• LINEARE INDIPENDENZA DI FUNZIONI

ES: STABILIRE SE LE SEGUENTI FUNZIONI SONO LINEARMENTE INDIPENDENTI

$f(x) = |x|$ e $g(x) = x$ in $(-\infty, 0)$ $\Rightarrow$ $c_1 f(x) + c_2 g(x) = 0 \Rightarrow c_1 = c_2 = 0 \quad \forall x \in I$

con $x \in (-\infty, 0)$ $f(x) = -x$

CONDIZIONE DI INDIP.

CIÒ È VERIFICATA PER $c_1 = c_2$ QUINDI SONO LIN. DIPENDENTI

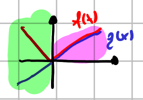
• METODO 2 CON MATRICE WRONSKIANA

$$\det\begin{pmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{pmatrix} = \begin{vmatrix} -x & x \\ -1 & 1 \end{vmatrix} = -x+x = 0 \Rightarrow \text{LIN. DIPENDENZA}$$

ES 2: LIN. INDIP. DI:

$f(x) = |x|$ e $g(x)$ in $\mathbb{R}$

$c_1 f(x) + c_2 g(x) = 0 \quad \forall x \in \mathbb{R}$



PER $x>0$: $x(c_1+c_2) = 0$, $c_1 = -c_2$

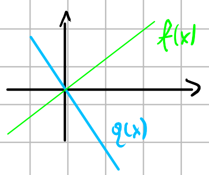
$x<0$: $-c_1 x + c_2 x = 0$, $x(-c_1+c_2) = 0$, $c_1 = c_2$

$\rightarrow$ NON C'È $x \in \mathbb{R} \neq 0$ t.c. SIANO SODDISFATTE ASSIEME

RIMANE SOLO $c_1 = c_2 = 0 \Rightarrow f(x)$ e $g(x)$ LIN. INDIP.

ES 3:

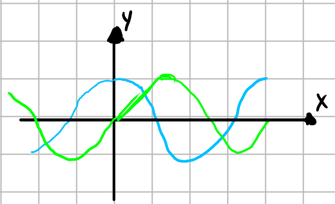
$f(x) = x$ e $g(x) = -2x$, $x \in \mathbb{R}$



$$c_1 f(x) + c_2 g(x) = c_1 x - 2c_2 x$$
$$= x(c_1 - 2c_2) = 0$$
$$c_1 = 2c_2 \quad \text{LIN. DIP.}$$

ES 4:

$f(x) = \sin(x)$ e $g(x) = \cos(x)$



$$c_1 f(x) + c_2 g(x) = 0$$
$$c_1 \sin(x) + c_2 \cos(x) = 0$$

in $\frac{\pi}{4} \Rightarrow c_1 \frac{\sqrt{2}}{2} + c_2 \frac{\sqrt{2}}{2} = 0 \Leftrightarrow \frac{\sqrt{2}}{2}(c_1 + c_2) = 0$, $c_1 = -c_2$

in $\frac{\pi}{2} \Rightarrow c_1 + 0 = 0 \quad c_1 = 0$

$\rightarrow$ ENTRAMBE $= 0$. LIN. INDIP.

ES 5:

$f(x) = \sin(x)$, $g(x) = \cos(x)$, $h(x) = 1$

$$\begin{pmatrix} \sin(x) & \cos(x) & 1 \\ \cos(x) & -\sin(x) & 0 \\ -\sin(x) & -\cos(x) & 0 \end{pmatrix} = -\cos(x)^2 - \sin(x)^2 = -1 \quad \text{LIN. INDIP.}$$

6.

$f(x) = \sin^2 x \quad g(x) = 2 \quad h(x) = 3\cos^2(x)$

$$c_1 \sin^2 x + 2c_2 + 3c_3(1 - \sin^2 x) = 0$$
$$c_1 \sin^2 x + 2c_2 + 3c_3 - 3c_3 \sin^2 x = 0$$
$$\sin^2(x)(c_1 - 3c_3) + 2c_2 + 3c_3 = 0$$

$c_2 = 1 \quad \sin^2(x)(c_1 - 3c_3) + 2 + 3c_3 = 0$

$\sin^2(x)(c_1 - 3c_3) = -3c_3 - 2$

$c_3 = -\frac{2}{3} \quad \sin^2(x)(c_1 + 2) = 0$

$c_1 = -2 \quad \sin^2(x) = 0 \quad$ SI' LIN. DIP.

7. RISOLVERE PROBLEMA CAUCHY

$$\begin{cases} y'' + \frac{y'}{x} - \frac{y}{x^2} = 0 \\ y(1) = 1 \\ y'(1) = 0 \end{cases}$$

$x \neq 0 \Rightarrow$ L'INTEGRALE FOND. È DEL TIPO: $c_1 y_1(x) + c_2 y_2(x)$ CIOÈ $L(y_1(x), y_2(x))$ LIN. INDIP.

$\rightarrow$ SOL. LIN. INDIP. DELL' ED

L'INTERVALLO DA ANALIZZARE (CONTENENTE x_0) È $(0, +\infty)$

PRENDO POLINOMI PER y_1 e y_2

PER ORA MI LIMITO AI MONOMI $\longrightarrow y^* = x^\alpha \Rightarrow y' = \alpha x^{\alpha-1}$

$y'' = \alpha(\alpha-1)x^{\alpha-2}$

SOSTITUISCO: (e PRIMA MOLTIP. PER x^2)

$$x^2\alpha(\alpha-1)x^{\alpha-2}+\alpha x\cdot x^{\alpha-1}-x^\alpha$$

$$x^\alpha(\alpha^2-\cancel{\alpha}+\cancel{\alpha}-1)=0$$

$$\alpha=\pm 1$$

QUINDI $y_1=x$ $\quad y_2=x^{-1}$ TUTTE LE SOL SONO $\mathcal{L}(y=x,\ y=\frac{1}{x})$ CIOÈ: $y(x)=c_1x+c_2\frac{1}{x}$ $\quad x\in(0,+\infty)$

IMPONGO CONDIZIONI INIZIALI $\qquad y'(x)=c_1-\frac{c_2}{x^2}$

$$\begin{cases}1=c_1+c_2\\0=c_1-c_2\end{cases}\Rightarrow\begin{cases}c_1=1-c_2\\0=1-c_2-c_2\end{cases}\Rightarrow\begin{matrix}c_1=\frac{1}{2}\\c_2=\frac{1}{2}\end{matrix}\quad\Rightarrow \text{SOL}=\ y(x)=\frac{1}{2}x+\frac{1}{2x}\quad x\in(0,+\infty)$$

NUMERI COMPLESSI

DENVELL

- $2z^2-2z+5=0 \qquad z\in\mathbb{C}$

△ 1° METODO: FORMULA RISOLUTIVA

$$z=\frac{-\frac{b}{2}\pm\sqrt{\left(\frac{b}{2}\right)^2-ac}}{a}=\frac{+1\pm\sqrt{1-10}}{2}\ \notin\mathbb{C}\ \Rightarrow\ \frac{1\pm\{3i,-3i\}}{2}\ \begin{matrix}\nearrow\ \frac{1+3i}{2}\\ \searrow\ \frac{1-3i}{2}\end{matrix}$$

△ 2° METODO: $z=a+ib$

$$2z^2-2z+5=0\Rightarrow 2(a+ib)^2-2(a+ib)+5=0\Rightarrow 2(a^2+2abi+\underbrace{i^2b^2}_{-b^2})-2a-2bi+5=0$$

$$=2a^2+4abi-2b^2-2a-2bi+5=0$$

$$(2a^2-2b^2-2a+5)+i(4ab-2b)=0=0+0i$$

$$\begin{cases}2a^2-2b^2-2a+5=0\\4ab-2b=0\end{cases}\underset{b\neq0}{\Rightarrow}\begin{cases}2\frac{1}{4}-2b^2-2\frac{1}{2}+5=0\\a=\frac{1}{2}\end{cases}\Rightarrow\quad \frac{1}{2}-2b^2-1+5\Rightarrow 2b^2=\frac{3}{2}\quad b=\pm\frac{3}{2}$$

$$z=\frac{1}{2}\pm\frac{3}{2}i$$

OK!