Tavole applicative

Corso di Controllo dei Robot

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Delta robot

Delta robot

The Delta robot is a 3-DOF parallel kinematic machine developed by Reymond Clavel[1] in 1991. It mainly consists of three actuated kinematic chains linked at a common moving platform. Each chain is a serial connection of a revolute actuator, a rear-arm and a forearm (composed of two parallel rods forming a parallelogram). The rear-arms and the forearms are linked through ball-and-socket passive joints. The parallelogram structure of the forearms ensures that the moving platform stays always parallel to the fixed base. Figure 1 shows a schematic view of the Delta robot with its main elements highlighted.

[1] Reymond Clavel. *Conception d'un robot parallele rapide à 4 degres de liberté*. 1991.

Delta robot - Schematic view

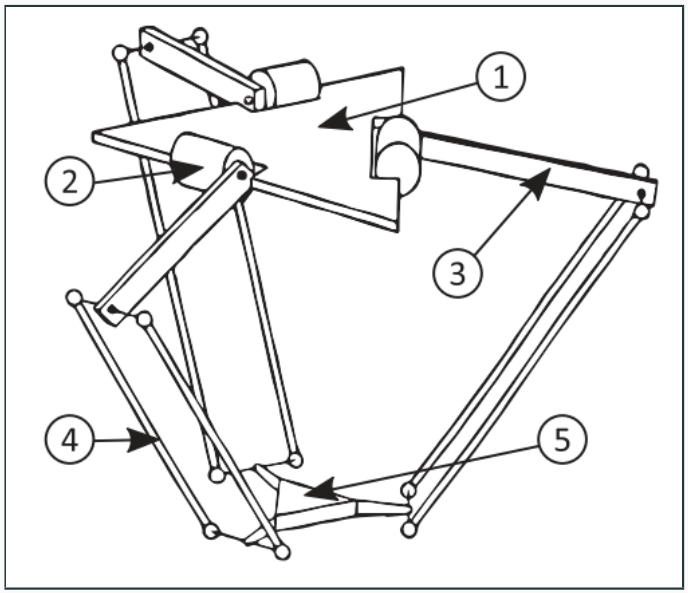


Figure 1: Schematic view of Delta robot

1. Fixed base-plate
2. Actuator
3. Rear-arm
4. Forearm
5. Moving platform

Delta robot - Parameters

Analytical studies on the working volume of the Delta robot[2] demonstrated that:

- A ratio $r = R/l_A < 0.63$ gives the most regular shape for the surface of the lower part of the working volume, with R as the distance between the center of the base plate and the rotation axis of the actuator and l_A as the length of the rear-arm.
- If $r > 0.0484$ and $b = l_A/l_B > 1.75$ there is no singularity occurrance within the robot working volume.

Thus the parameters shown in table 1 have been chosen for the Delta model used in this project.

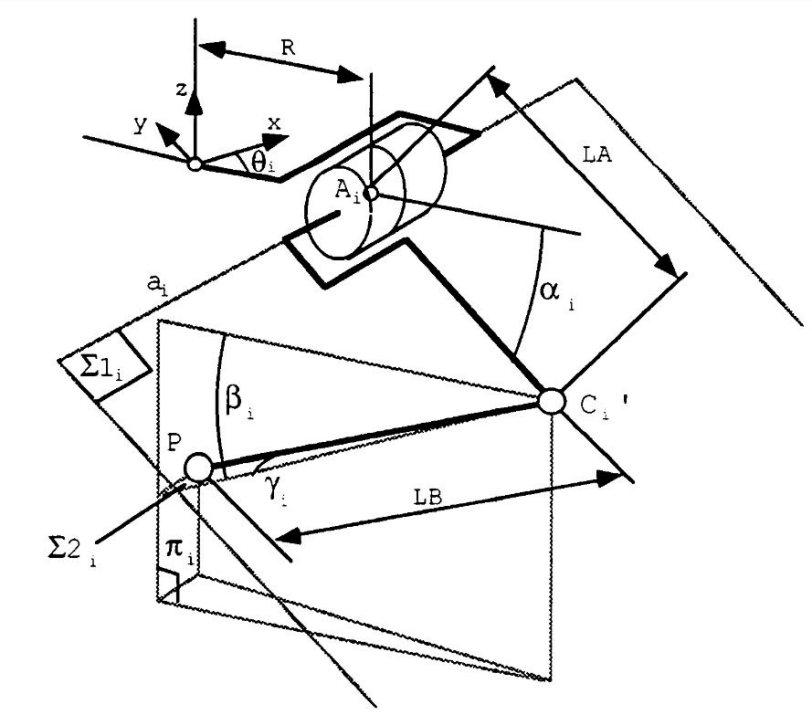
[2] L Rey and Reymond Clavel. "The Delta Parallel Robot". In: *Parallel Kinematic Machines. Advanced Manufacturing. Springer, London* (1999).

Delta robot - Parameters

Parameter	Description	Value
l_A	Rear-arm length	$0.2m$
m_A	Rear-arm mass	$0.1Kg$
R	Base platform parameter	$0.126m$
l_B	Forearm length	$0.4m$
m_B	Forearm mass	$0.045Kg$
m_c	Elbow mass	$0.018Kg$
m_n	Moving platform mass	$0.1Kg$
I_{bi}	Rear-arm inertia	$Kg \times m^2$

Table 1: Delta robot geometric and dynamic parameters

Delta robot - Direct kinematic



Delta robot - Direct kinematic

C_i coordinates are given by the intersection of three circles of radius L_A belonging to the plane π_i and the sphere centered in P having radius L_B.

$$C_i = \begin{pmatrix} (R + L_A cos\alpha_i)cos\theta_i \\ (R + L_A cos\alpha_i)sin\theta_i \\ -L_A sin\alpha_i \end{pmatrix}$$

Equation of the sphere centered in P:

$$(x - p_x)^2 + (y - p_y)^2 + (z - p_z)^2 = L_B \tag{1}$$

$$((R+L_A cos\alpha_i)cos\theta_i - x^2) + ((R+L_A cos\alpha_i)sin\theta_i - y)^2 + (L_A sin\alpha_i + z)^2 = L_B^2 \tag{2}$$

Delta robot - Direct kinematic

$$D_i = R^2 + 2 \cos q_i R l_A + {l_A}^2 - {l_B}^2 \tag{3}$$

$$E_i = \cos\theta_i \, (2R + 2 l_A \cos q_i) \tag{4}$$

$$F_i = \sin\theta_i \, (2R + 2 l_A \cos q_i) \tag{5}$$

$$G_i = -2 l_A \sin(q_i) \tag{6}$$

$$H_1 = E_1 G_2 - E_2 G_1 - E_1 G_3 + E_3 G_1 + E_2 G_3 - E_3 G_2 \tag{7}$$

$$H_2 = E_2 F_1 - E_1 F_2 + E_1 F_3 - E_3 F_1 - E_2 F_3 + E_3 G_2 \tag{8}$$

$$H_3 = D_1 E_2 - D_1 E_1 - D_1 E_3 + D_3 E_1 + D_2 E_3 - D_3 E_2 \tag{9}$$

$$H_4 = D_2 F_1 - D_1 F_2 + D_1 F_3 - D_3 F_1 - D_2 F_3 + D_3 F_2 \tag{10}$$

$$H_5 = F_2 G_1 - F_1 G_2 + F_1 G_3 - F_3 G_1 - F_2 G_3 + F_3 G_2 \tag{11}$$

Delta robot - Direct kinematic

$$L = \frac{{H_1}^2 + {H_5}^2}{{H_2}^2} + 1 \tag{12}$$

$$M = G_1 - \frac{E_1\, H_5 + F_1\, H_1}{H_2} + \frac{2\, H_1\, H_3 + 2\, H_4\, H_5}{{H_2}^2} \tag{13}$$

$$N = D_1 - \frac{E_1\, H_4 + F_1\, H_3}{H_2} + \frac{2\, {H_3}^2 + 2\, {H_4}^2}{{H_2}^2} \tag{14}$$

Delta robot - Direct kinematic

End effector coordinates computation:

$$z_{1,2} = -\frac{M \pm \sqrt{M^2 - 4\,L\,N}}{2\,L} \tag{15}$$

Among the two solutions we pick the one with lower height that belongs to the Delta robot workspace.

$$x = \frac{H_4}{H_2} - \frac{H_5\,\left(M - \sqrt{M^2 - 4\,L\,N}\right)}{2\,H_2\,L} \tag{16}$$

$$y = \frac{H_3}{H_2} - \frac{H_1\,\left(M - \sqrt{M^2 - 4\,L\,N}\right)}{2\,H_2\,L} \tag{17}$$

Delta robot - Inverse kinematic

$$A = {L_A}^2 - {L_B}^2 - R^2 + {x_i}^2 + {y_i}^2 + {z_i}^2 \tag{18}$$

$$B = 2x_i - 2R \tag{19}$$

$$z = \frac{A - Bx}{2\,z_i} \tag{20}$$

where:

$$x = \frac{b + \sqrt{b^2 - a\,c}}{a} \tag{21}$$

with:

$$a = (2\,R - 2\,x_i)^2 + 4\,{z_i}^2 \tag{22}$$

$$b = 4\,R\,{z_i}^2 + A\,B \tag{23}$$

$$c = A^2 - 4\,{L_A}^2\,{z_i}^2 + 4\,R^2\,{z_i}^2 \tag{24}$$

$$q_i = -\mathrm{asin}\left(\frac{z}{L_A}\right) \tag{25}$$

Delta robot - Jacobian computation

$$\begin{pmatrix} p - R_b \left(\overline{R} - L_A \cos\left(\overline{q_i}\right)\right) \\ p \\ p + L_A R_b \sin\left(\overline{q_i}\right) \end{pmatrix} \tag{26}$$