

# Tavole applicative

Corso di Controllo dei Robot

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# Delta robot

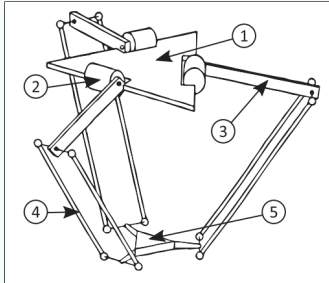
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The Delta robot is a 3-DOF parallel kinematic machine developed by Reymond Clavel<sup>1</sup> in 1991. It mainly consists of three actuated kinematic chains linked at a common moving platform. Each chain is a serial connection of a revolute actuator, a rear-arm and a forearm (composed of two parallel rods forming a parallelogram). The rear-arms and the forearms are linked through ball-and-socket passive joints. The parallelogram structure of the forearms ensures that the moving platform stays always parallel to the fixed base. Figure 1 shows a schematic view of the Delta robot with its main elements highlighted.

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<sup>1</sup>Reymond Clavel. *Conception d'un robot parallele rapide à 4 degres de liberté*. 1991.

# Delta robot - Schematic view



1. Fixed base-plate
2. Actuator
3. Rear-arm
4. Forearm
5. Moving platform

**Figure 1:** Schematic view of Delta robot

We consider a model with a ternary symmetric configuration with three kinematic chains disposed with a period of  $120^\circ$

# Delta robot - Parameters

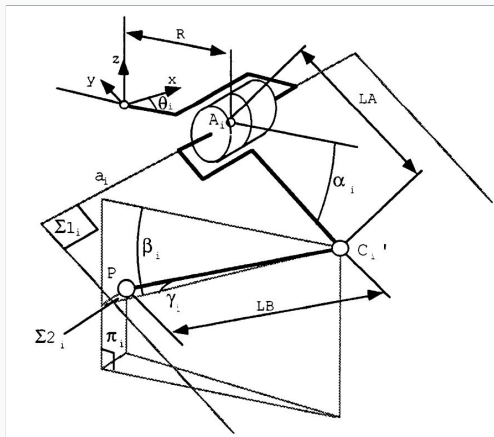


Figure 2: Delta robot length parameters and characteristic angles

# Delta robot - Parameters

Parameter	Description	Value
$l_A$	Rear-arm length	$0.2m$
$m_A$	Rear-arm mass	$0.1Kg$
$R$	Base platform dimension	$0.126m$
$l_B$	Forearm length	$0.4m$
$m_B$	Forearm mass	$0.045Kg$
$m_c$	Elbow mass	$0.018Kg$
$m_n$	Moving platform mass	$0.1Kg$
$I_{bi}$	Rear-arm inertia	$Kg \times m^2$

**Table 1:** Delta robot geometric and dynamic parameters

Analytical studies on the working volume of the Delta robot<sup>2</sup> demonstrated that:

- A ratio  $r = R/l_A < 0.63$  gives the most regular shape for the surface of the lower part of the working volume.
- If  $r > 0.0484$  and  $b = l_A/l_B > 1.75$  there is no singularity occurrence within the robot working volume.

Thus the parameters shown in table 1 have been chosen for the Delta model used in this project.

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<sup>2</sup>L Rey and Reymond Clavel. "The Delta Parallel Robot". In: *Parallel Kinematic Machines. Advanced Manufacturing*. Springer, London (1999).



Since the moving platform is only translating we can study the model in figure 2 without loss of generality.

In this model the moving platform is reduced to an ideal point with a translation of the three kinematic chains.

Scelta del sistema di riferimento, variabili.

Direct kinematic is found following the method presented by Clavel in 1991.

Taking in mind the Delta robot representation of figure 2 one can simply find that  $C_i$  coordinates are given by the intersection of three circles of radius  $L_A$  belonging to the plane  $\pi_i$  and the sphere centred in  $P$  having radius  $L_B$ . Those conditions give a three equations system that can be solved to find the coordinates of the end-effector.

# Delta robot - Direct kinematic

Coordinates of the point  $C_i$  in the base frame:

$$C_i = \begin{pmatrix} (R + L_A \cos \alpha_i) \cos \theta_i \\ (R + L_A \cos \alpha_i) \sin \theta_i \\ -L_A \sin \alpha_i \end{pmatrix} \quad (1)$$

Equation of the sphere centred in P:

$$\begin{aligned} & \left( (R + L_A \cos \alpha_i) \cos \theta_i - x \right)^2 + \left( (R + L_A \cos \alpha_i) \sin \theta_i - y \right)^2 \\ & + (L_A \sin \alpha_i + z)^2 = L_B^2 \end{aligned} \quad (2)$$

The system has two possible solutions. The one with negative  $z$  coordinate that belongs to the Delta robot workspace is selected.

The inverse kinematic model let calculate the joint angles  $q_i$  as functions of the position of the end effector. The model here presented have been developed by Codourey<sup>3</sup> and has the advantage of removing the points of singularity contained in the model previously introduced by Clavel.

The rationale is still the intersection of a sphere and three circles but the computation is made for each angle in a frame centred in the centre of the  $i - th$  joint and rotated with respect to the base frame of an angle  $\theta_i$ .

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<sup>3</sup>Alain Codourey. "Contribution à la commande des robots rapides et précis application au robot delta à entraînement direct". In: (1991), p. 188. DOI: 10.5075/epfl-thesis-922. URL: <http://infoscience.epfl.ch/record/31400>.

## Delta robot - Dynamic model assumptions

- Ideal joints are considered.
- The rotational inertia of the forearm is neglected.
- The mass of each forearm is split up into two point-masses located at both ends of the forearm.

$$\begin{pmatrix} p - R_b (\bar{R} - L_A \cos(\bar{q}_i)) \\ p \\ p + L_A R_b \sin(\bar{q}_i) \end{pmatrix} \quad (3)$$





## Delta robot - Working volume

## PD with gravity compensation

Control equation:

$$\tau_{PD} = K_P e + K_D \dot{e} + G(q) \quad (4)$$

with

$$K_P = \text{diag}(1500, 1500, 1500)$$

and

$$K_D = \text{diag}(60, 60, 60)$$





# Adaptive backstepping

## Ball and plate

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# Ball and plate

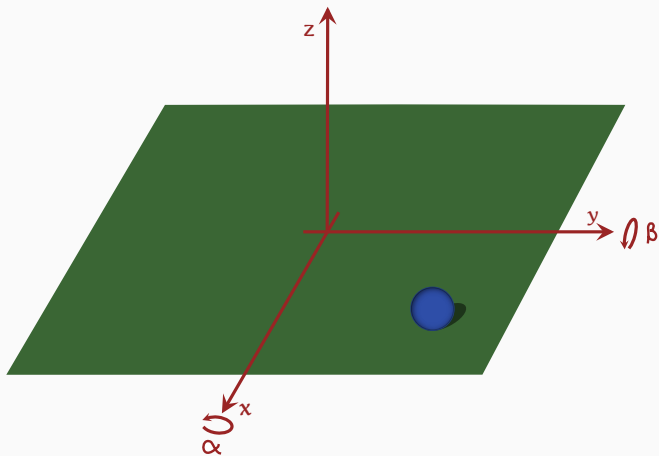


Figure 3: Coordinate frame of the ball and plate system



# Ball and plate - Parameters

Parameter	Description	Value
$m$	Mass of the ball	$0.0109 \text{ Kg}$
$r$	Radius of the ball	$0.01 \text{ m}$
$I_b$	Ball inertia	$4.3563e^{-7} \text{ Kg} \times \text{m}^2$
$l_p$	Plate side	$0.6 \text{ m}$
$I_p$	Plate inertia	$0.175 \text{ Kg} \times \text{m}^2$

**Table 2:** Ball and plate geometric and dynamic parameters

## Ball and plate - Dynamic model

The general form of Euler-Lagrange for dynamic equations is used to describe the system:

$$\frac{d}{dt} \frac{\delta T}{\delta \dot{q}_i} - \frac{\delta T}{\delta q_i} + \frac{\delta V}{\delta q_i} = Q_i \quad (5)$$

Where  $T$  is the kinetic energy,  $V$  is the potential energy,  $Q_i$  is the  $i$ -th generalized force and  $q_i$  is the  $i$ -th generalized coordinate. As generalized force we consider two torques acting on the plate ( $Q_\alpha = \tau_\alpha$ ,  $Q_\beta = \tau_\beta$ ). As generalized coordinates we select two ball position coordinates  $[x, y]$  on the frame fixed to the plate and two plate inclination  $[\alpha, \beta]$ .

## Ball and plate - Dynamic model

Kinetic energy of the ball:

$$T_b = \frac{1}{2}mv^2 + \frac{1}{2}I_b\omega^2 = \frac{1}{2} \left( m + \frac{I_b}{r^2} \right) (\dot{x}^2 + \dot{y}^2) \quad (6)$$

Kinetic energy of the plate:

$$T_p = \frac{1}{2} (I_b + I_p) (\dot{\alpha} + \dot{\beta})^2 + \frac{1}{2}m (\dot{\alpha}x + \dot{\beta}y)^2 \quad (7)$$

Potential energy:

$$V = mgh = mg(x \sin\alpha + y \sin\beta) \quad (8)$$

## Ball and plate - Dynamic model

After some derivations we find the following non-linear system:

$$\begin{aligned} \left(m + \frac{I_b}{r^2}\right) \ddot{x} - m \left(\dot{\alpha}\dot{\beta}y + \dot{\alpha}^2x\right) + mg \sin\alpha &= 0 \\ \left(m + \frac{I_b}{r^2}\right) \ddot{y} - m \left(\dot{\alpha}\dot{\beta}x + \dot{\beta}^2y\right) + mg \sin\beta &= 0 \end{aligned} \quad (9)$$

$$(I_p + I_b + mx^2) \ddot{\alpha} + m \left(\ddot{\beta}xy + \dot{\beta}(\dot{x}y + x\dot{y}) + 2\dot{\alpha}\dot{x}x\right) + mgx \cos\alpha = \tau_\alpha$$

$$(I_p + I_b + my^2) \ddot{\beta} + m \left(\ddot{\alpha}xy + \dot{\alpha}(\dot{x}y + x\dot{y}) + 2\dot{\beta}\dot{y}y\right) + mgy \cos\beta = \tau_\beta$$