Homework: Design program circuits for four states

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Task 1

State 1

We want to achieve the superposition $|+\rangle$, depicted in its circle notation in the following figure.

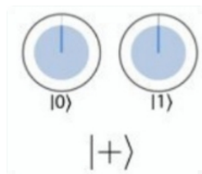

Figure 1: Circle notation for state 1

A circuit that does that is the following:

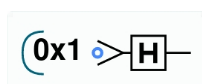

Figure 2: Circuit for state 1

In order to get this circuit, I used the following code:

```
qc.reset(1);
qc.write(0);
qc.had();
```

State 2

We want to achieve the superposition $|-\rangle$, depicted in its circle notation in the following figure.

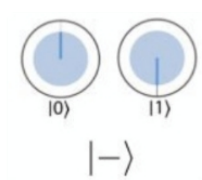

Figure 3: Circle notation for state 1

A circuit that does that is the following:

In order to get this circuit, I used the following code:

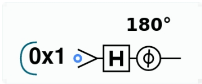

Figure 4: Circuit for state 1

```
qc.reset(1);
qc.write(0);
qc.had();
qc.phase(180);
```

State 3

We want to achieve the superposition $|+Y\rangle$, depicted in its circle notation in the following figure.

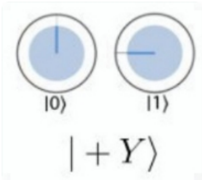

Figure 5: Circle notation for state 1

A circuit that does that is the following:

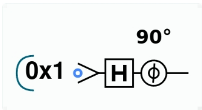

Figure 6: Circuit for state 1

In order to get this circuit, I used the following code:

```
qc.reset(1);
qc.write(0);
qc.had();
qc.phase(90);
```

State 4

We want to achieve the superposition $|-Y\rangle$, depicted in its circle notation in the following figure.

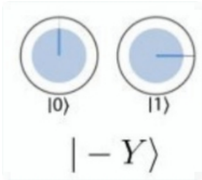

Figure 7: Circle notation for state 1

A circuit that does that is the following:

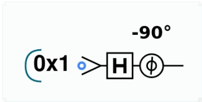

Figure 8: Circuit for state 1

In order to get this circuit, I used the following code:

```
qc.reset(1);
qc.write(0);
qc.had();
qc.phase(-90);
```

Task 2

Circuit 1

In this circuit we are applying only one Hadamard gate. This gate is defined as:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

Assuming that we have one qubit $\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$, the calculation is:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha+\beta\\ \alpha-\beta\end{pmatrix}$$

Circuit 2

In this circuit we are applying an Hadamard gate followed by a phase gate, with phase $\theta = 180 = \pi$; therefore, $e^{i\theta} = -1 + 0i = -1$. We then concatenate the two matrices which correspond to these operations and we calculate the final state:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha-\beta\\ \alpha+\beta\end{pmatrix}$$

Circuit 3

The procedure to calculate the final state for circuit 3 is similar to the one for circuit 2; the only exception is the relative phase, which is now $\theta = 90 = \frac{\pi}{2}$, and therefore $e^{i\theta} = 0 + 1i = i$. The calculation follows:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ 1 & -i\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\alpha+i\beta\\ \alpha-i\beta\end{pmatrix}$$

Circuit 4

The same applies also for circuit 4, where the relative phase is $\theta = -90 = \frac{3\pi}{2}$ and therefore $e^{i\theta} = 0 - 1i = -i$. The calculation follows:

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha - i\beta \\ \alpha + i\beta \end{pmatrix}$$