

Seebeck coefficient in classical fluids from equilibrium molecular dynamics

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Introduction

The thermo-electric effect in ionic conductors is the electric current induced by a temperature gradient. The computation of the corresponding off-diagonal Onsager coefficient is numerically ill-conditioned. We show how to calculate it as the numerically tractable product between a diagonal Onsager coefficient and an off-diagonal static susceptibility, both computed from equilibrium molecular dynamics. This static susceptibility is well defined even in the insulating case, where it provides a theoretically sound and numerically tractable formulation of the thermo-polarization coefficients, which have so far been computed only through cumbersome non-equilibrium simulations [1, 2].

Theory

The Seebeck coefficient, S , is defined as the ratio between the electric field and the temperature difference inducing it when no electric current flows (open circuit) [3]:

$$S = \left. \frac{\mathbf{E}}{\nabla T} \right|_{\mathbf{J}=0}$$

The Onsager equations for the electric and heat currents read:

$$\begin{aligned} \mathbf{J} &= \sigma \mathbf{E} - (K_{12}/T) \nabla T, \\ \mathbf{J}^q &= K_{12} \mathbf{E} - (L_{qq}T) \nabla T. \end{aligned}$$

The mechanical perturbation describing the temperature gradient is written as:

$$\mathcal{H}' = \int \frac{-1}{k_B T^2} \int \nabla T(\mathbf{r}) \cdot \mathbf{r} \left(\epsilon(\mathbf{r}) - \sum_i h_i n_i(\mathbf{r}) \right) d\mathbf{r},$$

where ϵ is the energy density, n_i is the particle density, h_i is the partial enthalpy, $q = \epsilon - \sum_i h_i n_i$ is the heat density [4]. The static response of the electric polarization to the temperature gradient reads: [5]:

$$S = \frac{K_{12}}{\sigma T} = \lim_{\mathbf{k} \rightarrow 0} \lim_{\omega \rightarrow 0} \frac{1}{T} \frac{\chi_{q,\rho}(\mathbf{k}, \omega)}{k^2},$$

where ρ is the charge density.

Bayesian inference

We fitted the Seebeck \mathbf{k} -dispersion as:

$$S(\mathbf{k}) \approx w_0 + w_1 k^2 + w_2 k^4 \dots$$

The M parameters, \mathbf{w} , and the regularization parameter, α , maximize the posterior distribution function, $p(\mathbf{w}, \alpha | \{S(\mathbf{k}_i)\}, M)$, computed from the Bayes Theorem $p(A|B)p(B) = p(B|A)p(A)$.

We evaluate the optimal number of parameters, M , maximizing the *evidence* function [6]

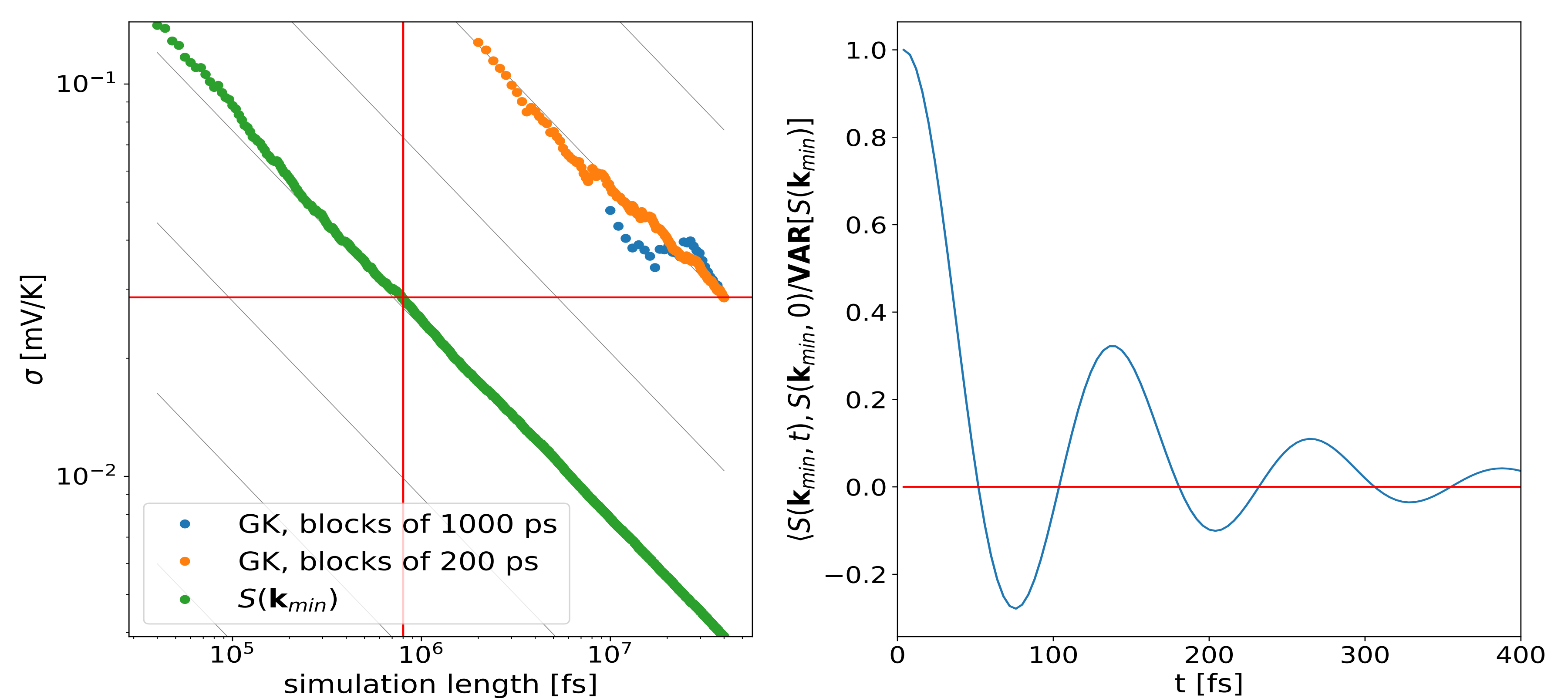
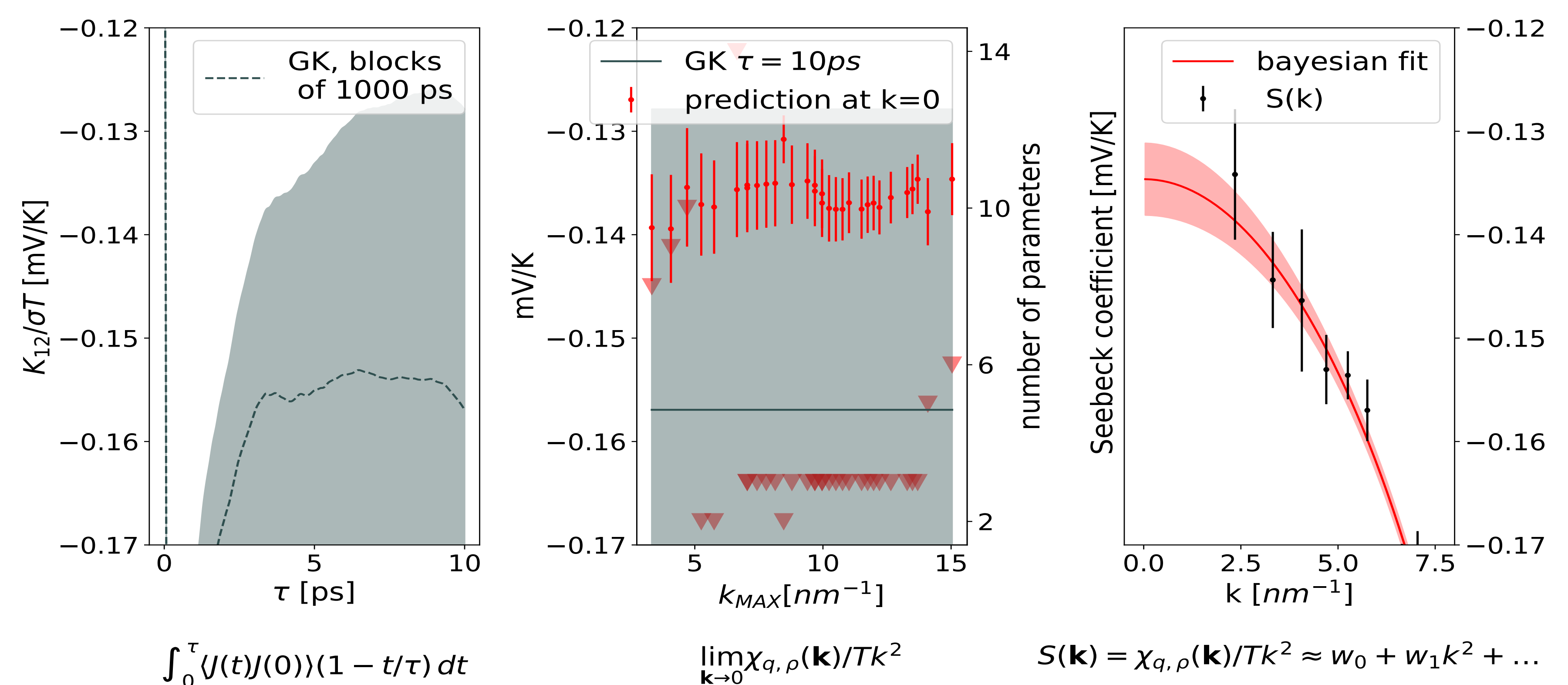
$$p(\{S(\mathbf{k}_i)\} | M) = \int d\mathbf{w} d\alpha p(\{S(\mathbf{k}_i)\} | \mathbf{w}) p(\mathbf{w} | \alpha) p(\alpha | \{S(\mathbf{k}_i)\}).$$

References

- [1] P. Wirmsberger, D. Fijan, R. A. Lightwood, A. Šarić, C. Dellago, and D. Frenkel. Numerical evidence for thermally induced monopoles. *Proc Natl Acad Sci U S A*, 114(19):4911–4914, April 2017.
- [2] Jeff Armstrong and Fernando Bresme. Temperature inversion of the thermal polarization of water. *Phys. Rev. E*, 92:060103, Dec 2015.
- [3] H. B. Callen. The application of onsager's reciprocal relations to thermoelectric, thermomagnetic, and galvanomagnetic effects. *Phys. Rev.*, 73:1349–1358, Jun 1948.
- [4] D. Forster. *Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions*. CRC Press, mar 2018.
- [5] P. C. Martin. Sum rules, kramers-kronig relations, and transport coefficients in charged systems. *Phys. Rev.*, 161:143–155, Sep 1967.
- [6] C. M. Bishop. *Pattern Recognition and Machine Learning*. Springer, 1 edition, 2006.

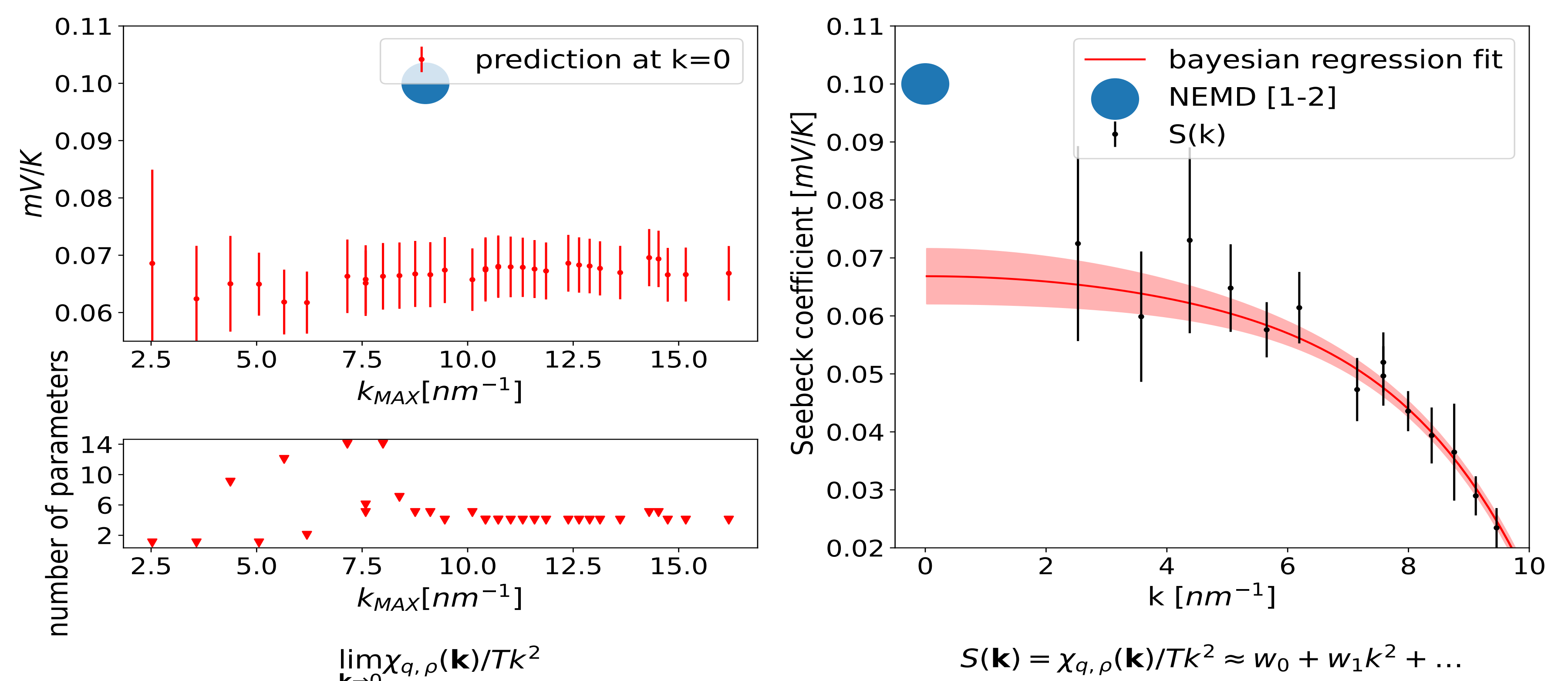
Test case: ionic conductor, molten salt Cs F

We performed a 40 ns-long classical MD simulation of 512 atoms of the molten salt Cs F of density 3.37 g/cm^3 at 1000K, we modeled the system with the Fumi-Tosi potential. The Seebeck coefficient computed as a static quantity on a 0.8 ns trajectory is as accurate as the Green-Kubo integral evaluated on a 40 ns simulation. **The static estimator is about one order of magnitude more efficient than the bare Green-Kubo (GK) calculation.**



Test case: insulating liquid, water

We performed a 1 ns-long classical MD simulation of 512 molecules of SPC/E water at room conditions. In the insulating case the two transport coefficient are 0 and their ratio would be numerically ill-conditioned.



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