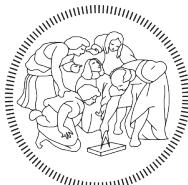
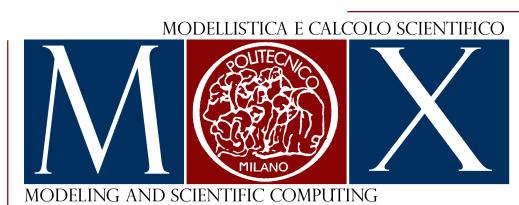


Flow and mechanics in fractured porous media: from high fidelity models to efficient reduced order solutions



POLITECNICO
MILANO 1863



PhD candidate: Enrico Ballini

Supervisor: Luca Formaggia

Co-supervisor: Alessio Fumagalli

Tutor: Filippo Gazzola

Milano, 6 February 2025

Introduction

Context → Models → ...

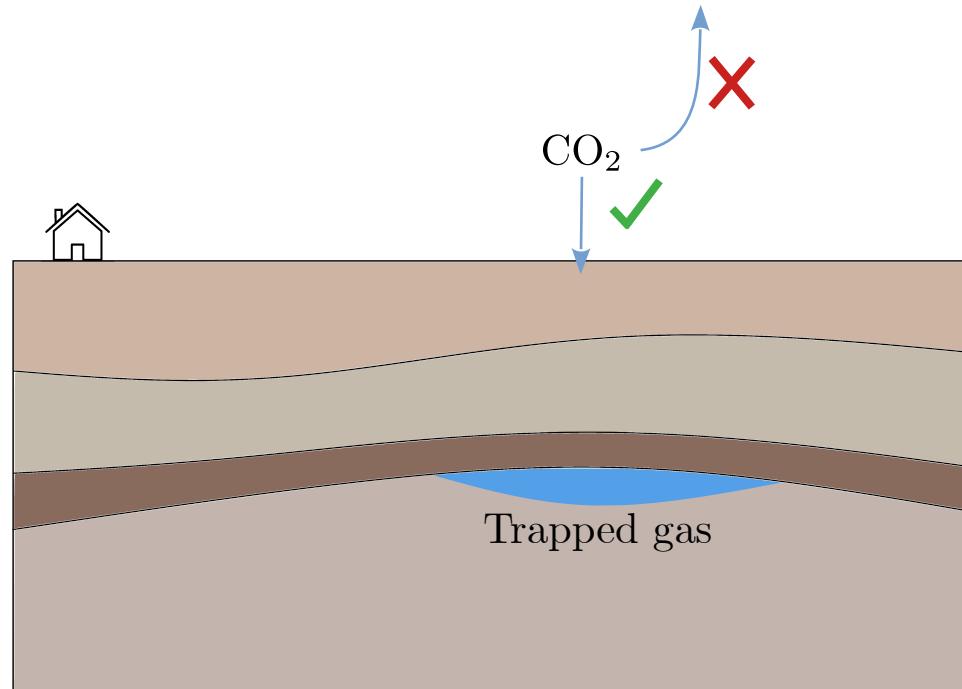
Goal: Greener economy

How: Carbon Capture Storage (CCS) to mitigate
climate change

Subsoil

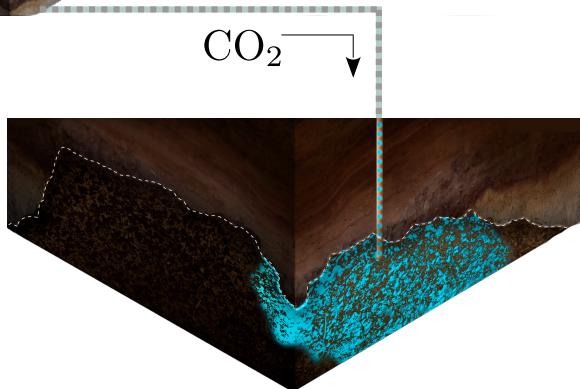
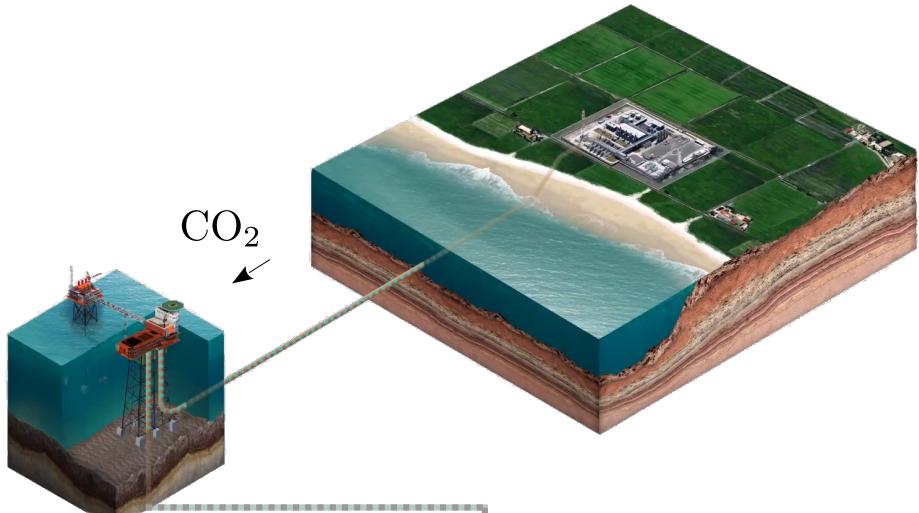
Porous medium
filled by fluids

Low permeable layer

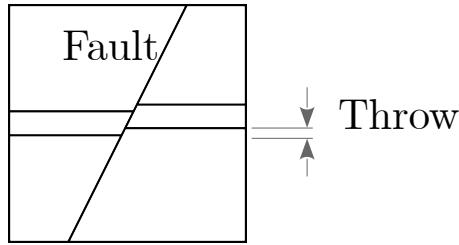


Ravenna CCS

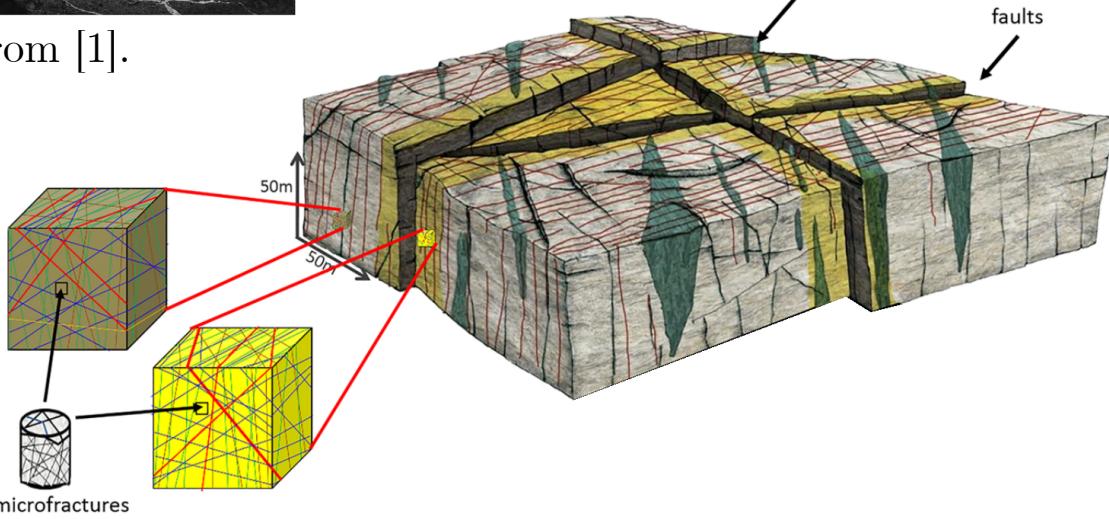
Context → Models → ...



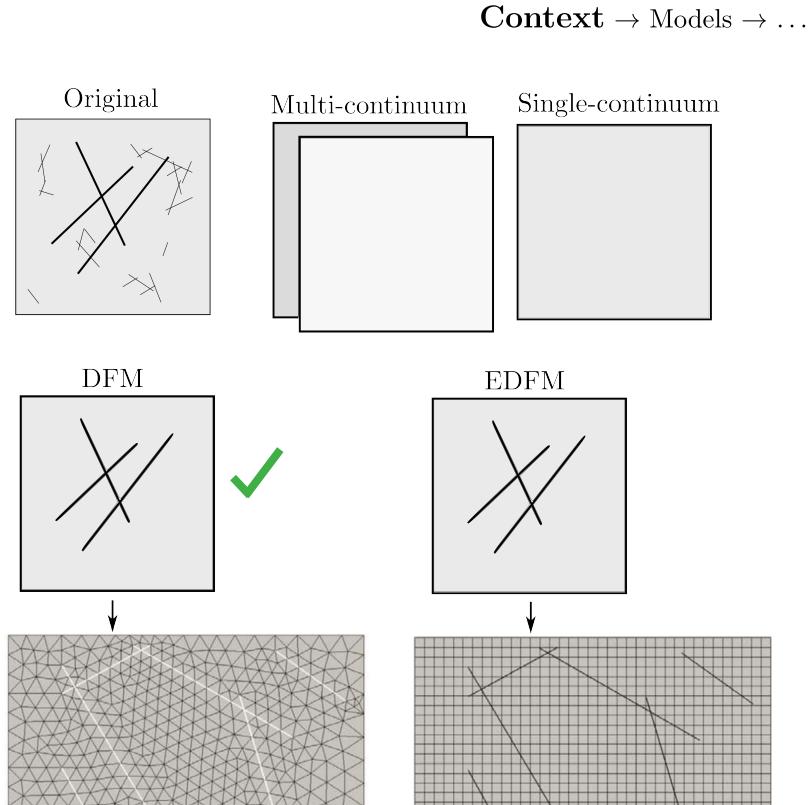
Fractures and faults



From [1].



From [2].



- [1] A. Belaidi, D. A. Bonter, C. Slightam, and R. C. Trice. *The lancaster field: progress in opening the UK's fractured basement play*. Geological Society, London, Petroleum Geology Conference Series, 2016.
[2] Peacock, D.C.P. and Sanderson, D.J. and Rotlevatn, A. *Relationships between fractures*, Journal of Structural Geology, 2018.

Simulating the subsoil phenomena

Interaction with subsoil



Undesirable scenarios:

Leakage

Subsidence

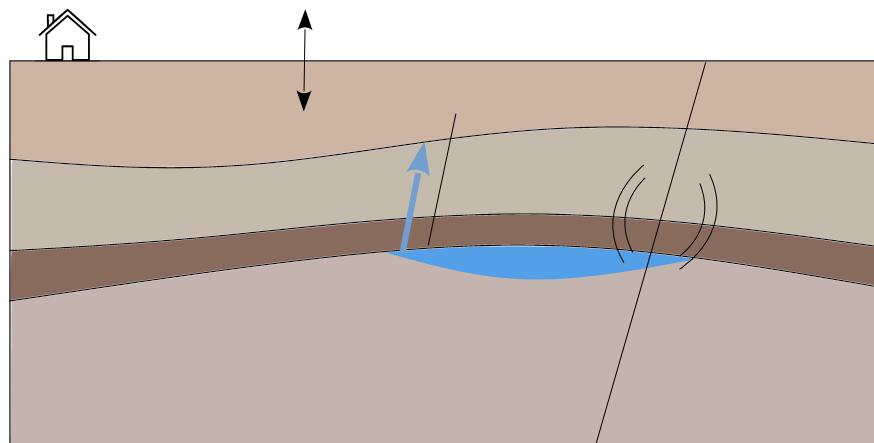
Fault reactivation (earthquakes)



Numerical simulations



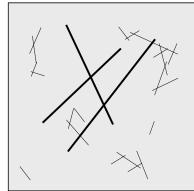
Challenges!



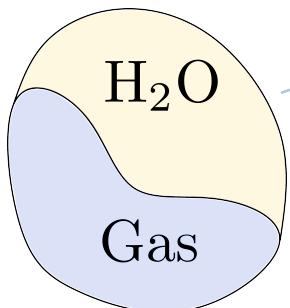
Simulating the subsoil phenomena

Context → Models → ...

Fractures, Faults



Multi-phase flow



Rock deformation and interaction rock–fluid

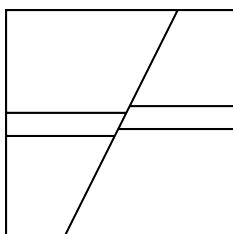


Uncertainties on parameters



Physical properties:
Permeability
Porosity
...

Geometry:
Layers position
Fault angle
...



Main contributions

- Integration of the reduced order models based on neural networks in the context of fractured porous media with physical and geometrical uncertainties to academic and realistic scenarios [3,4,5];
- The extension of a finite volume upwind scheme to a mixed-dimensional framework [6];
- A generalization to discontinuous functions of an interpolation method based on radial basis functions [3].

- [3] E. Ballini, L. Formaggia, A. Fumagalli, A. Scotti, P. Zunino. “*Application of deep learning reduced-order modeling for single-phase flow in faulted porous media*”, Computational Geosciences, 2024.
- [4] E. Ballini, A. Cominelli, L. Dovera, A. Forello, L. Formaggia, A. Fumagalli, S. Nardean, A. Scotti, P. Zunino. “*Enhancing computational efficiency of numerical simulation for subsurface fluid-induced deformation using Deep Learning Reduced Order Models*”. Accepted for SPE conference.
- [5] E. Ballini, A. Fumagalli, A. Scotti, P. Zunino. “*Reduced order modeling of time dependent PDE through manifold flattening*”. In preparation.
- [6] E. Ballini, L. Formaggia, A. Fumagalli, E. Keilegavlen, A. Scotti. “*A hybrid upwind scheme for two-phase flow in fractured porous media*”, Computer Methods in Applied Mechanics and Engineering, 2024

Contents

- Context
- Single-phase flow
- Two-phase flow
- Mesh deformation
- Fluid and solid
- Reduced order modeling to single and two-phase flow
- Industrial application

-
- Context
 - Single-phase flow
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Single phase flow – Model

Intro → Single-phase → Two-phase → ...

Mass balance

$$\text{Flux} + \text{Coupling} \\ \varepsilon_i \nabla \cdot \left(\frac{K_i}{\mu} \nabla p_i \right) + \sum_j \Xi \zeta_j = 0$$

In each subdomains
(hosting rock, fracture, fracture-fracture intersection, ...) $\Omega_i, i = 1, 2$

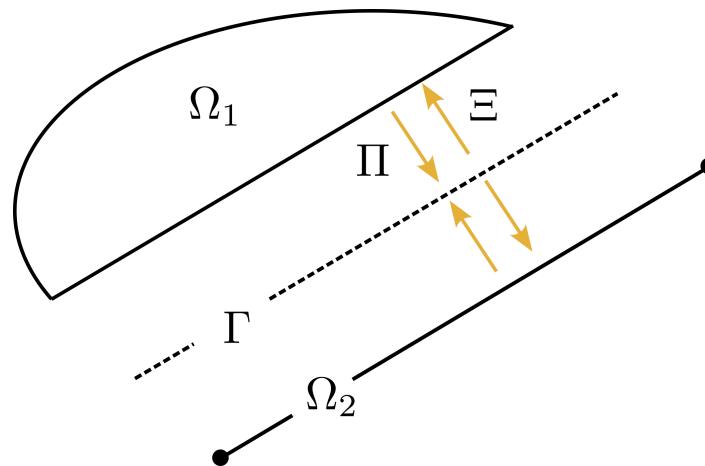
Interface equation

$$\zeta - \frac{k}{\mu} \left(\frac{1}{\varepsilon/2} (\Pi_{tr}(p_1) - \Pi p_2) \right) = 0$$

On each interface, Γ , in between subdomains

Unknowns: p_i, ζ

Coefficients (possibly uncertain): K, k, μ, ε

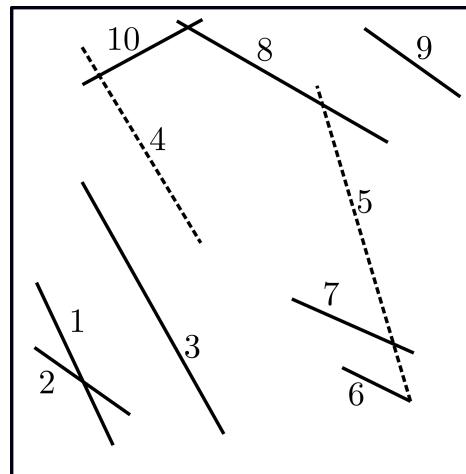
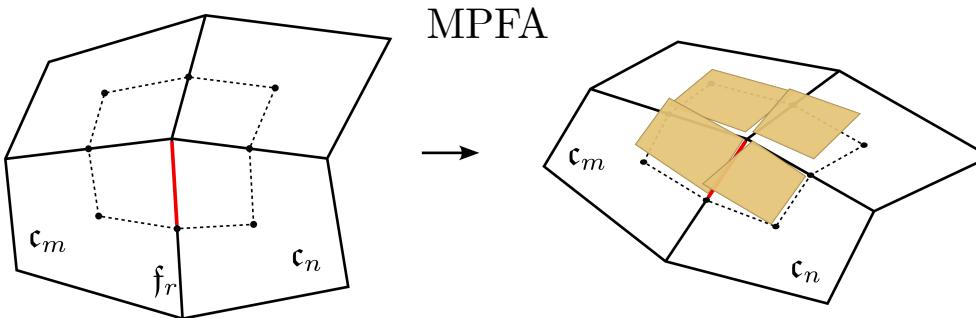


Single phase flow – Discretization

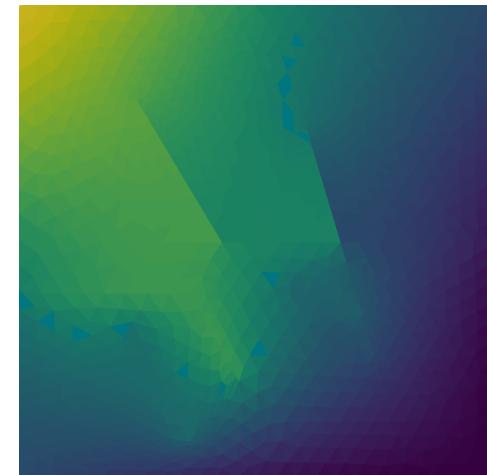
Intro → Single-phase → Two-phase → ...

Finite volume
Multi Point Flux
Approximation
(MPFA)

$$\underline{\underline{A}} \begin{bmatrix} p \\ \zeta \end{bmatrix} = \underline{b}$$

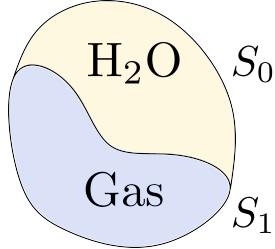


Pressure



-
- Context
 - Single-phase flow
 - Two-phase flow
 - Mesh deformation
 - Fluid and solid
 - Reduced order modeling to single and two-phase flow
 - Industrial application

Two phase flow – Model



Research mainly developed during a 6 months period ... → Single-phase → **Two-phase** → Mesh deformation → ...
at the University of Bergen, Norway.

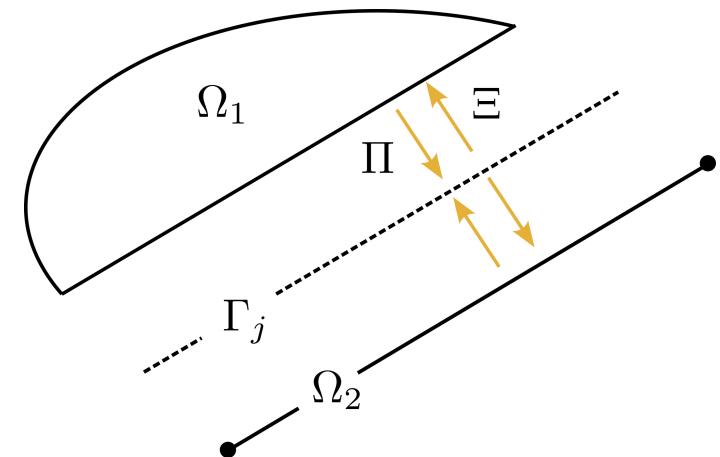
Mass balances

Accumulation	Flux	Coupling	Source
$\varepsilon\phi\partial_t(\rho_0S_0)$	$+ \varepsilon\nabla \cdot Q_0$	$+ \sum_j \Xi\mathcal{N}_j(\rho_0\lambda_0)\zeta_{0,j}$	$= f_0,$
$\varepsilon\phi\partial_t[\rho_0S_0 + \rho_1S_1]$	$+ \varepsilon\nabla \cdot Q_T$	$+ \sum_\ell \sum_j \Xi\mathcal{N}_j(\rho_\ell\lambda_\ell)\zeta_{\ell,j}$	$= f_T.$

Interface equation

$$\zeta_\ell - k \left\{ \frac{1}{\varepsilon/2} [\Pi tr(p_2) - \Pi p_1] - \mathcal{N}_j(\rho_\ell)g \right\} = 0.$$

Unknowns: $p_i, S_{0,i}, \zeta_{0,j}, \zeta_{1,j}$



Standard upwind method (PPU)

Mass balances

$$\begin{array}{cccc}
 \text{Accumulation} & \text{Flux} & \text{Coupling} & \text{Source} \\
 \varepsilon\phi\partial_t(\rho_0 S_0) & + \varepsilon\nabla \cdot Q_0 & + \sum_j \Xi \mathcal{N}_j(\rho_0 \lambda_0) \zeta_{0,j} & = f_0, \\
 \varepsilon\phi\partial_t[\rho_0 S_0 + \rho_1 S_1] & + \varepsilon\nabla \cdot Q_T & + \sum_\ell \sum_j \Xi \mathcal{N}_j(\rho_\ell \lambda_\ell) \zeta_{\ell,j} & = f_T.
 \end{array}$$

Interface equation

$$\zeta_\ell - k \left\{ \frac{1}{\varepsilon/2} [\Pi tr(p_2) - \Pi p_1] - \mathcal{N}_j(\rho_\ell) g \right\} = 0.$$

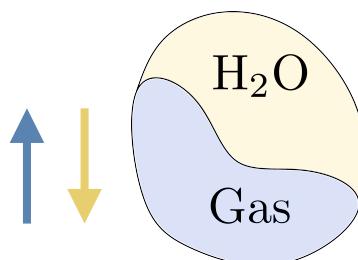
... → Single-phase → **Two-phase** → Mesh deformation → ...

Finite volume

Q_0, Q_T, \mathcal{N}_j discretized using
upwind method with
fluid velocities as upstream direction



$$\frac{1}{\Delta t} (\underline{U}^{n+1} - \underline{U}^n) + \underline{F}(\underline{U}^{n+1}) = 0$$



Issue:

Counter-current flow
because of gravity ⇒
⇒ No Newton convergence

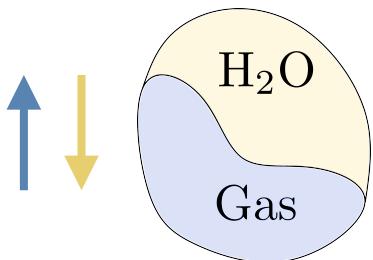
Towards hybrid upwind

... → Single-phase → **Two-phase** → Mesh deformation → ...

Mass balances

Accumulation	Flux	Coupling	Source
$\varepsilon\phi\partial_t(\rho_0 S_0)$	$+ \varepsilon\nabla \cdot Q_0$	$+ \sum_j \Xi \mathcal{N}_j(\rho_0 \lambda_0) \zeta_{0,j}$	$= f_0,$
$\varepsilon\phi\partial_t[\rho_0 S_0 + \rho_1 S_1]$	$+ \varepsilon\nabla \cdot Q_T$	$+ \sum_\ell \sum_j \Xi \mathcal{N}_j(\rho_\ell \lambda_\ell) \zeta_{\ell,j}$	$= f_T.$

Issue: countercurrent flow



$$Q_0 = V_0 + G_0$$

↓ ↓

Pressure Gravity
contribution contribution

$Q_T = Q_0 + Q_1$

Specific discretization
method

Hybrid Upwind [7] – Mixed dimensional [6]

Mass balances

Accumulation	Flux	Coupling	Source
$\varepsilon\phi\partial_t(\rho_0 S_0)$	$+ \varepsilon\nabla \cdot Q_0$	$+ \sum_j \Xi \mathcal{N}_j(\rho_0 \lambda_0) \zeta_{0,j}$	$= f_0,$
$\varepsilon\phi\partial_t[\rho_0 S_0 + \rho_1 S_1]$	$+ \varepsilon\nabla \cdot Q_T$	$+ \sum_\ell \sum_j \Xi \mathcal{N}_j(\rho_\ell \lambda_\ell) \zeta_{\ell,j}$	$= f_T.$

... → Single-phase → **Two-phase** → Mesh deformation → ...

$Q_0 = V_0 + G_0$. Upstrem directions:
 $V_0 \rightarrow$ **Mean flow**
 $G_0 \rightarrow$ **Gravity**

$\mathcal{N}_j \rightarrow$ **Fluid velocities** as upstream directions

Interface equation

$$\zeta_\ell - k \left\{ \frac{1}{\varepsilon/2} [\Pi tr(p_2) - \Pi p_1] - \mathcal{N}_j(\rho_\ell) g \right\} = 0.$$

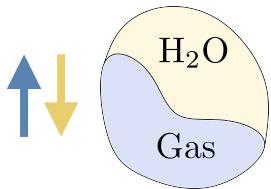
$Q_T \rightarrow$ discretized with
Blended upwind-cell average

Implemented and tested in PorePy [8]

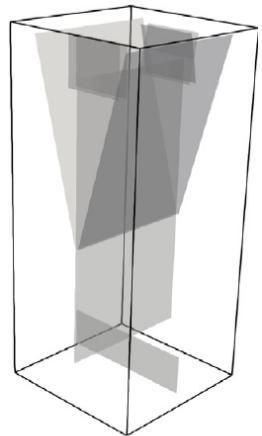
[6] E. Ballini, L. Formaggia, A. Fumagalli, E. Keilegavlen, A. Scotti. "A hybrid upwind scheme for two-phase flow in fractured porous media", Computer Methods in Applied Mechanics and Engineering, 2024.

[7] F. P. Hamon, H. Tchelepi. "Analysis of hybrid upwinding for fully-implicit simulation of three-phase flow with gravity". SIAM Journal on Numerical Analysis, 2016.

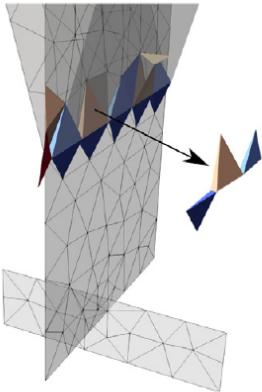
[8] E. Keilegavlen et. al. "PorePy: an open-source software for simulation of multiphysics processes in fractured porous media". Computational Geosciences, 2020.



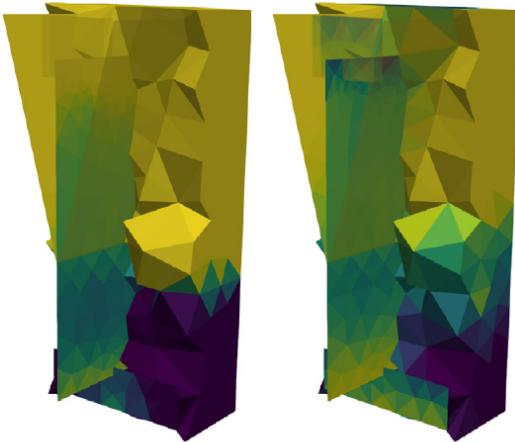
Mixed dimensional HU results



(a) Fracture network



(b) Ill-shaped cells



(c) $t = 1.3 \times 10^{-3}$

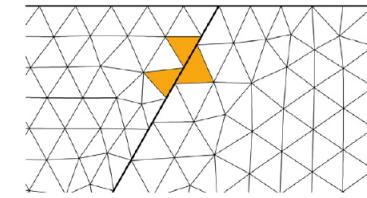
(d) $t = 2.86 \times 10^{-3}$

Standard method (PPU) fails.

Proposed method is robust and saves Newton iterations.

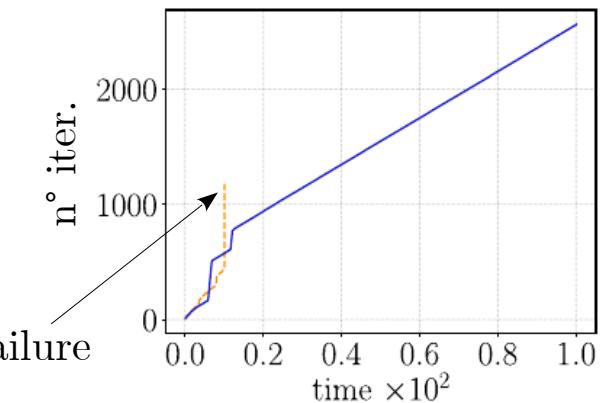
... → Single-phase → **Two-phase** → Mesh deformation → ...

Fracture



Partially non-conforming grids

PPU HU



Failure

-
- Context
 - Single-phase flow
 - Two-phase flow
 - Mesh deformation
 - Fluid and solid
 - Reduced order modeling to single and two-phase flow
 - Industrial application

Discontinuous mesh deformation

Mesh deformation using Radial Basis Function (RBF)

Main features:

- Algebraic equations
- Control points, x_{c_j}

Smooth deformation

$$s(x) = \sum_j g(x, x_{c_j}) z_j$$

Discontinuous deformation [3]

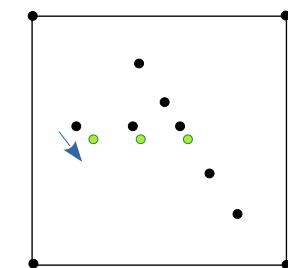
$$s(x) = \sum_j \mathcal{I}(x, \text{fracture}) g(x, x_{c_j}) z_j,$$

$$s(x_{c_j}) \cdot \nu_i = 0,$$

$$z_j \cdot t_i = 0,$$

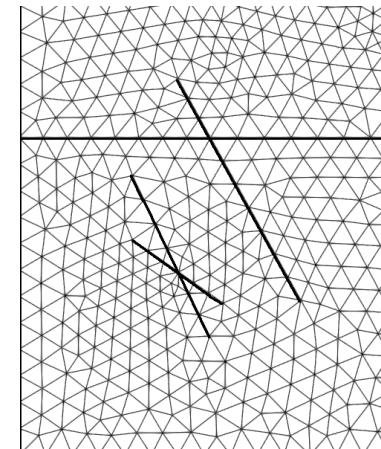
$$z_j \cdot b_i = 0$$

... → Two-phase → Mesh deformation → Fluid-solid → ...

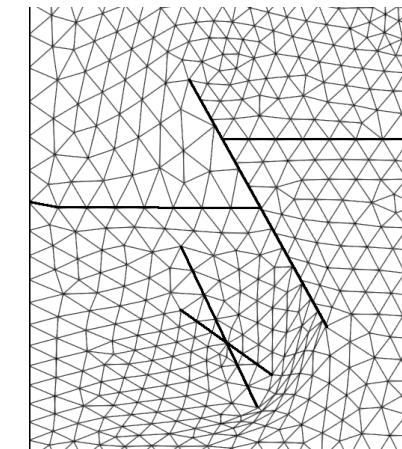


- Original control points
- Moved control points

Original



Deformed



-
- Context
 - Single-phase flow
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Fluid and solid – Model and discretization

... → Fluid-solid → ROM → Industrial application → ...

Two-phase
flow system

+

$$\begin{aligned} \nabla \cdot \sigma^{tot} + \bar{\rho}g &= 0, \\ \sigma^{tot} &= \sigma^{eff} - \beta \underline{p} \mathbb{I}, \\ \sigma^{eff} &= \mathbb{C}u, \\ \phi &= \phi_0(1 + C_p(\underline{p} - p_0)), \\ C_p &= \frac{(1 + \nu)(1 - 2\nu)}{\phi_0(1 - \nu)E}. \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

One-way coupling:
Fluid affects solid
Solid does not affect fluid

MPSA [9]

MPSA-MPFA-FV [10]

Solution procedure:
At each timestep:

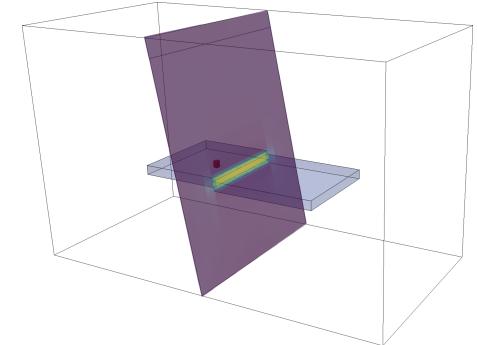
$$\underline{R}^{n+1}(p) = 0$$

$$\downarrow \underline{p}$$

$$\underline{A}\underline{u} = \underline{b}(\underline{p})$$

$$\downarrow$$

Displacement
Stress



[9] J. M. Nordbotten. Cell-centered finite volume discretizations for deformable porous media. International Journal for Numerical Methods in Engineering, 2014

[10] J. M. Nordbotten. Stable cell-centered finite volume discretization for biot equations. SIAM Journal on Numerical Analysis, 2016.

-
- Context
 - Single-phase flow
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Reduced Order Modeling

μ = parameter

Full order model (FOM)

$$\mathcal{P}_N(u_N; \mu) = 0$$

(Discrete equations of, e.g.,
single-phase flow, two-phase flow, ...)

where, e.g., $u_N = \begin{bmatrix} p_1 \\ p_2 \\ \zeta \end{bmatrix}$)

Replaced by


Reduced order model (ROM)

$$\mathcal{P}_n(u_n; \mu) = 0$$

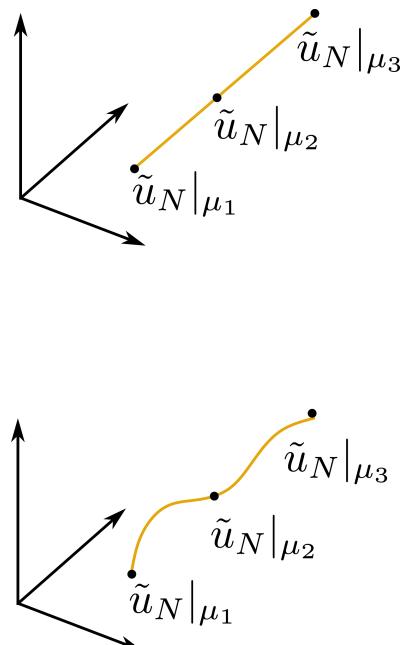
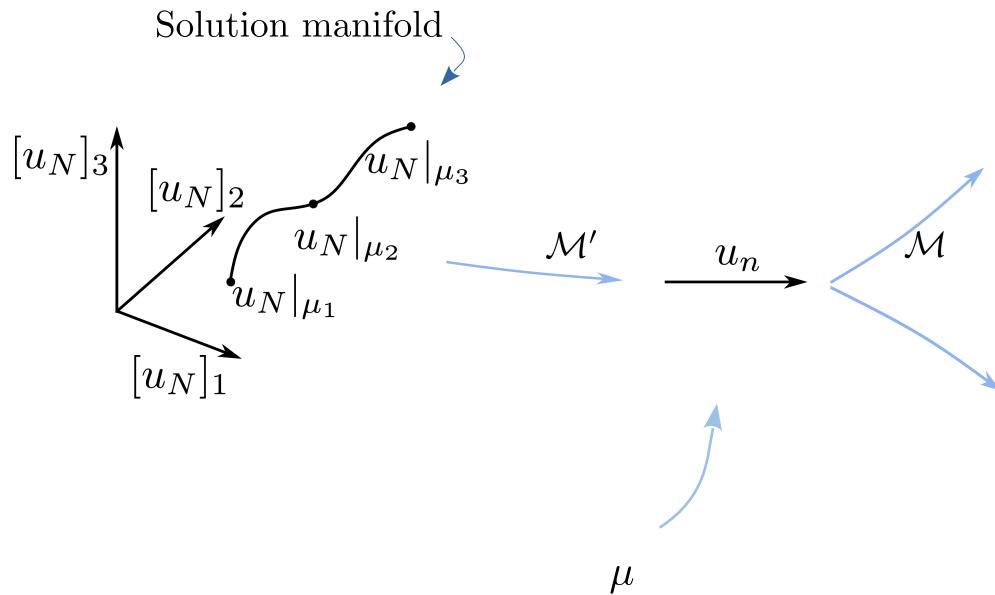
$$n \ll N$$

$$\text{then, } u_N = u_N(u_n)$$

Reduced Order Modeling

... → Fluid-solid → **ROM** → Industrial application → ...

Data-driven: **no** interaction
with FOM solver
Solution, u_N , parameterized by μ



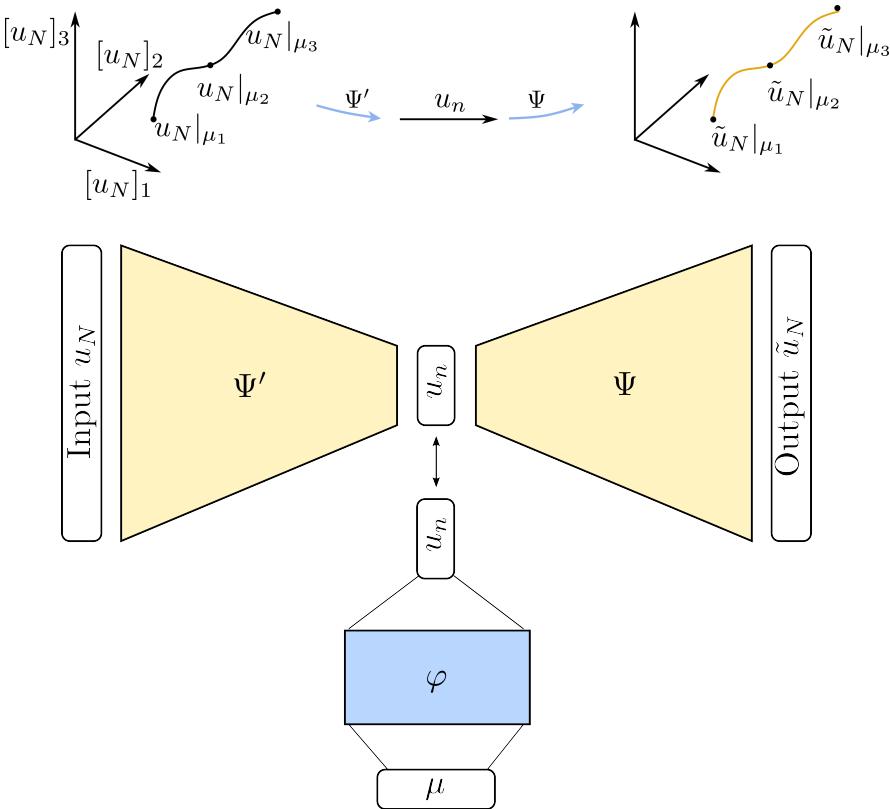
Proper Orthogonal Decomp.
 $\mathcal{M}', \mathcal{M}$ approx.
with matrices Φ^T, Φ

Inaccurate for highly curved
manifolds
Inefficient for non-affine
problems

Neural Networks
 $\mathcal{M}', \mathcal{M}$ approx. with NN

Deep Learning ROM (DL-ROM) [11]

... → Fluid-solid → ROM → Industrial application → ...



Size bottleneck autoencoder =
= Manifold dimension, n =
= Number of uncertain parameters [12]

Error bounds [12,13]

$$\varepsilon > 0$$

$$\|\Psi' - u_n^{\text{exact}}\| \lesssim f_1(\varepsilon)$$

$$\|\Psi - u_N^{\text{exact}}\| \lesssim f_2(\varepsilon)$$

$$\|\varphi - u_n^{\text{exact}}\| \lesssim f_3(\varepsilon)$$

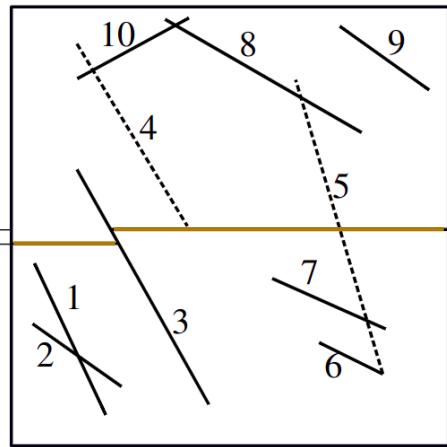
[11] S. Fresca, L. Dede', and A. Manzoni. "A comprehensive deep learning-based approach to reduced order modeling of nonlinear time-dependent parametrized PDEs". Journal of Scientific Computing, 2021.

[12] N. R. Franco, A. Manzoni, P. Zunino. "A Deep Learning approach to Reduced Order Modelling of Parameter Dependent Partial Differential Equations". Mathematics of Computation, 2023.

[13] T. Suzuki. "Adaptivity of deep ReLU network for learning in Besov and mixed smooth Besov spaces: optimal rate and curse of dimensionality", 2018. arXiv:1810.08033.

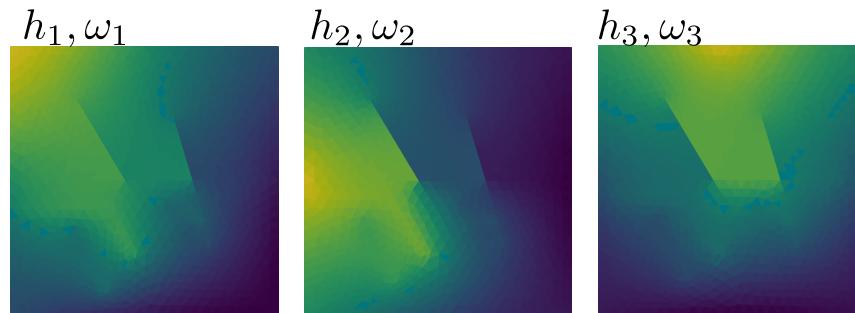
Single phase – Fracture network ROM

Mean flow

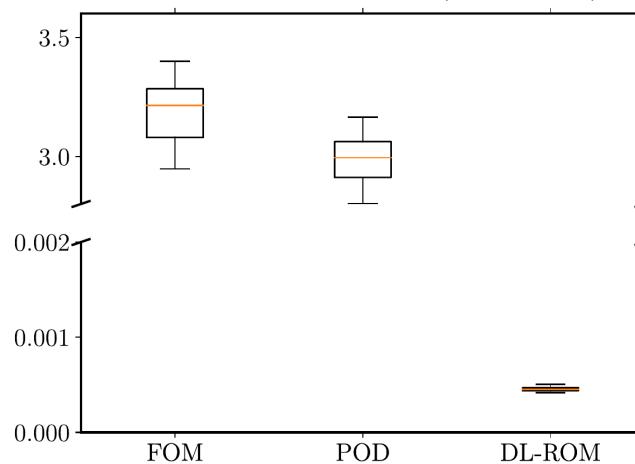


Parameters: h, ω

... → Fluid-solid → **ROM** → Industrial application → ...



Evaluation time (seconds)



Error test dataset:

e_{max}	2.3%
e_{min}	0.28%
e_{ave}	0.58%

Pruning

... → Fluid-solid → **ROM** → Industrial application → ...

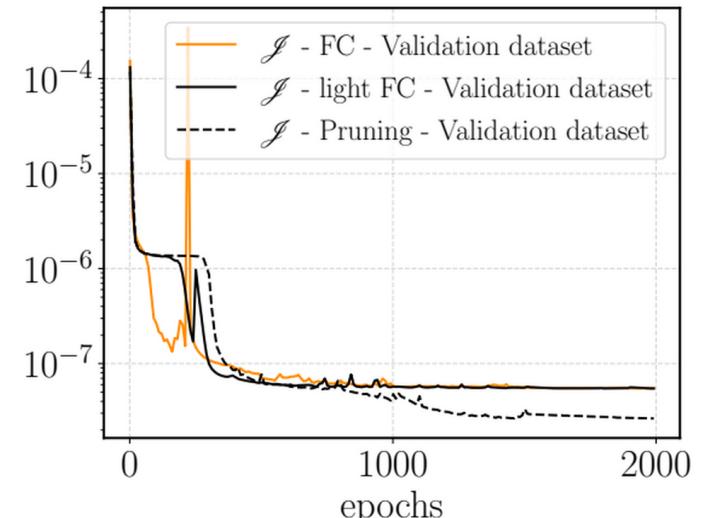
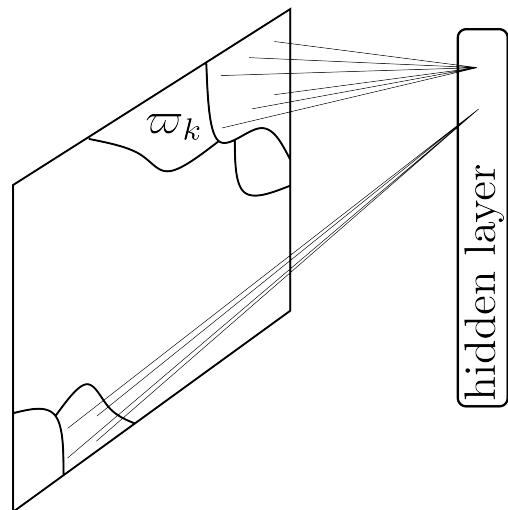
Problem: too many trainable weights.

Solution: pruning first layer of encoder
(resp. last layer of decoder):

$$\mathcal{I}_i = \sum_{j=1}^N W_{ij} u_{N,j} + b_i$$

↓ Pruning
 $M_{ij} \in \{0, 1\}$

$$\mathcal{I}_i = \sum_{j=1}^N M_{ij} W_{ij} u_{N,j} + b_i$$

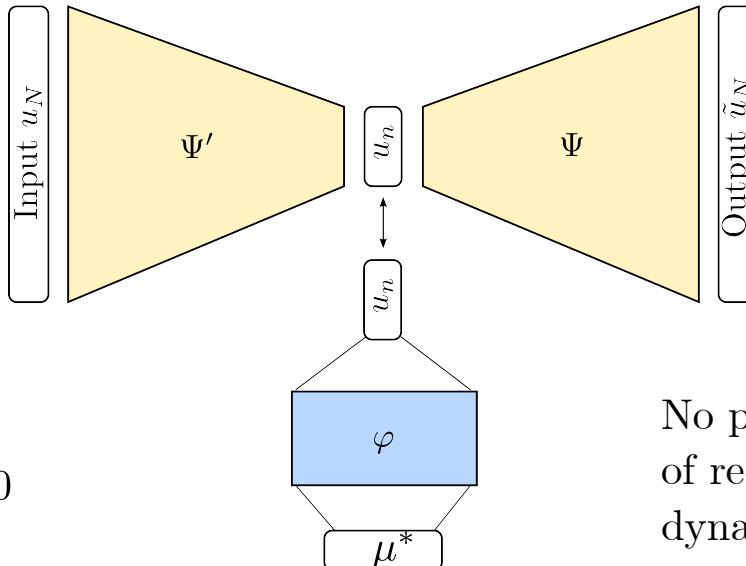


90% number of weight reduction

ROM to time dependent PDE

Stationary:

$$\underline{F}_\mu(\underline{U}^{n+1}) = 0$$



Time-dependent:

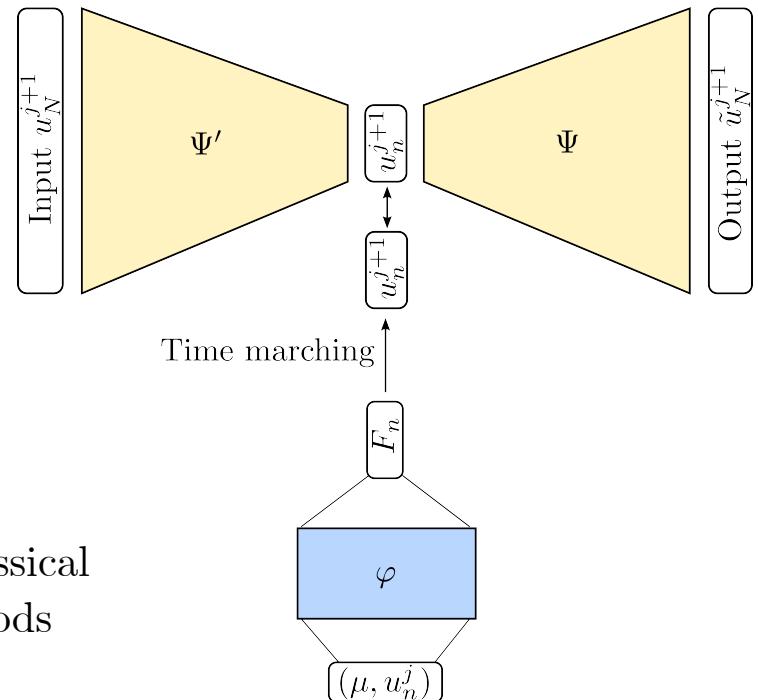
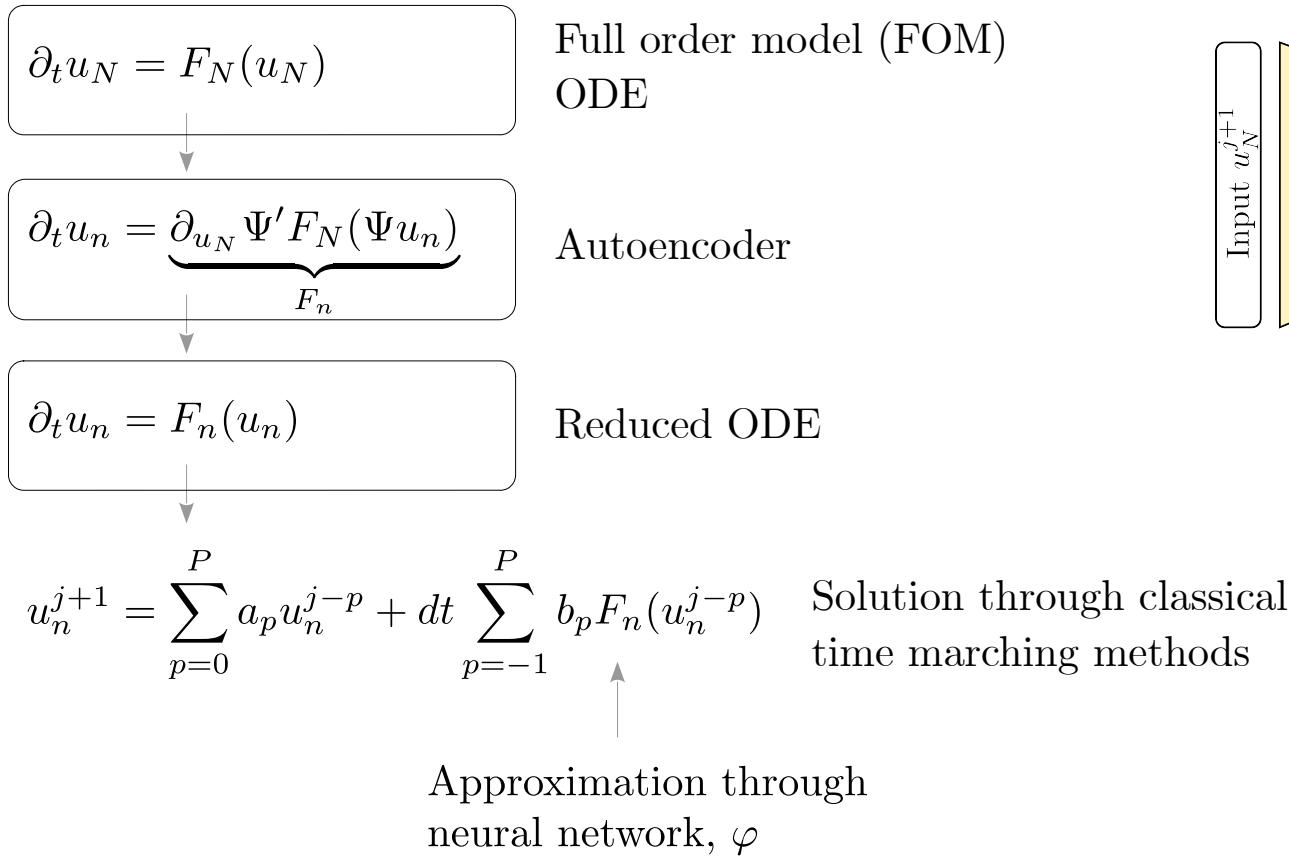
$$\frac{1}{dt}(\underline{U}^{n+1} - \underline{U}^n) + \underline{F}_\mu(\underline{U}^{n+1}) = 0$$

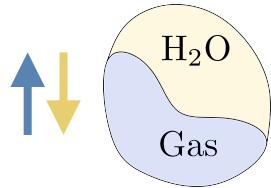
$$\mu^* = (\mu, t)$$

No proper treatment
of reduced solution
dynamics

DL-ROM – Recursive

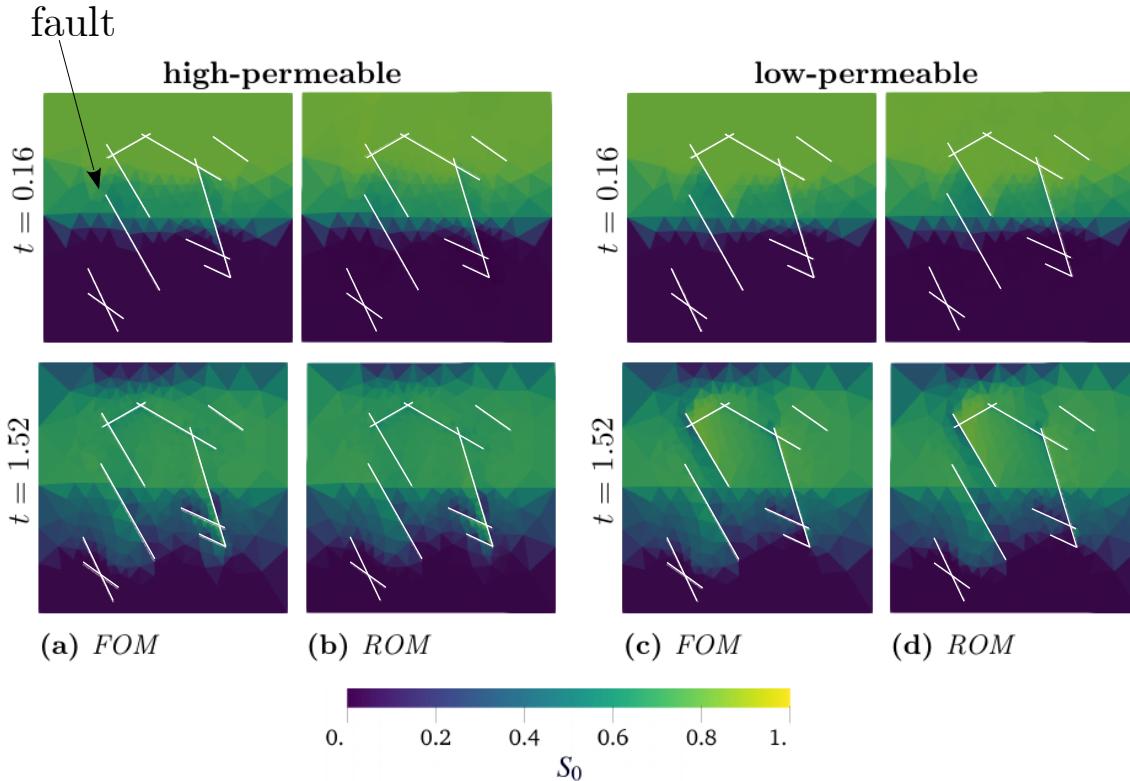
$\dots \rightarrow$ Fluid-solid \rightarrow ROM \rightarrow Industrial application $\rightarrow \dots$



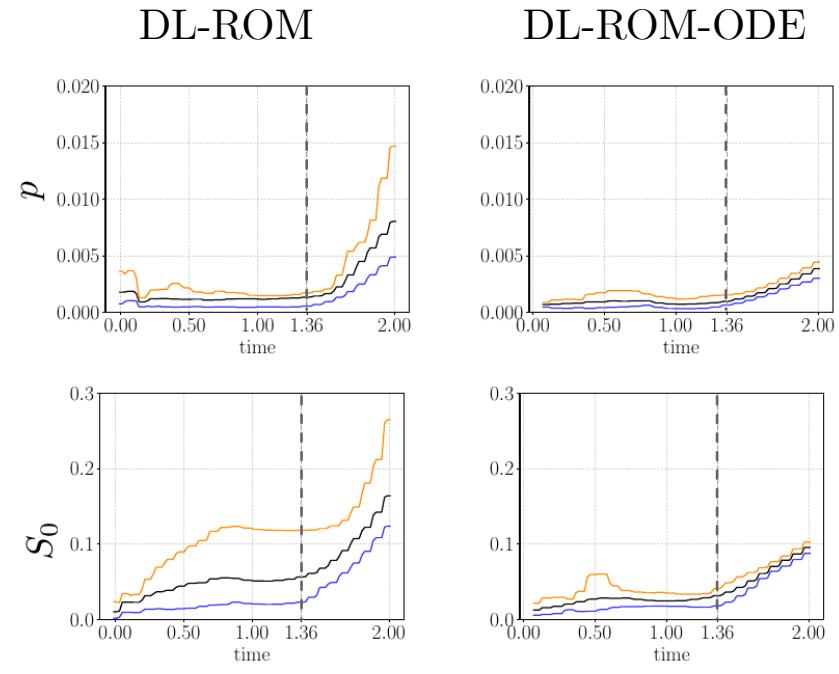


Two-phase flow – Fracture network

... → Fluid-solid → ROM → Industrial application → ...



Parameters: fault throw,
fracture permeability



-
- Context
 - Single-phase flow
 - Two-phase flow
 - Mesh deformation
 - Fluid and solid
 - Reduced order modeling to single and two-phase flow
 - Industrial application

Realistic scenario [4]

Research mainly developed during a 6 months period at Eni.

... → ROM → Eni application → end

- **Well control**

production,
injection

- **Two-phase flow equation**

$$\begin{aligned}\varepsilon\phi\partial_t(\rho_0S_0) + \varepsilon\nabla \cdot Q_0 + C_0 &= f_0, \\ \varepsilon\phi\partial_t[\rho_0S_0 + \rho_1S_1] + \varepsilon\nabla \cdot Q_T + \\ + C_T &= f_T.\end{aligned}$$

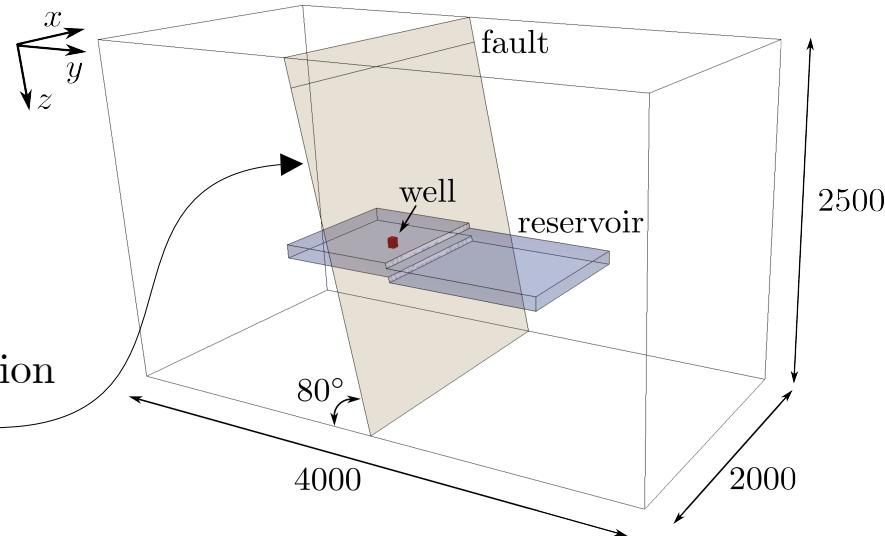
- **Solid mechanics equations**

$$\begin{aligned}\nabla \cdot \sigma^{tot} + \rho_b g &= 0, \\ \sigma^{tot} &= \sigma^{eff} + \beta p \mathbb{I},\end{aligned}$$

Traction on fault, T

Coulomb Failure Function

$$CFF = \|T_t\| - f T_n \cdot n$$



ROM to reproduce T

Parameters:

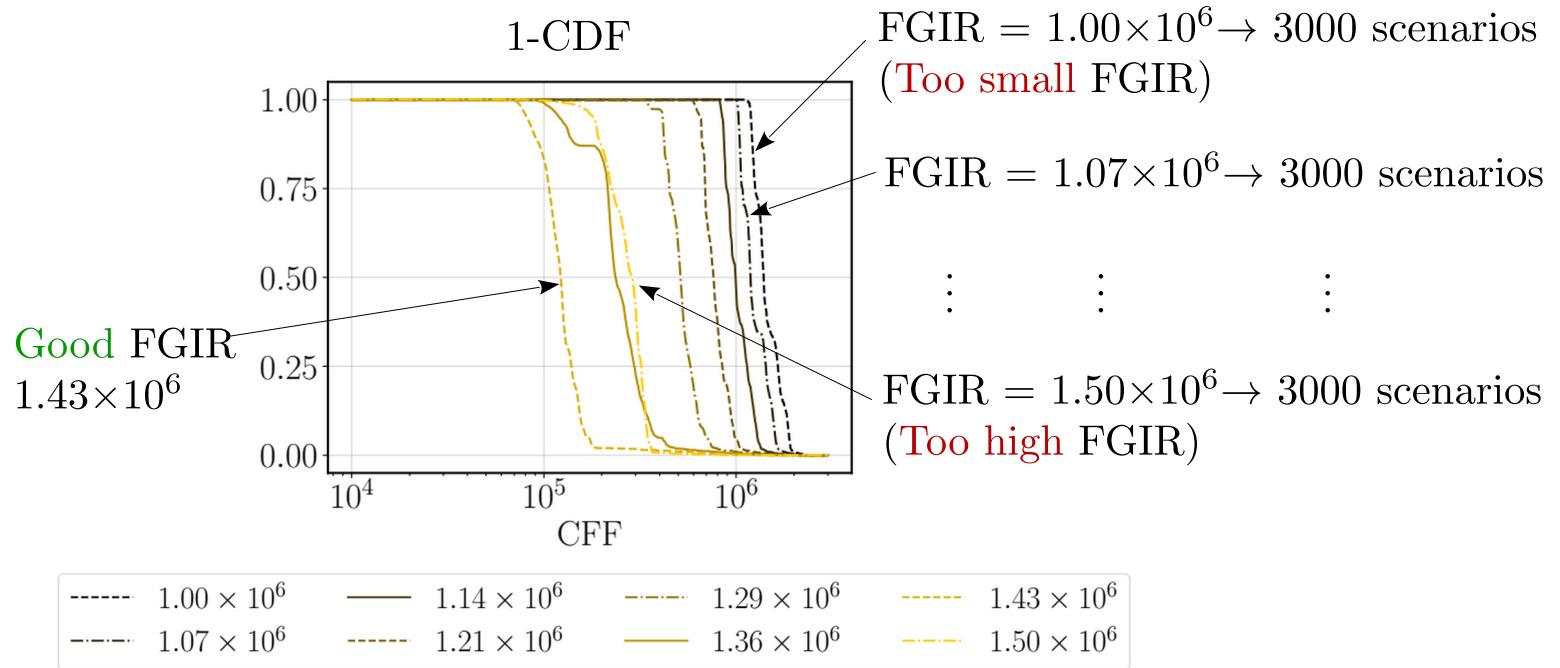
Fluid: field gas injection rate (FIGR), rock and fault permeability

Solid: Young's modulus reservoir and overburden

Realistic scenario

... → ROM → Eni application → end

Tot. ~ 25000
simulations



FGIR = Field gas injection rate, m^3/day

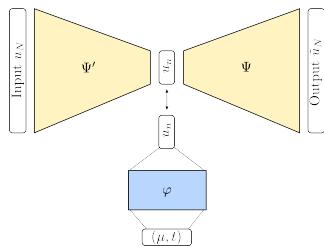
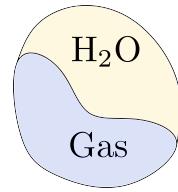
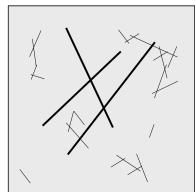
CFF = Coulomb failure function

CDF = Cumulative distribution function

Conclusion

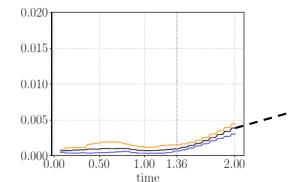
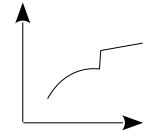
Summary:

Simulation **single/two-phase** flow in **fractured/faulted** subsoil →
Expensive simulation and uncertainties →
ROM based on NN for
steady and time dependent problem
in combination with **mesh deformation**
and **realistic scenario**.



Limitations and future developments

- Three phases, capillary pressure, chemical reactions, ...
- Integration of data from different sources and multi-fidelity approach and fast offline phase;
- Severely complicated scenarios showing discontinuous quantities of interests;
- Long-term extrapolation in time.



Thank you for your attention

Northen Lights

CO₂
from industry

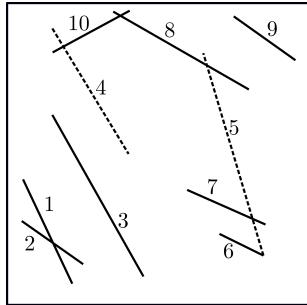


CO₂
temporary tanks

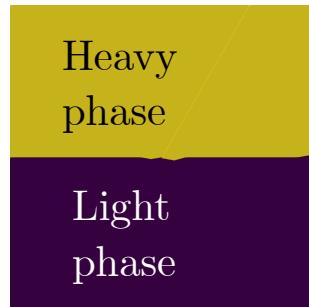
CO₂
towards reservoir

Mixed dimensional HU results

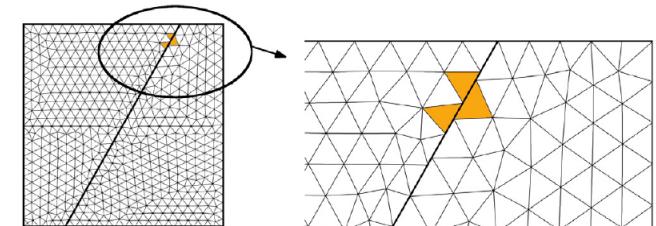
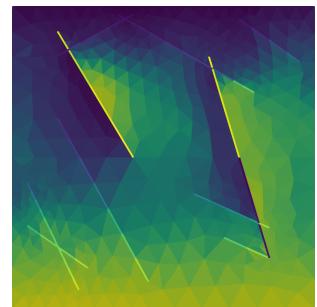
Domains



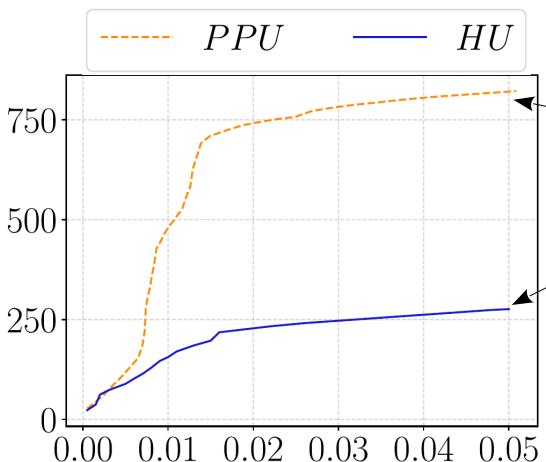
Initial condition



$t = 0.03$



Cumulative
number of Newton
iterations



Standard method (PPU):
Many iterations

HU: Few iterations

Fluid and solid

Two-phase
flow system

+

$$\nabla \cdot \sigma^{tot} + \bar{\rho}g = 0,$$

$$\sigma^{tot} = \sigma^{eff} - \beta \textcolor{blue}{p} \mathbb{I},$$

$$\sigma^{eff} = \mathbb{C}u,$$

$$\phi = \phi_0(1 + C_p(\textcolor{blue}{p} - p_0)),$$

$$C_p = \frac{(1 + \nu)(1 - 2\nu)}{\phi_0(1 - \nu)E}.$$

Assumptions:

- Small hydrostatic pressure contribution due to different fluid densities
- Small solid deformation

Training

Back propagation through time

$$\overline{\mathcal{J}} = \frac{1}{n_{train} N_{time}} \sum_{i=1}^{n_{train}} \sum_{j=1}^{N_{time}} \sum_{k=1}^K \mathcal{L}_{ij}^k$$

$$\mathcal{L}_{ij}^1 = \alpha_1 \|\overline{\Psi'(u_{N,i}^j)} - u_{n,i}^j\|^2$$

$$\mathcal{L}_{ij}^2 = \alpha_2 \|u_{N,i}^j - \Psi \circ \Psi'(u_{N,i}^j)\|^2$$

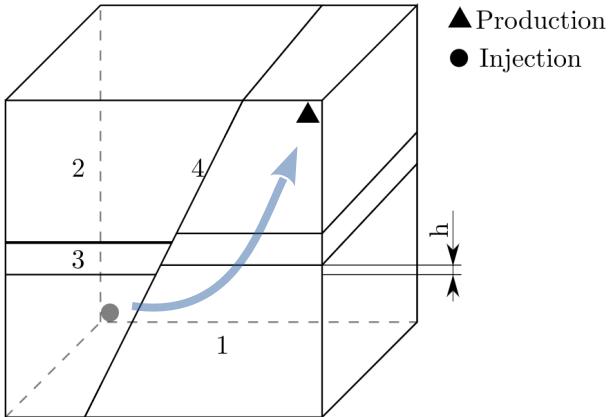
$$\mathcal{L}_{ij}^3 = \alpha_3 \left\| \sum_{p=-1}^P a_p \Psi'(u_{N,i}^{j-p}) - dt \sum_{p=-1}^P b_p \varphi(\mu_i, u_{n,i}^{j-p}) \right\|^2$$

$$\mathcal{L}_{ij}^4 = \alpha_4 \|\Psi'(u_{N,i}^j) - u_{n,i}^j\|^2$$

$$\mathcal{L}_{ij}^5 = \alpha_5 \|u_{N,i}^j - \Psi(u_{n,i}^j)\|^2$$

Multi-query application

... → Fluid-solid → **ROM** → Industrial application → ...



Inverse problem

$$\min_{\mu} (\Delta p(\mu) - \Delta p_d)^2$$

s.t. :

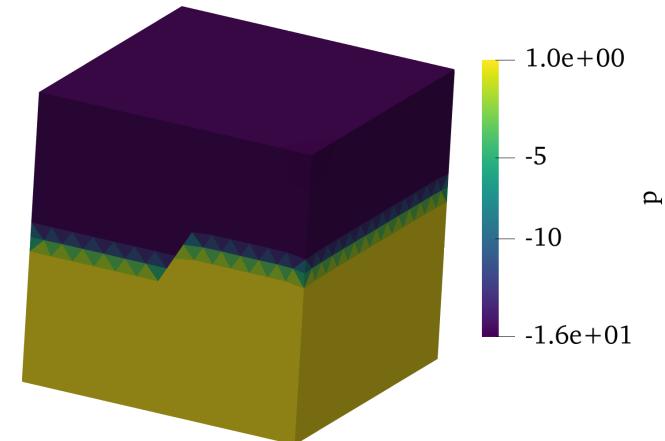
$$K_1 \in [10^{-4}, 1]$$

$$K_2 \in [10^2, 2 \times 10^2]$$

$$K_3 \in [10^{-6}, 10^{-4}]$$

$$K_4 \in [9 \times 10^{-4}, 10^{-3}]$$

$$h \in [0.01, 0.1]$$



Genetic algorithm, 7356 evaluations
in 18 s to find the optimum

Optimal value:

$$\Delta p_{ROM} = 15.5$$

$$\Delta p_{FOM} = 16.7$$

CFF

