

DET. INTEGRALE GENERALE DI: $y'' - 4y' + 4y = 25 \sin(x)$

$P_n(x) = 25 \sin x$ con $\alpha = 0$ e $\beta = 1$ ($25 \sin(x) = 25 \cdot e^{0x} \cdot \sin(1 \cdot x)$)
 $\Phi(x) = \lambda^2 - 4\lambda + 4 \Rightarrow (y-2)^2 = 0$ $y = +2$ UNICA RADICE $\in \mathbb{R}$ con $h=2$ e $\Delta=0$
 QUINDI $y_{om}(x) = C_1 e^{2x} + C_2 x e^{2x}$

• CERCHIAMO SOL PARTICOLARE

$y_*(x) = X^{h=0} \cdot e^{0x} (P_0(x) \sin(1 \cdot x) + Q_0(x) \cos(x)) = P_0(x) \sin(x) + Q_0(x) \cos(x)$ e Trovo P_0 e Q_0

$y'_*(x) = -A \sin(x) + B \cos(x) \rightarrow -A \cos(x) - B \sin(x) - 4(-A \sin(x) + B \cos(x)) + 4(A \cos(x) + B \sin(x)) = 25 \sin(x) = 3A \cos(x) + 3B \sin(x) + 4A \sin(x) - 4B \cos(x)$
 $y''_*(x) = -A \cos(x) - B \sin(x) = (3A - 4B) \cos(x) + (3B + 4A) \sin(x)$

$\begin{cases} 3A - 4B = 0 \\ 3B + 4A = 25 \end{cases} \rightarrow A = \frac{4}{3}B \rightarrow A = 4$
 $3B + \frac{16}{3}B = 25 \rightarrow B = 3$
 $y_*(x) = 4 \cos(x) + 3 \sin(x)$ e $y = 4 \cos(x) + 3 \sin(x) + C_1 e^{2x} + C_2 x e^{2x} \quad x \in \mathbb{R}$

INTEGRALE GENERALE DI: $y'' - 2y' + y = x e^x$

$\Phi(x) = y^2 - 2y + 1 \rightarrow (y-1)^2 \Rightarrow +1$ RADICE $\in \mathbb{R}$ con $m=2$ e $\Delta=0$
 $\alpha=1$ e $\beta=0$ $\alpha, \beta \in \text{RADICE}$ SOL GENERALE: $C_1 e^x + C_2 x e^x$

$y_*(x) = X^{m=2} \cdot e^{0x} [(Ax+B) \cos(0x) + \sin(0x) \dots]$

$y_*(x) = (Ax^3 + Bx^2) \cdot e^x$

TROVO: $y'_*(x) = (3Ax^2 + 2Bx) e^x + (Ax^3 + Bx^2) e^x = (Ax^3 + (3A+B)x^2 + 2Bx) e^x$

$y''_*(x) = (6Ax + (6A+B)x^2 + (6A+4B)x + 2B) e^x$

$\dots \rightarrow \begin{cases} 0=0 \\ 6A+6A+2B=0 \rightarrow 0=0 \\ 6A+12A+2B=1 \rightarrow A=\frac{1}{6} \\ 12A=0 \rightarrow B=0 \end{cases} \quad y_* = x^2 e^x (\frac{1}{6} x) = \frac{x^3 e^x}{6}$

$y_*(x) = \frac{x^3 e^x}{6} + C_1 e^x + C_2 x e^x$

SOLUZIONE DI (PC)

(PC) $\begin{cases} y'' - y = \frac{2(1+e^{2x})}{e^x} = \frac{2}{e^x} + 2e^x = 2e^{-x} + 2e^x \rightarrow \text{DEVO VEDERLO COME SOMMA DI FUNZIONI } f_1, f_2, f_3 \\ y'(0) = 0 \\ y(0) = 0 \end{cases}$

$\begin{cases} f_1 \rightarrow \alpha = -1 \text{ e } \beta = 0 \quad 2 = P_0 \\ f_2 \rightarrow \alpha = 0 \text{ e } \beta = 0 \quad 4 = P_0 \\ f_3 \rightarrow \alpha = 1 \text{ e } \beta = 0 \quad 2 = P_0 \end{cases} \quad \Phi = x^2 - 1 \rightarrow x = \pm 1 \quad \lambda_1 = +1 \text{ e } \lambda_2 = -1 \quad \Delta > 0 \rightarrow y_{om} = C_1 e^x + C_2 x e^x$

$y_{*1} = x^1 \cdot e^{-x} \cdot [A \cos(0x) + B \sin(0x)] = A x e^{-x}$

$y'_{*1} = A e^{-x} - A x e^{-x}$

$y''_{*1} = -A e^{-x} - A e^{-x} + A x e^{-x} = -2A e^{-x} + A x e^{-x}$

QUINDI $-2A e^{-x} + A x e^{-x} - A x e^{-x} = 2e^{-x} \rightarrow A = -1$ CIOE' $y_{*1}(x) = -x e^{-x}$

1 $y_{*3}(x) = x^1 \cdot e^{1x} \cdot [A \cos(0x) + B \sin(0x)] = A x e^x$

$y'_{*3} = A e^x + A x e^x$

$y''_{*3} = 2A e^x + A x e^x \rightarrow 2A e^x - A x e^x + A x e^x = 2e^x \rightarrow A = 1$

2 $y_{*2} = x e^x$

$y'' - y = 4$ DEVE ESSERE COSTANTE
 FACENDO CON ALGORITMO... $\rightarrow y_{*2}(x) = -4$

$y_* = \frac{-x e^{-x}}{1} + \frac{-4}{2} + \frac{x e^x}{3} + \frac{C_1 e^x + C_2 x e^x}{y_{om}(x)}$ e IMPONGO LE CONDIZ. INIZIALI:

$y(0) = 0 \rightarrow \begin{cases} 0 - 4 + 0 + C_1 + C_2 = 0 \rightarrow C_1 + C_2 = 4 \rightarrow C_1 = 2 = C_2 \\ y'(0) = 0 \rightarrow C_1 e^0 - C_2 e^0 - e^{-0} + 0 \cdot e^{-0} + e^0 + 0 \cdot e^0 = 0 \rightarrow C_1 - C_2 = 0 \rightarrow C_1 = C_2 \end{cases}$

SOLUZIONE: $y = -x e^{-x} - 4 + x e^x + 2 e^x + 2 e^{-x} \quad \forall x \in \mathbb{R}$

INTEGRALE GENERALE: $y'' + 4y = \sin(2x)$

$$\alpha = 0 \quad \beta = 12$$

$$\bar{y} = y^2 + 4 = 0 \rightarrow y^2 = -4 \rightarrow y = \pm 2i$$

$$y_0 = C_1 \cos(2x) + C_2 \sin(2x)$$

$$y_* = x^2 \cdot e^{0 \cdot x} [A \sin(2x) + B \cos(2x)] \rightarrow Ax \sin(2x) + Bx \cos(2x)$$

$$y'_* = (A \cdot \sin(2x) + Ax \cdot \cos(2x) \cdot 2) + (B \cos(2x) + Bx \cdot -\sin(2x) \cdot 2)$$

$$= A \sin(2x) + 2Ax \cos(2x) + B \cos(2x) - 2Bx \sin(2x) = (A - 2Bx) \sin(2x) + (2Ax + B) \cos(2x)$$

$$y''_* = -4A \sin(2x) - 4Ax \cos(2x) + 4B \cos(2x) - 4Bx \sin(2x)$$

SOUSTITUISCO:

$$-4A \sin(2x) - 4Ax \cos(2x) + 4B \cos(2x) - 4Bx \sin(2x) + 4Ax \cos(2x) = \sin(2x) + 4B \cos(2x)$$

$$\rightarrow \begin{cases} -4A = 1 \\ 4B = 0 \end{cases} \quad \begin{matrix} A = -\frac{1}{4} \\ B = 0 \end{matrix}$$

$$y_*(x) = -\frac{1}{4}x \cos(2x)$$

$$y(x) = C_1 \cos(2x) + C_2 \sin(2x) - \frac{1}{4}x \cos(2x), \quad x \in \mathbb{R}$$