• LINEARE INDIPENDENZA DI FUNZIONI

ES: STABILIRE SE LE SEGUENTI FUNZIONI SONO LINEARMENTE INDIPENDENTI

$f(x) = |x|$ e $g(x) = x$ in $(-\infty, 0) \Rightarrow c_1 f(x) + c_2 g(x) = 0 \Rightarrow c_1 = c_2 = 0 \quad \forall x \in I$

con $x \in (-\infty, 0)$: $f(x) = -x$

CONDIZIONE DI INDIP.

↳ È VERIFICATA PER $C_1 = C_2$ QUINDI SONO LIN. DIPENDENTI

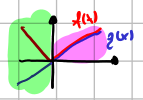
• METODO 2 CON MATRICE WRONSKIANA

$$\det\begin{pmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{pmatrix} = \begin{vmatrix} -x & x \\ -1 & 1 \end{vmatrix} = -x + x = 0 \Rightarrow \text{LIN. DIPENDENZA}$$

ES 2: LIN. INDIP. DI:

$f(x) = |x|$ e $g(x)$ in $\mathbb{R}$

$c_1 f(x) + c_2 g(x) = 0 \quad \forall x \in \mathbb{R}$



PER $x > 0$: $x(c_1 + c_2) = 0$, $c_1 = -c_2$

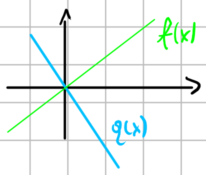
$x < 0$: $-c_1 x + c_2 x = 0$, $x(-c_1 + c_2) = 0$, $c_1 = c_2$

→ NON C'È $n \in \mathbb{R} \neq 0$ t.c. SIANO SODDISFATTE ASSIEME

RIMANE SOLO $C_1 = C_2 = 0 \Rightarrow f(x)$ e $g(x)$ LIN. INDIP.

ES 3:

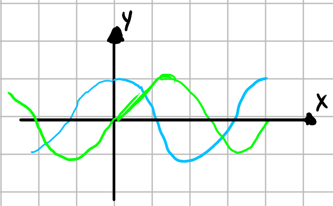
$f(x) = x$ e $g(x) = -2x$, $x \in \mathbb{R}$



$$C_1 f(x) + C_2 g(x) = C_1 x - 2C_2 x$$
$$= x(C_1 - 2C_2) = 0$$
$$C_1 = 2C_2 \quad \text{LIN. DIP.}$$

ES 4:

$f(x) = \sin(x)$ e $g(x) = \cos(x)$



$$C_1 f(x) + C_2 g(x) = 0$$
$$C_1 \sin(x) + C_2 \cos(x) = 0$$

in $\frac{\pi}{4} \Rightarrow C_1 \frac{\sqrt{2}}{2} + C_2 \frac{\sqrt{2}}{2} = 0 \Leftrightarrow \frac{\sqrt{2}}{2}(C_1 + C_2) = 0$, $C_1 = -C_2$

in $\frac{\pi}{2} \Rightarrow C_1 + 0 = 0$, $C_1 = 0$

→ ENTRAMBE $= 0$. LIN. INDIP.

ES 5:

$f(x) = \sin(x)$, $g(x) = \cos(x)$, $h(x) = 1$

$$\begin{pmatrix} \sin(x) & \cos(x) & 1 \\ \cos(x) & -\sin(x) & 0 \\ -\sin(x) & -\cos(x) & 0 \end{pmatrix} = -\cos(x)^2 - \sin(x)^2 = -1 \quad \text{LIN. INDIP.}$$

6.

$f(x) = \sin^2 x \quad g(x) = 2 \quad h(x) = 3\cos^2(x)$

$$C_1 \sin^2 x + 2C_2 + 3C_3(1 - \sin^2 x) = 0$$
$$C_1 \sin^2 x + 2C_2 + 3C_3 - 3C_3 \sin^2 x = 0$$
$$\sin^2(x)(C_1 - 3C_3) + 2C_2 + 3C_3 = 0$$

$C_2 = 1$: $\sin^2(x)(C_1 - 3C_3) + 2 + 3C_3 = 0$

$\sin^2(x)(C_1 - 3C_3) = -3C_3 - 2$

$C_3 = -\frac{2}{3}$: $\sin^2(x)(C_1 + 2) = 0$

$C_1 = -2$: $\sin^2(x) = 0$ SI' LIN. DIP.

7. RISOLVERE PROBLEMA CAUCHY

$$\begin{cases} y'' + \frac{y'}{x} - \frac{y}{x^2} = 0 \\ y(1) = 1 \\ y'(1) = 0 \end{cases}$$

$x \neq 0 \Rightarrow$ L'INTEGRALE FOND. È DEL TIPO: $C_1 y_1(x) + C_2 y_2(x)$ CIOÈ $L(y_1(x), y_2(x))$ LIN. INDIP.

→ SOL. LIN. INDIP. DELL' ED

L'INTERVALLO DA ANALIZZARE (CONTENENTE x_0) È $(0, +\infty)$

PRENDO POLINOMI PER y_1 e y_2

PER ORA MI LIMITO AI MONOMI $\longrightarrow y^* = x^\alpha \Rightarrow y' = \alpha x^{\alpha-1}$

$y'' = \alpha(\alpha-1)x^{\alpha-2}$

SOSTITUISCO: (e PRIMA MOLTIP. PER x^2)

$$x^2\alpha(\alpha-1)x^{\alpha-2} + \alpha x \cdot x^{\alpha-1} - x^\alpha$$

$$x^\alpha(\alpha^2 - \cancel{\alpha} + \cancel{\alpha} - 1) = 0$$

$$\alpha = \pm 1$$

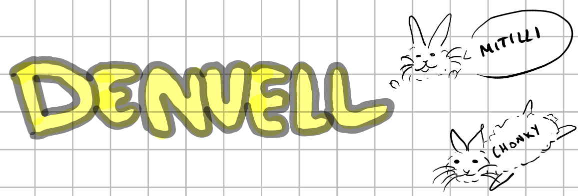
QUINDI $y_1 = x$ $y_2 = x^{-1}$ TUTTE LE SOL SONO $L(y=x, y=\frac{1}{x})$ CIOÈ: $y(x) = c_1 x + c_2 \frac{1}{x}$ $x \in (0,+\infty)$

IMPONGO CONDIZIONI INIZIALI

$$y'(x) = c_1 - \frac{c_2}{x^2}$$

$$\begin{cases} 1 = c_1 + c_2 \\ 0 = c_1 - c_2 \end{cases} \Rightarrow \begin{cases} c_1 = 1 - c_2 \\ 0 = 1 - c_2 - c_2 \end{cases} \Rightarrow \begin{matrix} c_1 = \frac{1}{2} \\ c_2 = \frac{1}{2} \end{matrix} \Rightarrow \text{SOL}: y(x) = \frac{1}{2}x + \frac{1}{2x} \quad x \in (0,+\infty)$$

NUMERI COMPLESSI



- $2z^2 - 2z + 5 = 0$ $\quad z \in \mathbb{C}$

1° METODO: FORMULA RISOLUTIVA

$$z = \frac{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - ac}}{a} = \frac{+1 \pm \sqrt{1-10}}{2} \in \mathbb{C} \Rightarrow \frac{1 \pm \{3i, -3i\}}{2} \to \frac{1+3i}{2}, \quad \frac{1-3i}{2}$$

2° METODO: $z = a + ib$

$$2z^2 - 2z + 5 = 0 \Rightarrow 2(a+ib)^2 - 2(a+ib) + 5 = 0 \Rightarrow 2(a^2 + 2abi + \underbrace{i^2b^2}_{-b^2}) - 2a - 2bi + 5 = 0$$

$$= 2a^2 + 4abi - 2b^2 - 2a - 2bi + 5 = 0$$

$$(2a^2 - 2b^2 - 2a + 5) + i(4ab - 2b) = 0 = 0 + 0i$$

$$\begin{cases} 2a^2 - 2b^2 - 2a + 5 = 0 \\ 4ab - 2b = 0 \end{cases} \xRightarrow{b \neq 0} \begin{cases} 2\frac{1}{4} - 2b^2 - 2\frac{1}{2} + 5 = 0 \\ a = \frac{1}{2} \end{cases} \Rightarrow \frac{1}{2} - 2b^2 - 1 + 5 \Rightarrow 2b^2 = \frac{9}{2} \quad b = \pm\frac{3}{2}$$

$$z = \frac{1}{2} \pm \frac{3}{2}i$$

OK!