Tavole applicative

Corso di Controllo dei Robot

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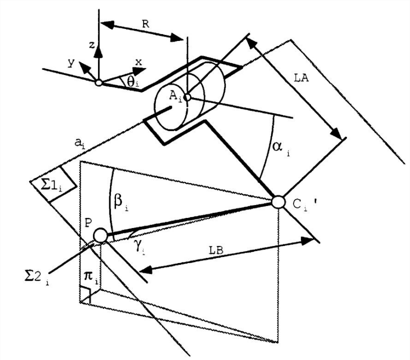

Delta robot - Direct kinematic

C_i coordinates are given by the intersection of three circles of radius L_A belonging to the plane π_i and the sphere centered in P having radius L_B.

$$C_i = \begin{pmatrix} (R + L_A cos\alpha_i) cos\theta_i \\ (R + L_A cos\alpha_i) sin\theta_i \\ -L_A sin\alpha_i \end{pmatrix}$$

Equation of the sphere centered in P:

$$(x - p_x)^2 + (y - p_y)^2 + (z - p_z)^2 = L_B \tag{1}$$

$$((R+L_A cos\alpha_i) cos\theta_i - x^2) + ((R+L_A cos\alpha_i) sin\theta_i - y)^2 + (L_A sin\alpha_i + z)^2 = L_B^2 \tag{2}$$

Delta robot - Direct kinematic

$$D_i = R^2 + 2\,\cos\,q_i\,R\,l_A + {l_A}^2 - {l_B}^2 \tag{3}$$

$$E_i = \cos\theta_i\,\left(2\,R + 2\,l_A\,\cos\,q_i\right) \tag{4}$$

$$F_i = \sin\theta_i\,\left(2\,R + 2\,l_A\,\cos\,q_i\right) \tag{5}$$

$$G_i = -2\,l_A\,\sin\left(q_i\right) \tag{6}$$

$$H_1 = E_1\,G_2 - E_2\,G_1 - E_1\,G_3 + E_3\,G_1 + E_2\,G_3 - E_3\,G_2 \tag{7}$$

$$H_2 = E_2\,F_1 - E_1\,F_2 + E_1\,F_3 - E_3\,F_1 - E_2\,F_3 + E_3\,G_2 \tag{8}$$

$$H_3 = D_1\,E_2 - D_1\,E_1 - D_1\,E_3 + D_3\,E_1 + D_2\,E_3 - D_3\,E_2 \tag{9}$$

$$H_4 = D_2\,F_1 - D_1\,F_2 + D_1\,F_3 - D_3\,F_1 - D_2\,F_3 + D_3\,F_2 \tag{10}$$

$$H_5 = F_2\,G_1 - F_1\,G_2 + F_1\,G_3 - F_3\,G_1 - F_2\,G_3 + F_3\,G_2 \tag{11}$$

Delta robot - Direct kinematic

$$L = \frac{{H_1}^2 + {H_5}^2}{{H_2}^2} + 1 \tag{12}$$

$$M = G_1 - \frac{E_1\, H_5 + F_1\, H_1}{H_2} + \frac{2\, H_1\, H_3 + 2\, H_4\, H_5}{{H_2}^2} \tag{13}$$

$$N = D_1 - \frac{E_1\, H_4 + F_1\, H_3}{H_2} + \frac{2\, {H_3}^2 + 2\, {H_4}^2}{{H_2}^2} \tag{14}$$

Delta robot - Direct kinematic

End effector coordinates computation:

$$z_{1,2} = -\frac{M \pm \sqrt{M^2 - 4\,L\,N}}{2\,L} \tag{15}$$

Among the two solutions we pick the one with lower height that belongs to the Delta robot workspace.

$$x = \frac{H_4}{H_2} - \frac{H_5\,\left(M - \sqrt{M^2 - 4\,L\,N}\right)}{2\,H_2\,L} \tag{16}$$

$$y = \frac{H_3}{H_2} - \frac{H_1\,\left(M - \sqrt{M^2 - 4\,L\,N}\right)}{2\,H_2\,L} \tag{17}$$

Delta robot - Inverse kinematic

$$A = {L_A}^2 - {L_B}^2 - R^2 + {x_i}^2 + {y_i}^2 + {z_i}^2 \tag{18}$$

$$B = 2x_i - 2R \tag{19}$$

$$z = \frac{A - Bx}{2\, z_i} \tag{20}$$

where:

$$x = \frac{b + \sqrt{b^2 - a\, c}}{a} \tag{21}$$

with:

$$a = \left(2\, R - 2\, x_i\right)^2 + 4\, {z_i}^2 \tag{22}$$

$$b = 4\, R\, {z_i}^2 + A\, B \tag{23}$$

$$c = A^2 - 4\, {L_A}^2\, {z_i}^2 + 4\, R^2\, {z_i}^2 \tag{24}$$

$$q_i = -\mathrm{asin}\left(\frac{z}{\ }\right) \tag{25}$$