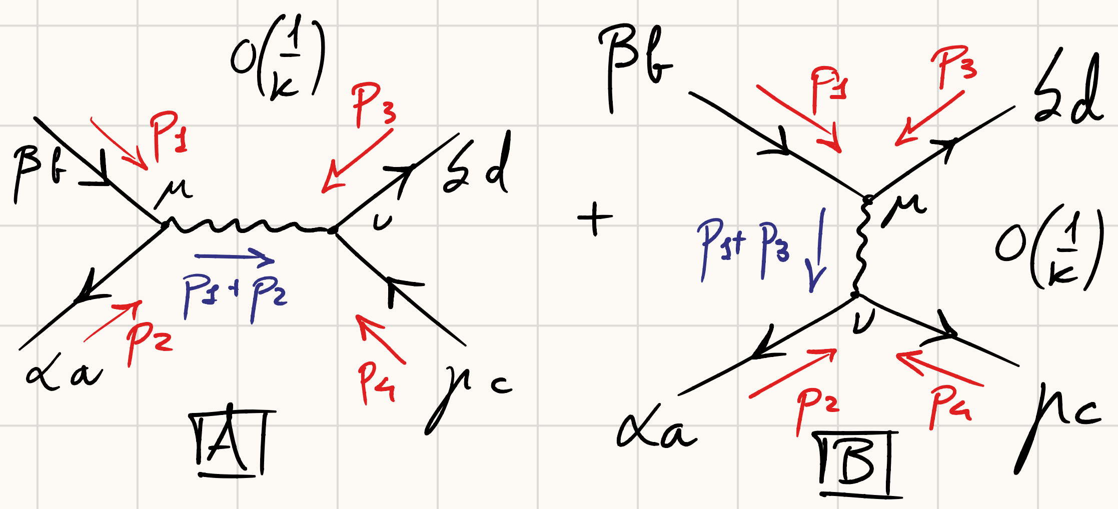
T_S AT TREE-LEVEL

28 FEBBRAIO 2025



$$\psi_b(p_1) + A^a(p_2) \to \psi_d(-p_3) + A^c(-p_4)$$

$\alpha, \beta, \gamma, \delta$: SPINOR INDICES; a, b, c, d: COLOR IND.

$$S = \left(\delta^c_b \delta^a_d - \frac{\delta^a_b \delta^c_d}{N}\right) S_T + \frac{\delta^a_b \delta^c_d}{N} S_S$$

$\boxed{A}$: $\bar{v}^a_\alpha(p_2, s_2)\,[-i\gamma^\mu_{\alpha\beta}(T^A)_{ab}]\,u^b_\beta(p_1, s_1)\,\frac{(-1)}{(p_1+p_2)^2}\frac{4\pi}{k}\epsilon_{\mu\nu\rho}(p_1+p_2)^\rho\,\delta^{AB}\,\bar{u}^d_\delta(-p_3, s_3)[-i\gamma^\nu_{\delta\gamma}(T^B)_{dc}]\,v^c_\gamma(-p_4, s_4)$

$\boxed{B}$: $\bar{u}^d_\delta(-p_3, s_3)\,[-i\gamma^\mu_{\delta\beta}(T^A)_{db}]\,u^b_\beta(p_1, s_1)\,\frac{(-1)}{(p_1+p_3)^2}\frac{4\pi}{k}\epsilon_{\mu\nu\rho}(p_1+p_3)^\rho\,\delta^{AB}\,\bar{v}^a_\alpha(p_2, s_2)[-i\gamma^\nu_{\alpha\gamma}(T^B)_{ac}]\,v^c_\gamma(-p_4, s_4)$

$\circledast$ $(T^A)^i_j (T^A)^l_m = \frac{1}{2}\left(\delta^i_m \delta^l_j - \frac{\delta^i_j \delta^l_m}{N}\right)$ is consist. with $T^A T^A = \left(\delta^i_m \cdot N - \frac{\delta^i_m}{N}\right)\frac{1}{2} = \frac{N^2-1}{2N}\delta^i_m = C_2(N)\delta^i_m$ in our normalization convent.

Which would mean, for:

$$\boxed{A} \quad (t^A_R)^a{}_b (t^A_R)^c{}_d = \frac{1}{2}\Big(\underbrace{\delta^a_d \delta^c_b - \frac{\delta^a_b \delta^c_d}{N}}_{\text{projector to the adjoint}}\Big)$$

$$\boxed{B} \quad (t^A_R)^d{}_b (t^A_R)^a{}_c = \frac{1}{2}\Big(\delta^d_c \delta^a_b - \underbrace{\frac{\delta^d_b \delta^a_c}{N}}_{\text{projector to the singlet channel!}}\Big)$$

$$(T^a)_k{}^i (T^a)^l_j : \quad (T^a \otimes T^a)^{il}_{kj}$$

$$= \frac{1}{2}\left[(T^a\otimes 1 + 1\otimes T^a)(T^a\otimes 1 + 1\otimes T^a) - (T^a T^a \otimes 1) - (1 \otimes T^a T^a)\right]^{il}_{kj}$$

Meaning: ONLY THE SECOND DIAGRAM CONTRIBUTES TO THE SINGLET CHANNEL

$$\longrightarrow \quad T_S = \frac{2\pi}{k\,(p_1+p_3)^2}\,\epsilon_{\mu\nu\rho}\,(\bar{u}(-p_3)\gamma^\mu u(p_1))(\bar{v}(p_2)\gamma^\nu v(-p_4))(p_1+p_3)^\rho$$

Needs to be simplified using:

GORDON IDENTITIES

Reviewing Dirac equations in Lorentz:

$$\begin{pmatrix} -m & p\cdot\sigma \\ p\cdot\bar{\sigma} & -m \end{pmatrix} u_s(p) = 0 \;\to\; (\gamma^\mu p_\mu - m)\,u(p) = 0 \,.$$

$$\begin{pmatrix} -m & -p\cdot\sigma \\ -p\cdot\bar{\sigma} & -m \end{pmatrix} v_s(p) = 0 \;\to\; (\gamma^\mu p_\mu + m)\,v_s(p) = 0 \,.$$

- $\bar{u}_s(p) u_{s'}(p) = -\bar{v}_s(p) v_{s'}(p) = 2m\,\delta_{ss'}$
- $\sum_{s=1}^{2} u_s(p)\bar{u}_s(p) = \not{p} + m$
- $\sum_s u_s(p)\bar{v}_s(p) = \not{p} - m$

$$\gamma^0 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \gamma^{0\,2} = 1$$

$$\gamma^1 = i\sigma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$\gamma^2 = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\gamma^\mu\gamma^\nu = g^{\mu\nu}\mathbb{1} - i\epsilon^{\mu\nu\rho}\gamma_\rho$$

And deriving the gordon identities $\quad \cdot\ \gamma^0 \gamma^{\mu\dagger} \gamma^0 = \gamma^\mu !$ $\quad \epsilon^{abc}\epsilon_{mnc} = \delta^a_m \delta^b_n - \delta^a_n \delta^b_m$

IN EUCLIDEAN:

* $(i\gamma^\mu p_\mu + m)\, u(p) = 0$
* $g^{\mu\nu} - \epsilon^{\mu\nu\rho}\gamma_\rho = \gamma^\mu \gamma^\nu$
* $\bar{u}(p)\,(i\gamma^\mu p_\mu + m) = 0$

$$-\bar{u}(p_1)\gamma^\mu u(p_2) = -\frac{1}{2}\,\bar{u}(p_1)\frac{\gamma^\mu}{m}\left(m\, u(p_2)\right) - \frac{1}{2}\left(m\,\bar{u}(p_1)\right)\frac{\gamma^\mu}{m}\, u(p_2)$$

$$= -\bar{u}(p_1)\frac{\gamma^\mu}{2m}\left(-i\gamma^\nu p_{2\nu}\right)u(p_2) - \frac{1}{2}\bar{u}(p_1)\left(-i\gamma^\nu p_{1\nu}\right)\frac{\gamma^\mu}{2m}u(p_2)$$

$$= i\left[\bar{u}(p_1)\frac{1}{2m}\left(g^{\mu\nu} - \epsilon^{\mu\nu\rho}\gamma_\rho\right)p_{2\nu}\, u(p_2) + \bar{u}(p_1)\frac{1}{2m}\left(g^{\nu\mu} - \overbrace{\epsilon^{\nu\mu\rho}}^{= +\epsilon^{\mu\nu\rho}}\gamma_\rho\right)p_{1\nu}\, u(p_2)\right]$$

$$= i\left[\bar{u}(p_1)\frac{(p_1+p_2)^\mu}{2m}u(p_2) - \epsilon^{\mu\nu\rho}\frac{(-p_1+p_2)_\nu}{2m}\,\bar{u}(p_1)\gamma_\rho\, u(p_2)\right]$$

$$\blacksquare \quad -\bar{u}(p_1)\gamma^\mu u(p_2) = i\left[\frac{(p_1+p_2)^\mu}{2m}\bar{u}(p_1)u(p_2) - \epsilon^{\mu\nu\rho}\frac{(-p_1+p_2)_\nu}{2m}\,\bar{u}(p_1)\gamma_\rho u(p_2)\right]$$

$$\bar{u}(p_1)\gamma^\mu u(p_2) = -i\left\{\frac{(p_1+p_2)^\mu}{2m}\bar{u}(p_1)u(p_2) - \epsilon^{\mu\nu\rho}\frac{(-p_1+p_2)_\nu}{2m}\left[-i\left(\frac{(p_1+p_2)_\rho}{2m}\bar{u}(p_1)u(p_2) - \epsilon_{\rho\alpha\beta}\frac{(-p_1+p_2)^\alpha}{2m}\bar{u}(p_1)\gamma^\beta u(p_2)\right)\right]\right.$$

$$i\bar{u}(p_1)\gamma^\mu u(p_2) = \frac{(p_1+p_2)^\mu}{2m}\bar{u}_1 u_2 + i\frac{\epsilon^{\mu\nu\rho}}{4m^2}\underbrace{(-p_1+p_2)_\nu(p_1+p_2)_\rho}_{-p_{1\nu}p_{1\rho} - p_{1\nu}p_{2\rho} + p_{2\nu}p_{1\rho} + p_{2\nu}p_{2\rho}}\bar{u}_1 u_2 - i\frac{\overbrace{\epsilon^{\mu\nu\rho}\epsilon_{\rho\alpha\beta}}^{\delta^\mu_\alpha\delta^\nu_\beta - \delta^\mu_\beta\delta^\nu_\alpha}}{4m^2}(-p_1+p_2)_\nu(-p_1+p_2)^\alpha\,\bar{u}_1\gamma^\beta u_2$$

ANTISIMM.

$$= \frac{(p_1+p_2)^\mu}{2m}\bar{u}_1 u_2 + i\frac{\epsilon^{\mu\nu\rho}}{2m^2}(p_1)_\rho(p_2)_\nu\,\bar{u}_1 u_2 - i\frac{1}{4m^2}(-p_1+p_2)^\mu(-p_1+p_2)_\nu\,\bar{u}_1\gamma^\nu u_2 + \frac{i}{4m^2}(-p_1+p_2)^2\,\bar{u}_1\gamma^\mu u_2$$

$$-i\frac{1}{4m^2}(-p_1+p_2)^\mu(-p_1+p_2)_\nu\,\bar{u}_1\gamma^\nu u_2 = \frac{(-i)}{4m^2}(p_2-p_1)^\mu(p_2-p_1)_\nu\frac{(-i)}{2m}(p_1+p_2)^\nu\bar{u}_1 u_2 = \ldots (p_2^2 - p_1^2) \underset{\text{SHELL}}{\overset{\text{ON}}{=}} 0$$

$$\longrightarrow\ i\left(1 - \left(\frac{p_1-p_2}{2m}\right)^2\right)\bar{u}_1\gamma^\mu u_2 = \frac{(p_1+p_2)^\mu}{2m}\bar{u}_1 u_2 - i\frac{\epsilon^{\mu\nu\rho}}{2m^2}(p_1)_\nu(p_2)_\rho\,\bar{u}_1 u_2$$

$$\blacksquare \quad \bar{u}(p_1)\gamma^\mu u(p_2) = \frac{1}{1 - \left(\frac{p_1-p_2}{2m}\right)^2}\left[-i\frac{(p_1+p_2)^\mu}{2m} - \frac{1}{2m^2}\epsilon^{\mu\nu\rho}(p_1)_\nu(p_2)_\rho\right]\bar{u}(p_1)u(p_2)$$

WRONG SIGN

In Lorentz:

* $(\gamma^\mu p_\mu - m)\,u(p) = 0 \longrightarrow$ $\quad u^\dagger(p)\left(\gamma^{\mu\dagger}p_\mu - m\right) = 0$ $\quad u^\dagger(p)\left(\gamma^0\gamma^\mu p_\mu - \gamma^0 m\right)\gamma^0 = 0$ $\quad \bar{u}(p)\left(\gamma^\mu p_\mu - m\right) = 0$
* $\gamma^\mu\gamma^\nu = g^{\mu\nu}\mathbb{1} - i\epsilon^{\mu\nu\rho}\gamma_\rho$ $\quad \bar{u}(p)\left(\gamma^\mu p_\mu - m\right) = 0$

$$\blacksquare\ -\bar{u}(p_1)\gamma^\mu u(p_2) = -\frac{1}{2}\,\bar{u}(p_1)\frac{\gamma^\mu}{m}\left(m\,u(p_2)\right) - \frac{1}{2}\left(\bar{u}(p_1)m\right)\frac{\gamma^\mu}{m}u(p_2)$$

$$= -\bar{u}(p_1)\frac{\gamma^\mu\gamma^\nu}{2m}p_{2\nu}\,u(p_2) - \bar{u}(p_1)\frac{\gamma^\nu\gamma^\mu}{2m}p_{1\nu}\,u(p_2)$$

$$= -\frac{1}{2m}\left(\bar{u}(p_1)\,p_2^\mu\, u(p_2) - i\epsilon^{\mu\nu\rho}p_{2\nu}\,\bar{u}(p_1)\gamma_\rho u(p_2) + \ldots\right)$$

$$= -\frac{1}{2m}\left(\bar{u}(p_1)(p_1+p_2)^\mu u(p_2) - i\epsilon^{\mu\nu\rho}(-p_1+p_2)_\nu\,\bar{u}(p_1)\gamma_\rho u(p_2)\right)$$