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COMMITTEES, HIERARCHIES AND POLYARCHIES

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This dangerous fallacy I shall now illumine:
To committees, nothing alien is human.

Ogden Nash

Committees represent a widespread form of modern decision making. This is because 'to err is human': on questions of importance, we are often reluctant to delegate the decision making authority to any single individual. There is an implicit belief that the wisdom of a committee might be greater than that of any single member, that collective decision making avoids some of the worst errors that might otherwise occur. But at the same time, some of the negative features of committees (for instance, the manhours devoted to decision making, or the delays in getting agreements) are also well known.

In this paper we study the decision making of committees and contrast this to the decision making of certain stylised forms of centralised versus decentralised organisations (which we respectively call hierarchies and polyarchies). Our analysis focuses on two economic trade-offs involved in organisational decision making. The first trade-off is between the individuals' errors of not approving good projects (Type-I errors) and the errors of approving bad projects (Type-II errors). For instance, by increasing the size of the consensus required for project acceptance in a committee of a fixed size, one can decrease the Type-II errors, but only at the expense of increasing Type-I errors. The second trade-off is between the gains from a more extensive evaluation of projects and the extra resources spent on evaluating projects. For example, by increasing the size of a committee (and changing the decision rule in an optimal way corresponding to the enlarged committee), one may increase the mean quality of the projects accepted, but one also increases evaluation costs. In fact, evaluation costs play a central role in our analysis; if these costs were absent then perfect decisions (that is, error free selection of projects) can be easily achieved. By the same token, the fact that evaluation costs are not zero implies not only that all organisations are fallible in their decisions, but also that the economic consequences of individuals' errors depend on the overall organisation of decision making.

In the committees which we study here, each member evaluates every project, and the project is accepted by the committee if approved by the number of members equal to or larger than the required level of consensus. In the centralised (hierarchical) organisations we study, a project is evaluated by a higher level individual (or bureau) only if approved by the lower levels, and only those projects are accepted by the organisation which are approved by the highest level. Thus, though an n level hierarchy accepts a project if it is

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approved by n individuals, just as an n member committee requiring unanimity would, the sequence of decision making is quite different in these two organisations, and this difference can lead to markedly different evaluation costs. Another form of organisation we study has decentralised (polyarchical) decision making where a project is accepted by the organisation as soon as it is approved by any one member. Thus, an n member polyarchy has the same project acceptance rule as an n member committee where the required consensus is one; but the evaluation costs entailed in these two organisations can, once again, be quite different. For each of the above three organisational forms, our focus is on developing qualitative results concerning the optimal organisational choices. We therefore characterise the optimal level of consensus in a committee and the optimal organisational size for committees, hierarchies and polyarchies. We then subject these optima to comparative statics with respect to the parameters representing the organisational environment (such as the quality of project portfolio under consideration and the abilities of individuals in selecting or rejecting different types of projects). This analysis, as we shall see, yields a number of qualitative insights. We also analyse how the relative performance of alternative organisational forms (for example, the performance of a committee compared to that of a hierarchy or polyarchy) alters when the organisational environment changes. Such an analysis might contribute to an understanding of the circumstances under which one organisational form might be more likely to emerge compared to another.

An individual's approval errors (that is, his probabilities of approving bad projects or not approving good projects) are represented in our analysis as reduced form parameters reflecting the limited abilities of homogeneous individuals. We also assume that the costs of evaluation depend primarily on how many individuals evaluate a project within an organisation. In this sense, the focus of our analysis is on analysing how different organisations aggregate different types of individuals' errors, and what the resulting trade-off is between the costs of increasing an organisation's size and the gains from the corresponding reduction in aggregate errors.

Underlying our analysis is the presumption that it is costly – or more accurately, impossible – for all individuals to share completely all of their information, and for the collectivity to reprocess all the information thus assembled to form an optimal decision. The group decision making we envisage is one in which there is not only considerable decentralisation in information processing but also limited communication; the information gathered by an individual is processed into a single binary signal, and it is only these signals which are communicated. For the present purposes, then, it makes little difference whether the binding constraint arises from abilities to communicate, or abilities to process information. Obviously, our analysis is meant to be a prototype for the much richer analysis where the dimensionality of what can be centrally processed and what can be communicated is endogenous.[1] Still, the

[1] A fuller analysis would obviously take into account the costs and benefits of communicating different kinds of signals. In general, communications involve not only resource and time costs (e.g. if it takes τ units of time for one individual to communicate his information, then the total time taken by an n member

decentralised information processing and limited communication which we model here reflect well some important aspects of organisational structures.

Though we have assumed that only binary signals (approval/disapproval) are communicated, the conditions under which each signal is communicated will, in general, be affected by the organisational structure, as we have argued elsewhere.[2] This paper abstracts from these and other sources of endogeneity of individuals' approval errors. We do show, however, that our simplifying assumption (that an individual's approval errors are not significantly different under alternative organisations under consideration) is consistent with certain types of Bayesian individual decision making, even when account is taken of those strategic considerations which might arise when each individual chooses his own approval rule independently of others in the organisation.

Related Literatures. There are two literatures to which the problems studied in this paper have some semblance. One is that on voting rules in the theory of social choice. There, the emphasis has been on identifying rules to 'aggregate' different preferences of individuals, which satisfy certain desiderata. Here, *values* (objectives) of the members of the organisation are the same, but their *judgments* differ (because of the incompleteness and the differences in information). Though, in an abstract sense, these differences in judgments can be represented as differences in preferences, the particular models which we study (including the decision and cost structure of committees, hierarchies, and polyarchies) are motivated by our interest in analysing and comparing alternative forms of organisations. Also, these models allow us to ask questions and obtain qualitative insights which are not emphasised in that literature.

There is also a resemblance between our formulation of a committee's decision making and some of the problems studied in reliability theory (where a relay network's components are subject to failures) and the analysis of jury decision making. However, the results we present here concerning committee's decision making are, to our knowledge, not available in these literatures.[3]

The paper is organised as follows. The basic model for analysing the central trade-offs is presented in section I. We use this model, in sections II and III, to characterise and interpret the optimal level of consensus in a committee, and

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committee is $n\tau$ if each person's communication is simultaneous to all committee members, and the total time taken is $n\tau(\tau-1)$ if communications are bilateral), but also there are costs of errors in communication (e.g. the information received by one person is almost never the same as the one which is intended by the person communicating it). Some aspects of benefits of communication have been analysed by Klevorick *et al.* (1984) in the context of jury decision making, where they compare majority rule without communication to the unanimity rule with specific types of communication.

[2] In Sah and Stiglitz (1986*a*), where the focus is on comparing the performance of a two-unit hierarchy to a two-unit polyarchy, we show how the performance of an organisation can be evaluated taking into account the effect of organisational structure on an individual's approval errors. In particular, we examine the specification where an individual observes a noisy signal, s, of the project's value, and the project is approved if s exceeds a reservation level R. By increasing R one decreases the probability of accepting a bad project but increases the probability of rejecting a good project. Clearly, the nature of an individual's errors may also depend on the expenditures on information acquisition, and the optimal level of those expenditures may differ across organisational forms.

[3] See Barlow and Proschan (1981) and Harrison (1965), among others, on reliability theory. Some of the differences between our analysis and reliability theory can be seen in Sah and Stiglitz (1987). See Klevorick *et al.* (1984), and references therein, on jury decision making.

the optimal committee size. An analysis of optimal hierarchies and polyarchies is presented in Section IV. In Section V, we analyse the relative performance of these three organisational forms. Some of the possible ways to extend the present analysis are briefly discussed in Section VI. In this section, we (i) describe some of the circumstances under which our assumption concerning individuals' approval errors are consistent with Bayesian individual decision making, (ii) discuss some of the implications of delays in decision making, and (iii) note some of the insights which our analysis might suggest for organisations more complex than those which we have examined. A brief summary is presented at the end of the paper.

I. THE BASIC MODEL

There are n members in an organisation, whose task is to accept or reject projects. The size of minimum consensus required for accepting a project is denoted by k. That is, a project is accepted by the organisation only if k or more members accept it; otherwise it is rejected. It is assumed throughout that $n \geqslant k \geqslant 0$, and $n > 1$. There are two kinds of projects, good and bad, with respective (net expected) profits z_1 and $-z_2$, where z_1 and z_2 are positive. α is the proportion of good projects; $1 > \alpha > 0$.[4] Individuals are homogeneous in their decision making abilities, and each individual has some, but not perfect, ability to distinguish between good and bad projects. If p_1 and p_2 respectively represent the probabilities that an individual approves a good and a bad project, then $1 > p_1 > p_2 > 0$. One can thus interpret $1-p_1$ and p_2 as the Type-I and Type-II errors entailed in an individual's approvals.

The probability that a project of type i is accepted by the organisation is

$$h_i = h(k, n, p_i) = \sum_{j=k}^{n} \binom{n}{j} p_i^j (1-p_i)^{n-j}, \tag{1}$$

where $i = 1$ and 2, and $1-h_1$ and h_2 can be respectively interpreted as the Type-I and Type-II errors entailed in the organisation's decision making. The (expected) profit of the organisation is represented by

$$I = \sum_{i=1}^{2} \gamma_i h_i - E, \tag{2}$$

where $\gamma_1 = \alpha z_1 > 0$, $\gamma_2 = -(1-\alpha) z_2 < 0$, and E is the (expected) evaluation cost per project. The evaluation cost depends, as we shall see, on the organisational form as well as the organisational size. Also note that in (2) and in the rest of the paper, we suppress the number of projects in the project portfolio.

Three intuitive properties of (1) which we shall use later are as follows. First, an organisation of a given size is less likely to accept a project (good or bad)

[4] We assume that these parameters describing the quality of project portfolio are known. A more general portfolio, consisting of a continuum of projects, can be modelled along the lines of Sah and Stiglitz (1986*a*).

if it requires a larger consensus. This can be seen directly from (1) which yields

$$h(k+1, n) - h(k, n) = -\binom{n}{k} p^k (1-p)^{n-k} < 0. \tag{3}$$

Second, for a given level of consensus, a larger organisation is more likely to accept a project. Specifically, it can be shown from (1) that

$$h(k, n+1) - h(k, n) = \binom{n}{k-1} p^k (1-p)^{n-k+1} > 0. \tag{4}$$

A third, obvious, property of expression (1) is that a project is more likely to be accepted by an organisation if the probability of its approval by individuals is higher. In particular

$$\partial h_i / \partial p_i = \binom{n}{k} k p_i^{k-1} (1-p_i)^{n-k} > 0. \tag{5}$$

II. ACCEPTANCE RULES FOR COMMITTEES OF FIXED SIZE

Our objective in this section is to identify some of the properties of the optimal acceptance rule for a committee of a given size. We delineate the circumstances under which the majority rule or the marginal majority rule is optimal, and under which the optimal acceptance rule entails a larger, or a smaller, consensus. Also, we ascertain the effects of exogenous parameters on the optimal level of consensus.

The evaluation cost E is a fixed parameter in the present case because the evaluation is simultaneous in a committee and, therefore, E depends on n but not on k. Maximisation of (2) is then equivalent to maximising

$$Y(k) = h_1 - \beta h_2 \tag{6}$$

where $\beta = (1-\alpha) z_2 / \alpha z_1 > 0$ is a summary parameter representing the portfolio quality. A better portfolio implies a smaller β; this is obvious because $\partial\beta/\partial\alpha < 0$, $\partial\beta/\partial z_1 < 0$, and $\partial\beta/\partial z_2 > 0$. Also, the parameter β has the following natural interpretation: if $\beta = 1$, then the expected value of a project selected randomly (without screening) equals zero. We refer to portfolios with $\beta < 1$ as high quality; with $\beta > 1$ as low quality; and $\beta = 1$ as *neutral* quality.

A. Optimal Level of Consensus

It is straightforward to show, from (1) and (6), that Y is single peaked in k.[5] At an interior optimum (that is, where $n > k > 0$), thus, it must be the case that

$$Y(k) - Y(k-1) \geqslant 0, \tag{7}$$

and $\qquad Y(k) - Y(k+1) \geqslant 0, \quad$ with at least one strict inequality. $\qquad$ (8)

The above expressions, in combination with (1), (3) and (6), yield the following characterisation of the optimal k:

$$r^{n-k} q^k \geqslant \beta \geqslant r^{n-k+1} q^{k-1}, \quad \text{for} \quad n > k > 0; \tag{9}$$

[5] For a proof, see an earlier version (1985) of the present paper.

where, for brevity, we have used the notation: $q = p_1/p_2$, and $r = (1-p_1)/(1-p_2)$. To characterise corner solutions, note that if $Y(k=0) \geqslant Y(k=1)$, then all projects should be accepted; and if $Y(k=n) \geqslant Y(k=n-1)$, then only those projects should be accepted for which there is complete unanimity.[6] Correspondingly, from (1) and (6), we obtain

$$k = 0, \quad \text{if} \quad r^n \geqslant \beta; \tag{10}$$

and

$$k = n, \quad \text{if} \quad \beta \geqslant rq^{n-1}. \tag{11}$$

A rearrangement of (9), derived in Appendix I(a), yields[7]

$$k < \frac{n}{2} + 1, \quad \text{if} \quad \beta < 1, \quad \text{and} \quad p_2 \leqslant 1 - p_1. \tag{12}$$

$$k > \frac{n}{2}, \quad \text{if} \quad \beta > 1, \quad \text{and} \quad p_2 \geqslant 1 - p_1. \tag{13}$$

We thus obtain

Proposition 1. *A sufficient condition for the optimal consensus to be smaller (larger) than the majority rule is that the parameter β is smaller (larger) than unity and that an individual's Type-II error is smaller (larger) than his Type-I error. In the special case where an individual's Type-I and Type-II errors are equal and where the portfolio is of neutral quality (that is, $\beta = 1$), the majority rule is optimal.*

This result is easily understood. An increase in k lowers the proportion of projects (good or bad) accepted by the committee, and whether such an increase improves or worsens committee's profit depends on the quality of portfolio and the nature of individuals' errors. To see this, note that if the portfolio is extrememly good (that is, if β is close to zero) then any scrutiny is entirely undesirable, and if the project portfolio is relatively bad (that is, β is large) then more scrutiny is desirable. Analogously, if individuals' Type-II errors are relatively negligible (that is, $1-p_1 > p_2 \to 0$) then it is not desirable to have k larger than one, but if Type-I errors are relatively negligible (that is, $p_2 > 1-p_1 \to 0$) then additional scrutiny can only improve committee's profit.

B. Comparative Statics

In the above model, the optimal consensus depends on the parameters (n, β, p_1, p_2). For a comparative statics analysis of an interior optimum, with respect to these parameters, we treat k and n as continuous variables, and employ a standard normal approximation to the binomial distribution entailed in (1). That is

$$h_i = 1 - \phi(z_i) \tag{14}$$

[6] By restricting k to be no larger than n, we are assuming that there is some acceptance rule within this range (that is, the range $n \geqslant k \geqslant 0$) for which the committee's profit, (2), is positive. Consequently, we do not consider the case where all projects are systematically turned down. Analogous assumptions concerning the viability of organisations underlie our analysis of hierarchies and polyarchies.

[7] Note that conditions such that p_2 is larger or smaller than $1-p_1$ imply certain restrictions on the magnitudes of p_1 and p_2. This is because of our assumption that $p_1 > p_2$. Specifically, $p_2 \geqslant 1-p_1$ means that $p_1 > \frac{1}{2}$, whereas $p_2 \leqslant 1-p_1$ means that $p_2 < \frac{1}{2}$. Naturally then, $p_2 = 1-p_1$ means that $p_1 > \frac{1}{2} > p_2$.

where ϕ is the unit normal distribution function, and $z_i = (k-np_i)/[np_i(1-p_i)]^{\frac{1}{2}}$.[8] The derivatives of (14) with respect to k and n, which we will use later, are[9]

$$h_{ik} = -\phi_z(z_i)\,[np_i(1-p_i)]^{-\frac{1}{2}} < 0, \tag{15}$$

and

$$h_{in} = \phi_z(z_i)\frac{1}{2n}(k+np_i)\,[np_i(1-p_i)]^{-\frac{1}{2}} > 0, \tag{16}$$

where $\phi_z(z_i)$ is the unit normal probability density at z_i, and it is always positive. From (6), then, the optimum is characterised by

$$Y_k = h_{1k} - \beta h_{2k} = 0, \quad \text{and} \quad n > k > 0. \tag{17}$$

Now, if θ represents an exogenous parameter, then a perturbation in (17) yields

$$\frac{dk}{d\theta} = -Y_{k\theta}/Y_{kk}. \tag{18}$$

We evaluate Y_{kk} at the optimum, using (15) and (17), as[10]

$$Y_{kk} = bh_{1k}(p_1-p_2)/[np_1p_2(1-p_1)(1-p_2)] < 0, \tag{19}$$

where $b = k(1-p_1)(1-p_2)+(n-k)p_1p_2 > 0$. The negative sign of Y_{kk} is obvious from (15) and (19).

Effect of Committee Size. We first evaluate Y_{kn} at the optimum, and substitute the resulting expression,[11] along with (19), into (18). This yields

$$\frac{dk}{dn} = [k^2(1-p_1-p_2)+n^2p_1p_2]/2nb. \tag{20}$$

This expression provides a basis for a number of qualitative observations.

First, the numerator in the right hand side of (20) can be reexpressed as $k^2(1-p_1)(1-p_2)+(n^2-k^2)p_1p_2 > 0$. Therefore

$$\frac{dk}{dn} > 0. \tag{21}$$

That is: *A larger committee has a larger optimal consensus.* This is intuitive because if the size of consensus is left unchanged but the committee size is increased, then the scrutiny becomes slacker than before. To restore the desired tightness in screening, therefore, it is necessary that the required consensus should be increased.

Second, expression (20) yields

$$1-\frac{dk}{dn} = [k(2n-k)(1-p_1)(1-p_2)+(n-k)^2p_1p_2]/2nb > 0. \tag{22}$$

[8] As is well known, there are several other approximations of a binomial cumulative density which can be more accurate than the one employed here. The qualititative results we derive, however, are unlikely to change if other approximations were to be employed.

[9] All subscripts, other than 1, 2 and i, denote the variables with respect to which a partial derivative is being taken.

[10] A simple derivation of (19) is as follows. Expression (17) can be reexpressed as $Y_k = h_{2k}[(h_{1k}/h_{2k})-\beta] = 0$ which, upon differentiation, yields $Y_{kk} = h_{2k}\partial(h_{1k}/h_{2k})/\partial k$. Expression (15) and the properties of unit normal density function are then used to obtain (19). The same method is also helpful in deriving the expressions for Y_{kn} and $\partial Y_k/\partial p_1$, which are stated below in footnotes 11 and 13 respectively.

[11] $Y_{kn} = -h_{1k}(p_1-p_2)[k^2(1-p_1-p_2)+n^2p_1p_2]/[2n^2p_1p_2(1-p_1)(1-p_2)]$.

Thus

$$\frac{dk}{dn} < 1. \tag{23}$$

That is: *The optimal consensus increases less than the increase in the committee size.*

Third, we use (20) to obtain

$$\frac{dk}{dn} - \tfrac{1}{2} = k(k-n)\,(1-p_1-p_2)/2nb. \tag{24}$$

Therefore

$$\frac{dk}{dn} \gtrless \tfrac{1}{2} \quad \text{if} \quad p_2 \gtrless 1-p_1. \tag{25}$$

Clearly, $dk/dn = \frac{1}{2}$, in the special case when $p_2 = 1-p_1$. This case could be called the *marginal majority rule* since the increase in the optimal consensus is one-half of the increase in the committee size. From (25), we conclude therefore that: (i) *The marginal majority rule is optimal if the two types of individuals' errors are equal*, and (ii) *The increase in the optimal consensus in response to an increased committee size is greater (smaller) than that in the marginal majority rule if individuals are less (more) likely to reject a good project than to accept a bad project.*

The last set of results are parallel to those concerning majority rule (see Proposition 1), but there is one critical difference. The majority rule results are global, but they hold only when the portfolio quality satisfies certain conditions. In contrast, the marginal majority results hold only in the neighbourhood of an interior optimum, but they do not depend on the quality of the project portfolio.

Finally, we present a result concerning the effect of committee size on the optimal k/n. The general relationship between the variation of optimal k/n when n varies, and the corresponding variation of optimal k is straightforward:

$$\frac{d(k/n)}{dn} = \left(\frac{dk}{dn} - \frac{k}{n}\right)\frac{1}{n}. \tag{26}$$

Now consider the case where the two types of errors are equal, that is, $p_2 = 1-p_1$. Then, from (25), $dk/dn = \frac{1}{2}$. Also, when (k, n) are treated as continuous variables, then (12) and (13) imply

$$\frac{k}{n} \lesseqgtr \frac{1}{2}, \quad \text{if} \quad \beta \lesseqgtr 1, \quad \text{and} \quad p_2 = 1-p_1. \tag{27}$$

The last two observations, along with (26), yield:

$$\frac{d(k/n)}{dn} \gtrless 0, \quad \text{if} \quad \beta \lesseqgtr 1.$$[12]

Combining this result with (27) it follows that: *When the two types of individuals' errors are equal, the optimal acceptance rule becomes closer to the majority rule as the size of committee increases.*

[12] Another way to express this result is to let $\epsilon_{kn} = d\ln k/d\ln n$ denote the elasticity of the optimal consensus with respect to the committee size. Then: $\epsilon_{kn} \gtrless 1$ if $\beta \lesseqgtr 1$, and $p_2 = 1-p_1$. In fact, a lower bound on ϵ_{kn} can be identified by establishing from (20) that $(dk/dn)-(k/2n) > 0$. Thus, $\epsilon_{kn} > \frac{1}{2}$; that is: *The elasticity of optimal consensus with respect to the committee size is greater than one-half.* Note that the preceding result holds regardless of the relative magnitude of the two types of individuals' errors.

Effect of Portfolio Quality: Recall that a worse portfolio implies a larger β. Also, from (17), $\partial Y_k/\partial\beta > 0$. Using (18) and recalling that $Y_{kk} < 0$, therefore, the effect of portfolio quality is immediately ascertained.

PROPOSITION 2. *The optimal size of consensus is larger if a committee faces a worse portfolio.*

Effect of Managerial Quality: An improvement in the individuals' decision making abilities is represented in our model by a larger p_1 (in which case an individual rejects fewer good projects), and a smaller p_2 (in which case an individual rejects more bad projects). The impact of such improvements on the optimal consensus is ambiguous in general. This is because it may be desirable to reduce k under some circumstances, so that a yet larger proportion of good projects can be accepted; whereas in other circumstances, it may be better to increase k, so that the acceptance of bad projects is lowered even further.

To see this, consider the special case in which the two types of individual errors are equal: that is, $p_2 = 1-p_1$. A larger p_1 now represents not only a lower Type-I error but also a lower Type-II error. In this case, it can be ascertained from (17) that $\partial Y_k/\partial p_1 \gtrless 0$, if $k/n \lesseqgtr \frac{1}{2}$.[13] Combining the last expression with (18) and (27), we obtain

$$\frac{dk}{dp_1} \gtrless 0, \quad \text{if} \quad \beta \lesseqgtr 1, \quad \text{and} \quad p_2 = 1-p_1. \tag{28}$$

This result has an interesting implication. If the project portfolio is relatively bad (that is $\beta > 1$), then we know from (27) that a consensus larger than the majority rule is desirable. Now, if the managerial quality improves, then according to (28), the scrutiny should be slackened so that more good projects can be accepted. On the other hand, if the portfolio is relatively good (that is, $\beta < 1$) then a consensus smaller than the majority rule is desirable, and if the managerial quality improves in this case, then the scrutiny should be tightened according to (28), so that a larger number of bad projects can be rejected. Thus: *When the two types of individuals' errors are equal, the optimal decision rule becomes closer to the majority rule as the managerial quality improves.*

III. THE OPTIMAL SIZE OF A COMMITTEE

In this section, we briefly look at the effect of evaluation costs on the simultaneous determination of the optimal committee size and the optimal size of consensus. Once again, we adopt the approximation (14), and focus on an interior (k,n). This is characterised by (17), and by

$$I_n = \textstyle\sum_{i=1}^{2} \gamma_i h_{in} - E_n = 0, \tag{29}$$

where $E_n = \partial E(n)/\partial n > 0$ denotes the marginal cost of a committee member.

Let θ denote a parameter such that a larger θ implies a larger marginal cost of committee members; that is $E_{n\theta} > 0$. Then, it is obvious that the optimal k is affected by θ only through the change in n, that is, $dk/d\theta = (dk/dn)(dn/d\theta)$.

[13] Since $\partial Y_k/\partial p_1 = -h_{1k}(n-2k)[(1-p_1)^2+p_1^2]/2p_1^2(1-p_1)^2$.

Also if I is strictly concave in (k,n), then a perturbation in (17) and (29) yields $dn/d\theta < 0$.[14] In this case, therefore, the results obtained above can be translated immediately to ascertain the effect of evaluation cost on the optimal consensus, when the committee size is optimal. For instance, multiplying (21) and (23) by $dn/d\theta$, we obtain the following result.

PROPOSITION 3. *A larger marginal cost of committee members leads to a smaller committee size as well as to a smaller size of consensus. But, the former reduction is larger than the latter reduction.*

IV. HIERARCHIES AND POLYARCHIES

A. *Hierarchies*

We consider a hierarchy consisting of n bureaux, in which a higher bureau (or individual) evaluates only those projects which have been approved by the bureau below it, and the organisation finally accepts only those projects which are approved by the highest bureau. The project acceptance rule in such a hierarchy is, therefore, the same as that in a committee in which unanimity is required. The probability of projects' acceptance by a hierarchy can be obtained by substituting $k = n$ into (1), which yields $h_i^H = p_i^n$. (Note that the superscript H refers to a hierarchy, and the superscript P will refer to a polyarchy in the analysis below.)

A central difference between the two organisational forms, however, is that the number of evaluations that a project goes through in a hierarchy depends not only on n (as it does in a committee) but also on individuals' approval probabilities for different projects (because the latter determines which bureau will evaluate how many projects). Specifically, the expected number of evaluations for a project of type i is: $\sum_{j=1}^{n} p_i^{j-1} = (1-p_i^n)/(1-p_i)$. If a single evaluation costs e, then the expected evaluation cost per project is

$$E^H = e\left[\alpha\frac{1-p_1^n}{1-p_1}+(1-\alpha)\frac{1-p_2^n}{1-p_2}\right]. \tag{30}$$

It is easily verified that $\partial E^H/\partial p_i > 0$. This is what one would expect, because a larger approval probability of a bureau implies that all bureaux (except the lowest) must evaluate a larger number of projects.

Substitution of (30), and of $h_i^H = p_i^n$, into (2) shows that the maximisation of expected profit in such a hierarchy is equivalent to maximising

$$Y^H = p_1^n - \beta^H p_2^n, \tag{31}$$

where
$$\beta^H = (1-\alpha)\left(z_2-\frac{e}{1-p_2}\right)\Big/\alpha\left(z_1+\frac{e}{1-p_1}\right) > 0. \tag{32}$$

In (32), β^H can be viewed as a summary parameter representing the 'effective' portfolio quality; it is the relative loss in accepting a bad project (when the gain

[14] The assumption that I is strictly concave in (k,n) would obviously not always be satisfied. An analysis which partitions the parameter space into cases where this assumption does or does not hold is beyond the scope of the present paper.

from accepting a good project is 1), taking into account the cost of evaluating good and bad projects. As one would expect, β^H is smaller if the actual portfolio is better (that is, $\partial\beta^H/\partial\alpha < 0$, $\partial\beta^H/\partial z_1 < 0$, and $\partial\beta^H/\partial z_2 > 0$), and β^H is smaller if the cost per evaluation is larger, or if individuals' approval probabilities are larger (that is, $\partial\beta^H/\partial e < 0$, and $\partial\beta^H/\partial p_i < 0$).[15]

Treating n as a continuous variable, the first order condition of optimality of (31), with respect to n, yields the following expressions for the optimal number of levels in a hierarchy.[16]

$$n^H = \frac{\ln(\beta^H \ln p_2/\ln p_1)}{\ln(p_1/p_2)} \quad \text{and} \quad dn^H/d\beta^H > 0. \tag{33}$$

Recalling our interpretation of the parameter β^H, therefore, it is straightforward to ascertain the effects of evaluation cost or the portfolio quality on the size of a hierarchy.

PROPOSITION 4. *A better project portfolio, or a larger evaluation cost, implies a smaller number of levels in a hierarchy.*

The quality of managerial decision making (represented by p_i's) has a direct effect on the selection of projects, and also an indirect effect on evaluation costs. The corresponding implications on n^H can be separated, respectively, as

$$\frac{dn^H}{dp_i} = \frac{\partial n^H}{\partial p_i} + \frac{\partial n^H}{\partial \beta^H}\frac{\partial \beta^H}{\partial p_i}. \tag{34}$$

The assessment of the indirect effect is straightforward from (33) and from $\partial\beta^H/\partial p_i < 0$: a larger p_i lowers the optimal number of levels in a hierarchy.

The direct effect (through the selection of projects) is ambiguous however, and the reason for this ambiguity is parallel to the one which we noted earlier concerning the effect of managerial quality on the optimal consensus in a committee. Sufficient conditions can be obtained, however, under which the effect of p_i on n^H is predictable. For instance, we show in Appendix I(b) that: $\partial n^H/\partial p_1 > 0$ if $\beta^H < 1$; and $\partial n^H/\partial p_2 > 0$ if $\beta^H > 1$. That is: The direct effect of a larger Type-I (Type-II) managerial error is to lower (raise) the optimal number of levels in a hierarchy, if the effective quality of the portfolio is high (low).

B. Polyarchies

The hierarchical decision structure examined above requires complete unanimity. At the opposite extreme are decentralised polyarchical organisations in which little or no consensus is required. We consider here a polyarchy in which a project is undertaken if any one of the units accepts it.

[15] We assume that $\beta^H > 0$ which, from (32), implies that $z_2 > z_2 p_2 + e$. That is, the expected loss (including the evaluation cost) from evaluating a bad project for the first time is smaller than the loss if the same bad project were to be accepted without evalution. The evaluation of projects is clearly unnecessary if this condition is not met.

[16] The second order condition is satisfied at this optimum as well as for the optimal number of units in a polyarchy to be examined below.

The rule for project acceptance, therefore, is similar to that in a committee in which acceptance of a project requires only one member's approval. If n is the number of units in a polyarchy, then substitution of $k = 1$ into (1) yields $h_i^P = 1-(1-p_i)^n$.

The particular flow of projects on which we focus here is the one in which a project arrives randomly at one of the units which evaluates the project. The project is evaluated by another unit only if the first unit rejects the project, and this chain of evaluation continues until the project is approved by any one of the units, or until it is rejected by all units. The same project is not evaluated more than once by any one unit. The expected number of evaluations for a project of type i then is:

$$\sum_{j=1}^{n} (1-p_i)^{j-1} = [1-(1-p_i)^n]/p_i,$$

and
$$E^P = \{e\alpha[1-(1-p_1)^n]/p_1\}+\{e(1-\alpha)[1-(1-p_2)^n]/p_2\}. \tag{35}$$

In contrast to (30), now $\partial E^P/\partial p_i < 0$. This is intuitive because if one unit accepts more projects, then other units evaluate fewer projects.

Substituting the cost function (35), and $h_i^P = 1-(1-p_i)^n$, into (2), it follows that the expected profit maximisation is equivalent to maximising

$$Y^P = -(1-p_1)^n+\beta^P(1-p_2)^n, \tag{36}$$

where
$$\beta^P = (1-\alpha)\left(z_2+\frac{e}{p_2}\right)\Big/\alpha\left(z_1-\frac{e}{p_1}\right). \tag{37}$$

Once again, β^P summarises the effective portfolio quality, taking into account the evaluation cost; but now β^P is the relative gain in rejecting a bad project, when the loss in rejecting a good project is 1. A smaller β^P implies a higher effective quality of the portfolio, and if the actual portfolio quality is better, or if the cost per evaluation is smaller, then β^P is smaller. Parallel to (33), therefore, we obtain the optimal number of units in a polyarchy as

$$n^P = \frac{\ln[\beta^P \ln(1-p_2)/\ln(1-p_1)]}{\ln(1-p_1)/(1-p_2)}, \quad \text{and} \quad \partial n^P/\partial\beta^P < 0. \tag{38}$$

The interpretation of the above expression is analogous to that of (33); we therefore omit the details, and summarise the main results.[17]

PROPOSITION 5. *A better portfolio, or a smaller evaluation cost, implies a larger number of units in a polyarchy.*

V. COMPARISON OF COMMITTEES, HIERARCHIES AND POLYARCHIES

In this section, we compare the performances of the three organisational forms analysed earlier. Within our model, the key differences between a committee and a hierarchy or a polyarchy are that: (i) while the project acceptance rule

[17] Given the similarity between (33) and (38), it is also obvious that the direct and indirect effects of p_i on n^P are precisely opposite to those on n^H. Specifically, $\partial\beta^P/\partial p_i < 0$, and the indirect effect of a larger p_i is to raise n^P. An evaluation of direct effect yields: $\partial n^P/\partial p_1 < 0$ if $\beta^P < 1$, and $\partial n^P/\partial p_2 < 0$ if $\beta^P > 1$.

can be varied in a committee by altering k, it is fixed in a hierarchy (where $k = n$) or in a polyarchy (where $k = 1$); and (ii) while the evaluations are simultaneous in a committee, they follow specific sequential patterns in the other two organisations. Our interest, then, is in investigating circumstances under which the relative performance of one organisation might improve or worsen compared to another.

Recall that the superscripts H and P respectively denote the variables corresponding to a hierarchy and a polyarchy. Let the superscript C denote the variables corresponding to a committee. Then from (2), the relative performance (expected profit) of two different organisations, u and v, can be represented as

$$\Delta(u, v) = \sum_{i=1}^{2} \gamma_i(h_i^u - h_i^v) - (E^u - E^v), \tag{39}$$

where u and v are C, H or P; and $u \neq v$. We therefore ascertain how the above expression (defined for specific pairs of organisations) alters when exogenous parameters change. In the comparisons below, it is assumed that: (i) the size of alternative organisations, n, is the same (though, as we shall see, the envelope theorem permits us to obtain somewhat weaker results for those comparisons in which each of the organisations under consideration is of optimal size and, because of this, the size of alternative organisations can be different), (ii) alternative organisations face the same set of parameters (α, z_1, z_2, p_1, p_2), and (iii) the evaluation cost per evaluation, e, is fixed. Thus, $E^C = ne$, whereas (30) and (35) respectively represent E^H and E^P.

First, note that a committee entails a larger number of evaluations than either a polyarchy or a hierarchy. Therefore, $\partial E^C/\partial e = n$, $\partial E^H/\partial e < n$, and $\partial E^P/\partial e < n$. Expression (39) therefore yields

$$\partial\Delta(H, C)/\partial e > 0, \quad \text{and} \quad \partial\Delta(P, C)/\partial e > 0. \tag{40}$$

That is: *The relative performance of a hierarchy or a polyarchy improves, compared to that of a committee, if evaluations are costlier.*[18,19] This is what we would expect. However, as we shall comment below, the above conclusion may need to be modified if there are important delay costs.

Second, it is apparent from our earlier analysis that any project (good or bad) has a higher probability of being accepted in a polyarchy than in a

[18] To extend this result to comparisons among organisations with optimal sizes, let n^u denote the optimal size of organisation u. Using the envelope therorem, it can be verified then that:

$$\partial\Delta(H, C)/\partial e = n^C - \textstyle\sum_{j=1}^{n^H} [\alpha p_1^{j-1} + (1-\alpha) p_2^{j-1}],$$

and

$$\partial\Delta(P, C)/\partial e = n^C - \textstyle\sum_{j=1}^{n^P} [\alpha(1-p_1)^{j-1} + (1-\alpha)(1-p_2)^{j-1}].$$

The signs of these expressions are the same as those in (40), if $n^C \geqslant n^P$ and $n^C \geqslant n^H$. The same conclusions hold even if n^C is smaller, but not very much smaller, than n^P and n^H.

[19] These and other results in this section can also help in comparing the absolute performance of alternative organisations. It is apparent for instance that if the evaluation cost, e, is negligible then a committee cannot perform worse than a hierarchy or a polyarchy (because, in this case, the latter organisational forms are special cases of a committee), and the reverse would be the case if e is sufficiently large. A comparison between a hierarchy and a polyarchy, on the other hand, is less clear because the expected number of evaluations is larger for a hierarchy in some circumstances (for example, if p_1 and p_2 are larger than one-half) but smaller in others (for example, if p_1 and p_2 are smaller than one-half).

committee, and the probability of acceptance in a committee is in turn higher than that in a hierarchy. That is

$$h_i^P > h_i^C > h_i^H. \tag{41}$$

Since higher probabilities of acceptance are relatively more beneficial when the project portfolio is better, it follows that: *The relative performance of a polyarchy in comparison to a committee, or the relative performance of a committee in comparison to a hierarchy, improves if the portfolio quality is better.* This can be ascertained by noting from (39) and (41) that

$$\partial\Delta(P,C)/\partial\alpha > 0 \quad \text{and} \quad \partial\Delta(C,H)/\partial\alpha > 0, \tag{42}$$

and that the signs of the corresponding derivatives with respect to z_1 and z_2 have the same meaning.

Finally consider the effect of managerial quality on the relative performance of organisations. From (5) and (39), we obtain

$$\partial\Delta(C,H)/\partial p_i = \gamma_i\left[\binom{n}{k}kp_i^{k-1}(1-p_i)^{n-k} - np_i^{n-1}\right] + \partial E^H/\partial p_i, \tag{43}$$

and

$$\partial\Delta(P,H)/\partial p_i = \gamma_i n[(1-p_i)^{n-1} - p_i^{n-1}] + \partial E^H/\partial p_i - \partial E^P/\partial p_i, \tag{44}$$

where it will be recalled that $\gamma_1 > 0$ and $\gamma_2 < 0$. It is obvious that the signs of the expressions such as those above depend in part on what the current managerial abilities, p_i's, are. Within specific ranges of p_i's, however, results of the following type can be obtained. Note that $\binom{n}{k}k \geqslant n$ for an interior k (that is, for $n > k > 0$); and recall that $\partial E^H/\partial p_i > 0$, and $\partial E^P/\partial p_i < 0$. Expressions (43) and (44) therefore yield:

$$\partial\Delta(C,H)/\partial p_1 > 0 \quad \text{and} \quad \partial\Delta(P,H)/\partial p_1 > 0 \quad \text{if} \quad p_1 \leqslant \tfrac{1}{2}. \tag{45}$$

That is: *A smaller Type-I error improves the relative performance of either a committee or a polyarchy, compared to a hierarchy, provided an individual's probability of approving a good project is no more than one-half.*[20]

VI. EXTENSIONS

Our analysis has abstracted from several aspects of decision making. Some of these may easily be incorporated; others would require substantial modifications in the analysis and the results. In this section, we discuss two aspects: the determination of individuals' approval errors (including the strategic considerations which individuals might face), and the effects of delays in decision making.[21] At the end of this section, we briefly describe some of the

[20] If the sizes of alternative organisations are optimal then, using the envelope therorem once again, it can be verified that these results hold provided p_1 is sufficiently small (for instance, when $p_1 \to 0$).

[21] We have studied elsewhere several other aspects of organisational decision making. In Sah and Stiglitz (1986*b*), for instance, we have analysed the consequences of heterogeneity among managers (concerning their ability to choose projects as well as their successors) on the dynamic selection and performance of managers in centralised versus decentralised organisations. Sah (1987) has analysed the sensitivity of centralised systems' performance to the top managers' capabilities. See Sah and Stiglitz (1988) for an overview.

insights from our analysis which might be useful in the context of complex organisations.

A. Individuals' Errors and Strategic Considerations

As we indicated in the introductory section, the individuals' approval errors (that is, p_i's) are, in general, endogenous. Among the features on which these errors may depend are the organisation's structure and the nature of individuals' information processing. In addition, these errors may also depend on whether or not individuals take into account others' errors in choosing their own decision rules; that is, whether or not they attempt to accept or reject projects strategically to offset others' errors.

An analysis of various possible determinants of individuals' errors is beyond the scope of the present paper. What we do here, instead, is to show that our formulation (where p_i's are treated as exogenous parameters which are the same for all individuals) is consistent, under certain circumstances, with the case in which individuals act as strategic Bayesians in determining their rules for project approval. We begin with the simple case where individuals' decisions are based on binary information, and where the determination of individuals' decision is coordinated (equivalently, individuals are told what decision rule to use). We then examine a Nash equilibrium in which each committee member determines his own decision rule.

For each project, an individual receives a binary signal s which can be either s_1 or s_2. If G and B respectively denote a good project and a bad project, then $1 > p_1 = \Pr(s_1 \mid G) > p_2 = \Pr(s_1 \mid B) > 0$. Obviously, $1-p_1 = \Pr(s_2 \mid G) < 1-p_2 = \Pr(s_2 \mid B)$. If an individual's vote is to be informative then there are two possible decision rules: $d_1 \equiv$ {approve if $s = s_1$, disapprove otherwise}, or $d_2 \equiv$ {approve if $s = s_2$, disapprove otherwise}.[22] d^m denotes the decision rule for individual m, where $m = 1, \ldots, n$. $t_i^m(d^m)$ denotes the probability that a project of type i is approved by the individual m when his decision rule is d^m. Thus, $t_i^m(d_1) = p_i$, and $t_i^m(d_2) = 1-p_i$.

Let the vector $\mathbf{D}^m = (d^1, \ldots, d^{m-1}, d^{m+1}, \ldots, d^n)$ denote the decision rules of individuals other than the m-th individual. The probability that these $(n-1)$ individuals will cast $(k-1)$ affirmative votes for a project of type i is denoted by $\eta_i(\mathbf{D}^m)$. Clearly, this probability is positive regardless of what subsets of individuals follow d_1 or d_2. Let $\epsilon_i(\mathbf{D}^m)$ denote the probability that these $(n-1)$ individuals will cast k or more affirmative votes for a project of type i. The committee's profit, then, is represented by

$$I(d^m, \mathbf{D}^m) = \sum_{i=1}^{2} \gamma_i \left[t_i^m(d^m)\eta_i(\mathbf{D}^m) + \epsilon_i(\mathbf{D}^m)\right] - E^C. \tag{46}$$

The above expression yields a relationship which is quite useful for later analysis. For any given $\mathbf{D}^m$, the difference in the committee's profit when the

[22] Non-informative decision rules (such as approve regardless of the signal, or disapprove regardless of the signal) would obviously not be chosen in a coordinated determination of decision rules, because it would be more profitable to reduce the committee size instead and alter the rule accordingly.

decision rule of person m is d_1 versus d_2 is given by

$$I(d_1, \mathbf{D}^m) - I(d_2, \mathbf{D}^m) = \sum_{i=1}^{2} \gamma_i \eta_i(\mathbf{D}^m)(2p_i - 1). \tag{47}$$

Since, $\gamma_1 > 0$, $\gamma_2 < 0$, and $\eta_i(\mathbf{D}^m) > 0$, the expression (47) is positive for any $\mathbf{D}^m$, provided[23]

$$p_1 \geqslant \tfrac{1}{2} \geqslant p_2, \quad \text{with at least one strict inequality.} \tag{48}$$

Now, consider a coordinated determination of individuals' decision rules. It follows that if condition (48) is satisfied, then there is a unique optimum which entails a symmetric decision rule for all individuals ($d^m = d_1$, for all m) no matter what the committee size or the level of consensus might be.

The above observations apply also to the uncoordinated determination of decision rules in a Nash equilibrium. The reason is simple. d_1 is a dominant strategy because (47) is positive when (48) holds. Every individual will thus choose the decision rule d_1 in this case, regardless of what others' choices might be. ($d^m = d_1$, for all m), in other words, is the unique Nash equilibrium.[24]

We have therefore identified a set of simple conditions under which individuals' decision rules (and hence the nature of their errors) are not only symmetric across individuals but they also remain unchanged for committees of different sizes with different levels of consensus. Under plausible conditions, it is straightforward to show that an analogous conclusion holds in the context of hierarchies and polyarchies.[25]

B. Delays in Decision Making

An important difference between simultaneous evaluations (as in a committee) and sequential evaluations (as in a hierarchy or a polyarchy) from which the above analysis has abstracted is that sequential decisions might entail additional time delays. The economic cost of the time delays is not only a reduction (due to discounting) in the present value of projects undertaken, but also a possible reduction in the value of projects due to competitors' actions (e.g., in patent races).

Assume, for instance, that time delays impose a cost on every accepted project (good or bad) which is proportional to the length of the sequence of evaluations.[26] Then, it is apparent that a committee entails the smallest time delay cost, whereas a hierarchy entails the largest time delay cost. In between these two extremes is the time delay cost entailed in a polyarchy; also this cost increases less and less rapidly as the organisation's size increases.

[23] This is clearly a sufficient condition. Much weaker conditions will yield the required result.

[24] In fact, this result holds even if the beliefs of individuals (concerning the parameters of the economy) differ within a range. If (α^m, z_1^m, z_2^m, p_1^m, p_2^m) denote the beliefs of individual m, then it is obvious that this result does not require homogeneity of beliefs; it only requires that for each m: $1 > p_1^m \geqslant \frac{1}{2} \geqslant p_2^m > 0$, with at least one strict inequality; and that α^m, z_1^m and z_2^m are positive.

[25] This can be seen by noting that, in general, there is an additional term: $E^u(d^m = d_2, \mathbf{D}^m) - E^u(d^m = d_1, \mathbf{D}^m)$, in the right hand side of (47). This term is zero for a committee but non-zero in a hierarchy or a polyarchy because the evaluation costs in these organisations are affected by whether a particular individual chooses decision rule d_1 or d_2. But so long as this additional term is positive, or it is negative but negligible compared to the right hand side of (47), our conclusions remain unchanged.

[26] This specification would obviously be modified if there were delays arising from communication among committee members (see footnote 1).

Next, note that our models of committees and polyarchies also point out that the resources spent on project evaluation in a polyarchy are likely to be smaller compared to a committee, when the organisational sizes are large. This is because $E^C = en$, whereas $E^P < e[\alpha/p_1 + (1-\alpha)/p_2]$ from (35). These two aspects of costs taken together then suggest that polyarchies might have an advantage, compared to committees or hierarchies, in those circumstances where large organisational sizes are desirable.

C. Complex Organisations

Our analysis of previous sections can, in principle, be extended to complex organisations. One could examine, for instance, a committee consisting of subcommittees (rather than individuals) where constituents of subcommittees are even smaller subcommittees, and so on. Similarly, one can examine hierarchies or polyarchies in which the constituent units are combinations of hierarchies, polyarchies and committees.[27] We have focused on analysing simpler organisational forms because this analysis can serve as a basis for studying more complex organisations, and also because many organisations which one encounters do indeed have simpler forms.

The fact that we observe simple organisations points to an apparent puzzle because, given a set of parameters representing the organisation's environment, there might always be *specific* complex organisations which can do better than simpler organisations. This is simply because complexity of organisations provides an additional dimension of choice. To see an example, consider a committee of fixed size n, where k is the level of consensus. If n is not very small, then it is always possible to construct a 'complex' committee consisting of two subcommittees of sizes n^1 and $n-n^1$ with respective levels of consensus k^1 and k^2, such that a project is accepted only if approved by both subcommittees. Since the number of individuals remains unchanged, and since the simple committee is a special case of the complex committee (with $n^1 = k^1 = 0$, and $k^2 = k$), one would expect that for any given set of parameters, a particular choice of k^1 and n^1 could always do better (or, at least, no worse) than the simple committee.

A positive question which arises then is, why do we observe simple organisations at all. One possible explanation, which appears worth investigating in future research, is that simple organisations might be robust to changes in the environment; that is, a particular complex organisation might be better than a simple one under one environment (for example, for a particular project portfolio to be evaluated), but it might be worse under another environment which the organisation also expects to face.

It is perhaps also important to point out that though organisational complexity can in certain circumstances (such as those indicated above) ameliorate the economic consequences of human fallibility, it can not remove

[27] Even though a firm is typically viewed as a hierarchy, it is often the case that there are committees within a large firm which report to various members of the hierarchy who, in turn, sometimes act alone and at other times act as members of various committees. Similarly, an economy with many firms can be viewed, at this crude level of approximation, as a polyarchy of hierarchies.

these consequences. In fact, it is neither feasible in an economic sense, nor desirable, to seek error free (first-best) organisational systems. This viewpoint can be illustrated by the following economic reinterpretation of the celebrated Moore-Shannon theorem concerning the design of perfect relay networks from imperfect components.[28] If we choose an appropriate (k, n) committee, treat this committee as a constituent element of another (k, n) committee, and go on repeating this process, then, in the limit, the system becomes error free. From an economic viewpoint, however, the error-free decisions become feasible because this theorem requires infinite components, and not because of the particular decision structure it advocates. In fact, many different and simpler organisational forms will yield the same result. For instance, a simple committee, where $p_1 > k/n > p_2$, will yield (from the law of large numbers) perfect decisions if the number of observations were to be increased without bound.[29]

It becomes obvious, therefore, that the reason why we do not see perfect decision making, nor should we expect to see it in economic settings, is that there are costs associated with evaluation, and perfection is economically infeasible. That is also why we have emphasised evaluation costs in our analysis; an essential implication of these costs is that not only are all organisations fallible in their decisions (like the individuals of which they are composed) but also (even taking, as we have done here, the individuals' errors as exogenous) the economic consequences of organisational errors are endogenously determined by how the decision making is organised.

VII. CONCLUSIONS

This paper has been concerned with exploring some of the organisational consequences of three facts:

(1) Information gathering, transmission and processing is costly.

(2) All human decision making is fallible.

(3) The errors made by an organisation depend on the structure of the organisation. In particular, the structure of the organisation and its decision rules determine how individuals' errors within an organisation are aggregated.

The economic trade-offs in decision making which our analysis has emphasised are: (i) The trade-off between the errors of rejecting good projects (or ideas) versus the errors of accepting bad projects; and (ii) the trade-off between the resource costs of extra evaluation of projects versus the gains from an improved selection. The organisational forms we have analysed are committees (where evaluation of projects is simultaneous, and those projects are accepted which are approved by at least the minimum consensus), hierarchies (which accepts only those projects which all levels of the hierarchy approve, and where a higher level bureau evaluates only the projects approved by the lower bureaus), and polyarchies (where the evaluation of projects is also

[28] See Harrison (1965, pp. 255–62) for a description of the theorem.
[29] We thank a referee for suggesting this interpretation.

sequential, but a project is undertaken by the organisation as soon as it is approved by any one member). The last two organisations respectively capture certain features of centralised versus decentralised systems.

We have been concerned both with the optimal design of each of these organisational forms, and with the comparison of performance across organisational forms. For committees, our paper provides a framework for ascertaining the optimal committee size as well as the optimal degree of consensus. We have shown, for instance, that the optimal increase in the level of consensus, in response to an increased committee size, is larger (smaller) than the marginal majority rule – the rule where the level of consensus is increased by one whenever the committee size increases by two – if an individual is less (more) likely to reject a good project than to accept a bad project. This result does not depend on the quality of the project portfolio; that is, on what the proportions of good and bad projects are in the set of projects from which the organisation has to choose, and on how good the good projects are compared to how bad the bad projects are. If, in addition, the project portfolio is of bad (good) quality then the optimal consensus is larger (smaller) than the majority rule.

For hierarchies, our analysis provides a framework for determining the optimal number of levels, and for analysing how the optimal number is affected by changes in the underlying economic parameters. Also, the differences in the decision structures of hierarchies and polyarchies turn out to be quite important in determining the nature of trade-offs within these two organisational forms. We show, for instance, that a better project portfolio reduces the optimal number of levels in a hierarchy but increases the optimal number of units in a polyarchy.

We have also attempted to analyse how the relative performances of these three organisational forms change under different sets of parameters of the economy. Such an analysis can help in ascertaining circumstances under which one might be more likely to observe one particular organisational form compared to another. We show, for example, that: (i) if the project portfolio is better, then the relative performance of a polyarchy improves compared to a committee, and the relative performance of a committee improves compared to a hierarchy, and (ii) if evaluation costs are larger, then the relative performance of either a hierarchy or a polyarchy improves compared to a committee.

An analysis of the kind developed in the present paper might also provide insights on why there is such a widespread sense of powerlessness in modern societies, even among individuals who occupy seemingly important decision making positions. One interpretation of this phenomenon is that an individual feels powerless if the collective decision is contrary to his judgment; for example if a project or an idea is accepted (rejected) when this individual disapproves (approves) of the project. The analysis which we have developed suggests that when the nature of human fallibility is recognised, and when alternative ways and costs of ameliorating the consequences of this fallibility are recognised, then this form of powerlessness is an essential counterpart of the economic organisation of decision making: the more important the decision, the larger the number of individuals whose approval is required and, in this sense, the less important the role of any one individual.

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APPENDIX I

(*a*) Since, $q = p_1/p_2$, and $r = (1-p_1)/(1-p_2)$, it follows that

$$rq \gtrless 1, \quad \text{if} \quad p_2 \lessgtr 1-p_1. \tag{49}$$

Next, rewrite (9) as

$$(n-k)\ln rq + (2k-n)\ln q \geqslant \ln\beta \geqslant (n-k+1)\ln rq + (2k-n-2)\ln q. \tag{50}$$

Now, suppose (12) is not true: that is, $k \geqslant n/2+1$, when $\beta < 1$, and $p_2 \leqslant 1-p_1$. Then, the right hand side of (50) is nonnegative. This is because $rq \geqslant 1$ from (49), $(n-k+1) > 0$ at an interior optimum, $q = p_1/p_2 > 1$, and $(2k-n-2) \geqslant 0$. On the other hand, $\ln\beta < 0$. Expression (50) is thus contradicted. An analogous argument shows that (50) is contradicted if (13) is not true.

(*b*) Expression (33) yields

$$\partial n^H/\partial p_1 = g_1(-1-\ln\beta^H w + w), \tag{51}$$

and

$$\partial n^H/\partial p_2 = g_2(-1+\ln\beta^H w + 1/w), \tag{52}$$

where g_1 and g_2 are positive numbers, and $w = \ln p_2/\ln p_1 > 0$. Next, the strict concavity of ln(.) in its argument implies

$$\beta^H w - 1 \geqslant \ln\beta^H w \geqslant 1 - 1/\beta^H w. \tag{53}$$

Substitution of the left part of inequality (53) into (51) yields: $\partial n^H/\partial p_1 > 0$, if $1 > \beta^H$. Similarly, substitution of the right part of inequality (53) into (52) yields: $\partial n^H/\partial p_2 > 0$, if $\beta^H > 1$.

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