Tavole applicative

Corso di Controllo dei Robot

E. Puglisi   A. Ryals

January 15, 2019

Ingegneria robotica e dell'automazione
Università di Pisa

Table of contents

Delta robot

Delta robot

The Delta robot is a 3-DOF parallel kinematic machine developed by Reymond Clavel[1] in 1991. It mainly consists of three actuated kinematic chains linked at a common moving platform. Each chain is a serial connection of a revolute actuator, a rear-arm and a forearm (composed of two parallel rods forming a parallelogram). The rear-arms and the forearms are linked through ball-and-socket passive joints. The parallelogram structure of the forearms ensures that the moving platform stays always parallel to the fixed base. Figure 1 shows a schematic view of the Delta robot with its main elements highlighted.

[1] Reymond Clavel. *Conception d'un robot parallele rapide à 4 degres de liberté*. 1991.

Delta robot - Schematic view

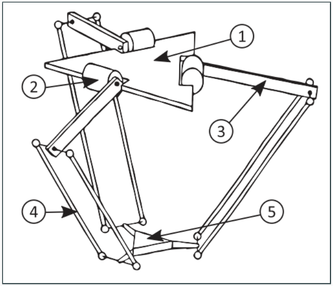

Figure 1: Schematic view of Delta robot

1. Fixed base-plate
2. Actuator
3. Rear-arm
4. Forearm
5. Moving platform

We consider a model with a ternary symmetric configuration with three kinematic chains disposed with a period of 120°

Delta robot - Parameters

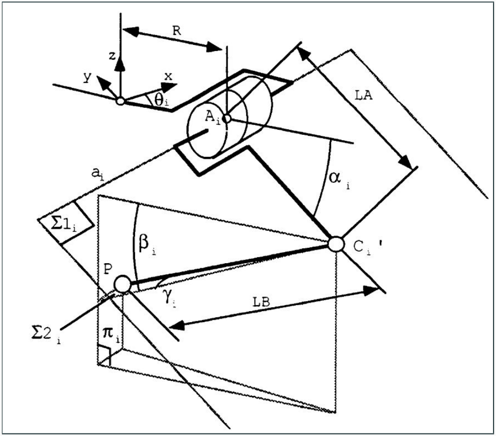

Figure 2: Delta robot length parameters and characteristic angles

Delta robot - Parameters

Parameter	Description	Value
l_A	Rear-arm length	$0.2m$
m_A	Rear-arm mass	$0.1Kg$
R	Base platform dimension	$0.126m$
l_B	Forearm length	$0.4m$
m_B	Forearm mass	$0.045Kg$
m_c	Elbow mass	$0.018Kg$
m_n	Moving platform mass	$0.1Kg$
I_{bi}	Rear-arm inertia	$Kg \times m^2$

Table 1: Delta robot geometric and dynamic parameters

Delta robot - Parameters

Analytical studies on the working volume of the Delta robot[2] demonstrated that:

- A ratio $r = R/l_A < 0.63$ gives the most regular shape for the surface of the lower part of the working volume.
- If $r > 0.0484$ and $b = l_A/l_B > 1.75$ there is no singularity occurrence within the robot working volume.

Thus the parameters shown in table 1 have been chosen for the Delta model used in this project.

[2] L Rey and Reymond Clavel. "The Delta Parallel Robot". In: *Parallel Kinematic Machines. Advanced Manufacturing. Springer, London* (1999).

Delta robot - Reference system and state variables

The position of the End-effector

$$(x, y, z)^T$$

is described in a reference frame fixed to the base plate, as shown in figure 2.

The joint variables α_i have been selected as state-variables to describe the robot dynamic:

$$q = (\alpha_1, \alpha_2, \alpha_3)^T$$

Delta robot - Direct kinematic

Since the moving platform is only translating we can study the model in figure 2 without loss of generality.
In this model the moving platform is reduced to an ideal point with a translation of the three kinematic chains.

Delta robot - Direct kinematic

Direct kinematic is found following the method presented by Clavel in 1991.
Taking in mind the Delta robot representation of figure 2 one can simply find that C_i coordinates are given by the intersection of three circles of radius L_A belonging to the plane π_i and the sphere centred in P having radius L_B. Those conditions give a three equations system that can be solved to find the coordinates of the end-effector.

Delta robot - Direct kinematic

Coordinates of the point C_i in the base frame:

$$C_i = \begin{pmatrix} (R + L_A cos\alpha_i) cos\theta_i \\ (R + L_A cos\alpha_i) sin\theta_i \\ -L_A sin\alpha_i \end{pmatrix} \tag{1}$$

Equation of the sphere centred in P:

$$\left((R + L_A cos\alpha_i)\, cos\theta_i - x^2\right) + ((R + L_A cos\alpha_i)\, sin\theta_i - y)^2 + (L_A sin\alpha_i + z)^2 = L_B^2 \tag{2}$$

The system has two possible solutions. The one with negative z coordinate that belongs to the Delta robot workspace is selected.

Delta robot - Inverse kinematic

The inverse kinematic model let calculate the joint angles q_i as functions of the position of the end effector. The model here presented have been developed by Codourey[3] and has the advantage of removing the points of singularity contained in the model previously introduced by Clavel.
The rationale is still the intersection of a sphere and three circles but the computation is made for each angle in a frame centred in the centre of the $i-th$ joint and rotated with respect to the base frame of an angle θ_i.

[3] Alain Codourey. "Contribution à la commande des robots rapides et précis application au robot delta à entraînement direct". In: (1991), p. 188. DOI: 10.5075/epfl-thesis-922. URL: http://infoscience.epfl.ch/record/31400.

Delta robot - Dynamic model assumptions

- Ideal joints are considered.
- The rotational inertia of the forearm is neglected.
- The mass of each forearm is split up into two point-masses located at both ends of the forearm.

Delta robot - Dynamic model

We express the dynamic of the delta robot in the classical matrix formulation:

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) \tag{3}$$

Where:

$$M(q) = \left(I_b + m_{nt} J^T J\right), \quad C(q,\dot{q}) = \left(J^T m_n t J\right), \quad G(q) = -\Gamma_{Gb} - \Gamma_{Gn} \tag{4}$$

- I_b is the inertia matrix of the arms in joint space.
- m_{nt} is the totalling mass acting on the travelling plate.
- J is the Jacobian matrix.
- Γ_{Gn} is the gravity force acting on the moving platform.
- Γ_{Gb} is the gravity force acting on the rear-arms.

Delta robot - Working volume

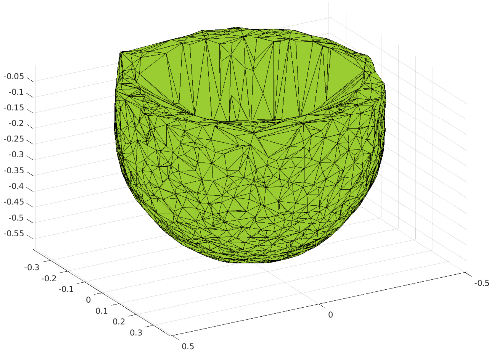

Figure 3: A convex hull of the workspace of the Delta robot

Delta robot - Working volume

In figure 3 a covex hull of the workspace of the Delta robot is reported.
The surface has been generated as an $\alpha - \textit{shape}$[4] with $r_\alpha = 0.2$.
The geometric figure gives an analytical instrument to validate a sound reference trajectory generation for the Delta kinematic.

[4] H. Edelsbrunner, D. Kirkpatrick, and R. Seidel. "On the Shape of a Set of Points in the Plane". In: *IEEE Trans. Inf. Theor.* 29.4 (Sept. 2006), pp. 551–559. ISSN: 0018-9448. DOI: 10.1109/TIT.1983.1056714. URL: http://dx.doi.org/10.1109/TIT.1983.1056714.

PD with gravity compensation

Control equation:

$$\tau_{PD} = K_P e + K_D \dot{e} + G(q) \tag{5}$$

with

$$K_P = diag(1500, 1500, 1500)$$

and

$$K_D = diag(60, 60, 60)$$

Computed torque

Backstepping

Adaptive backstepping

Ball and plate

Ball and plate

Ball and plate

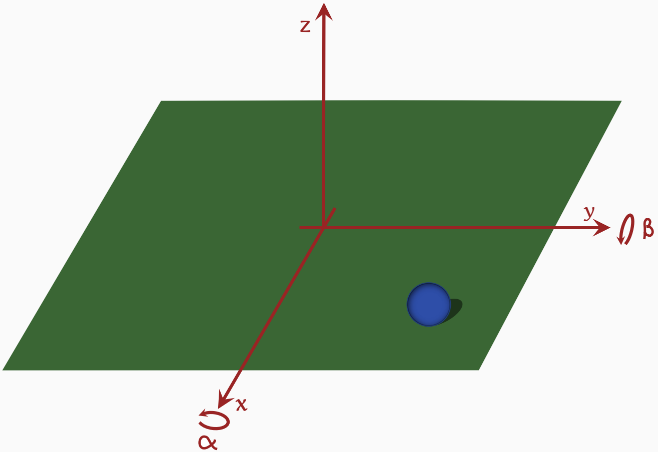

Figure 4: Coordinate frame of the ball and plate system

Ball and plate - Parameters

Parameter	Description	Value
m	Mass of the ball	$0.0109\,Kg$
r	Radius of the ball	$0.01\,m$
I_b	Ball inertia	$4.3563e^{-7}\,Kg \times m^2$
l_p	Plate side	$0.6\,m$
I_p	Plate inertia	$0.175\,Kg \times m^2$

Table 2: Ball and plate geometric and dynamic parameters

Ball and plate - Dynamic model

The general form of Euler-Lagrange for dynamic equations is used to describe the system:

$$\frac{d}{dt}\frac{\delta T}{\delta q_i} - \frac{\delta T}{\delta q_i} + \frac{\delta V}{\delta q_i} = Q_i \tag{6}$$

Where T is the kinetic energy, V is the potential energy, Q_i is the i-th generalized force and q_i id the i-th generalized coordinate. As generalized force we consider two torques acting on the plate ($Q_\alpha = \tau_\alpha, Q_\beta = \tau_\beta$). As generalized coordinates we select two ball position coordinates $[x, y]$ on the frame fixed to the plate and two plate inclination $[\alpha, \beta]$.

Ball and plate - Dynamic model

Kinetic energy of the ball:

$$T_b = \frac{1}{2}mv^2 + \frac{1}{2}I_b\omega^2 = \frac{1}{2}\left(m + \frac{I_b}{r^2}\right)\left(\dot{x}^2 + \dot{y}^2\right) \tag{7}$$

Kinetic energy of the plate:

$$T_p = \frac{1}{2}\left(I_b + I_p\right)\left(\dot{\alpha} + \dot{\beta}\right) + \frac{1}{2}m\left(\dot{\alpha}x + \dot{\beta}y\right)^2 \tag{8}$$

Potential energy:

$$V = mgh = mg(x\, sin\alpha + y\, sin\beta) \tag{9}$$

Ball and plate - Dynamic model

After some derivations we find the following non-linear system of equations:

$$\begin{aligned}
\left(m + \frac{I_b}{r^2}\right)\ddot{x} - m\left(\dot{\alpha}\dot{\beta}y + \dot{\alpha}^2 x\right) + mg\, sin\alpha = 0 \\
\left(m + \frac{I_b}{r^2}\right)\ddot{y} - m\left(\dot{\alpha}\dot{\beta}x + \dot{\beta}^2 y\right) + mg\, sin\beta = 0 \\
\left(I_p + I_b + mx^2\right)\ddot{\alpha} + m\left(\ddot{\beta}xy + \dot{\beta}\left(\dot{x}y + x\dot{y}\right) + 2\dot{\alpha}\dot{x}x\right) + mgx\, cos\alpha = \tau_\alpha \\
\left(I_p + I_b + my^2\right)\ddot{\beta} + m\left(\ddot{\alpha}xy + \dot{\alpha}\left(\dot{x}y + x\dot{y}\right) + 2\dot{\beta}\dot{y}y\right) + mgy\, cos\beta = \tau_\beta
\end{aligned} \tag{10}$$

Ball and plate - Dynamic model

We express the dynamic in matrix form:

$$
M(q) = \begin{bmatrix} \left(m + \frac{I_b}{r^2}\right) & 0 & 0 & 0 \\ 0 & \left(m + \frac{I_b}{r^2}\right) & 0 & 0 \\ 0 & 0 & \left(I_b + I_p + mx^2\right) & mxy \\ 0 & 0 & mxy & \left(I_b + I_p + my^2\right) \end{bmatrix}
$$

$$
C(q, \dot{q}) = \begin{bmatrix} 0 & 0 & -\dot{\alpha}x & -\dot{\alpha}y \\ 0 & 0 & -\dot{\beta}x & -\dot{\beta}y \\ 2\dot{\alpha}x & 0 & 0 & (\dot{x}y + x\dot{y}) \\ 0 & 2\dot{\beta}y & (\dot{x}y + x\dot{y}) & 0 \end{bmatrix}
$$

$$
G(q) = \begin{bmatrix} mg\ sin\alpha \\ mg\ sin\beta \\ mgx\ cos\alpha \\ mgx\ cos\beta \end{bmatrix} \tag{11}
$$