- $\lim_{x,y \to 0,0} \frac{x^3 - 2xy + y^2}{x^2+y^2}$ $D = \mathbb{R}^2 - \{(0,0)\}$. QUINDI CONSIDERO UNA RESTRIZIONE SU $(0,0)$

① $y=0,\ x \neq 0$ $\frac{x^3 + \cancel{2x0} + \cancel{0 \cdot 0}}{x^2 + \cancel{0 \cdot 0}} = x$. $\lim_{x\to 0} x = 0$. QUINDI SE $\exists$, DEVE ESSERE $= 0$

② $y \neq 0,\ x = 0$ $\frac{0^3 + 2 \cdot 0 \cdot y + y^2}{0^2 + y^2} = 1$ $\lim_{y \to 0} 1 = 1$

$1 \neq 0$, LIMITE $\nexists$

- $\lim_{x,y \to 0,2} \frac{xy - 2x}{x^2 + (y-2)^2}$ $D = \mathbb{R}^2 - \{(0,2)\}$

① $x=0,\ y \neq 2$ $\frac{0 \cdot y - 2 \cdot 0}{0^2 + (y-2)^2} = \frac{0}{(y-2)^2} = 0$ $\lim_{y \to 2} 0 = 0$

$=$

② $y = 2,\ x \neq 0$ $\frac{x \cdot 2 - 2x}{x^2 + (2-2)^2} = \frac{0}{x^2} = 0$ $\lim_{x \to 0} 0 = 0$

$\neq$ $\to \nexists$ LIMITE

③ $y = x+2$ $\frac{x \cdot (x+2) - 2x}{x^2 + (x+2-2)^2} = \frac{x^2}{2x^2} = \frac{1}{2}$ $\lim_{x \to 0} \frac{1}{2} = \frac{1}{2}$ $\to$ POSSO USARE UNA RETTA QUALSIASI PASSANTE PER $(0,2)$
$\to f(x, mx+2)$

- $\lim_{x,y \to 0,0} \frac{x^4 y}{x^8 + y^2}$ $D: \mathbb{R}^2 - \{(0,0)\}$

① $x=0,\ y \neq 0$ $\lim_{y \to 0} \frac{0}{y^2} = 0$ ② $y=0,\ x \neq 0$ $\lim_{x \to 0} \frac{0}{x^8} = 0$ ③ $f(x, mx) = \frac{x^4 \cdot mx}{x^8 + (mx)^2} = \frac{mx^5}{x^8 + m^2 x^2}$, $\lim_{x \to 0} \frac{\cancel{x^2}(mx^3)}{\cancel{x^2}(x^6 + m^2)} = 0$

④ $y = x^4$ $\frac{x^4 \cdot x^4}{x^8 + x^8} = \frac{x^8}{2x^8} = \frac{1}{2} \neq 0$ LIMITE $\nexists$

- $\lim_{x,y \to 0,0} \frac{x^3 + xy^4}{x^2 + 2y^4}$ ① $x = 0 \to 0$ ② $y=0 \to 0$ ③

TEOREMA CONFRONTO: $0 \leq |f(x,y)| \to \frac{|x| \cdot (x^2 + y^4)}{x^2 + 2y^4} \leq$

LUOGO DI PUNTI DI: $4x^2 + 9y^2 + 8x - 32 = 0$

$= 4(x+1)^2 + 9(y-0)^2 = 36\ (32+4)$

$= \frac{(x+1)^2}{9} + \frac{(y-0)^2}{4} = 1 \Rightarrow$ FORMA CANONICA

$\boxed{\begin{array}{l} c = (-1, 0) \\ a, b = 3, 2 \end{array}}$