PhD program in Mathematical Models and Methods in Engineering

Flow and mechanics in fractured porous media: from high fidelity models to efficient reduced order solutions

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MODELLISTICA E CALCOLO SCIENTIFICO
MOX
MODELING AND SCIENTIFIC COMPUTING

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Co-supervisor: Alessio Fumagalli
Tutor: Filippo Gazzola

Milano, 6 February 2025

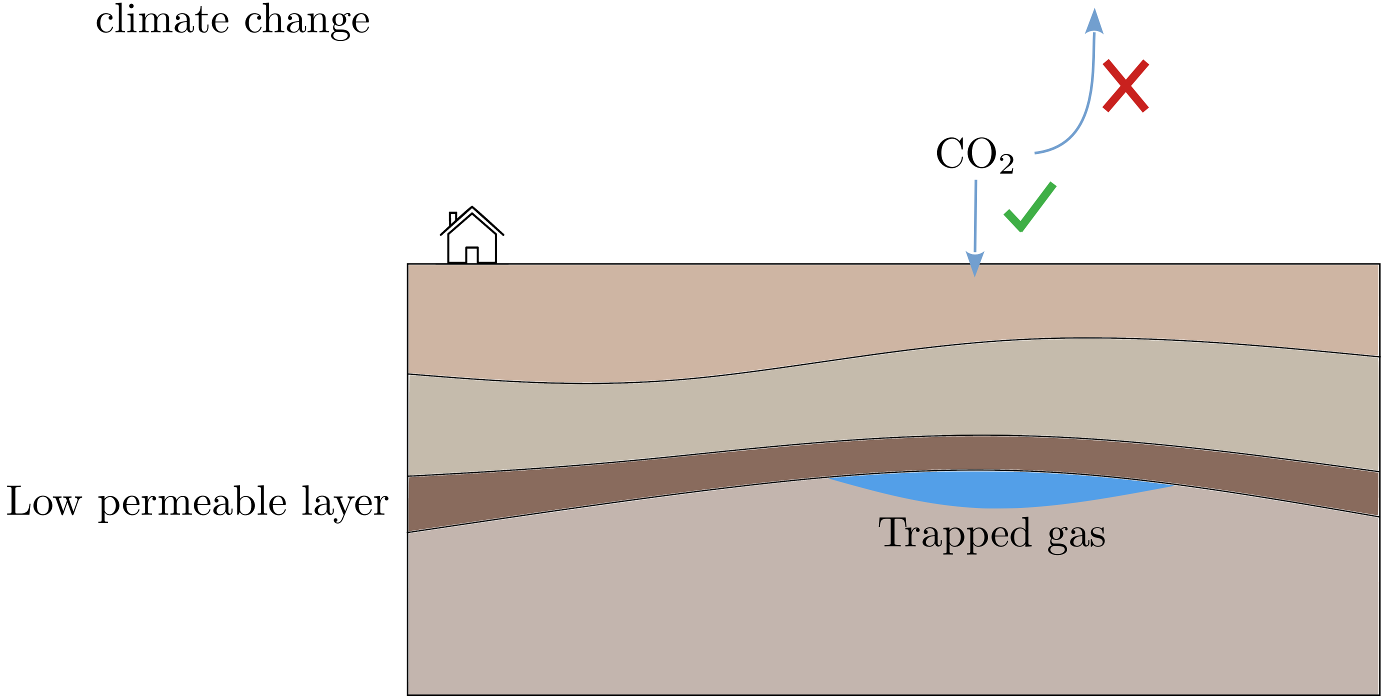
Introduction

Goal: Greener economy

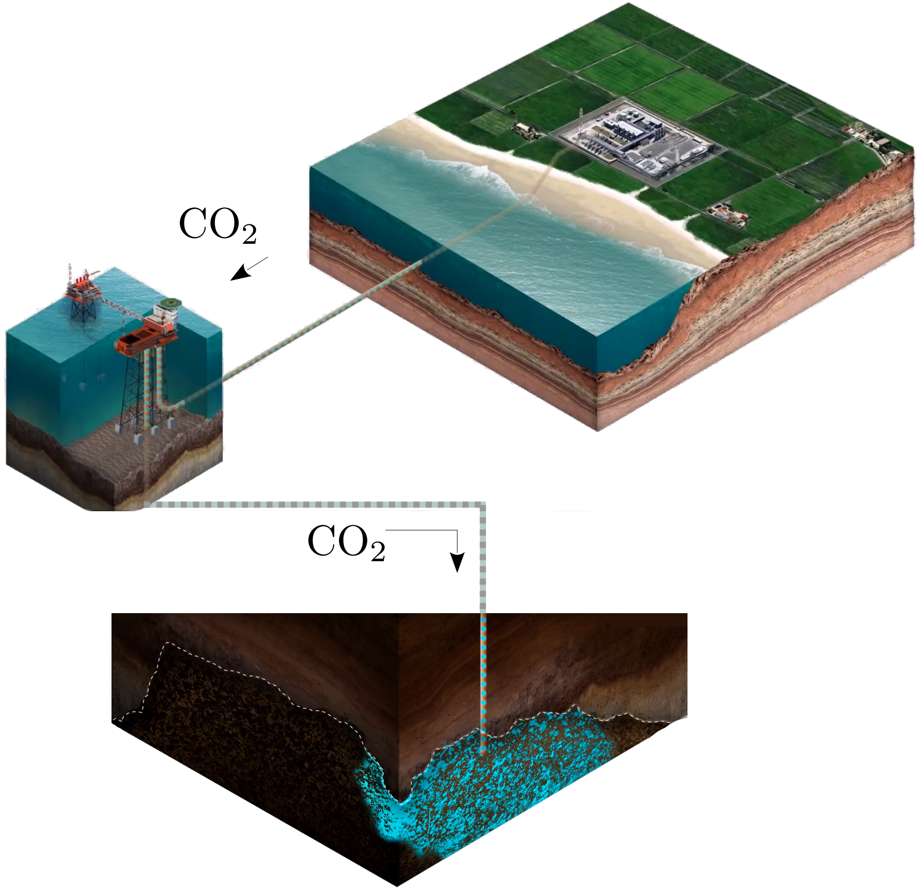
How: Carbon Capture Storage (CCS) to mitigate climate change

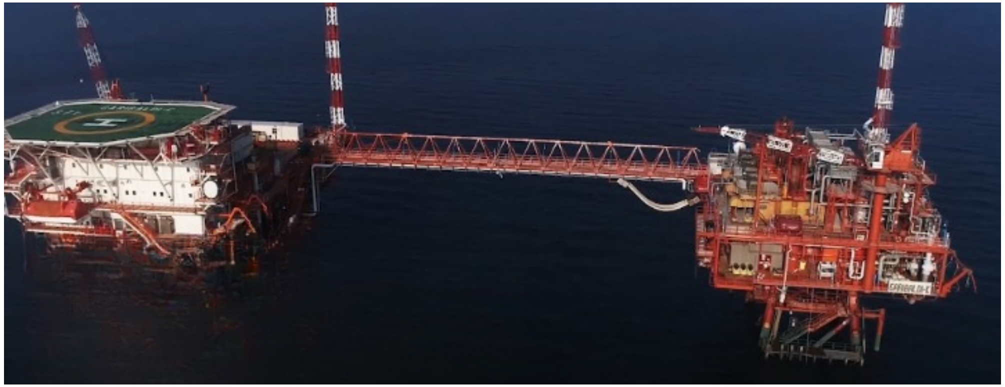
Subsoil

Porous medium filled by fluids



Ravenna CCS

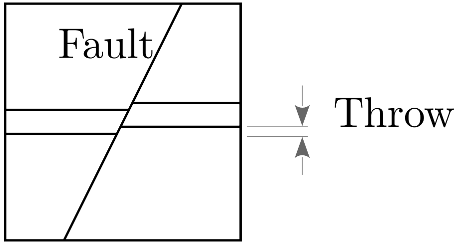


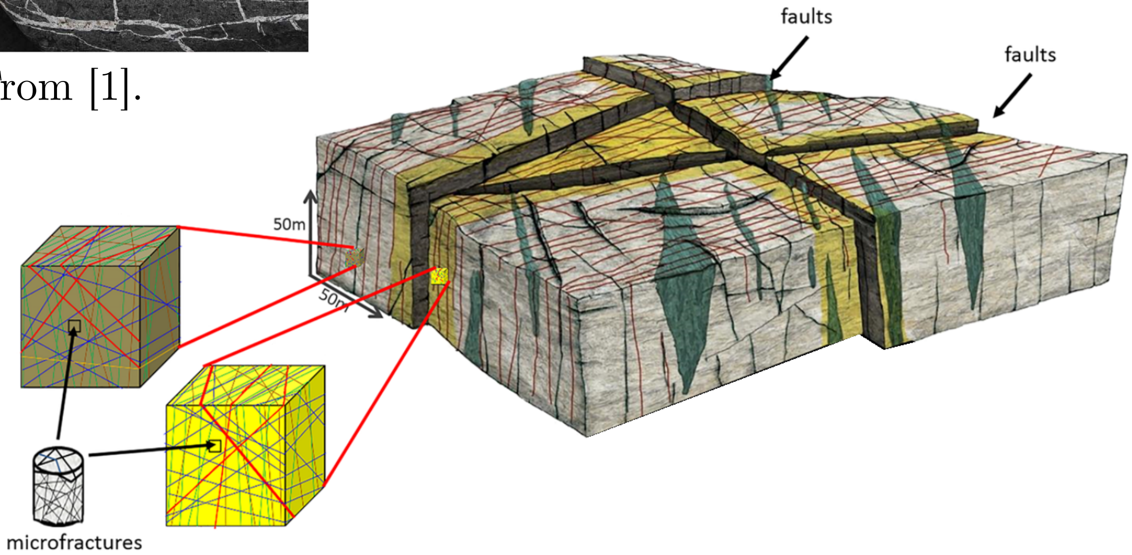


Ravenna CCS. Progetto ravenna CCS. https://ravennaccs.com/it-IT/progetto. Accessed: 2024-09-10.

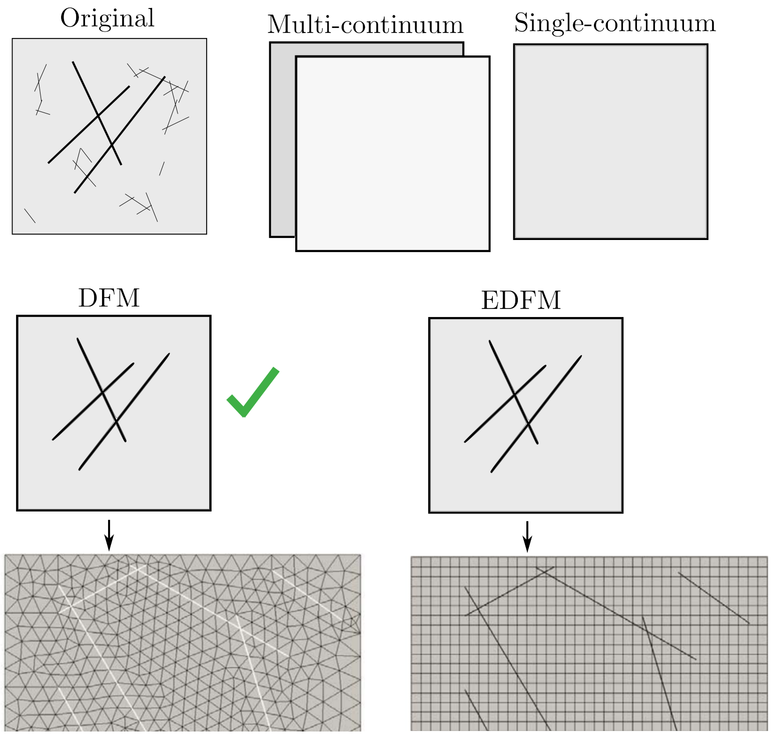
Fractures and faults

From [1].





From [2].



[1] A. Belaidi, D. A. Bonter, C. Slightam, and R. C. Trice. *The lancaster field: progress in opening the UK's fractured basement play*. Geological Society, London, Petroleum Geology Conference Series, 2016.
[2] Peacock, D.C.P. and Sanderson, D.J. and Rotevatn, A. *Relationships between fractures,* Journal of Structural Geology, 2018.

Simulating the subsoil phenomena

Interaction with subsoil

↓

Undesirable scenarios:

Leakage

Subsidence

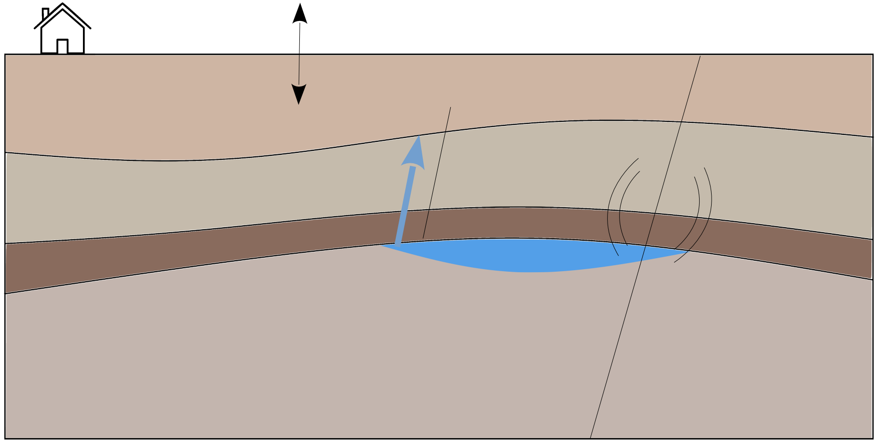
Fault reactivation (earthquakes)

↓

Numerical simulations

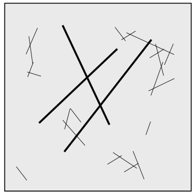
↓

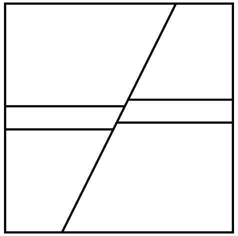
Challenges!



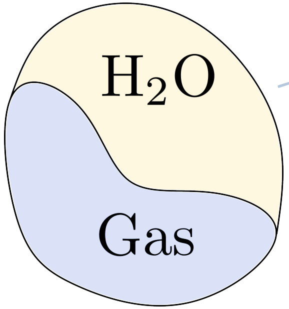
Simulating the subsoil phenomena

Fractures, Faults

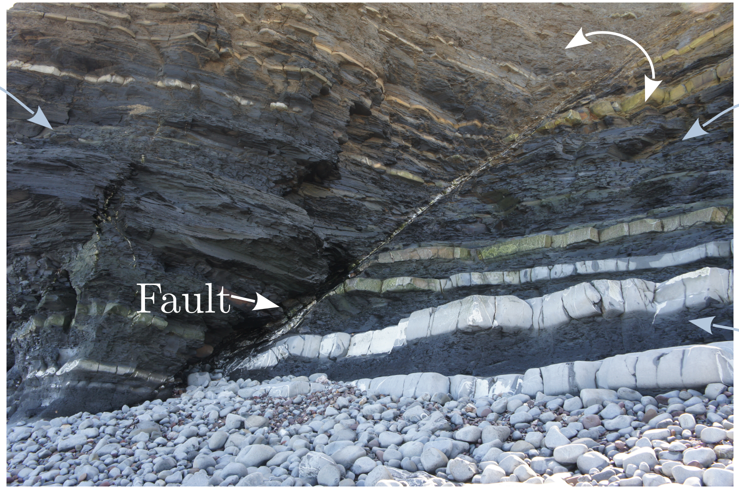




Multi-phase flow



Rock deformation and interaction rock–fluid



Uncertainties on parameters

↓

Physical properties:
Permeability
Porosity
...

Geometry:
Layers position
Fault angle
...

Main contributions

- Integration of the reduced order models based on neural networks in the context of fractured porous media with physical and geometrical uncertainties [3,5];

- Validation on realistic scenarios in collaboration with Eni SpA [4];

- Extension of a finite volume upwind scheme for two phase flows in the presence of gravity to the mixed-dimensional formulation [6];

- Mesh deformation strategy based on radial basis function to handle complex and discontinuous grid deformations [3].

[3] E. Ballini, L. Formaggia, A. Fumagalli, A. Scotti, P. Zunino. *"Application of deep learning reduced-order modeling for single-phase flow in faulted porous media"*, Computational Geosciences, 2024.
[4] E. Ballini, A. Cominelli, L. Dovera, A. Forello, L. Formaggia, A. Fumagalli, S. Nardean, A. Scotti, P. Zunino. "*Enhancing computational efficiency of numerical simulation for subsurface fluid-induced deformation using Deep Learning Reduced Order Models".* Accepted for SPE conference.
[5] E. Ballini, A. Fumagalli, A. Scotti, P. Zunino. *"Reduced order modeling of time dependent PDE through manifold flattening"*. In preparation.
[6] E. Ballini, L.Formaggia, A. Fumagalli, E. Keilegavlen, A. Scotti. "*A hybrid upwind scheme for two-phase flow in fractured porous media*", Computer Methods in Applied Mechanics and Engineering, 2024

Contents

- Single-phase flow
- Two-phase flow
- Mesh deformation
- Fluid and solid
- Reduced order modeling to single and two-phase flow
- Industrial application

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Single phase flow – Model

Mass balance

Flux + Coupling

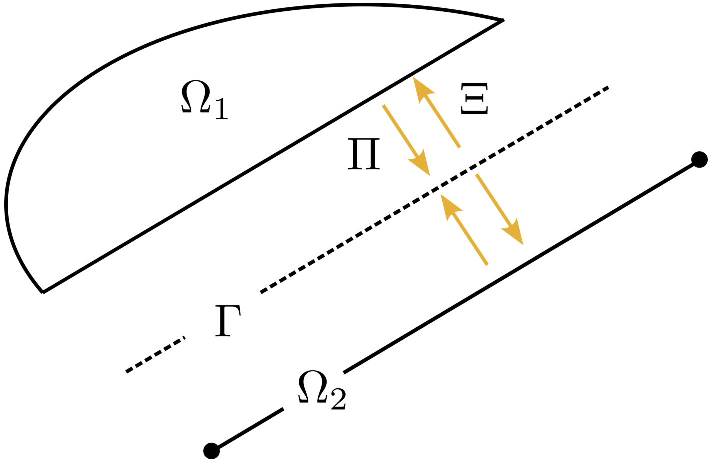
$$\varepsilon_i \nabla \cdot \left(\frac{K_i}{\mu} \nabla p_i \right) + \sum_j \Xi \zeta_j = 0$$

In each subdomains
(hosting rock, fracture, fracture-fracture intersection, …)
$\Omega_i, \quad i = 1, 2$

Interface equation

$$\zeta - \frac{k}{\mu} \left(\frac{1}{\varepsilon/2} (\Pi tr(p_1) - \Pi p_2) \right) = 0$$

On each interface, Γ, in between subdomains



Unknowns: p_i, ζ

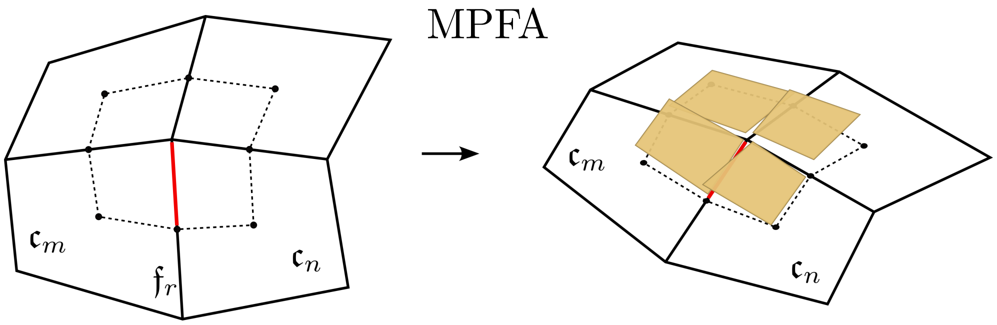
Coefficients (possibly uncertain): K, k, μ, ε

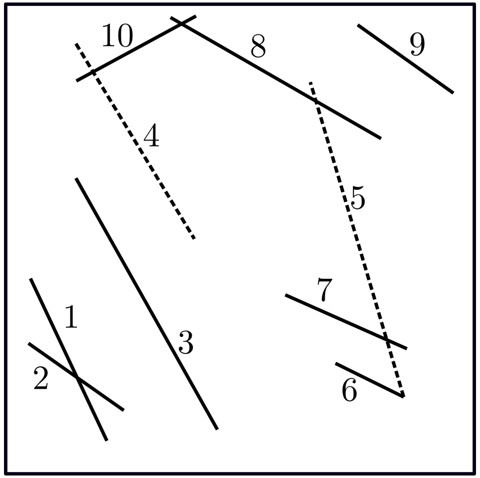
Single phase flow – Discretization

Finite volume
Multi Point Flux
Approximation
(MPFA)

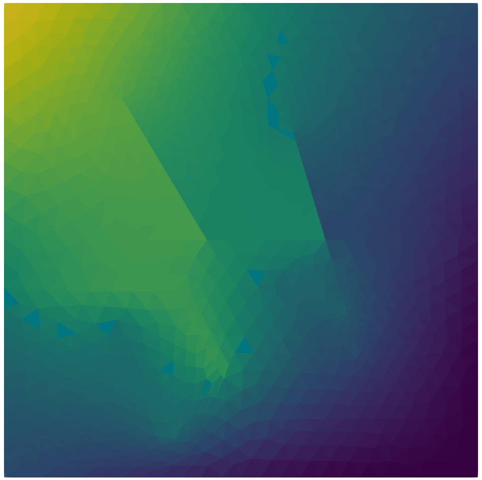
↓

$$\underline{\underline{A}}\begin{bmatrix} p \\ \zeta \end{bmatrix} = \underline{b}$$





Pressure



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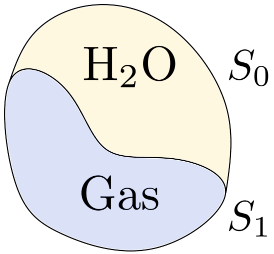
Two phase flow – Model



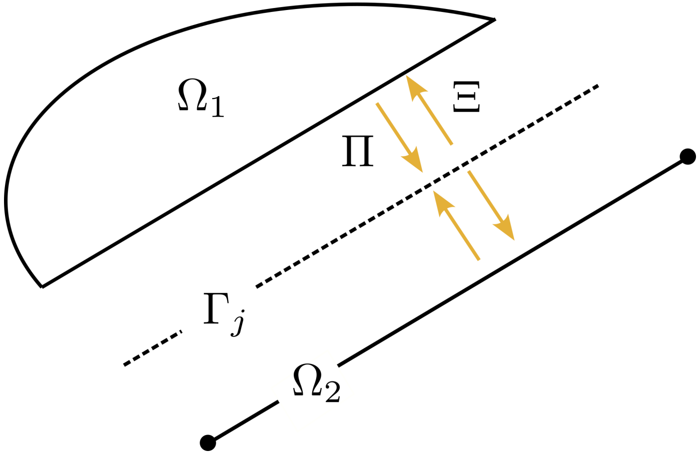
Research mainly developed during a 6 months period at the University of Bergen, Norway.

Mass balances

Accumulation	Flux	Coupling	Source
$\varepsilon\phi\partial_t(\rho_0 S_0)$	$+\,\varepsilon\nabla\cdot Q_0$	$+\sum_j \Xi\mathcal{N}_j(\rho_0\lambda_0)\zeta_{0,j}$	$= f_0,$
$\varepsilon\phi\partial_t[\rho_0 S_0 + \rho_1 S_1]$	$+\,\varepsilon\nabla\cdot Q_T$	$+\sum_\ell\sum_j \Xi\mathcal{N}_j(\rho_\ell\lambda_\ell)\zeta_{\ell,j}$	$= f_T.$

Interface equation

$$\zeta_\ell - k\left\{\frac{1}{\varepsilon/2}[\Pi tr(p_2) - \Pi p_1] - \mathcal{N}_j(\rho_\ell)g\right\} = 0.$$



Unknowns: $p_i, S_{0,i}, \zeta_{0,j}, \zeta_{1,j}$

Standard upwind method (PPU)

Mass balances

Accumulation	Flux	Coupling	Source
$\varepsilon\phi\partial_t(\rho_0 S_0)$	$+\,\varepsilon\nabla\cdot Q_0$	$+\sum_j \Xi\mathcal{N}_j(\rho_0\lambda_0)\zeta_{0,j}$	$= f_0,$
$\varepsilon\phi\partial_t[\rho_0 S_0 + \rho_1 S_1]$	$+\,\varepsilon\nabla\cdot Q_T$	$+\sum_\ell\sum_j \Xi\mathcal{N}_j(\rho_\ell\lambda_\ell)\zeta_{\ell,j}$	$= f_T.$

Interface equation

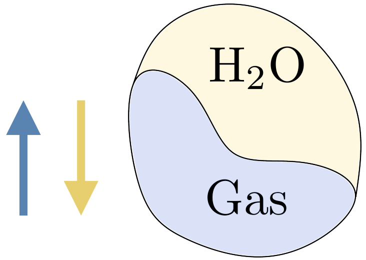
$$\zeta_\ell - k\left\{\frac{1}{\varepsilon/2}[\Pi tr(p_2) - \Pi p_1] - \mathcal{N}_j(\rho_\ell)g\right\} = 0.$$

Finite volume

Q_0, Q_T, $\mathcal{N}_j$ discretized using **upwind method** with **fluid velocities** as upstream direction

$\downarrow$

$$\frac{1}{\Delta t}(\underline{U}^{n+1} - \underline{U}^n) + \underline{F}(\underline{U}^{n+1}) = 0$$



Issue:

Counter-current flow because of gravity $\Rightarrow$

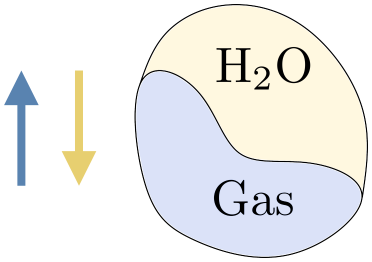
$\Rightarrow$ No Newton convergence

Towards hybrid upwind

Mass balances

Accumulation	Flux	Coupling	Source
$\varepsilon\phi\partial_t(\rho_0 S_0)$	$+\,\varepsilon\nabla\cdot Q_0$	$+\sum_j \Xi\mathcal{N}_j(\rho_0\lambda_0)\zeta_{0,j}$	$= f_0,$
$\varepsilon\phi\partial_t[\rho_0 S_0 + \rho_1 S_1]$	$+\,\varepsilon\nabla\cdot Q_T$	$+\sum_\ell\sum_j \Xi\mathcal{N}_j(\rho_\ell\lambda_\ell)\zeta_{\ell,j}$	$= f_T.$

Issue: countercurrent flow



$Q_0 = V_0 \quad + \quad G_0$

Pressure contribution

Gravity contribution

$Q_T = Q_0 + Q_1$

Specific discretization method

Hybrid Upwind [7] – Mixed dimensional [6]

... → Single-phase → **Two-phase** → Mesh deformation → ...

Mass balances

Accumulation	Flux	Coupling	Source
$\varepsilon\phi\partial_t(\rho_0 S_0)$	$+\,\varepsilon\nabla\cdot Q_0$	$+\sum_j \Xi\mathcal{N}_j(\rho_0\lambda_0)\zeta_{0,j}$	$= f_0,$
$\varepsilon\phi\partial_t[\rho_0 S_0 + \rho_1 S_1]$	$+\,\varepsilon\nabla\cdot Q_T$	$+\sum_\ell\sum_j \Xi\mathcal{N}_j(\rho_\ell\lambda_\ell)\zeta_{\ell,j}$	$= f_T.$

Interface equation

$$\zeta_\ell - k\left\{\frac{1}{\varepsilon/2}[\Pi tr(p_2) - \Pi p_1] - \mathcal{N}_j(\rho_\ell)g\right\} = 0.$$

$Q_0 = V_0 + G_0$. Upstream directions:
$V_0 \rightarrow$ **Mean flow**
$G_0 \rightarrow$ **Gravity**

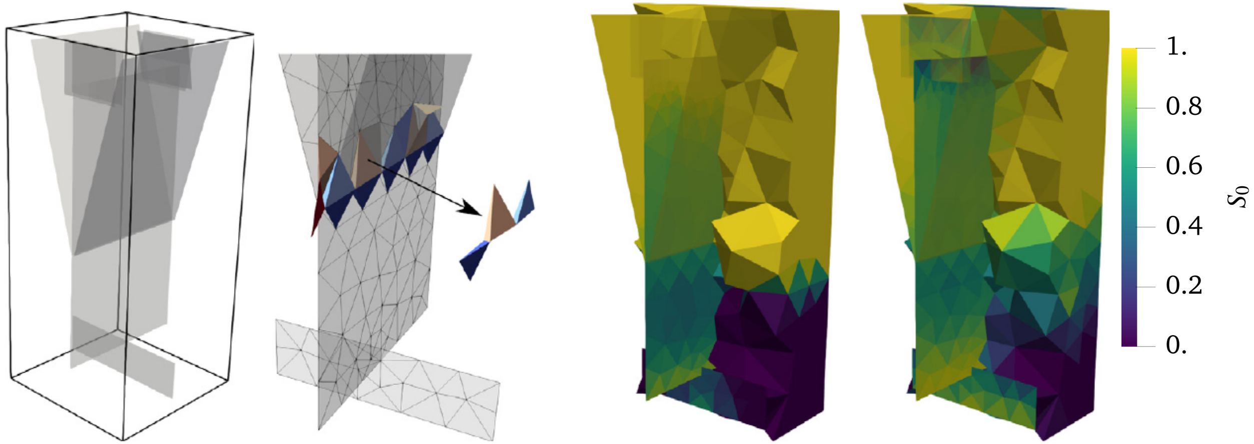
$\mathcal{N}_j \rightarrow$ **Fluid velocities** as upstream directions

$Q_T \rightarrow$ discretized with
Blended upwind-cell average

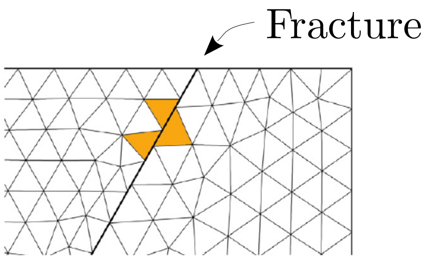
Implemented and tested in PorePy [8]

[6] E. Ballini, L.Formaggia, A. Fumagalli, E. Keilegavlen, A. Scotti. "*A hybrid upwind scheme for two-phase flow in fractured porous media*", Computer Methods in Applied Mechanics and Engineering, 2024.
[7] F. P. Hamon, H. Tchelepi. "*Analysis of hybrid upwinding for fully-implicit simulation of three-phase flow with gravity*". SIAM Journal on Numerical Analysis, 2016.
[8] E. Keilegavlen et. al. "*PorePy: an open-source software for simulation of multiphysics processes in fractured porous media*". Computational Geosciences, 2020.

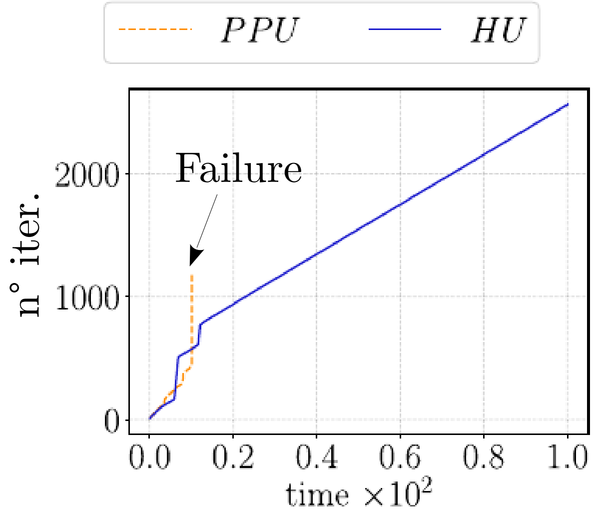
Mixed dimensional HU results



(a) Fracture network (b) Ill-shaped cells (c) $t = 1.3 \times 10^{-3}$ (d) $t = 2.86 \times 10^{-3}$



Partially non-conforming grids



Standard method (PPU) fails.

Proposed method is robust and saves Newton iterations.

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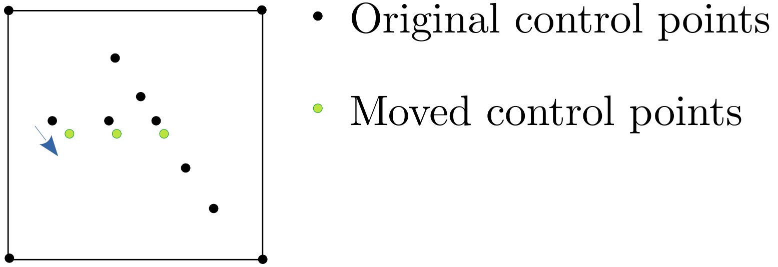
Discontinuous mesh deformation

Mesh deformation using Radial Basis Function (RBF)
Main features:

- Algebraic equations
- Control points, x_{c_j}



Smooth deformation

$$s(x) = \sum_j g(x, x_{c_j}) z_j$$

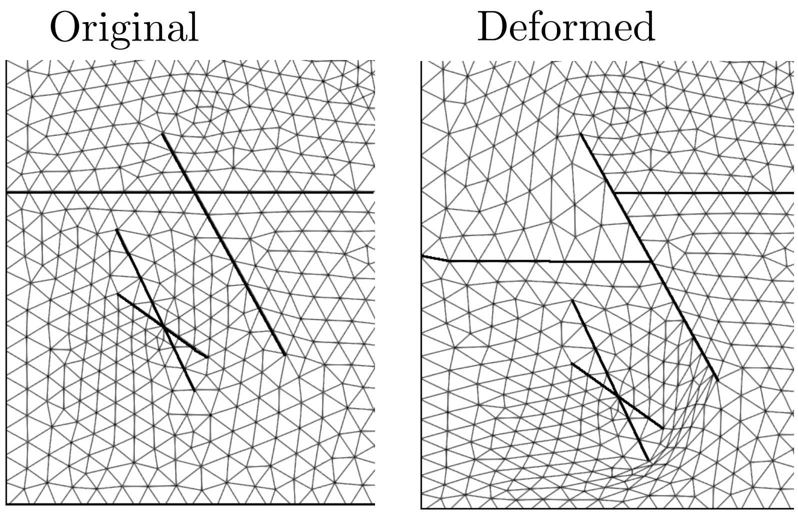
Discontinuous deformation [3]

$$s(x) = \sum_j \mathcal{I}(x, \text{fracture}) g(x, x_{c_j}) z_j,$$

$$s(x_{c_j}) \cdot \nu_i = 0,$$

$$z_j \cdot t_i = 0,$$

$$z_j \cdot b_i = 0$$



[3] E. Ballini, L. Formaggia, A. Fumagalli, A. Scotti, P. Zunino. *"Application of deep learning reduced-order modeling for single-phase flow in faulted porous media"*, Computational Geosciences, 2024.

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Fluid and solid – Model and discretization

Two-phase
flow system

$+$

$$\left.\begin{aligned}&\nabla \cdot \sigma^{tot} + \overline{\rho} g = 0,\\ &\sigma^{tot} = \sigma^{eff} - \beta p \mathbb{I},\end{aligned}\right\}$$

$$\sigma^{eff} = \mathbb{C}u,$$

$$\phi = \phi_0(1 + C_p(p - p_0)),$$

$$C_p = \frac{(1+\nu)(1-2\nu)}{\phi_0(1-\nu)E}.$$

One-way coupling:
Fluid affects solid
Solid does not affect fluid

MPSA [9]

MPSA-MPFA-FV [10]

Solution procedure:
At each timestep:

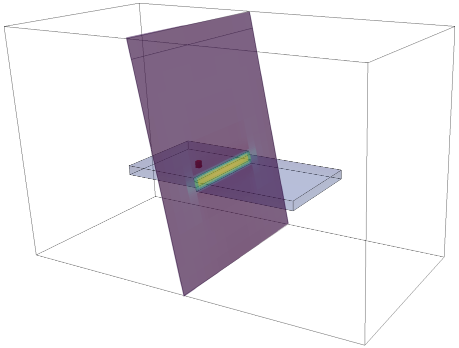
$$\underline{R}^{n+1}(p) = 0$$

$$\big\downarrow p$$

$$\underline{\underline{A}}\,\underline{u} = \underline{b}(p)$$

$$\big\downarrow$$

Displacement
Stress



[9] J. M. Nordbotten. Cell-centered finite volume discretizations for deformable porous media. International Journal for Numerical Methods in Engineering, 2014
[10] J. M. Nordbotten. Stable cell-centered finite volume discretization for biot equations. SIAM Journal on Numerical Analysis, 2016.

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Reduced Order Modeling

$\mu =$ parameter

Full order model (FOM)

$\mathcal{P}_N(u_N; \mu) = 0$

(Discrete equations of, e.g.,
single-phase flow, two-phase flow, ...
where, e.g., $u_N = \begin{bmatrix} p_1 \\ p_2 \\ \zeta \end{bmatrix}$)

$\xrightarrow{\text{Replaced by}}$

Reduced order model (ROM)

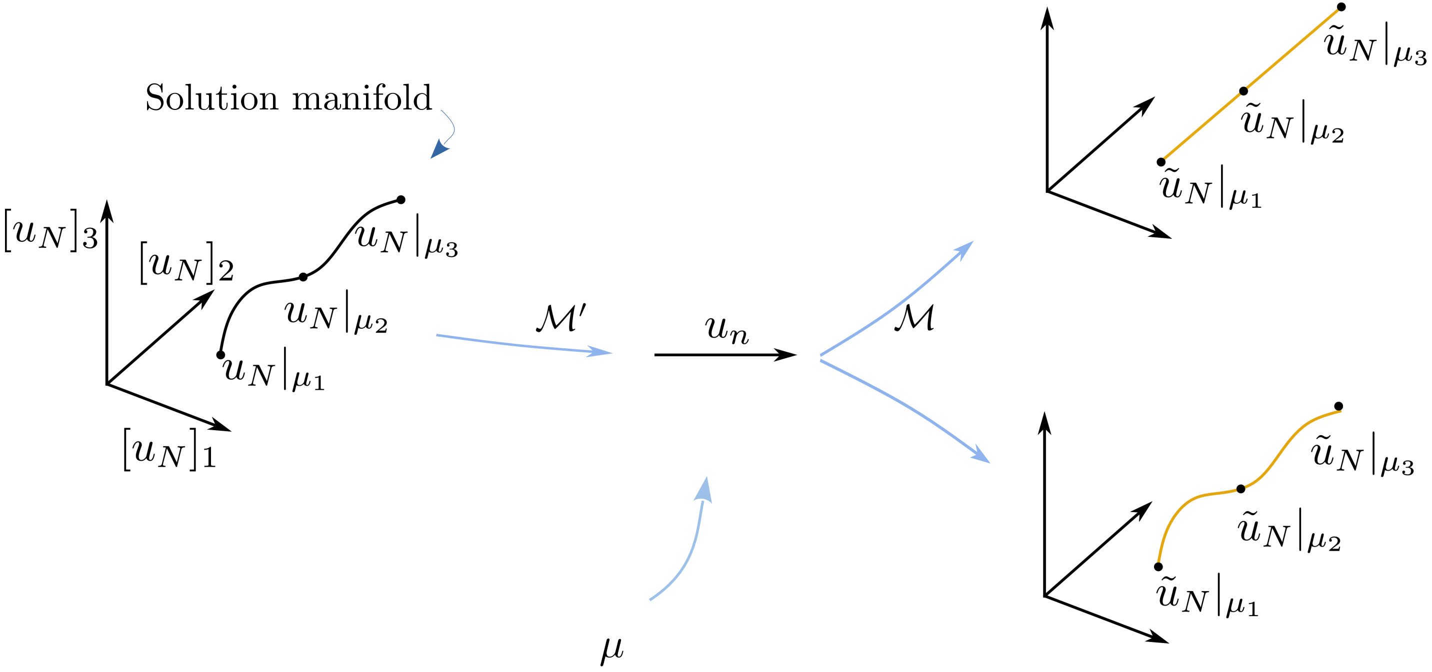
$\mathcal{P}_n(u_n; \mu) = 0$

$n \ll N$
then, $u_N = u_N(u_n)$

Reduced Order Modeling

Data-driven: **no** interaction
with FOM solver

Solution, u_N, paremeterized by μ



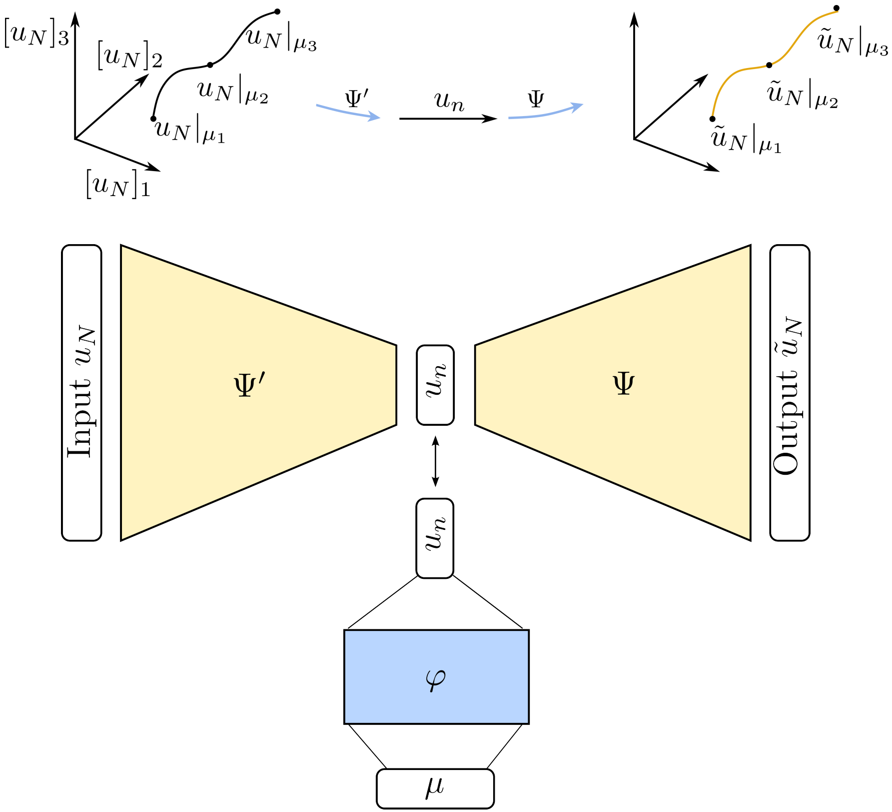
Proper Orthogonal Decomp.
$\mathcal{M}', \mathcal{M}$ approx.
with matrices Φ^T, Φ

Inaccurate for highly curved manifolds

Inefficient for non-affine problems

Neural Networks
$\mathcal{M}', \mathcal{M}$ approx. with NN

Deep Learning ROM (DL-ROM) [11]



Size bottleneck autoencoder =
= Manifold dimension, n =
= Number of uncertain parameters [12]

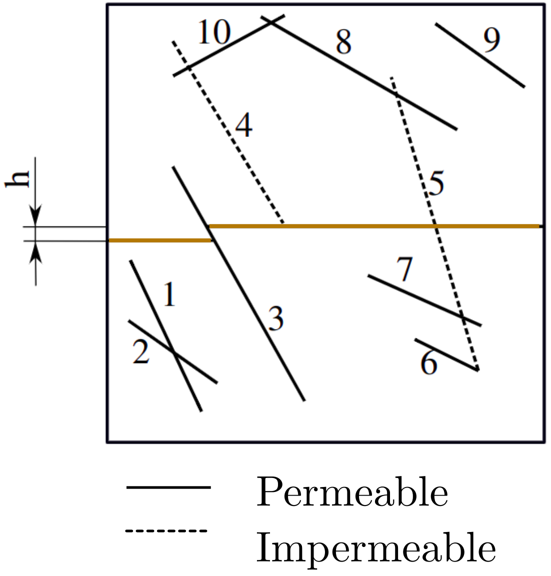
Error bounds [12,13]

$$\varepsilon > 0$$
$$\|\Psi' - u_n^{\text{exact}}\| \lesssim f_1(\varepsilon)$$
$$\|\Psi - u_N^{\text{exact}}\| \lesssim f_2(\varepsilon)$$
$$\|\varphi - u_n^{\text{exact}}\| \lesssim f_3(\varepsilon)$$

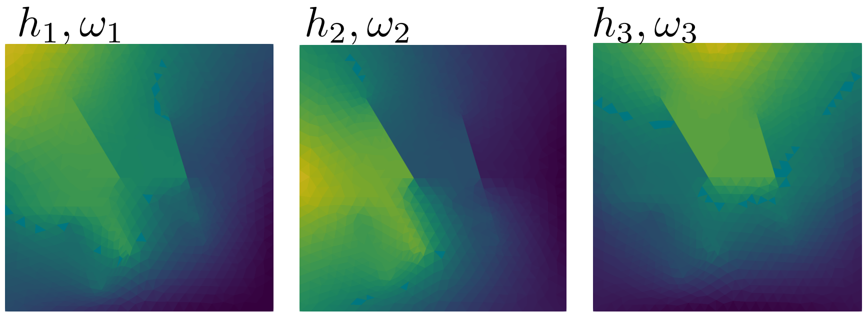
[11] S. Fresca, L. Dede', and A. Manzoni. "*A comprehensive deep learning-based approach to reduced order modeling of nonlinear time-dependent parametrized PDEs*". Journal of Scientific Computing, 2021.
[12] N. R. Franco, A. Manzoni, P. Zunino. "*A Deep Learning approach to Reduced Order Modelling of Parameter Dependent Partial Differential Equations*". Mathematics of Computation, 2023.
[13] T. Suzuki. "*Adaptivity of deep ReLU network for learning in Besov and mixed smooth Besov spaces: optimal rate and curse of dimensionality*", 2018. arXiv:1810.08033.

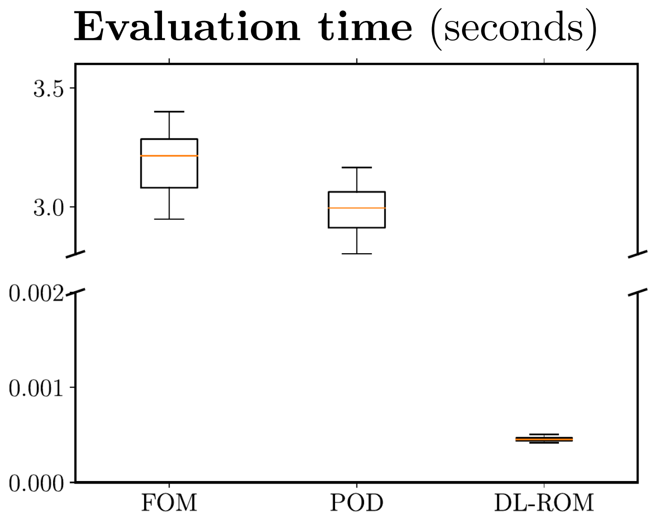
Single phase – Fracture network ROM

Mean flow



Parameters: h, ω





Error test dataset:

e_{max}	2.3%
e_{min}	0.28%
e_{ave}	0.58%

E. Ballini, L. Formaggia, A. Fumagalli, A. Scotti, P. Zunino. *"Application of deep learning reduced-order modeling for single-phase flow in faulted porous media"*, Computational Geosciences, 2024.

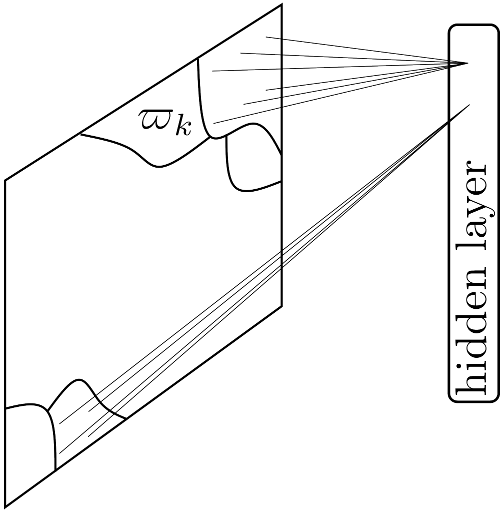
Pruning

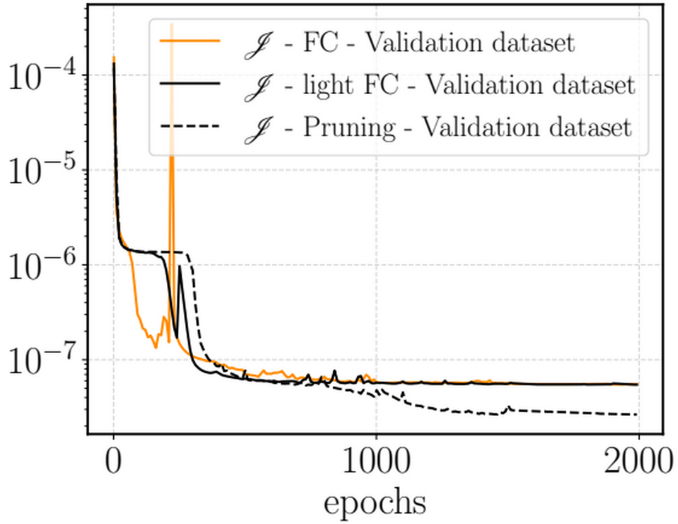
Problem: too many trainable weights.
Solution: pruning first layer of encoder (resp. last layer of decoder):

$$\mathcal{I}_i = \sum_{j=1}^{N} W_{ij} u_{N,j} + b_i$$

$$\Big\downarrow \begin{array}{l}\text{Pruning} \\ M_{ij} \in \{0,1\}\end{array}$$

$$\mathcal{I}_i = \sum_{j=1}^{N} M_{ij} W_{ij} u_{N,j} + b_i$$





90% number of weight reduction

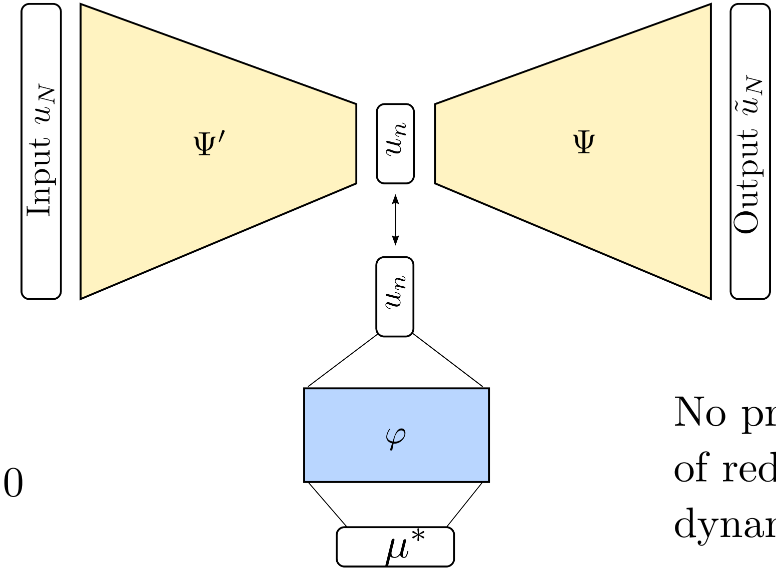
ROM to time dependent PDE

Stationary:

$$\underline{F}_\mu(\underline{U}^{n+1}) = 0$$

Time-dependent:

$$\frac{1}{dt}(\underline{U}^{n+1} - \underline{U}^n) + \underline{F}_\mu(\underline{U}^{n+1}) = 0$$



$$\mu^* = (\mu, t)$$

No proper treatment of reduced solution dynamics

DL-ROM – Recursive

$$\partial_t u_N = F_N(u_N)$$

Full order model (FOM) ODE

$$\partial_t u_n = \underbrace{\partial_{u_N} \Psi' F_N(\Psi u_n)}_{F_n}$$

Autoencoder

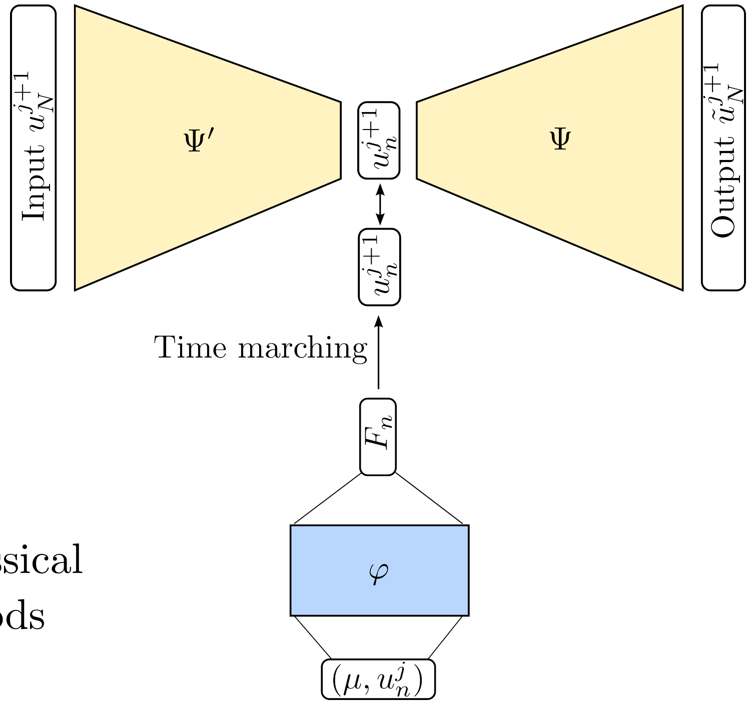
$$\partial_t u_n = F_n(u_n)$$

Reduced ODE

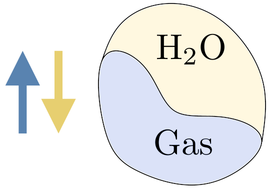
$$u_n^{j+1} = \sum_{p=0}^{P} a_p u_n^{j-p} + dt \sum_{p=-1}^{P} b_p F_n(u_n^{j-p})$$

Solution through classical time marching methods

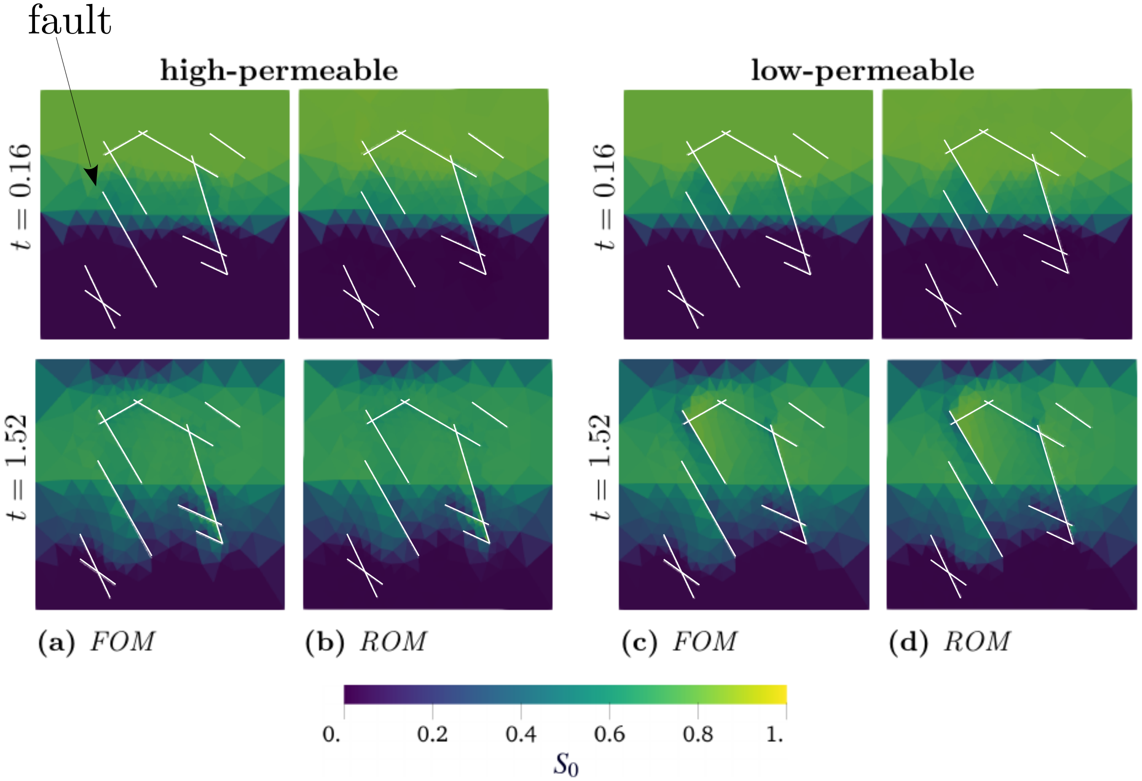
Approximation through neural network, φ

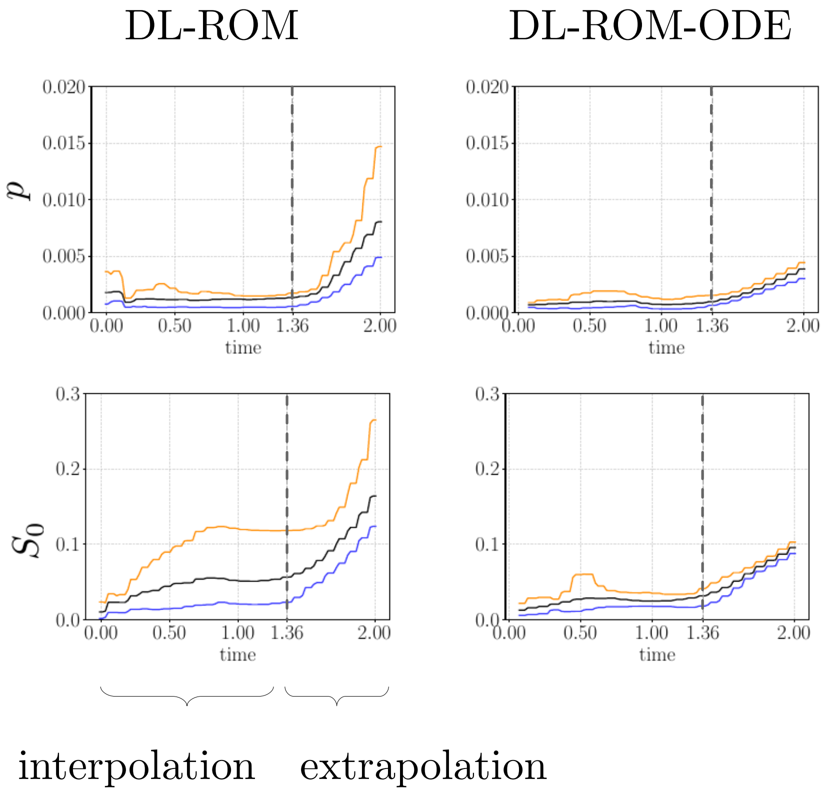


E. Ballini, A. Fumagalli, A. Scotti, P. Zunino. *"Reduced order modeling of time dependent PDE through manifold flattening"*. In preparation.



Two-phase flow – Fracture network





Parameters: fault throw,
fracture permeability

- Single-phase flow
- Two-phase flow
- Mesh deformation
- Fluid and solid
- Reduced order modeling to single and two-phase flow
- Industrial application

Realistic scenario [4]

Research mainly developed during a 6 months period at Eni.

- **Well control**
  production,
  injection

- **Two-phase flow equation**

$$\varepsilon\phi\partial_t(\rho_0 S_0) + \varepsilon\nabla\cdot Q_0 + C_0 = f_0,$$
$$\varepsilon\phi\partial_t[\rho_0 S_0 + \rho_1 S_1] + \varepsilon\nabla\cdot Q_T + $$
$$+ C_T = f_T.$$

- **Solid mechanics equations**

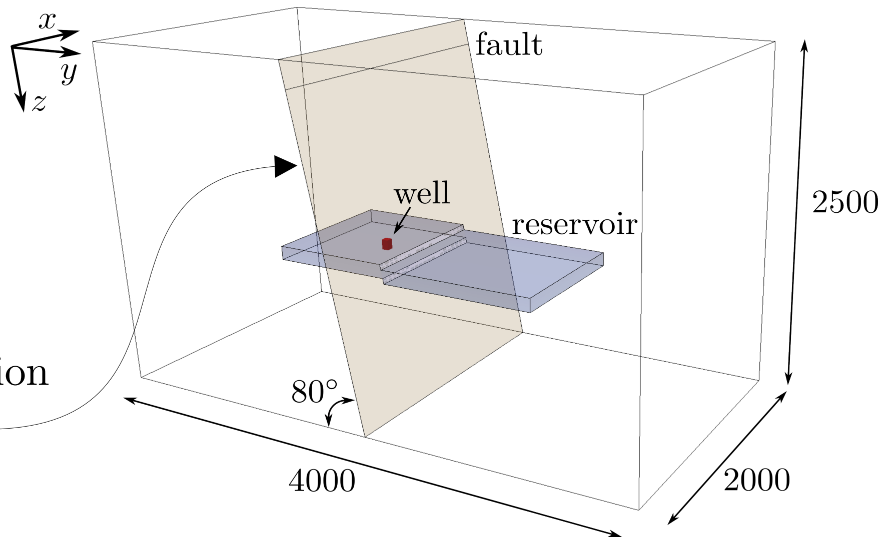
$$\nabla\cdot\sigma^{tot} + \rho_b g = 0,$$
$$\sigma^{tot} = \sigma^{eff} + \beta p\mathbb{I},$$

Traction on fault, T

$\downarrow$

Coulomb Failure Function

$$\mathrm{CFF} = \|T_t\| - f\, T_n\cdot n$$



ROM to reproduce T

Parameters:

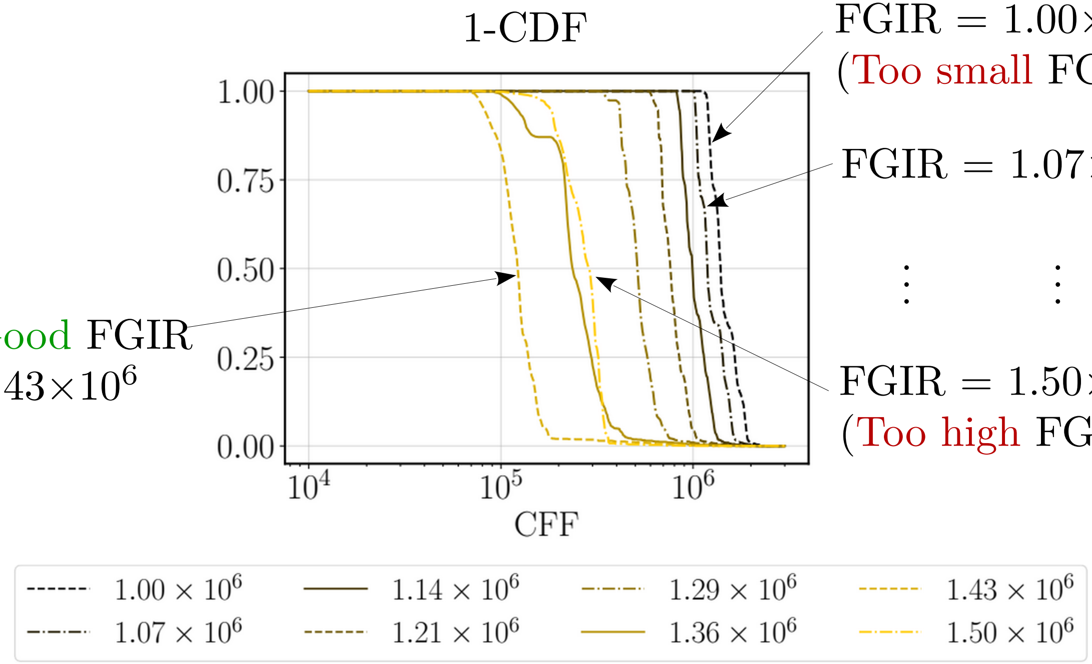
Fluid: field gas injection rate (FIGR), rock and fault permeability

Solid: Young's modulus reservoir and overburden

[4] E. Ballini, A. Cominelli, L. Dovera, A. Forello, L. Formaggia, A. Fumagalli, S. Nardean, A. Scotti, P. Zunino. "*Enhancing computational efficiency of numerical simulation for subsurface fluid-induced deformation using Deep Learning Reduced Order Models*". Accepted for SPE conference.

Realistic scenario

Tot. ∼ 25000 simulations



Good FGIR
1.43×10^6

FGIR = $1.00 \times 10^6 \rightarrow$ 3000 scenarios
(Too small FGIR)

FGIR = $1.07 \times 10^6 \rightarrow$ 3000 scenarios

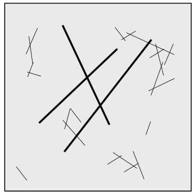
⋮ ⋮ ⋮

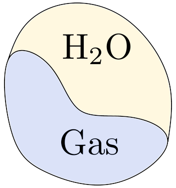
FGIR = $1.50 \times 10^6 \rightarrow$ 3000 scenarios
(Too high FGIR)

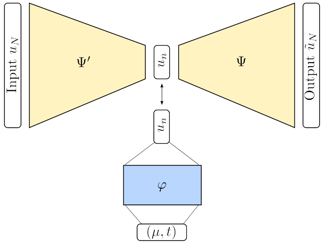
FGIR = Field gas injection rate, m^3/day
CFF = Coulomb failure function
CDF = Cumulative distribution function

Conclusion

Summary:
Simulation **single/two-phase** flow in **fractured/faulted** subsoil $\rightarrow$
Expensive simulation and uncertaintes $\rightarrow$
ROM based on NN for
steady and time dependent problem
in combination with **mesh deformation**
and **realistic scenario**.

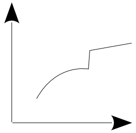


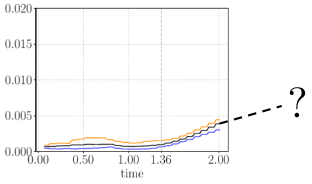




Limitations and future developments

- Three phases, capillary pressure, chemical reactions, ...
- Integration of data from different sources and multi-fidelity approach and fast offline phase;
- Severely complicated scenarios showing discontinuous quantities of interests;
- Long-term extrapolation in time.





Thank you for your attention

Northen Lights

CO_2
from industry

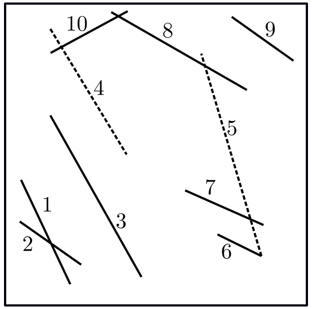
CO_2
temporary tanks

CO_2
towards reservoir

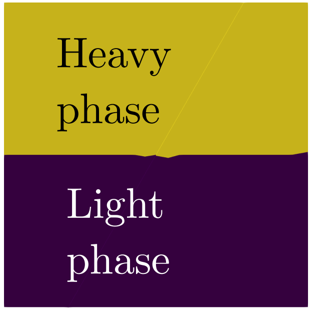
Equinor: Energy provider and innovator. https://www.equinor.com/.Accessed: 2024-08-10.

Mixed dimensional HU results

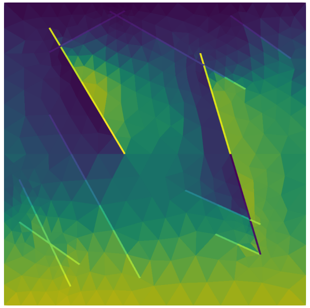
Domains

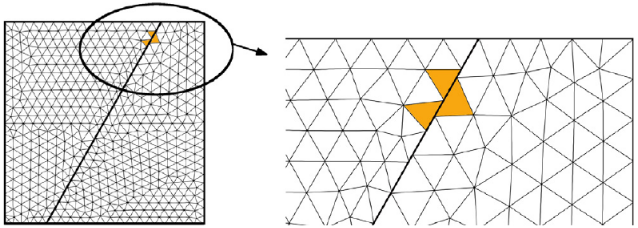


Initial condition

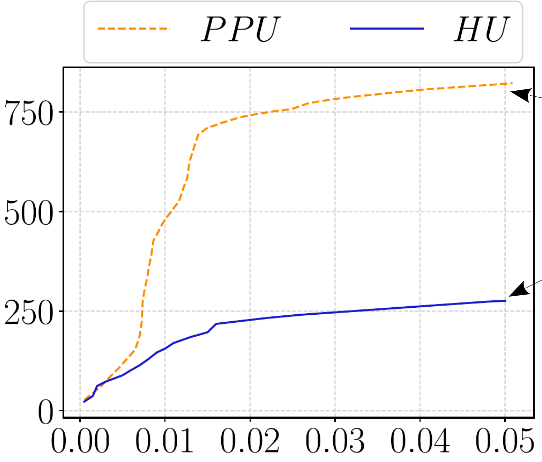


$t = 0.03$





Cumulative number of Newton iterations



Standard method (PPU): Many iterations

HU: Few iterations

Fluid and solid

Two-phase
flow system

$+$

$$\nabla \cdot \sigma^{tot} + \overline{\rho} g = 0,$$

$$\sigma^{tot} = \sigma^{eff} - \beta p \mathbb{I},$$

$$\sigma^{eff} = \mathbb{C} u,$$

$$\phi = \phi_0 (1 + C_p (p - p_0)),$$

$$C_p = \frac{(1+\nu)(1-2\nu)}{\phi_0 (1-\nu) E}.$$

Assumptions:

- Small hysdrostatic pressure contribution due to different fluid densities
- Small solid deformation

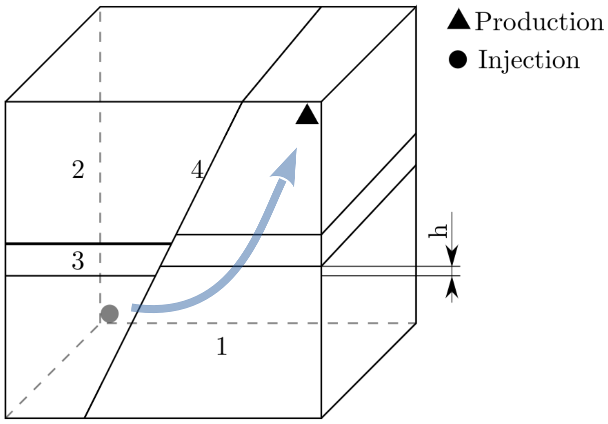
Training

Back propagation through time

$$\overline{\mathscr{J}} = \frac{1}{n_{train} N_{time}} \sum_{i=1}^{n_{train}} \sum_{j=1}^{N_{time}} \sum_{k=1}^{K} \mathscr{L}_{ij}^{k}$$

$$\begin{aligned}
\mathscr{L}_{ij}^{1} &= \alpha_1 \| \overline{\Psi'(u_{N,i}^{j})} - u_{n,i}^{j} \|^2 \\
\mathscr{L}_{ij}^{2} &= \alpha_2 \| u_{N,i}^{j} - \Psi \circ \Psi'(u_{N,i}^{j}) \|^2 \\
\mathscr{L}_{ij}^{3} &= \alpha_3 \left\| \sum_{p=-1}^{P} a_p \Psi'(u_{N,i}^{j-p}) - dt \sum_{p=-1}^{P} b_p \varphi(\mu_i, u_{n,i}^{j-p}) \right\|^2 \\
\mathscr{L}_{ij}^{4} &= \alpha_4 \| \Psi'(u_{N,i}^{j}) - u_{n,i}^{j} \|^2 \\
\mathscr{L}_{ij}^{5} &= \alpha_5 \| u_{N,i}^{j} - \Psi(u_{n,i}^{j}) \|^2
\end{aligned}$$

Multi-query application



Inverse problem

$$\min_{\mu}(\Delta p(\mu) - \Delta p_d)^2$$

$s.t.:$

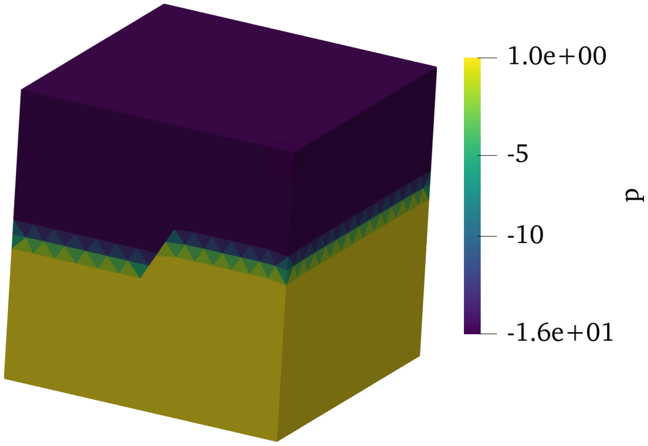
$K_1 \in [10^{-4}, 1]$

$K_2 \in [10^2, 2 \times 10^2]$

$K_3 \in [10^{-6}, 10^{-4}]$

$K_4 \in [9 \times 10^{-4}, 10^{-3}]$

$h \in [0.01, 0.1]$



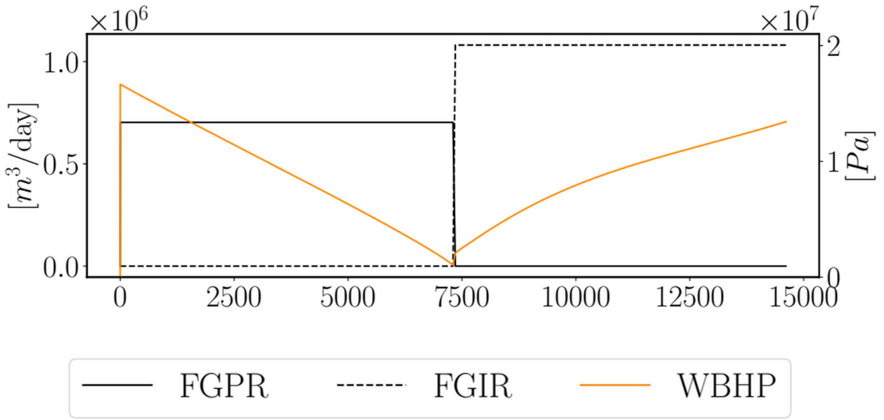
Genetic algorithm, 7356 evaluations in 18 s to find the optimum

Optimal value:

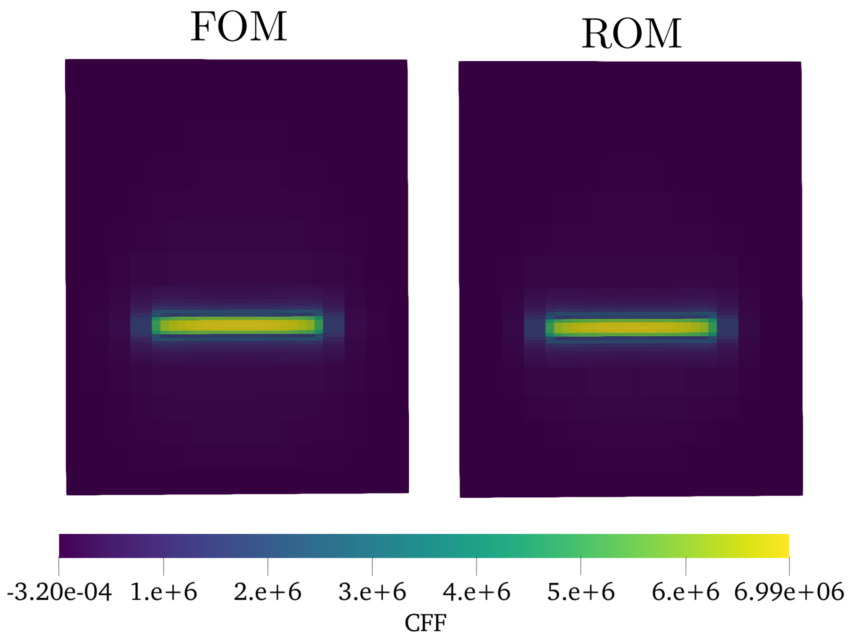
$\Delta p_{ROM} = 15.5$

$\Delta p_{FOM} = 16.7$

CFF



×10^6
×10^7
1.0
0.5
0.0
[m^3/day]
2
1
0
[Pa]
0
2500
5000
7500
10000
12500
15000
FGPR
FGIR
WBHP



FOM
ROM
-3.20e-04
1.e+6
2.e+6
3.e+6
4.e+6
5.e+6
6.e+6
6.99e+06
CFF