Tavole applicative

Corso di Controllo dei Robot

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Delta robot

Delta robot

The Delta robot is a 3-DOF parallel kinematic machine developed by Reymond Clavel[1] in 1991. It mainly consists of three actuated kinematic chains linked at a common moving platform. Each chain is a serial connection of a revolute actuator, a rear-arm and a forearm (composed of two parallel rods forming a parallelogram). The rear-arms and the forearms are linked through ball-and-socket passive joints. The parallelogram structure of the forearms ensures that the moving platform stays always parallel to the fixed base. Figure 1 shows a schematic view of the Delta robot with its main elements highlighted.

[1] **Clavel1991**.

Delta robot - Schematic view

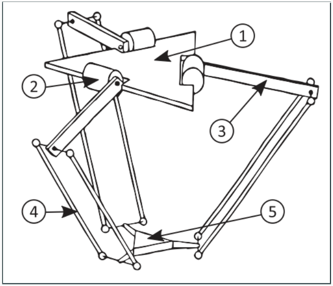

Figure 1: Schematic view of Delta robot

1. Fixed base-plate
2. Actuator
3. Rear-arm
4. Forearm
5. Moving platform

We consider a model with a ternary symmetric configuration with three kinematic chains disposed with a period of 120°

Delta robot - Parameters

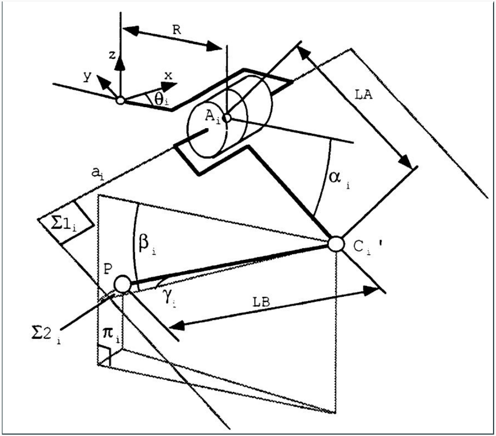

Figure 2: Delta robot length parameters and characteristic angles

Delta robot - Parameters

Parameter	Description	Value
l_A	Rear-arm length	$0.2m$
m_A	Rear-arm mass	$0.1Kg$
R	Base platform dimension	$0.126m$
l_B	Forearm length	$0.4m$
m_B	Forearm mass	$0.045Kg$
m_c	Elbow mass	$0.018Kg$
m_n	Moving platform mass	$0.1Kg$
I_{bi}	Rear-arm inertia	$Kg \times m^2$

Table 1: Delta robot geometric and dynamic parameters

Delta robot - Parameters

Analytical studies on the working volume of the Delta robot[2] demonstrated that:

- A ratio $r = R/l_A < 0.63$ gives the most regular shape for the surface of the lower part of the working volume.
- If $r > 0.0484$ and $b = l_A/l_B > 1.75$ there is no singularity occurrence within the robot working volume.

Thus the parameters shown in table 1 have been chosen for the Delta model used in this project.

[2] **Rey**.

Delta robot - Direct kinematic

Since the moving platform is only translating we can study the model in figure 2 without loss of generality.
In this model the moving platform is reduced to an ideal point with a translation of the three kinematic chains.

Delta robot - Direct kinematic

Scelta del sistema di riferimento, variabili.

Delta robot - Direct kinematic

Direct kinematic is found following the method presented by Clavel in 1991.

Taking in mind the Delta robot representation of figure 2 one can simply find that C_i coordinates are given by the intersection of three circles of radius L_A belonging to the plane π_i and the sphere centred in P having radius L_B. Those conditions give a three equations system that can be solved to find the coordinates of the end-effector.

Delta robot - Direct kinematic

Coordinates of the point C_i in the base frame:

$$C_i = \begin{pmatrix} (R + L_A cos\alpha_i)cos\theta_i \\ (R + L_A cos\alpha_i)sin\theta_i \\ -L_A sin\alpha_i \end{pmatrix} \quad (1)$$

Equation of the sphere centred in P:

$$\left((R + L_A cos\alpha_i)\, cos\theta_i - x^2\right) + ((R + L_A cos\alpha_i)\, sin\theta_i - y)^2 + (L_A sin\alpha_i + z)^2 = L_B^2 \quad (2)$$

The system has two possible solutions. The one with negative z coordinate that belongs to the Delta robot workspace is selected.

Delta robot - Inverse kinematic

The inverse kinematic model let calculate the joint angles q_i as functions of the position of the end effector. The model here presented have been developed by Codourey[3] and has the advantage of removing the points of singularity contained in the model previously introduced by Clavel.
The rationale is still the intersection of a sphere and three circles but the computation is made for each angle in a frame centred in the centre of the $i-th$ joint and rotated with respect to the base frame of an angle θ_i.

[3] **Codourey_thesis**.

Delta robot - Dynamic model assumptions

- Ideal joints are considered.
- The rotational inertia of the forearm is neglected.
- The mass of each forearm is split up into two point-masses located at both ends of the forearm.

Delta robot - Dynamic model

$$\begin{pmatrix} p - R_b \left(\overline{R} - L_A \cos\left(\overline{q_i}\right)\right) \\ p \\ p + L_A \, R_b \sin\left(\overline{q_i}\right) \end{pmatrix} \tag{3}$$

Delta robot - Dynamic model

Delta robot - Working volume

PD with gravity compensation

Control equation:

$$\tau_{PD} = K_P e + K_D \dot{e} + G(q) \tag{4}$$

with

$$K_P = diag(1500, 1500, 1500)$$

and

$$K_D = diag(60, 60, 60)$$

Computed torque

Backstepping

Adaptive backstepping

Ball and plate

Ball and plate

Ball and plate

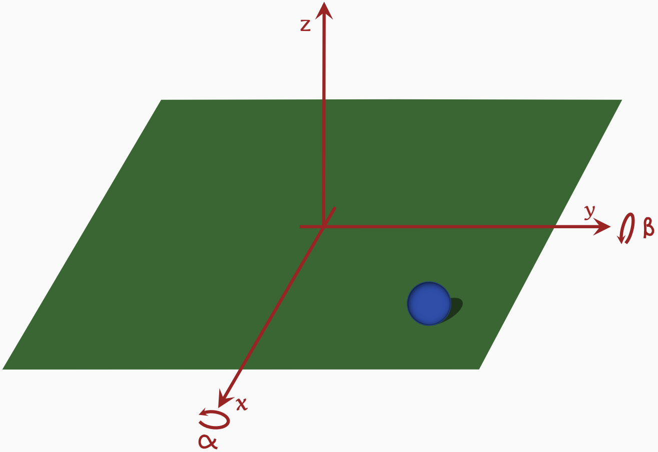

Figure 3: Coordinate frame of the ball and plate system

Ball and plate - Parameters

Parameter	Description	Value
m	Mass of the ball	$0.0109\,Kg$
r	Radius of the ball	$0.01\,m$
I_b	Ball inertia	$4.3563e^{-7}\,Kg \times m^2$
l_p	Plate side	$0.6\,m$
I_p	Plate inertia	$0.175\,Kg \times m^2$

Table 2: Ball and plate geometric and dynamic parameters

Ball and plate - Dynamic model

The general form of Euler-Lagrange for dynamic equations is used to describe the system:

$$\frac{d}{dt}\frac{\delta T}{\delta q_i} - \frac{\delta T}{\delta q_i} + \frac{\delta V}{\delta q_i} = Q_i \tag{5}$$

Where T is the kinetic energy, V is the potential energy, Q_i is the i-th generalized force and q_i id the i-th generalized coordinate. As generalized force we consider two torques acting on the plate ($Q_\alpha = \tau_\alpha, Q_\beta = \tau_\beta$). As generalized coordinates we select two ball position coordinates $[x, y]$ on the frame fixed to the plate and two plate inclination $[\alpha, \beta]$.

Ball and plate - Dynamic model

Kinetic energy of the ball:

$$T_b = \frac{1}{2}mv^2 + \frac{1}{2}I_b\omega^2 = \frac{1}{2}\left(m + \frac{I_b}{r^2}\right)\left(\dot{x}^2 + \dot{y}^2\right) \tag{6}$$

Kinetic energy of the plate:

$$T_p = \frac{1}{2}\left(I_b + I_p\right)\left(\dot{\alpha} + \dot{\beta}\right) + \frac{1}{2}m\left(\dot{\alpha}x + \dot{\beta}y\right)^2 \tag{7}$$

Potential energy:

$$V = mgh = mg(x\,sin\alpha + y\,sin\beta) \tag{8}$$

Ball and plate - Dynamic model

After some derivations we find the following non-linear system of equations:

$$\begin{aligned}
\left(m + \frac{I_b}{r^2}\right)\ddot{x} - m\left(\dot{\alpha}\dot{\beta}y + \dot{\alpha}^2 x\right) + mg\,sin\alpha = 0 \\
\left(m + \frac{I_b}{r^2}\right)\ddot{y} - m\left(\dot{\alpha}\dot{\beta}x + \dot{\beta}^2 y\right) + mg\,sin\beta = 0 \\
\left(I_p + I_b + mx^2\right)\ddot{\alpha} + m\left(\ddot{\beta}xy + \dot{\beta}\left(\dot{x}y + x\dot{y}\right) + 2\dot{\alpha}\dot{x}x\right) + mgx\,cos\alpha = \tau_\alpha \\
\left(I_p + I_b + my^2\right)\ddot{\beta} + m\left(\ddot{\alpha}xy + \dot{\alpha}\left(\dot{x}y + x\dot{y}\right) + 2\dot{\beta}\dot{y}y\right) + mgy\,cos\beta = \tau_\beta
\end{aligned} \tag{9}$$

Ball and plate - Dynamic model

We express the dynamic in matrix form:

$$M(q) = \begin{bmatrix} \left(m + \frac{I_b}{r^2}\right) & 0 & 0 & 0 \\ 0 & \left(m + \frac{I_b}{r^2}\right) & 0 & 0 \\ 0 & 0 & \left(I_b + I_p + mx^2\right) & mxy \\ 0 & 0 & mxy & \left(I_b + I_p + my^2\right) \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} 0 & 0 & -\dot{\alpha}x & -\dot{\alpha}y \\ 0 & 0 & -\dot{\beta}x & -\dot{\beta}y \\ 2\dot{\alpha}x & 0 & 0 & (\dot{x}y + x\dot{y}) \\ 0 & 2\dot{\beta}y & (\dot{x}y + x\dot{y}) & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} mg\,sin\alpha \\ mg\,sin\beta \\ mgx\,cos\alpha \\ mgx\,cos\beta \end{bmatrix} \tag{10}$$