 2 huge spikes in volatility during this period namely Japanese asset price bubble in 1990-1991 and Global Financial crisis in 2008-2009.





Methodology

The analysis begins by calculating the historical volatility of monthly returns for Nikkei 225 Index over the starting month from Jan 1980 to December 1982 (36 months). This provides the starting volatility, which seeds the in-sample Exponentially Weighted Moving Average (EWMA) volatility estimates. Using this starting point, the EWMA volatility estimates are iteratively calculated for each subsequent month, from January 1980 to December 2024, for a given λ value. We then repeat the analysis for Hang Seng Index.

To evaluate the accuracy of these estimates, the 'error' between the realized volatility and the EWMA volatility estimate is computed for each month. In our analysis, we focus on 2 main statistical measures:

- Root Mean Square Error (RMSE)
- Mean Absolute Error (MAE)

For the RMSE, the error is calculated as follows:

$$\sqrt{\sum_{i=1}^{D}(x_i - y_i)^2}$$

For the MAE, the error is calculated as follows:

$$\sum_{i=1}^{D}|x_i - y_i|$$

Methodology for Optimization and Evaluation

In the optimization and evaluation process, we employ a rolling window approach to optimize and evaluate the performance of an Exponentially Weighted Moving Average (EWMA) volatility forecasting model. Each rolling window consists of a 36-month **in-sample period**, followed by a **one-month out-of-sample period**. The objective is to determine the optimal smoothing parameter λ for the EWMA model and assess its ability to forecast future volatility.

For each rolling window, the following steps are performed:

1. **In-Sample Optimization**:
    - The realized volatilities over the 36-month in-sample period are used as benchmarks.

- The EWMA model is applied iteratively with different values of λ to estimate monthly volatilities.
- λ is optimized using constrained minimization with $\lambda \in [0,1]$ to minimize a chosen error statistic between the EWMA volatility estimates and the realized volatilities. The error statistics considered are RMSE and MAE.
- The optimal λ for the in-sample period will then be recorded.

2. **Out-of-Sample Forecasting and Evaluation**:
   - The optimal λ derived from the in-sample period is used to forecast the volatility for the next month (out-of-sample period).
   - The forecasted volatility is compared to the realized volatility for the out-of-sample month, and the out-of-sample error is calculated using the same error statistic optimized in the in-sample period.

3. **Rolling Window Advancement**:
   - The rolling window is shifted forward by one month, and steps 1 and 2 are repeated. This process continues until forecasts are generated for all available data, covering both in-sample and out-of-sample periods.

4. **Forecasting**:
   - This process is repeated for each time point, resulting in a series of one-step-ahead EWMA volatility forecasts, each based on the optimized lambda value for the corresponding rolling window.

5. **Aggregation of Results**:
   - The out-of-sample errors from all rolling windows are aggregated to compute the overall performance of the EWMA model. The average RMSE and MAE across all out-of-sample periods are used to evaluate the forecasting accuracy.

This methodology ensures a robust evaluation of the EWMA model by testing its ability to generalize to unseen data (out-of-sample) while iteratively adapting to changing market conditions through rolling in-sample optimizations. The aggregated out-of-sample results provide an unbiased assessment of the model's predictive performance and allow for comparisons across different error metrics.

In this section, the important functions used in the analysis will be examined.

'calculate_ewma_volatility' helps us compute the EWMA volatility from our returns series. 'realized_volatility' is the rolling standard deviation of our returns series. These two vectors will serve as inputs to our minimization problem. 'calculate_errors' is the Python implementation of the formulas of the respective errors

```
# Calculate EWMA volatility for a given lambda
def calculate_ewma_volatility(returns, lambda_, start, end):
    ewma_volatility = [calculate_historical_volatility(returns, start - 36, start - 1)]  # Initial volatility seed
    for t in range(start, end + 1):
        sigma_t_minus_1 = ewma_volatility[-1]
        ewma = np.sqrt(lambda_ * sigma_t_minus_1**2 + (1 - lambda_) * returns[t - 1]**2)
        ewma_volatility.append(ewma)
    return ewma_volatility[1:]  # Exclude the seed value

def calculate_errors(realized_volatility, ewma_volatility):

    rmse = np.sqrt(np.mean((realized_volatility - ewma_volatility) ** 2))
    mae = np.mean(np.abs(realized_volatility - ewma_volatility))

    return rmse, mae
```

The function 'objective_function' minimizes these error metrics to optimize lambda using constrained minimization. This function will be used inside our optimization function.

```
def objective_function(lambda_, returns, realized_vol, start, end, error_type):
    ewma_volatility = calculate_ewma_volatility(returns, lambda_[0], start, end)
    rmse_errors, mae_errors = calculate_errors(realized_vol, ewma_volatility)

    # minimize the correct error statistic
    if error_type == "RMSE":
        return calculate_final_statistics(rmse_errors)
    elif error_type == "MAE":
        return calculate_average_statistics(mae_errors)
```

The function "optimize_lambda" is the function that we call to optimize the lambda in a given rolling window. We're able to specify which error we wish to minimize. The function begins with an initial guess for lambda (could be any values) and we introduced the constraint for lambda to be between 0 and 1.

```
# Optimize lambda using constrained minimization
def optimize_lambda(returns, realized_vol, start, end, error_type):
    # Initial guess for lambda
    initial_lambda = [0.5]

    # Constraints: lambda must be between 0 and 1
    bounds = [(0, 1)]

    # Minimize the objective function
    result = minimize(
        objective_function,
        x0=initial_lambda,  # Initial guess
        args=(returns, realized_vol, start, end, error_type),  # Additional arguments to the function
        bounds=bounds,  # Bounds for lambda
        method='L-BFGS-B'  # Optimization method
    )

    # Extract the optimal lambda
    optimal_lambda = result.x[0]
    return optimal_lambda
```

The 'forecast_ewma_one_step' function generates one-step-ahead EWMA volatility forecasts and calculates forecast errors by optimizing lambda for each rolling window by calling the function "optimize_lambda". It calculates the EWMA volatility for the one-step-ahead forecast using the calculate_ewma_volatility function and appends the result to forecasts. The forecast error is computed as the difference between the realized volatility and the last EWMA volatility, which is then appended to forecast_errors.

```python
# One-step ahead EWMA forecasting
def forecast_ewma_one_step(returns, start, end, realized_vol, error_type):
    forecasts = []
    list_of_optimal_lambda = []
    forecast_errors = []

    for t in range(start, end):
        rolling_start = t - 36
        rolling_end = t - 1

        # Optimize lambda for the rolling window
        best_lambda = optimize_lambda(returns, realized_vol[rolling_start:rolling_end + 1], rolling_start, rolling_end, error_type)
        list_of_optimal_lambda.append(best_lambda)

        # Calculate EWMA for the one-step-ahead forecast
        ewma_volatility = calculate_ewma_volatility(returns, best_lambda, rolling_start, t)
        forecasts.append(ewma_volatility[-1])  # One-step-ahead forecast

        forecast_error = realized_vol[t] - ewma_volatility[-1]
        forecast_errors.append(forecast_error)

    return forecasts, forecast_errors, list_of_optimal_lambda
```

Results

After running the analysis, we will have a vector of respective lambdas that minimizes each of the four statistics. In Table 1 below, we take the average of each optimal lambda and the average of the error value.

Unsurprisingly, we got a lambda that is different from 0.94. By minimizing RMSE, we got an average optimal lambda of 0.88 and 0.73 for minimizing MAE.

A small λ makes the average decay quickly, so it's more sensitive to recent changes. A large λ makes it decay slowly, so it's more stable and less sensitive to recent changes. Thus, our optimization analysis shows that we should place more weight to recent changes, as compared to 0.94 by RiskMetrics.

	Average Lambda	Average Error Value (Realized volatility – EWMA volatility)
RMSE	0.882	-0.009
MAE	0.731	-0.007

Table 1: Optimal Lambda for Nikkei 225

It is also important that we look at the distribution of the different lambda values attained via our optimization process. Table 2 displays the frequency of the various values of the optimal value for the RMSE and MAE statistics when conducting out-of-sample testing.

We observed that the spread of the optimal lambda differs quite significantly between RMSE and MAE. MAE has more optimal lambdas with lower values. This also means that when minimizing lambdas, MAE tends to give more weight to more recent data. On the other hand, the optimal lambdas for RMSE are heavily skewed towards higher values of lambda where most optimal lambdas are between 0.9 and 1.

	RMSE	MAE
(0.1, 0.2]	0	2
(0.2, 0.3]	0	1
(0.3, 0.4]	1	17
(0.4, 0.5]	3	58
(0.5, 0.6]	24	56
(0.6, 0.7]	30	60
(0.7, 0.8]	55	43
(0.8, 0.9]	82	54
(0.9, 1.0]	286	183

Table 2: Distribution of optimal lambda for Nikkei 225

In Table 3, we observe the RMSE and MAE when lambda is 0.94 in our sample data and period. A direct comparison of the RMSE and MAE statistics in Table 3 with those displayed in Table 1, and we see that higher error statistics when lambda = 0.94.

	Errors (lambda = 0.94)
RMSE	0.036
MAE	0.0298

Table 3: Nikkei average RMSE and MAE when lambda = 0.94

<u>Hang Seng Index (HSI) Analysis</u>

In this section, we explore the same analysis on Hang Seng Index. We carry out the same analysis for Hang Seng Index. Similarly, the data on HSI is pulled out using Yahoo Finance via its Python API. Unfortunately, the data is only available starting from 1987. Thus, our dataset for HSI is from 1987-2024.

	Average Lambda	Average Error Value (Realized volatility – EWMA volatility)
RMSE	0.854	-0.011
MAE	0.773	-0.008

Table 4: Optimal Lambda for HIS

	RMSE	MAE
(0.1, 0.2]	0	9
(0.2, 0.3]	0	2
(0.3, 0.4]	0	27
(0.4, 0.5]	11	26
(0.5, 0.6]	22	10
(0.6, 0.7]	18	31
(0.7, 0.8]	65	55
(0.8, 0.9]	82	84
(0.9, 1.0]	175	129

Table 5: Distribution of optimal lambda for HSI

	Errors (lambda = 0.94)
RMSE	0.0462
MAE	0.0365

Table 6: HSI average RMSE and MAE when lambda = 0.94

In Tables 4 and 5, we see similar results when we run the analysis on HSI, where we have lower optimal lambda values, compared to the recommended value by RiskMetrics. We too observed similar distribution of lambdas where RMSE is skewed towards higher values while MAE has more lower magnitude lambdas.

In Table 6, we observe the RMSE and MAE when lambda is 0.94 in our sample data and period. We observed higher error statistics in Table 6 when lambda = 0.94.

Analysis

In our findings, we found that optimal lambdas which produces lower RMSE and MAE are lower than the universal value recommended by RiskMetrics. The implications of a lower lambda are:

- More Responsive: A lower λ gives more weight to recent data. This makes the volatility estimate more responsive to recent changes in the market.

- Shorter Memory: The model "forgets" past data more quickly. This means that older observations have less influence on the current volatility estimate.

- Risk of Overfitting: May overfit recent fluctuations, thus misrepresenting true volatility patterns.

- Increased noise: While being more responsive, it can also make the volatility estimate more volatile, as it reacts strongly to recent market movements.

It's important to highlight that in both of our analyses, the mode of the distribution falls within the 0.9-1.0 range. Some might argue that using the mode as the optimal lambda, instead of the average would be more appropriate, and this perspective is valid. Our method of taking the average of the optimal lambda values helps accommodate a broader range of market conditions and investor behaviors. This is one of many methods to derive an optimal lambda. It is also crucial to note that the optimal lambda can vary across different markets and assets.

In the broader context, this analysis can be carried out for companies who are looking for better fitting lambda, specifically for the market or asset that they are covering. Once implement, the performance of the forecast can be compared with using 0.94 as their lambda.

Areas of improvement

Anderson et al. (1999) stated that the highly non-linear and heteroskedastic environment may render the usual measures based on root-mean squared errors (RMSE) unreliable. Following this point, further research could explore robust error statistics to better accommodate the heteroskedasticity in the forecast errors by using the corresponding heteroskedasticity adjusted statistics HRMSE and HMAE.

Further work could carry out the same analysis and include the use of two heteroskedasticity-adjusted statistics such as HRMSE and HMAE and coming up with a weighted lambda, placing more weight on this error statistics.

Conclusion

In this study, our goal is to find better fitting lambda values for the Japan and Hong Kong equities markets, as opposed to using the universal 0.94 recommended by RiskMetrics. We evaluated two error statistics and conducted optimization processes to minimize these errors, using the optimal lambda to forecast one-step ahead EWMA volatility. Following this, we reported the average of all optimal lambda values. We found that the optimal lambda for Nikkei 225 is 0.88 (RMSE) and 0.73 (MAE) and the optimal lambda for Hang Seng Index is 0.85 (RMSE) and 0.77 (MAE). These reported lambdas have lower RMSE and MAE, compared to when lambda is 0.94.

We also looked at the distributions of all optimal lambda and found that most optimal lambdas are between 0.9-1.0, which is aligned with RiskMetrics' recommendation of 0.94. Although the mode of the distribution in both analysis falls within the 0.9-1.0 range, aligning with RiskMetrics' recommendation, our analysis provides a more tailored solution, reflecting the unique characteristics and volatility patterns of these specific markets. Moreover, our approach highlights the variability of optimal lambda values across different markets, emphasizing the importance of context-specific analysis. Implementing this tailored lambda can enhance the performance of forecasting models compared to using a fixed lambda value, such as 0.94. This method empowers companies to adapt their strategies to the distinct dynamics of the markets they operate in, ultimately leading to more precise and reliable risk management practices.

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