DET. INTEGRALE GENERALE DI: $y'' - 4y' + 4y = 25\sin(x)$

$P_n(x) = 25\sin x$ con $\alpha = 0$ e $\beta = 1$ $\quad (25\sin(x) = 25 \cdot e^{0x} \cdot \sin(1 \cdot x))$

$\Phi(x) = \lambda^2 - 4y + 4 \Rightarrow (y-2)^2 = 0$ $\quad y = +2$ UNICA RADICE $\in \mathbb{R}$ con $h = 2$ e $\Delta = 0$

QUINDI $y_{om}(x) = C_1 e^{2x} + C_2 x e^{2x}$

- CERCHIAMO SOL PARTICOLARE

$y_*(x) = \underset{h=0}{x^h} \cdot \underset{\alpha=0}{\cancel{e^{\alpha x}}} \left(P_0(x)\sin(1 \cdot x) + Q_0(x)\cos(x) \right) = P_0 \sin(x) + Q_0 \cos(x)$ e TROVO P_0 e Q_0

$y_*'(x) = -A\sin x + B\cos(x)$

$y_*''(x) = -A\cos x - B\sin(x)$

$\rightarrow -A\cos(x) - B\sin(x) - 4(-A\sin(x) + B\cos(x)) + 4(A\cos x + B\sin(x)) = 25\sin x = 3A\cos(x) + 3B\sin(x) + 4A\sin(x) - 4B\cos(x)$

$= (3A - 4B)(\cos(x)) + (3B + 4A)(\sin(x))$

$$\begin{cases} 3A - 4B = 0 \\ 3B + 4A = 25 \end{cases} \rightarrow \quad A = \frac{4}{3}B \quad \longrightarrow A = 4 \qquad 3B + \frac{16}{3}B = 25 \rightarrow B = 3$$

$y_*(x) = 4\cos(x) + 3\sin(x)$ e $\boxed{y = 4\cos(x) + 3\sin(x) + C_1 e^{2x} + C_2 x e^{2x}}$ $x \in \mathbb{R}$

INTEGRALE GENERALE DI: $y'' - 2y' + y = \underbrace{x}_{P_1(x)} e^x$

$\Phi(x) = y^2 - 2y + 1 \rightarrow (y-1)^2 \Rightarrow +1$ RADICE $\in \mathbb{R}$ con $m = 2$ e $\Delta = 0$

$\alpha = 1$ e $\beta = 0$ $\qquad$ SOL GENERALE: $C_1 e^x + C_2 x e^x$

$\alpha + i\beta$ È RADICE

$y_*(x) = x^{2} \cdot e^x \left[(Ax+B) \underset{1}{\cancel{\cos(0x)}} + \underset{0}{\cancel{\sin(x)}} \ldots \right]$

$y_*(x) = (Ax^3 + Bx^2) \cdot e^x$

TROVO: $y_*'(x) = (3Ax^2 + 2Bx)e^x + (Ax^3 + Bx^2)e^x = (Ax^3 + (3A+B)x^2 + 2Bx)e^x$

$y_*''(x) = (Ax^3 + (6A+B)x^2 + (6A+4B)x + 2B)e^x$

$$\ldots \rightarrow \begin{cases} 0=0 \\ 6A+B-6A-2B+B=0 \rightarrow 0=0 \\ 6A+4B-2B=1 \rightarrow A = 1/6 \\ 2B = 0 \rightarrow B=0 \end{cases}$$

$y_* = x^2 e^x \left(\frac{1}{6}x \right) = \frac{x^3 e^x}{6}$ $\qquad \boxed{y_*(x) = \frac{x^3 e^x}{6} + C_1 e^x + C_2 x e^x}$

SOLUZIONE DI PC

$$PC \begin{cases} y'' - y = \frac{2(1+e^x)^2}{e^x} = \underset{f_1}{\frac{2}{e^x}} + \underset{f_2}{4} + \underset{f_3}{2e^x} = 2e^{-x} + 4 + 2e^x \\ y'(0) = 0 \\ y(0) = 0 \end{cases}$$

$\rightarrow$ DEVO VEDERLO COME SOMMA DI FUNZIONI f_1, f_2, f_3

- $f_1 \rightarrow \alpha = -1$ e $\beta = 0$ $\quad 2 = P_0$
- $f_2 \rightarrow \alpha = 0$ e $\beta = 0$ $\quad 4 = P_0$
- $f_3 \rightarrow \alpha = 1$ e $\beta = 0$ $\quad 2 = P_0$

$\Phi = x^2 - 1 \rightarrow x = \pm 1$ $\quad \lambda_1 = +1$ e $\lambda_2 = -1$ $\quad \Delta > 0 \rightarrow y_{om} = C_1 e^x + C_2 x e^{-x}$

1

$y_{*1} = x^1 \cdot e^{-x} \cdot [A\cos(0x) + \cancel{B\sin(0x)}] = Axe^{-x}$

$y_{*1}' = Ae^{-x} - Axe^{-x}$

$y_{*1}'' = -Ae^{-x} - Ae^{-x} + Axe^{-x} = -2Ae^{-x} + Axe^{-x}$

QUINDI $-2Ae^{-x} + \cancel{Axe^{-x}} - \cancel{Axe^{-x}} = 2e^{-x} \rightarrow A = -1$ $\quad$ CIOÈ $y_{*1}(x) = -xe^{-x}$

$y_{*3}(x) = x^1 \cdot e^{1x} \cdot [A \underset{1}{\cancel{\cos(0x)}} + \underset{0}{\cancel{B\sin(0x)}}] = Axe^x$

$y_{*3}' = Ae^x + Axe^x$

$y_{*3}'' = 2Ae^x + Axe^x$

$\rightarrow 2Ae^x - \cancel{Axe^x} + \cancel{Axe^x} = 2e^x$

$A = 1$

$y_{*3} = xe^x$

2

$y'' - y = 4$ DEVE ESSERE COSTANTE

FACENDO CON ALGORITMO ... $\rightarrow y_{*2}(x) = -4$

$y_* = \underbrace{-xe^{-x}}_{1} \underbrace{-4}_{2} + \underbrace{xe^x}_{3} + \underbrace{C_1 e^x + C_2 e^{-x}}_{y_0(x)}$ e IMPONGO LE CONDIZ. INIZIALI:

$$\begin{matrix} y(0) = 0 \\ y'(0) = 0 \end{matrix} \rightarrow \begin{cases} 0 - 4 + 0 + C_1 + C_2 = 0 \rightarrow C_1 + C_2 = 4 \rightarrow C_1 = 2 = C_2 \\ C_1 e^0 - C_2 e^{-0} - e^{-0} + 0 \cdot e^{-0} + e^0 + 0 \cdot e^0 = 0 \rightarrow C_1 - C_2 = 0 \rightarrow C_1 = C_2 \end{cases}$$

$\boxed{\text{SOLUZIONE: } y = -xe^{-x} - 4 + xe^x + 2e^x + 2e^{-x} \quad \forall x \in \mathbb{R}}$

INTEGRALE GENERALE: $y'' + 4y = \underbrace{1}_{P_0} \sin(2x)$

$\alpha = 0 \qquad \beta = +2$

$\Phi = y^2 + 4 = 0 \rightarrow y^2 = -4 \rightarrow y = \pm 2i$

$y_0 = C_1 \cos(2x) + C_2 \sin(2x)$

$y_* = x^1 \cdot e^{0 \cdot x} [A \sin(2x) + B\cos(2x)] \rightarrow Ax \sin(2x) + Bx \cos(2x)$

$y'_* = (A \cdot \sin(2x) + Ax \cdot \cos(2x) \cdot 2) + (B \cos(2x) + Bx \cdot -\sin(2x) \cdot 2)$

$= A \sin(2x) + 2Ax \cdot \cos(2x) + B\cos(2x) - 2Bx \sin(2x) = (A - 2Bx)(\sin(2x)) + (2Ax + B)(\cos(2x))$

$y''_* = -4A \sin(2x) - 4Ax \cos(2x) + 4B\cos(2x) - 4Bx \sin(2x)$

SOSTITUISCO:

$-4A\sin(2x) \cancel{-4Ax\cos(2x)} \cancel{+4B\cos(2x)} \cancel{-4Bx\sin(2x)} \cancel{+4Ax\cos(2x)} = \sin 2x$
$+4B\cos(2x)$

$\rightarrow \begin{cases} -4A = 1 \\ 4B = 0 \end{cases} \qquad A = -\frac{1}{4} \quad B = 0 \qquad y_*(x) = -\frac{1}{4} x \cos(2x)$

$y(x) = C_1 \cos(2x) + C_2 \sin(2x) - \frac{1}{4} x \cos(2x), \quad x \in \mathbb{R}$