$f(x) = \ln\sqrt{x^2+y^2} = \frac{1}{2}\ln(x^2+y^2)$

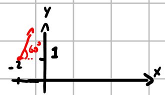
$\frac{\partial f}{\partial x} = \frac{1}{\cancel{2}} \cdot \frac{1}{x^2+y^2} \cdot \cancel{2}x = \frac{x}{x^2+y^2}$ e $\frac{\partial f}{\partial y} = \frac{1}{\cancel{2}} \cdot \frac{1}{x^2+y^2} \cdot \cancel{2}y = \frac{y}{x^2+y^2}$ $\qquad D = \mathbb{R}^2 - \{0\}$

ENTRAMBE HANNO DOMINIO

CALCOLO $\langle \nabla f(1,1), \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \rangle$ (DERIVATA DIREZIONALE IN 1,1)

$\langle \left(\frac{1}{1^2+1^2}, \frac{1}{1^2+1^2}\right), \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \rangle = \langle \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \rangle = \frac{\sqrt{2}}{2}$

ES: $f(x,y) = \frac{x-y}{x^2+y^2+5}$

① CALCOLA $\frac{\partial f}{\partial v}(-2,1)$, CON  $V = (\cos 60, \sin 60) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$D = \mathbb{R}^2$ (IL DENOM. NON SI ANNULLA MAI)

$\frac{\partial f}{\partial x} = \frac{1 \cdot (x^2+y^2+5) - (x-y) \cdot 2x}{(x^2+y^2+5)^2} = \frac{-x^2+y^2+2xy+5}{(x^2+y^2+5)^2}$

$\frac{\partial f}{\partial y} = \frac{(-1) \cdot (x^2+y^2+5) - (x-y)2y}{(x^2+y^2+5)^2} = \frac{-x^2+y^2-2xy-5}{(x^2+y^2+5)^2}$

$\rightarrow D = \mathbb{R}^2$, CONTINUE $\xRightarrow{\text{TEO. DIFF. TOTALE}}$ f È DIFFERENZIABILE OVUNQUE in $\mathbb{R}^2$

- FORMULA GRADIENTE:

$\frac{\partial f}{\partial v}(-2,1) = \langle \nabla f(-2,1), \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \rangle = \langle \left(\frac{\partial f}{\partial x}(-2,1), \frac{\partial f}{\partial y}(-2,1)\right), \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \rangle$

$= \langle \left(-\frac{2}{100}, -\frac{4}{100}\right), \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \rangle = -\frac{1}{50} \cdot \frac{1}{2} + -\frac{1}{25} \cdot \frac{\sqrt{3}}{2} = -\frac{1}{100} - \frac{\sqrt{3}}{50} = \frac{-1-2\sqrt{3}}{100} < 0$ ✓

ES: EQ. PIANO TANGENTE A G_f

– FORMULA PIANO TANGENTE: $f(x_0, y_0) + \frac{\partial f}{\partial x}(\qquad)(x-x_0) + \frac{\partial f}{\partial y}(\qquad)(y-y_0)$