① $\mathcal{E}^3$ $\quad A \equiv (-1, 1, 0) \quad B \equiv (3, 0, 1)$

$C \equiv (2, -2, 5) \quad D \equiv (3, 5, 7)$

(i) $$V = \frac{1}{3!} \sqrt{\begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 3 & 2 & 3 \\ 1 & 0 & -2 & 5 \\ 0 & 1 & 5 & 7 \end{vmatrix}}$$

$$\downarrow$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & 3 & 4 \\ 1 & 0 & -2 & 5 \\ 0 & 1 & 5 & 7 \end{vmatrix}$$

$$\downarrow$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & 3 & 4 \\ 0 & -1 & -3 & 4 \\ 0 & 1 & 5 & 7 \end{vmatrix} = \frac{1}{6} \cdot \begin{vmatrix} 4 & 3 & 4 \\ -1 & -3 & 4 \\ 1 & 5 & 7 \end{vmatrix} = -12 \cdot 7 + 12 - 20 + 12 - 16 \cdot 5 + 21$$

$$\boxed{= \frac{133}{6}}$$

(ii) $d(D, \pi_{ABC}) =$

$$\pi_{ABC} : \begin{vmatrix} a & b & c \\ 4 & -1 & 1 \\ 3 & -3 & 5 \end{vmatrix} = 0 \rightarrow -2a - 17b - 9c + d = 0$$

$$2 \cancel{+} - 17 + d = 0$$

$$d = +13$$

$\overrightarrow{AB}$ e $\overrightarrow{AC}$ $\Rightarrow$ $\overrightarrow{AB} = (4, -1, 1)$

$\overrightarrow{AC} = (3, -3, 5)$

$$\pi_{ABC} = -2x - 17y - 9z + 15 = 0$$

$$d(D, \pi_{ABC}) = \frac{|3(-2) + 5(-17) + 7(-8) + 15|}{\sqrt{4 + 17^2 + 8^2}} \simeq 7,18$$

(iii)

$$A_{A\hat{B}C} = \frac{1}{2}\sqrt{\begin{vmatrix} \langle 4,-1,1;4,-1,1\rangle & \langle 4,-1,1;3,-3,5\rangle \\ \langle 4,-1,1;3,-3,5\rangle & \langle 3,-3,5;3,-3,5\rangle \end{vmatrix}}$$

$$= \frac{1}{2}\sqrt{\begin{vmatrix} 18 & 20 \\ 20 & 43 \end{vmatrix}} = \frac{1}{2}\sqrt{374} = \frac{\sqrt{374}}{2}$$

$$C: 4x^2 + 2y^2 + 2xy + 10y + 3 = 0$$ ②

$$A = \begin{vmatrix} 3 & 0 & 5 \\ 0 & 4 & 1 \\ 5 & 1 & 2 \end{vmatrix} \qquad |A_{00}| = 7 \quad \text{ELLISSE}$$

$$\lambda A_{00} = \begin{pmatrix} \lambda - 4 & -1 \\ -1 & \lambda - 2 \end{pmatrix} = \lambda^2 - 6\lambda + 7 = 0$$

$$\lambda = 3 \pm \sqrt{2}$$

$$U_{\lambda_{1,2}} \begin{pmatrix} 3 \pm \sqrt{2} - 4 & -1 \\ -1 & 3 \pm \sqrt{2} - 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

(+) $$\begin{cases} (-1+\sqrt{2})x - y = 0 \Rightarrow y = (-1+\sqrt{2})x \\ -x \quad (1+\sqrt{2})y = 0 \end{cases}$$

(+) $L((1, -1+\sqrt{2})$

(−) $L((1, -1-\sqrt{2})$

$$A\begin{pmatrix}0\\ l\\ m\end{pmatrix} = S_m + (4l+m)x + (l+2m)y = 0$$

$\downarrow$

PUNTO IMPROPRIO GENERALE

$$\begin{cases} 4l+m=0 \\ l+2m=0 \end{cases} \rightarrow \begin{matrix} m=-4l \\ l\neq 0 \end{matrix}$$

$$\text{POLO} = \begin{array}{c}(0, 1, -4) \\ \quad\; l \;\; m\end{array}$$

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$$-20l - 7ly = 0$$

$$y = -\frac{20}{7}$$

(3)

$$2\alpha xy + (1-\alpha^2)y^2 - 4x + 2y + 4 = 0 \qquad (i)$$

$$A = \begin{pmatrix} 4 & -2 & 1 \\ -2 & 0 & \alpha \\ 1 & \alpha & 1-\alpha^2 \end{pmatrix} \qquad \det A = -2\alpha - 2\alpha - 4\alpha^2 - 4(1-\alpha^2)$$

$$= -4\alpha - 4$$

$\alpha = 1$ DEGENERE

$\alpha = -1$ NON DEGENERE

$\alpha = 0$ PARABOLA

$\alpha = 2$ IPERBOLE

(ii)

$\alpha = 2$

$$\begin{pmatrix} 4 & -2 & 1 \\ -2 & 0 & 2 \\ 1 & 2 & -3 \end{pmatrix} \Rightarrow \begin{cases} 4x_1x_2 - 3x_2^2 - \cancel{4x_0x_1} + \cancel{2x_0x_2} + \cancel{4x_0^2} = 0 \\ x_0 = 0 \end{cases}$$

$$x_2(4x_1 - 3x_2) = 0 \qquad x_2 = 0 \quad [0, 1, 0]$$

$$x_1 = \frac{3}{4}x_2 \quad [0, 3, 4]$$

ASINTOTI

(1) $A\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}-2\\0\\2\end{pmatrix} \Rightarrow -2x_0+2x_2=0$

(2) $A\begin{pmatrix}0\\3\\4\end{pmatrix}=\begin{pmatrix}-2\\8\\-6\end{pmatrix} \Rightarrow -2x_0+8x_1-6x_2=0$

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