

LINEARE INDIPENDENZA DI FUNZIONI

ES: STABILIRE SE LE SEGUENTI FUNZIONI SONO LINEARMENTE INDIPENDENTI

$$f(x) = |x| \quad g(x) = x$$

$$\text{in } (-\infty, 0) \Rightarrow C_1 f(x) + C_2 g(x) = 0 \Rightarrow C_1 = C_2 = 0 \quad \forall x \in I$$

CONDIZIONE DI INDIP.

È VERIFICATA PER $C_1 = C_2$ QUINDI SONO LIN. DIPENDENTI

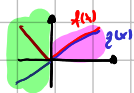
METODO 2 CON MATRICE WRONSKIANA

$$\det \begin{pmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{pmatrix} = \begin{vmatrix} -x & x \\ -1 & 1 \end{vmatrix} = -x + x = 0 \Rightarrow \text{LIN. DIPENDENZA}$$

ES 2: LIN. INDIP. DI:

$$f(x) = |x| \quad g(x) = x \quad \text{in } \mathbb{R}$$

$$C_1 f(x) + C_2 g(x) = 0 \quad \forall x \in \mathbb{R}$$



PER $x > 0$

$$x(C_1 + C_2) = 0$$

$$C_1 = -C_2$$

$$-Cx + C_2x = 0$$

$$x(-C_1 + C_2) = 0$$

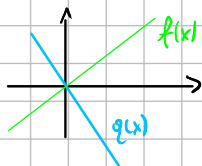
$$C_1 = C_2$$

NON L'È $\forall x \in \mathbb{R} \neq 0$ L. SIAMO SODDISFATTE ASSIEME

RIMANE SOLO $C_1 = C_2 = 0 \Rightarrow f(x) \text{ e } g(x) \text{ LIN. INDIP.}$

ES 3:

$$f(x) = x \quad g(x) = -2x, \quad x \in \mathbb{R}$$

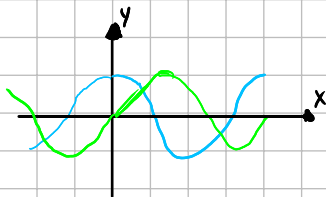


$$C_1 f(x) + C_2 g(x) = C_1 x - 2C_2 x = x(C_1 - 2C_2) = 0$$

$$C_1 = 2C_2 \quad \text{LIN. DIP.}$$

ES 4:

$$f(x) = \sin(x) \quad g(x) = \cos(x)$$



$$C_1 f(x) + C_2 g(x) = 0$$

$$C_1 \sin(x) + C_2 \cos(x) = 0$$

$$\text{in } \frac{\pi}{4} \Rightarrow C_1 \frac{\sqrt{2}}{2} + C_2 \frac{\sqrt{2}}{2} = 0 \Rightarrow \frac{\sqrt{2}}{2}(C_1 + C_2) = 0$$

$$C_1 = -C_2$$

ENTRANCE = 0. LIN. INDIP.

$$\text{in } \frac{\pi}{2} \Rightarrow C_1 + 0 = 0 \quad C_1 = 0$$

ES 5:

$$f(x) = \sin(x), \quad g(x) = \cos(x), \quad h(x) = 1$$

$$\begin{pmatrix} \sin(x) & \cos(x) & 1 \\ \cos(x) & -\sin(x) & 0 \\ -\sin(x) & -\cos(x) & 0 \end{pmatrix} = -\cos^2(x) - \sin^2(x) = -1 \quad \text{LIN. INDIP.}$$

6.

$$f(x) = \sin^2 x \quad g(x) = 2 \quad h(x) = 3 \cos^2(x)$$

$$C_1 \sin^2 x + 2C_2 + 3C_3(1 - \sin^2 x) = 0$$

$$C_1 \sin^2 x + 2C_2 + 3C_3 - 3C_3 \sin^2 x = 0$$

$$\sin^2 x(C_1 - 3C_3) + 2C_2 + 3C_3 = 0$$

$$C_2 = 2 \quad \sin^2 x(C_1 - 3C_3) + 2 - 3C_3 = 0$$

$$\sin^2 x(C_1 - 3C_3) = -2 + 3C_3$$

$$C_3 = -\frac{2}{3} \quad \sin^2 x(C_1 + 2) = 0$$

$$C_1 = -2 \quad \sin^2 x = 0 \quad \text{LIN. DIP.}$$

7. RISOLVERE PROBLEMA CAUCHY

$$\begin{cases} y'' + \frac{y'}{x} - \frac{y}{x^3} = 0 \\ y(1) = 1 \\ y'(1) = 0 \end{cases}$$

$x \neq 0 \Rightarrow$ L'INTEGRALE FOND. È DEL TIPO:

$C_1 y_1(x) + C_2 y_2(x)$ CIOÈ $L(y_1(x), y_2(x))$ LIN. INDIP.

SCR. LIN. INDIP. DELL'ED

L'INTERVALLO DA ANALIZZARE (CONTENENTE x_0) È $(0, +\infty)$

PRENDO POLINOMI PER y_1 E y_2

PER ORA MI LIMITO AI MONOMI $\rightarrow y^* = x^d \Rightarrow y' = dx^{d-1}$
 $y'' = d(d-1)x^{d-2}$

SOSTITUISCO: (1 PRIMA MOLTIPL. PER x^2)

$$x^2 \alpha (\alpha - 1) x^{\alpha-2} + \alpha x \cdot x^{\alpha-1} - x^\alpha$$

$$x^\alpha (\alpha^2 - \alpha + \alpha - 1) = 0$$

$$\alpha = \pm 1$$

QUINDI $y_1 = x$ $y_2 = x^{-1}$ TUTTE LE SOL. SONO $L(y=x, y=\frac{1}{x})$ CIOE': $y(x) = C_1 x + C_2 \frac{1}{x} \quad x \in (0, +\infty)$

IMPONGO CONDIZIONI INIZIALI

$$y'(x) = C_1 - \frac{C_2}{x^2}$$

$$\begin{cases} 1 = C_1 + C_2 \\ 0 = C_1 - C_2 \end{cases} \Rightarrow \begin{cases} C_1 = 1 - C_2 \\ 0 = 1 - C_2 - C_2 \end{cases} \Rightarrow \begin{cases} C_1 = \frac{1}{2} \\ C_2 = \frac{1}{2} \end{cases} \Rightarrow \text{SOL} = y(x) = \frac{1}{2}x + \frac{1}{2x} \quad x \in (0, +\infty)$$

NUMERI COMPLESSI

• $2z^2 - 2z + 5 = 0 \quad z \in \mathbb{C}$

Δ 1° METODO: FORMULA RISOLUTIVA

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 10}}{2} \in \mathbb{C} \Rightarrow \frac{1 \pm \{5i, -5i\}}{2} \Rightarrow \begin{cases} \frac{1+3i}{2} \\ \frac{1-3i}{2} \end{cases}$$

Δ 2° METODO: $z = a + ib$

$$2z^2 - 2z + 5 = 0 \Rightarrow 2(a+ib)^2 - 2(a+ib) + 5 = 0 \Rightarrow 2(a^2 + 2abi - b^2) - 2a - 2bi + 5 = 0$$

$$= 2a^2 + 4abi - 2b^2 - 2a - 2bi + 5 = 0$$

$$(2a^2 - 2b^2 - 2a + 5) + i(4ab - 2b) = 0 = 0 + 0i$$

$$\begin{cases} 2a^2 - 2b^2 - 2a + 5 = 0 \\ 4ab - 2b = 0 \end{cases} \Rightarrow \begin{cases} 2\frac{1}{4} - 2b^2 - \frac{1}{2} + 5 = 0 \\ b \neq 0 \Rightarrow a = \frac{1}{2} \end{cases} \Rightarrow \frac{1}{2} - 2b^2 - \frac{1}{2} + 5 = 0 \Rightarrow 2b^2 = \frac{9}{2} \Rightarrow b = \pm \frac{3}{2}$$

OK!

$$z = \frac{1}{2} \pm \frac{3}{2}i$$

DENVELL

