Convolutional Neural Networks

Giacomo Boracchi,
DEIB, Politecnico di Milano
October, 10th, 2022

giacomo.boracchi@polimi.it
https://boracchi.faculty.polimi.it/

Is Image Classification a Challenging Problem?

Yes, it is...

Is Image Classification a Challenging Problem?

What does prevent us from

- Stacking multiple hidden layers
- Training the network with a huge amount of images

to successfully classify images?

Would this work?

Is it a challenging problem?

First challenge: dimensionality

Images are very high-dimensional image data

CIFAR-10 dataset

The CIFAR-10 dataset contains 60000 images:

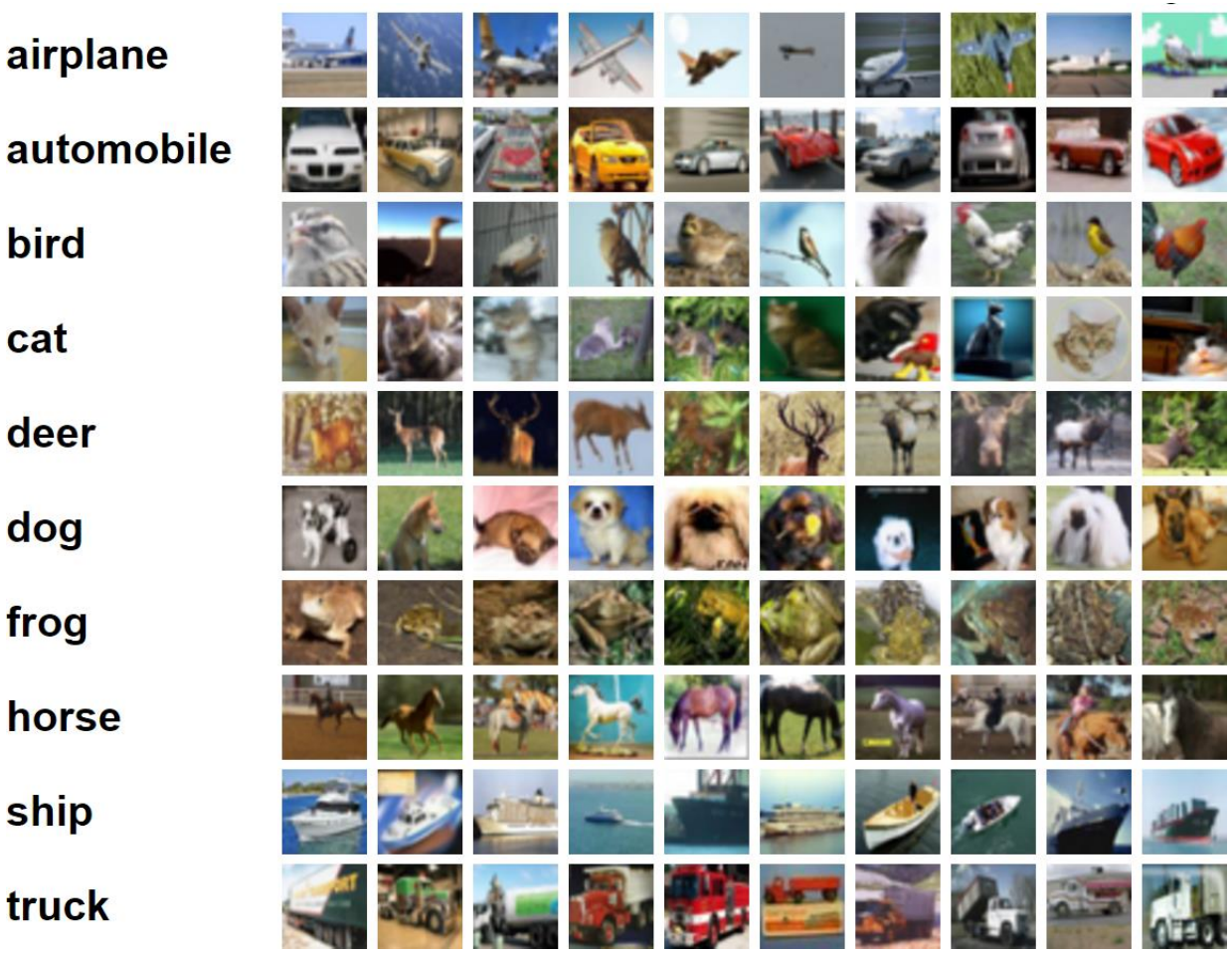
Each image is 32x32 RGB

Images are in 10 classes

6000 images per class

Extremely small images, but high-dimensional:

$$d = 32 \times 32 \times 3 = 3072$$



Learning Multiple Layers of Features from Tiny Images, Alex Krizhevsky, 2009.


This resolution is by far smaller than what we are used to
$d = 3072$

Former standard repository for ML research

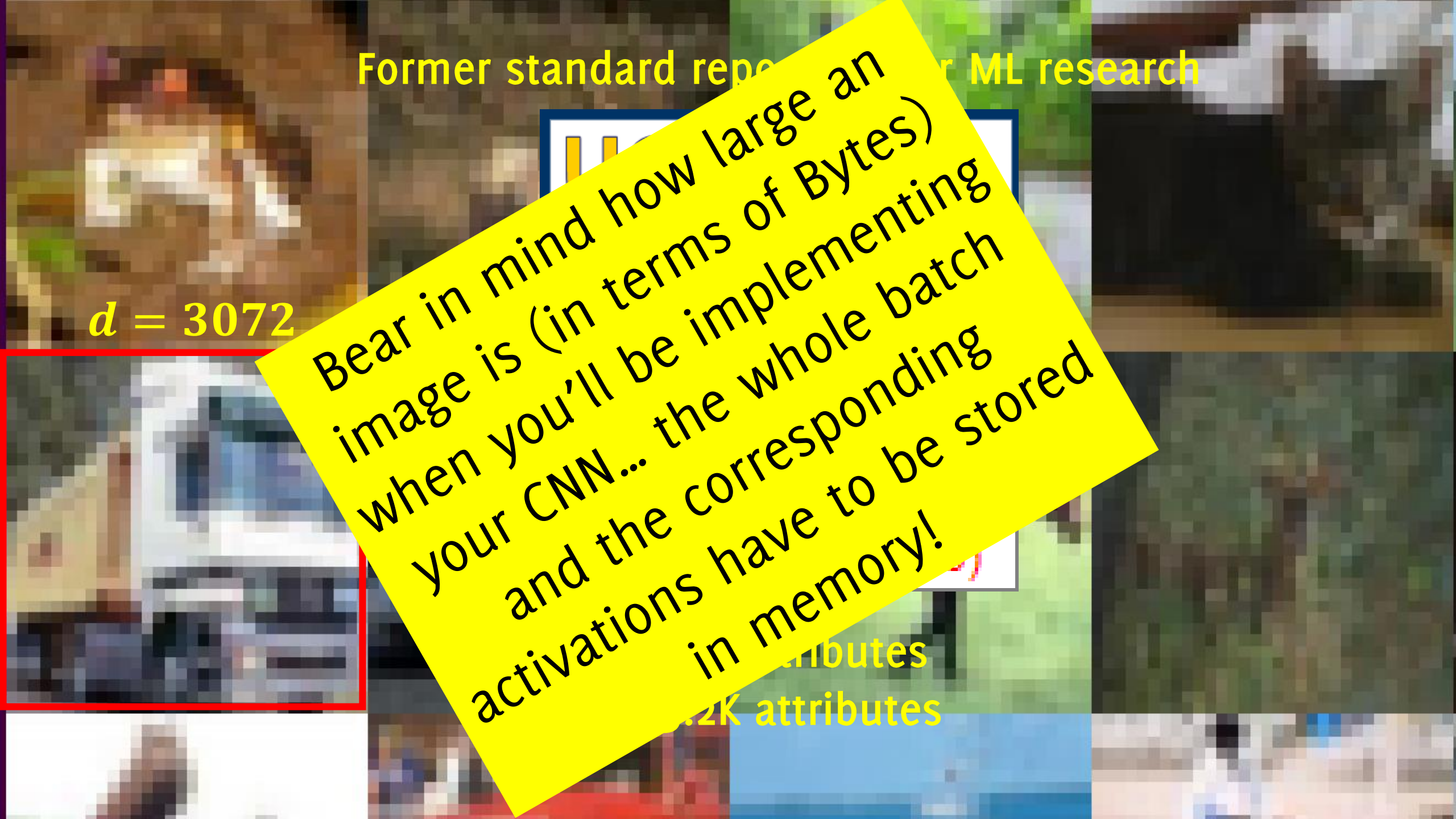
UCI Machine Learning Repository
Center for Machine Learning and Intelligent Systems

$d = 3072$

 # Attributes

Less than 10 (116)
10 to 100 (218)
Greater than 100 (86)

- 88% < 500 attributes
- 92% < 3.2K attributes


Former standard repe r ML research
$d = 3072$
Bear in mind how large an image is (in terms of Bytes) when you'll be implementing your CNN… the whole batch and the corresponding activations have to be stored in memory!
ributes
.2K attributes

Second challenge: label ambiguity

A label might not uniquely identify the image

Second challenge: label ambiguity

Man?

Beer?

Dinner?

Restaurant?

Sausages?

….

Third challenge: transformations

There are many transformations that change the image dramatically, while not its label

Changes in the Illumination Conditions






Giacomo Boracchi

Deformations

Copyright Christine Matthews

© Copyright Patrick Roper

View Point Change

… and many others

Occlusion

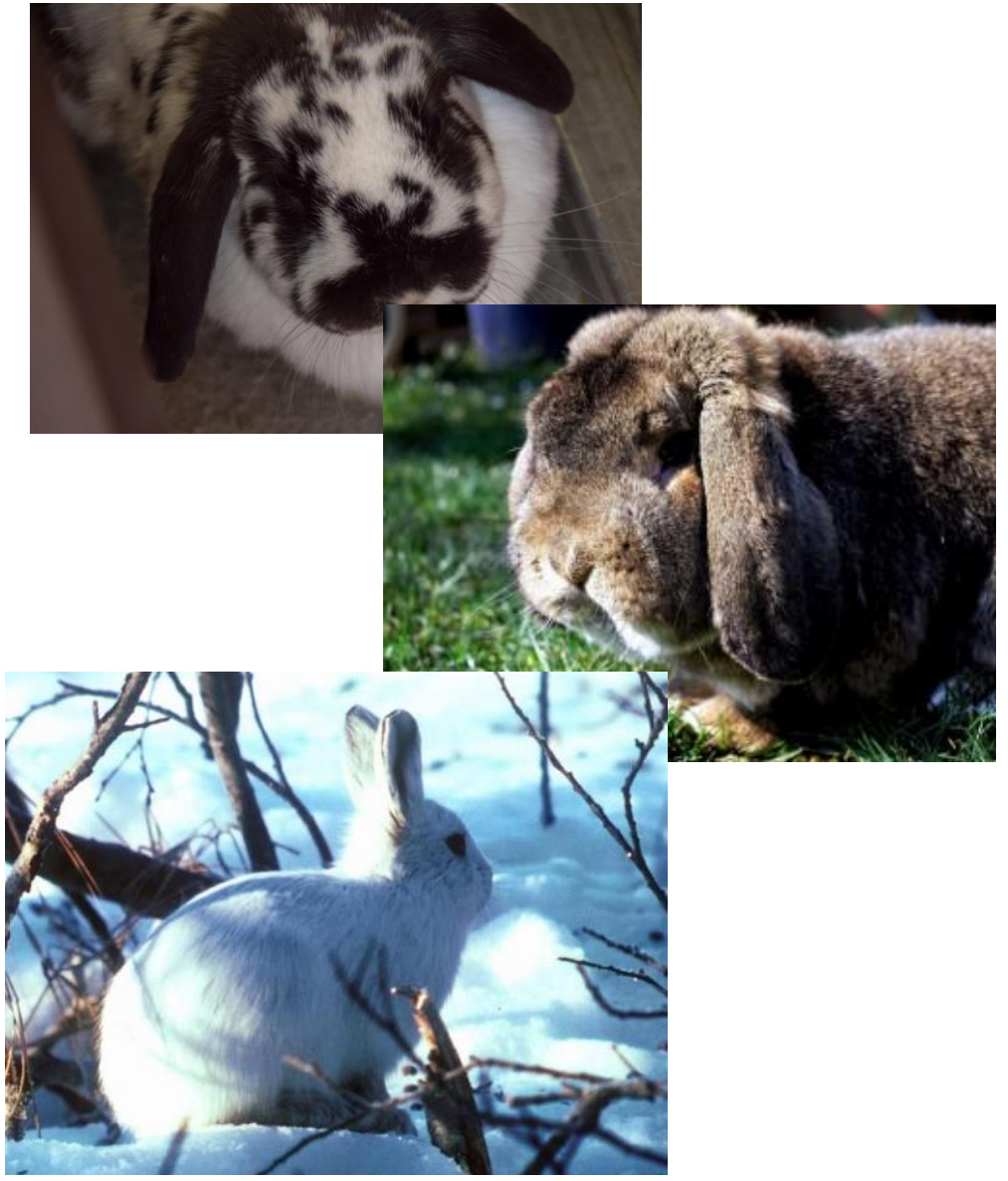
Background clutter

Scale variation

Fourth challenge: inter-class variability

Images in the same class might be dramatically different

Inter-class variability



Fifth problem: perceptual similarity

Perceptual similarity in images is not related to pixel-similarity

Nearest Neighborhood Classifiers for Images

Assign to a each test image, **the label of the closest image** in the training set

$$\hat{y}_j = y_{j^*}, \qquad \text{being} \quad j^* = \underset{i=1\ldots N}{\operatorname{argmin}}\, d(\boldsymbol{x}_j, \boldsymbol{x}_i)$$

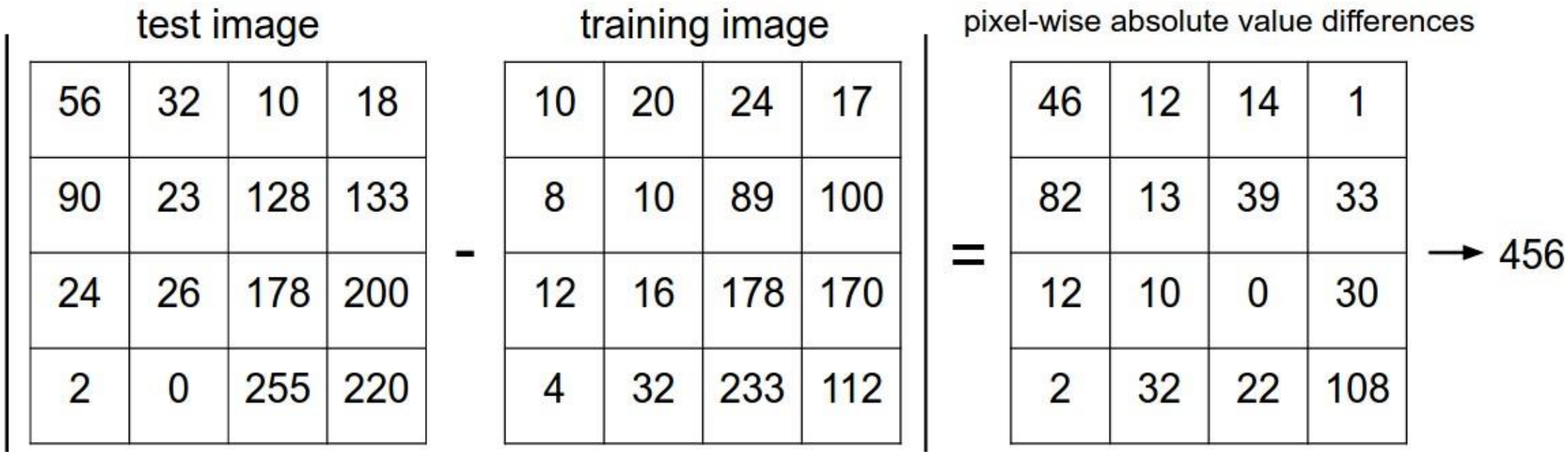
Distances are typically measured as

$$d(\boldsymbol{x}_j, \boldsymbol{x}_i) = \|\boldsymbol{x}_j - \boldsymbol{x}_i\|_2 = \sqrt{\sum_k \left([\boldsymbol{x}_j]_k - [\boldsymbol{x}_i]_k\right)^2}$$

Or

$$d(\boldsymbol{x}_j, \boldsymbol{x}_i) = |\boldsymbol{x}_j - \boldsymbol{x}_i| = \sum_k \left|[\boldsymbol{x}_j]_k - [\boldsymbol{x}_i]_k\right|$$

Pixel-wise distance among images



K-Nearest Neighborhood Classifiers for Images

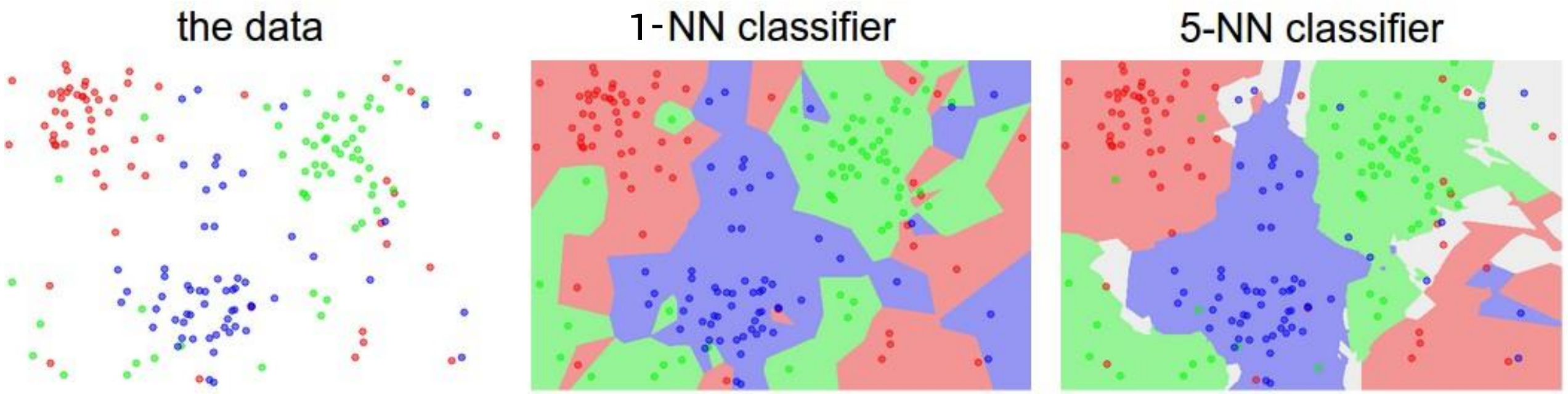
Assign to a each test image, **the most frequent label among the K −closest images in the training set**

$$\hat{y}_j = y_{j^*}, \qquad \text{being } \; j^* \text{ the mode of } \mathcal{U}_K(\boldsymbol{x}_j)$$

where $\mathcal{U}_K(\boldsymbol{x}_j)$ contains the K closest training images to $\boldsymbol{x}_j$

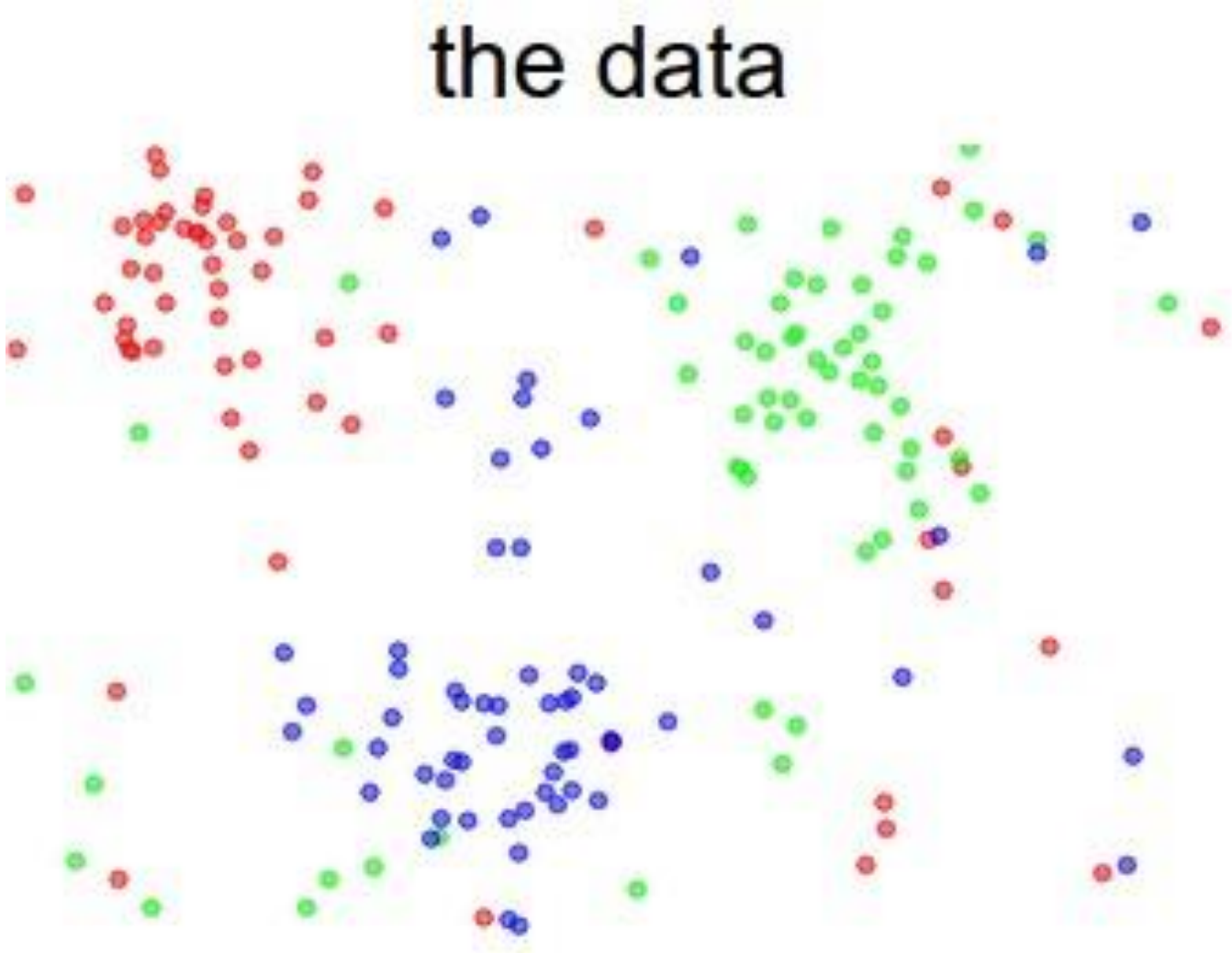
Setting the parameter K and the distance measure is an issue

Nearest Neighborhood Classifier (k-NN) for Images



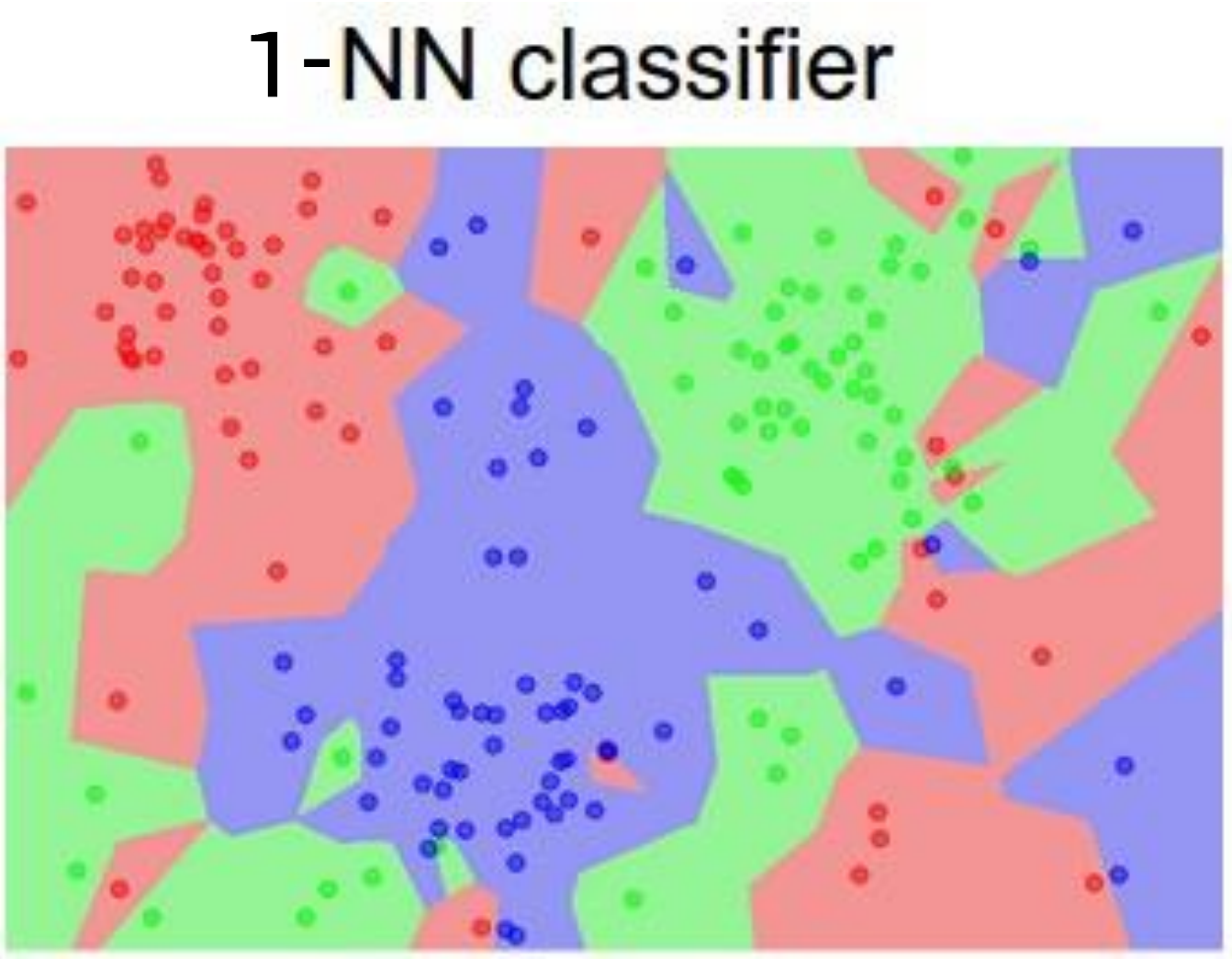
CS231n: Convolutional Neural Networks for Visual Recognition http://cs231n.github.io/

k-NN for Images



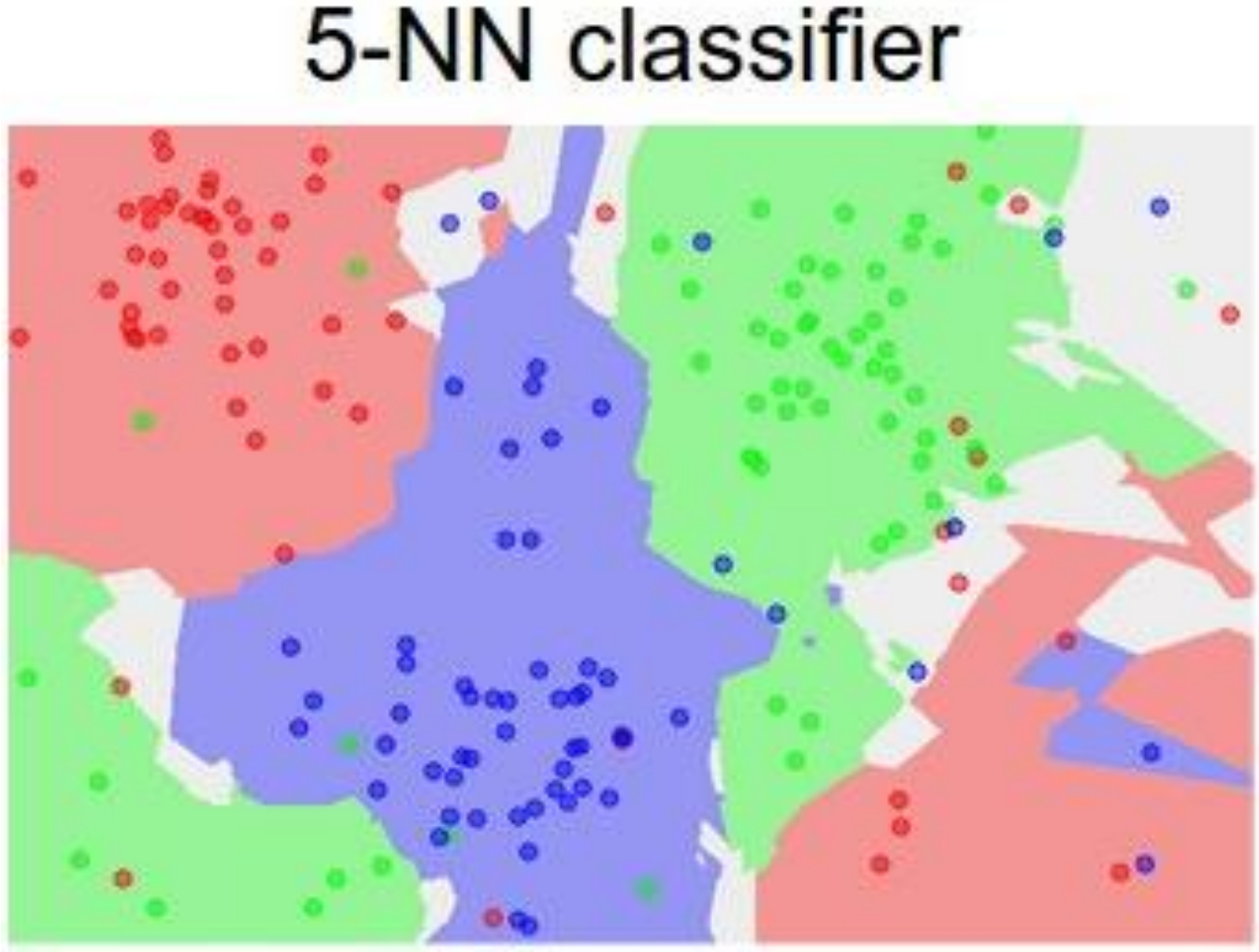
CS231n: Convolutional Neural Networks for Visual Recognition http://cs231n.github.io/

k-NN for Images



CS231n: Convolutional Neural Networks for Visual Recognition http://cs231n.github.io/

k-NN for Images



CS231n: Convolutional Neural Networks for Visual Recognition http://cs231n.github.io/

k-NN for Images

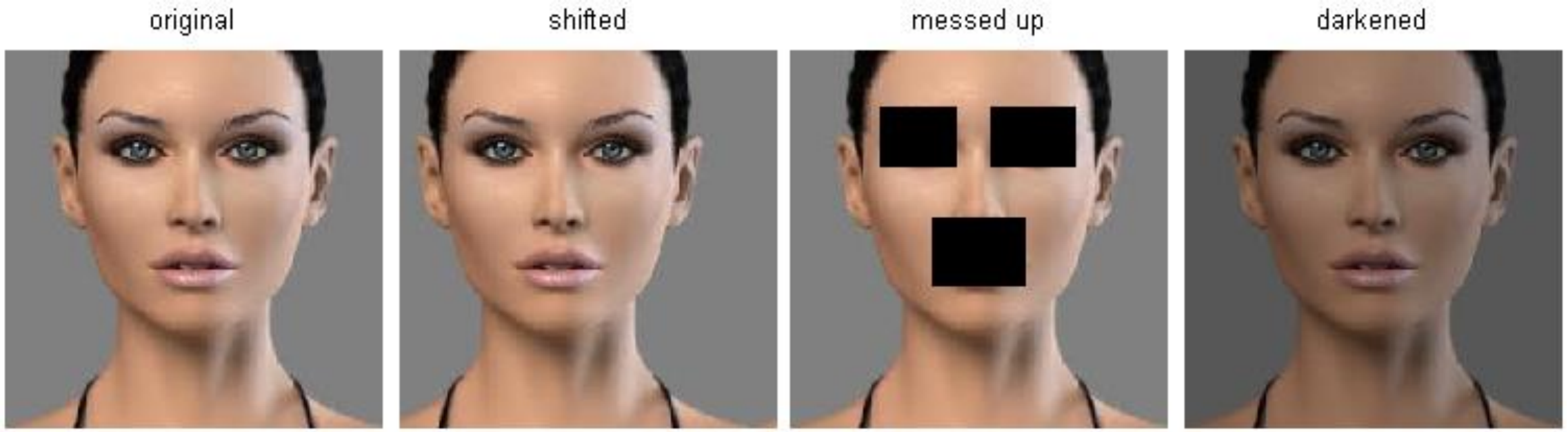
Pros:

- Easy to understand and implement
- It takes no training time

Cons:

- Computationally demanding at test time (in particular when TR is large and d is also large)
- Large training sets have to be stored in memory
- Rarely practical on images: distances on high-dimensional objects are difficult to interpret

Perceptual Similarity vs Pixel Similarity

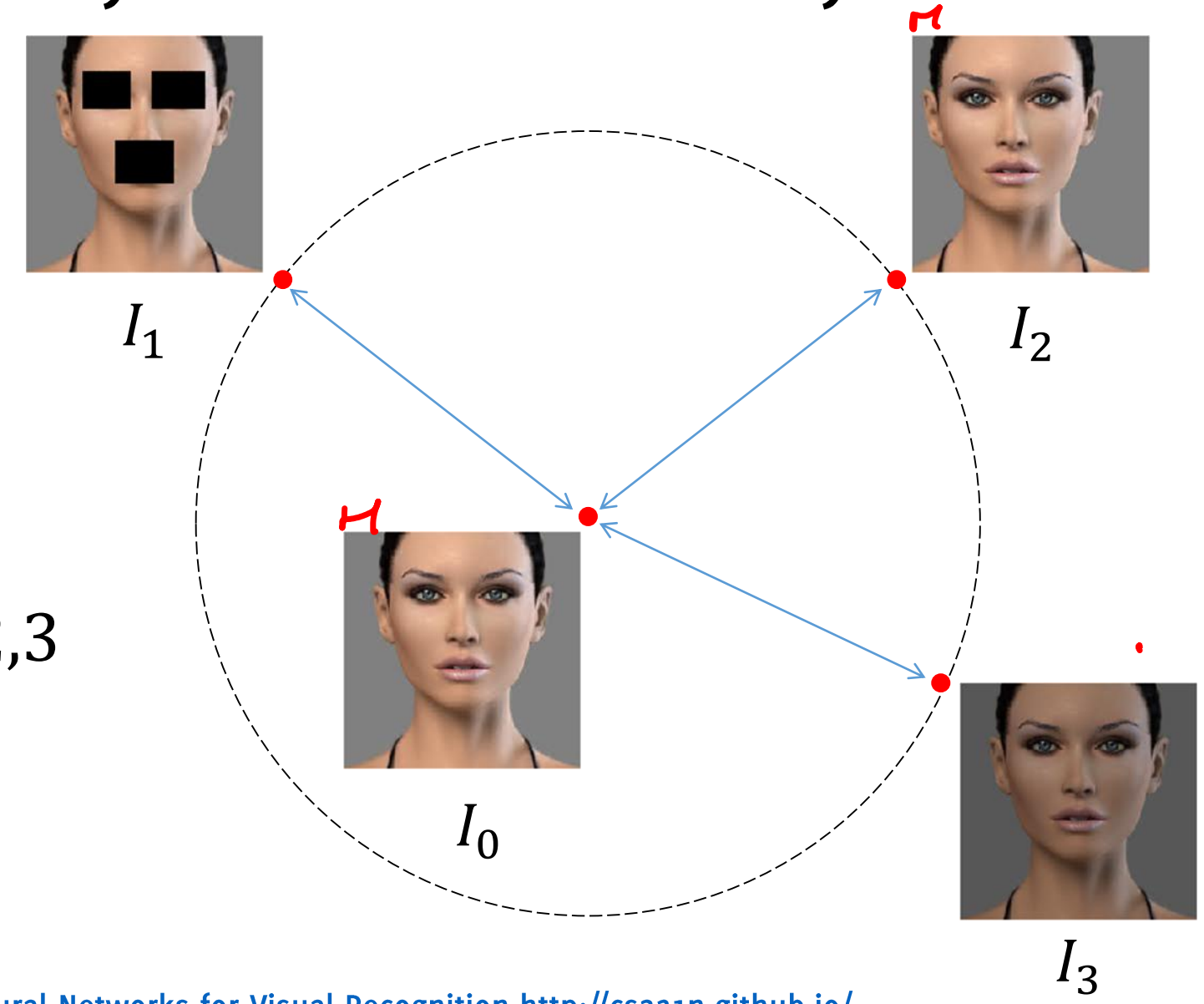


The three images have the same pixel-wise distance from the original one...

...but perceptually they are very different

Perceptual Similarity vs Pixel Similarity

$$\|I_j - I_0\|_2 \approx const\ j = 1,2,3$$



Let's see what happens on the whole CIFAR10

On CIFAR10 we see exactly this problem

CS231n: Convolutional Neural Networks for Visual Recognition http://cs231n.github.io/

On CIFAR10 we see exactly this problem

On CIFAR10 we see exactly this problem

The Feature Extraction Perspective

The Feature Extraction Perspective

Images can not be directly fed to a classifier

We need some intermediate step to:

- Extract meaningful information (to our understanding)
- Reduce data-dimension

We need to extract features:

- The better our features, the better the classifier

The Feature Extraction Perspective

Input image

$I_1 \in \mathbb{R}^{r_1 \times c_1}$

Feature Extraction Algorithm

$\mathbf{x} \in \mathbb{R}^d$

$(d \ll r_1 \times c_1)$

Classifier (NN)

"wheel"

$y \in \Lambda$

The Feature Extraction Perspective

Input image

$I_2 \in \mathbb{R}^{r_2 \times c_2}$

Feature Extraction Algorithm

$\mathbf{x} \in \mathbb{R}^d$

$(d \ll r_2 \times c_2)$

Classifier (NN)

"castle"

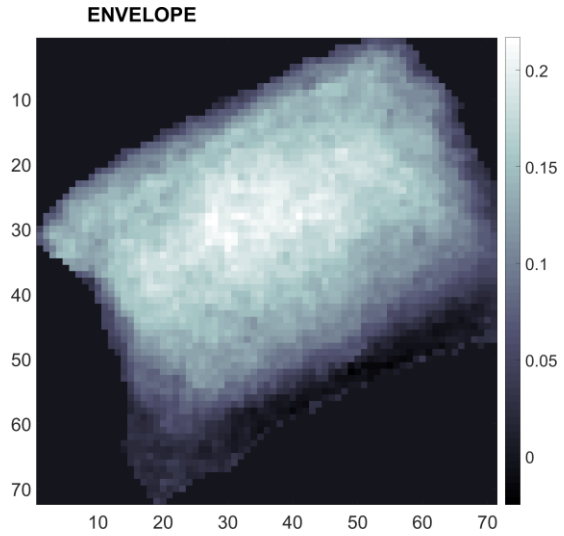
$y \in \Lambda$

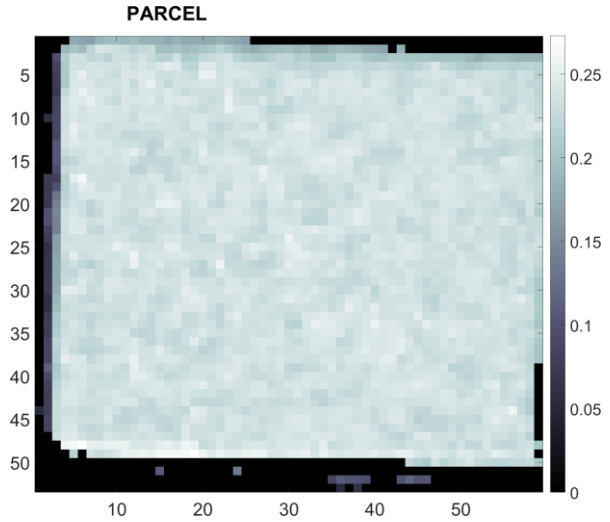
Hand-Crafted Features

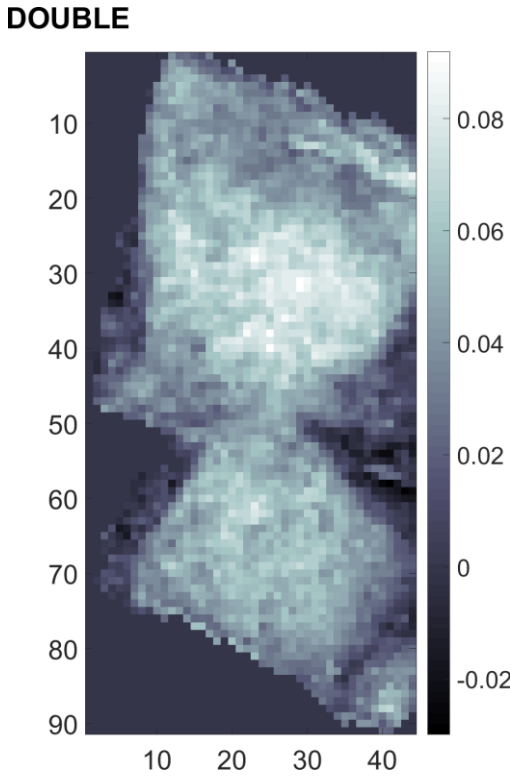
Example of Hand-Crafted Features

Example of features:

- Average height
- Area (coverage with nonzero measurements)
- Distribution of heights
- Perimeter
- Diagonals

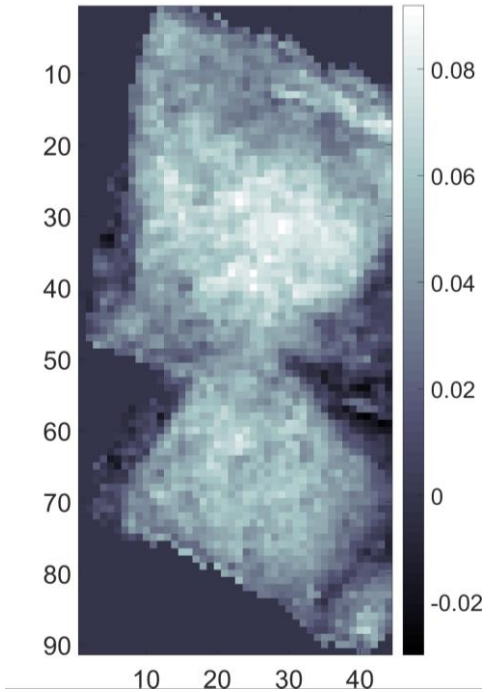






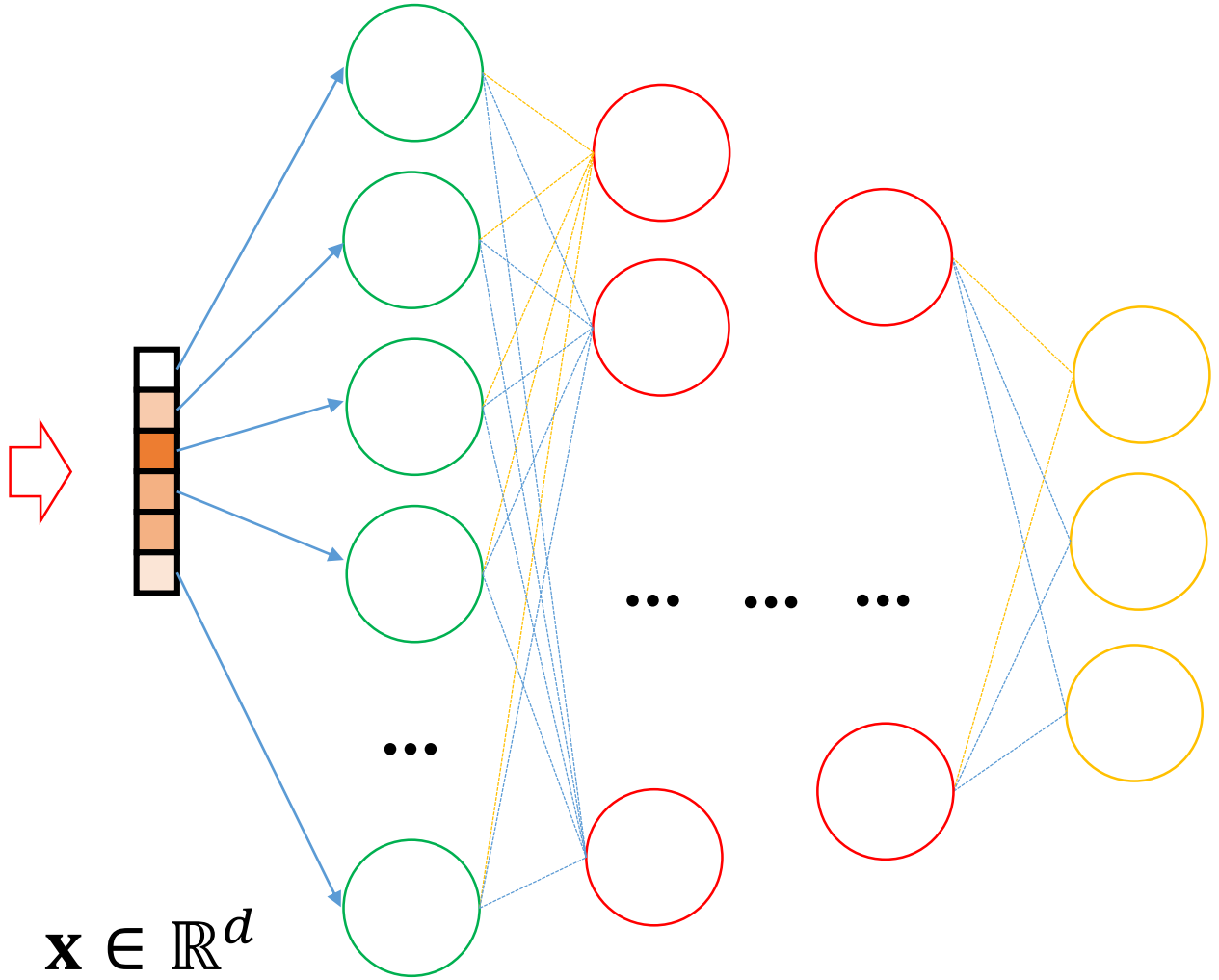
Neural Networks

Input image



$I_1 \in \mathbb{R}^{r_1 \times c_1}$

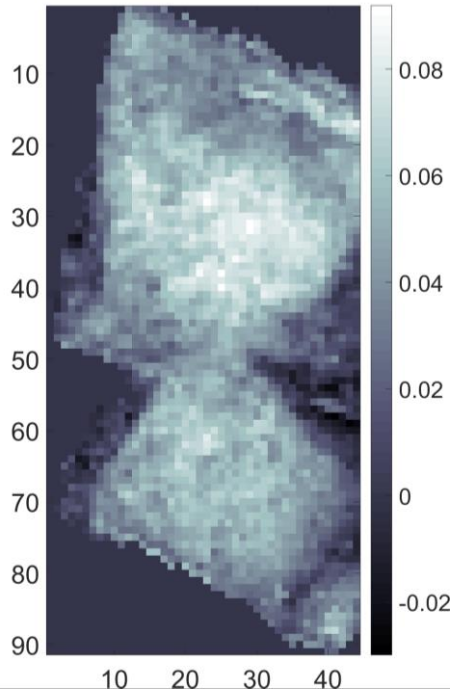
Feature Extraction Algorithm



Neural Networks

Input layer: Same size of the feature vector

Input image



$I_1 \in \mathbb{R}^{r_1 \times c_1}$

Feature Extraction Algorithm

$\mathbf{x} \in \mathbb{R}^d$

input layer

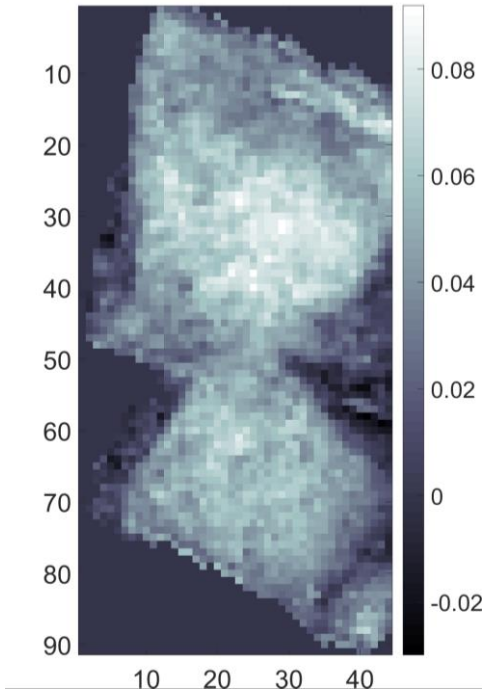
Hidden layer(s)

Output Layer

Giacomo Boracchi

Neural Networks

Input image



$I_1 \in \mathbb{R}^{r_1 \times c_1}$

Feature Extraction Algorithm

$\mathbf{x} \in \mathbb{R}^d$

Output layer: Same size as the number of classes

$P(l = \text{"doub."} | \boldsymbol{x})$

$P(l = \text{"env."} | \boldsymbol{x})$

$P(l = \text{"parc."} | \boldsymbol{x})$

input layer

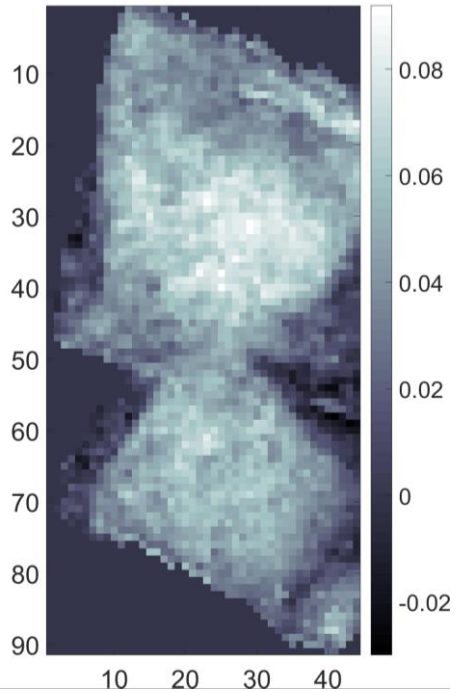
Hidden layer(s)

Output Layer

Neural Networks

Hidden layers: arbitrary size

Input image



$I_1 \in \mathbb{R}^{r_1 \times c_1}$

Feature Extraction Algorithm

$\mathbf{x} \in \mathbb{R}^d$

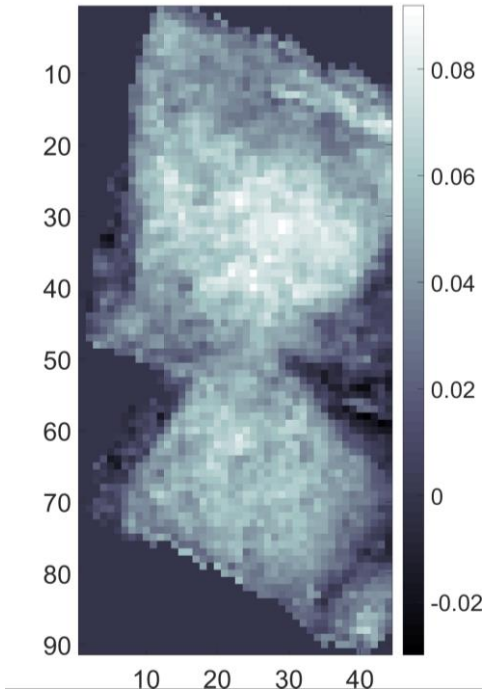
input layer

Hidden layer(s)

Output Layer

Neural Networks

Input image



$I_1 \in \mathbb{R}^{r_1 \times c_1}$

Feature Extraction Algorithm

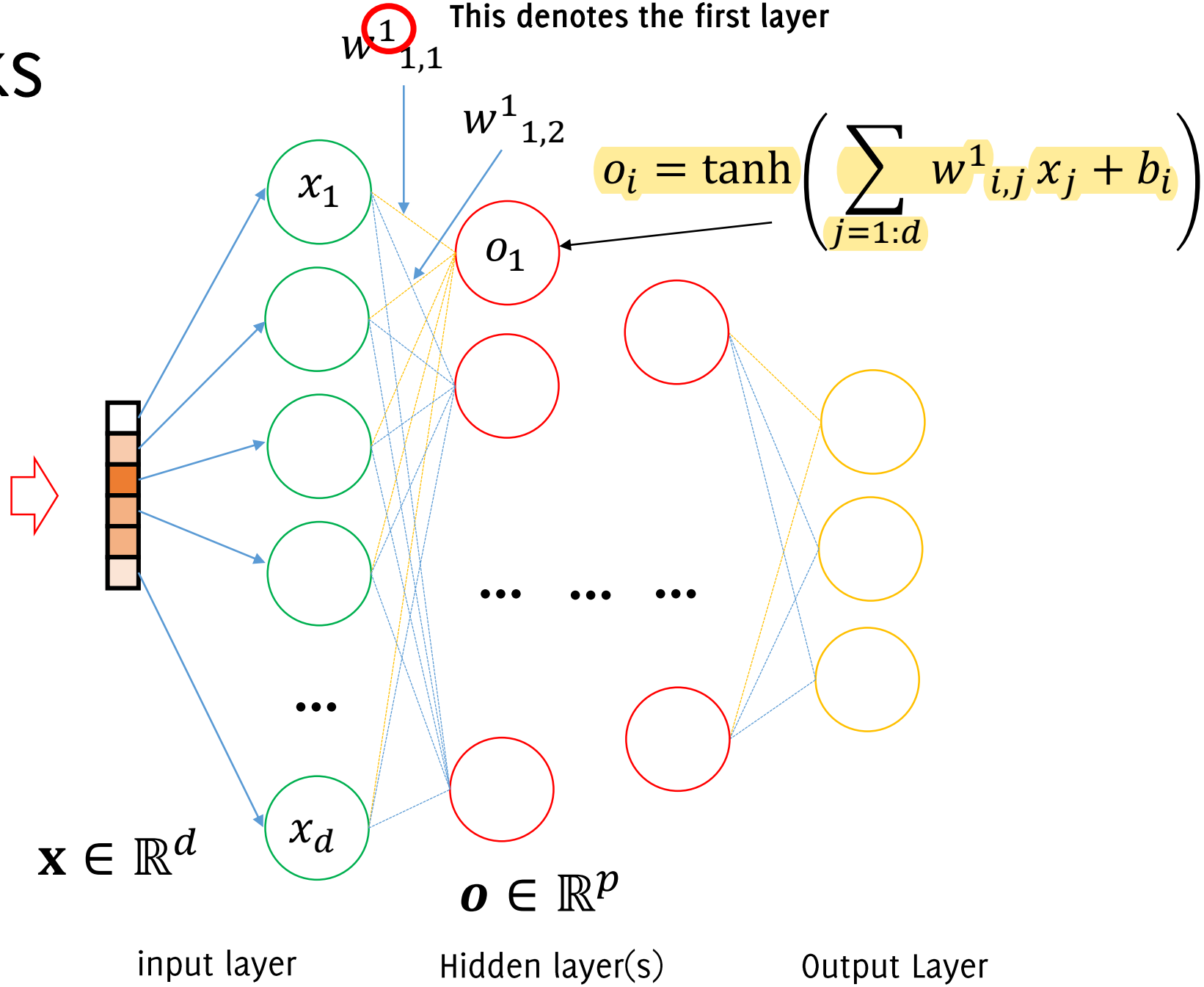
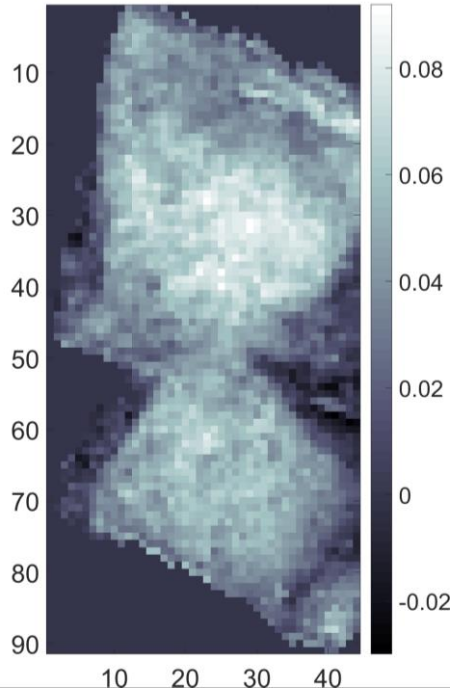


Image Classification by Hand Crafted Features

Input image



$I_1 \in \mathbb{R}^{r_1 \times c_1}$

Feature Extraction Algorithm

Hand Crafted

Data Driven

Giacomo Boracchi

Hand Crafted Featues, pros:

- **Exploit a priori** / expert **information**
- Features are **interpretable** (you might understand why they are not working)
- You can **adjust features** to improve your performance
- **Limited amount of training data** needed
- You can give more relevance to some features

Hand Crafted Featues, cons:

- Requires a lot of **design/programming efforts**
- **Not viable** in many **visual recognition** tasks (e.g. on natural images) which are easily performed by humans
- **Risk of overfitting** the training set used in the design
- **Not very general and "portable"**

Data-Driven Features

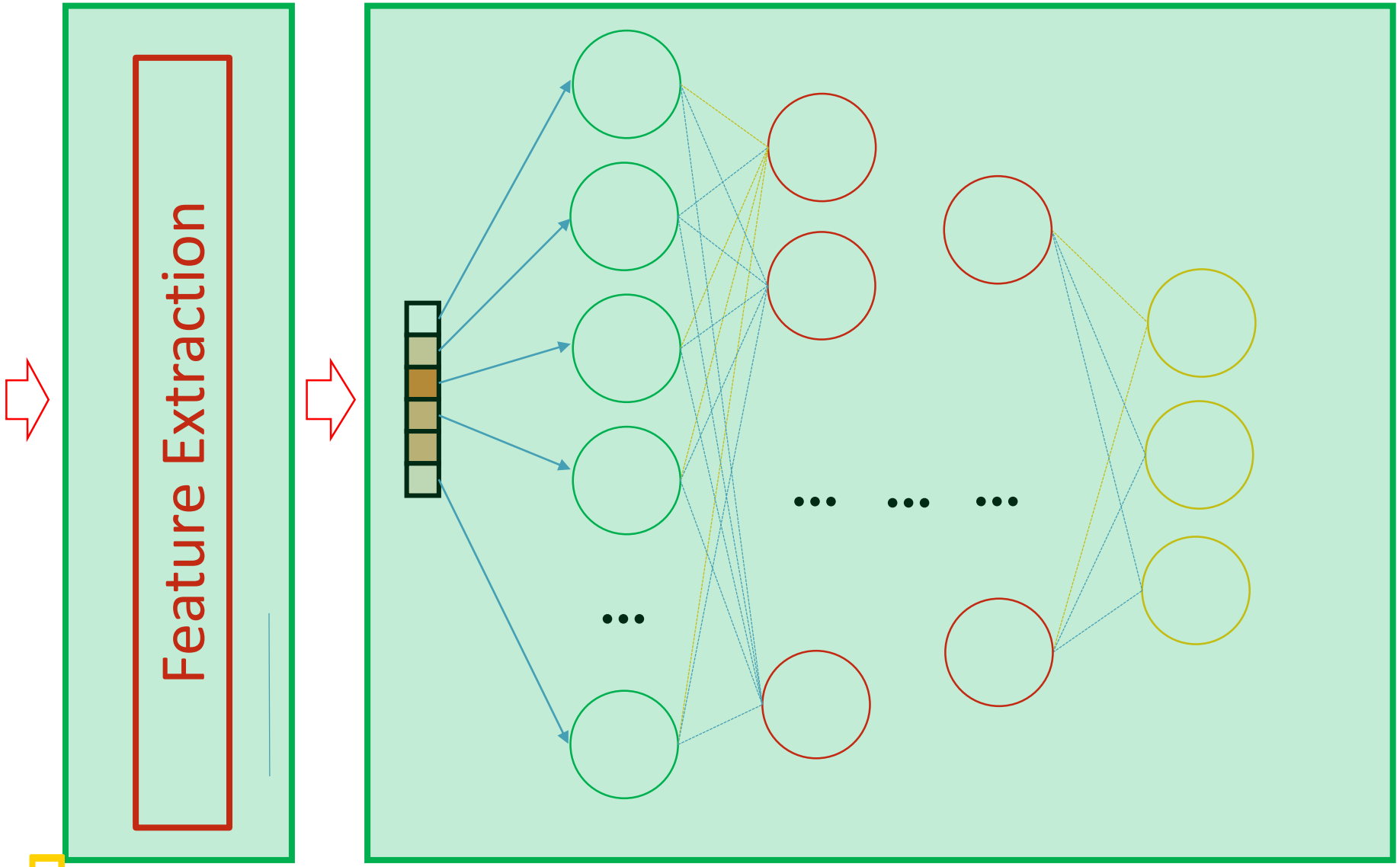
… the advent of deep learning

Giacomo Boracchi

Data-Driven Features

Input image

$I_1 \in \mathbb{R}^{r_1 \times c_1}$

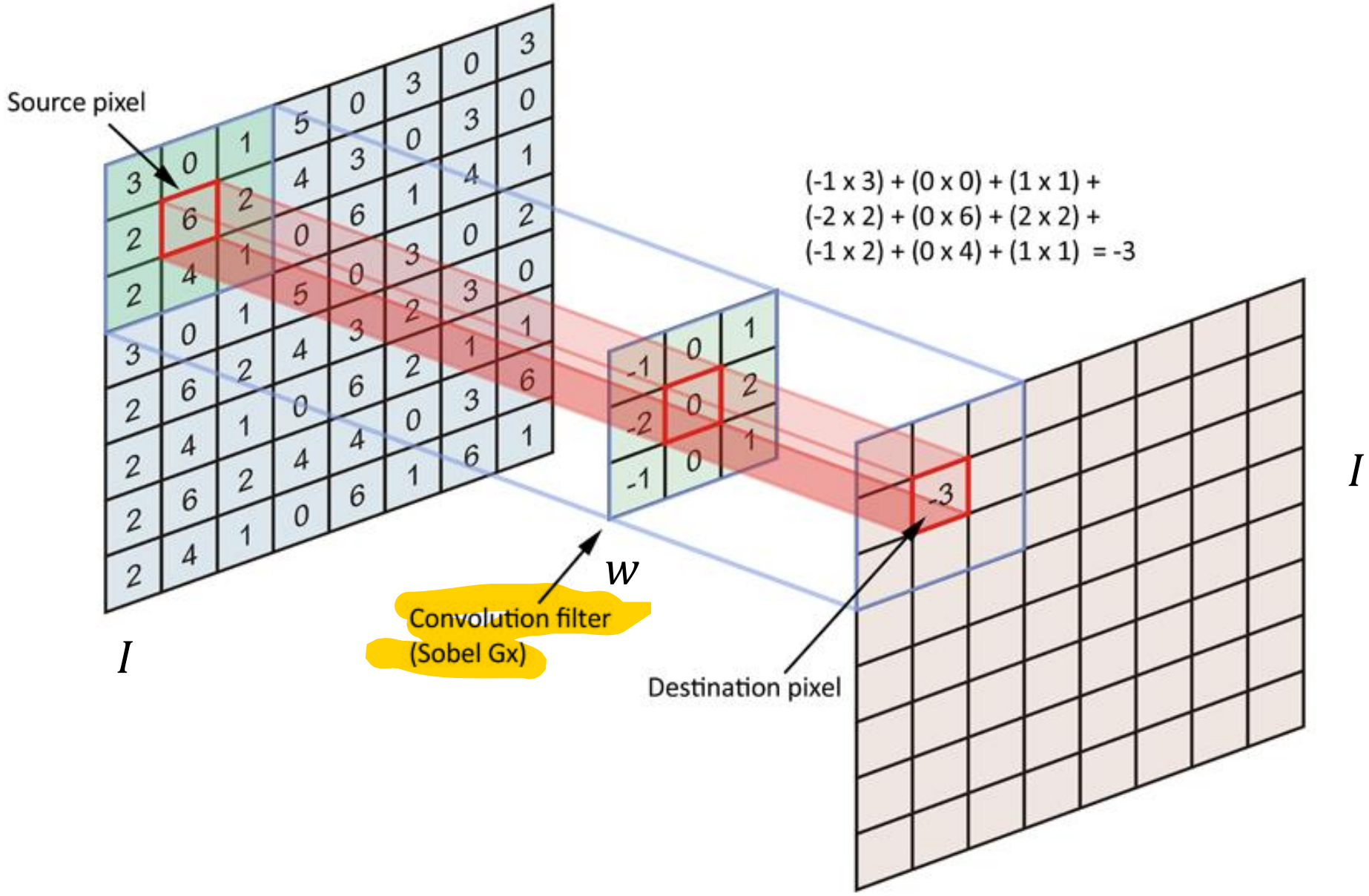


Convolutional Neural Networks

Setting up the stage

Giacomo Boracchi

What is a convolution?



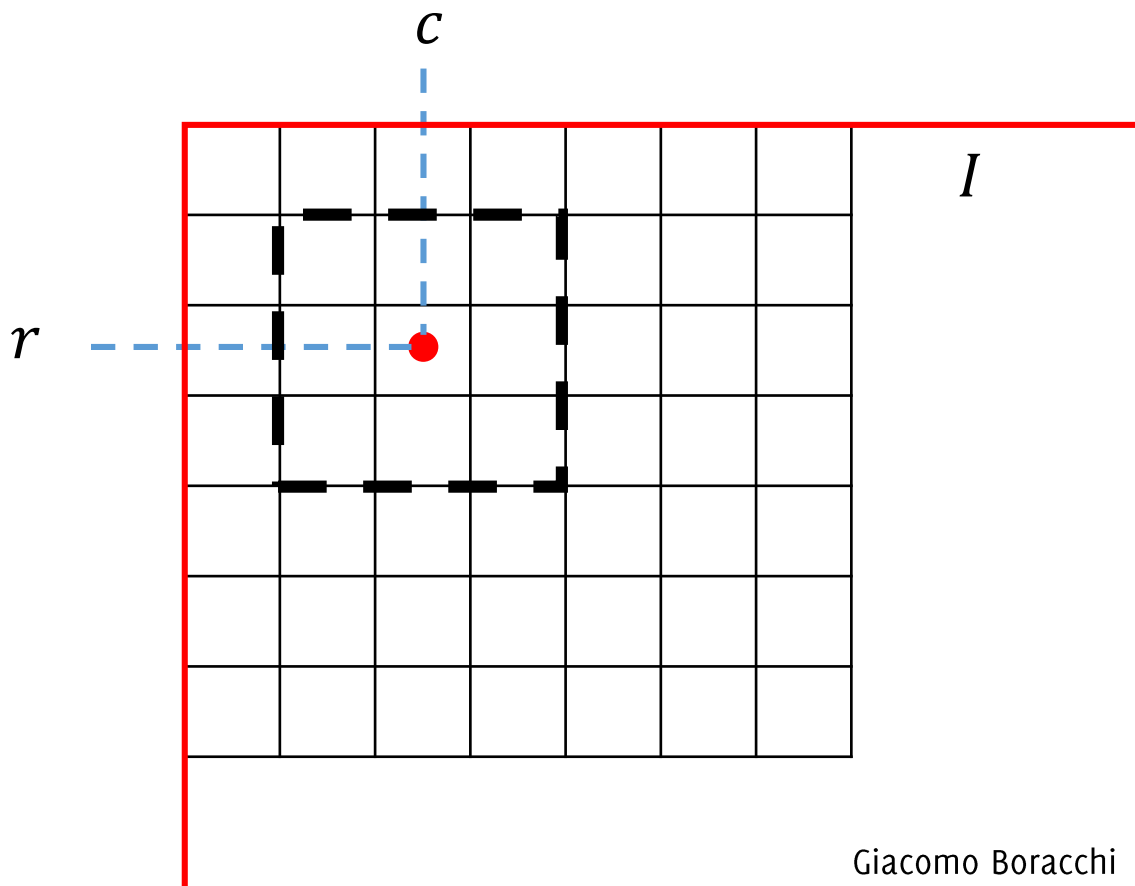
2D Correlation

Convolution is a linear transformation. Linearity implies that

$$(I \otimes w)(r,c) = \sum_{(u,v)\in U} w(u,v) * I(r+u, c+v)$$

w

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$



We can consider weights as **a filter** h

The filter h **entirely defines convolution**

Convolution operates the same in each pixel

2D Convolution

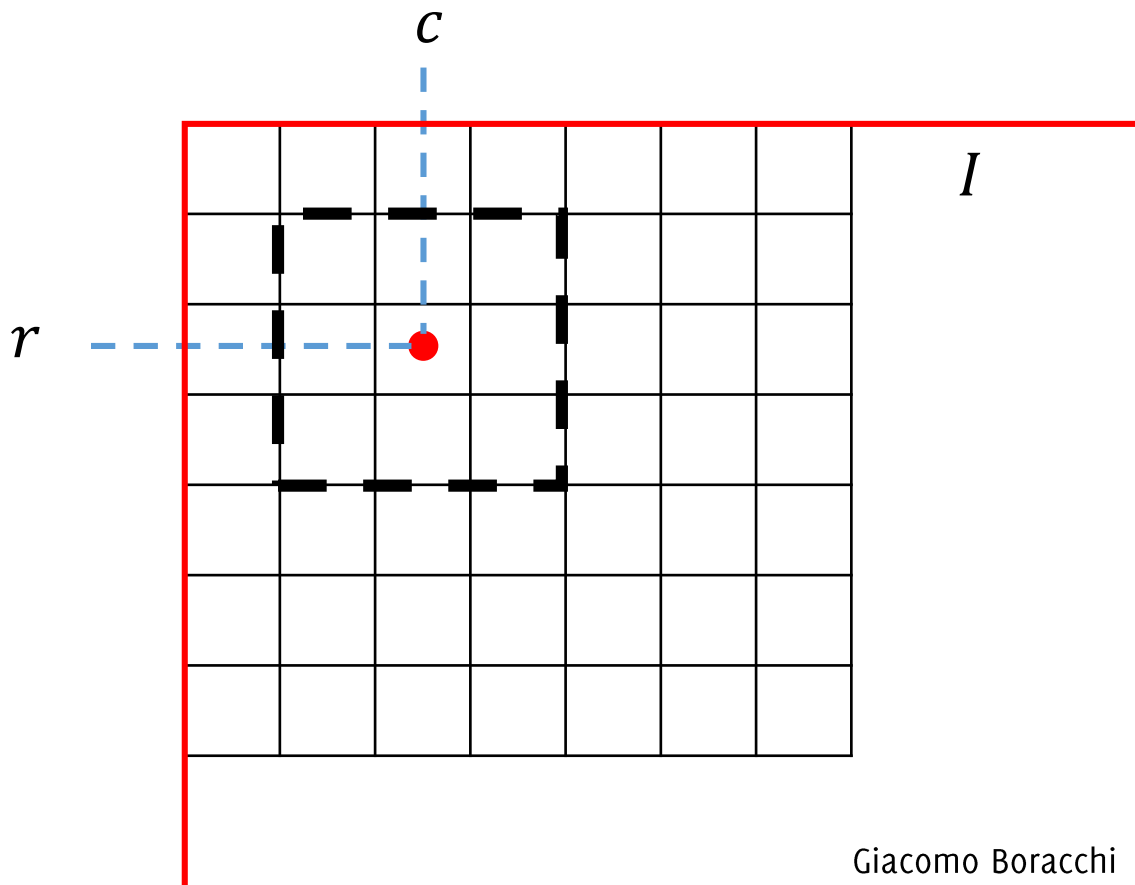
Convolution is a linear transformation. Linearity implies that

$$(I \circledast w)(r,c) = \sum_{(u,v)\in U} w(u,v) * I(r-u, c-v)$$

w

$w(1,1)$	$w(1,0)$	$w(1,-1)$
$w(0,1)$	$w(0,0)$	$w(0,-1)$
$w(-1,1)$	$w(-1,0)$	$w(-1,-1)$

Rmk: indexes have been shifted in the filter w



We can consider weights as **a filter**

The filter entirely defines convolution

Convolution operates the same in each pixel

2D Convolution

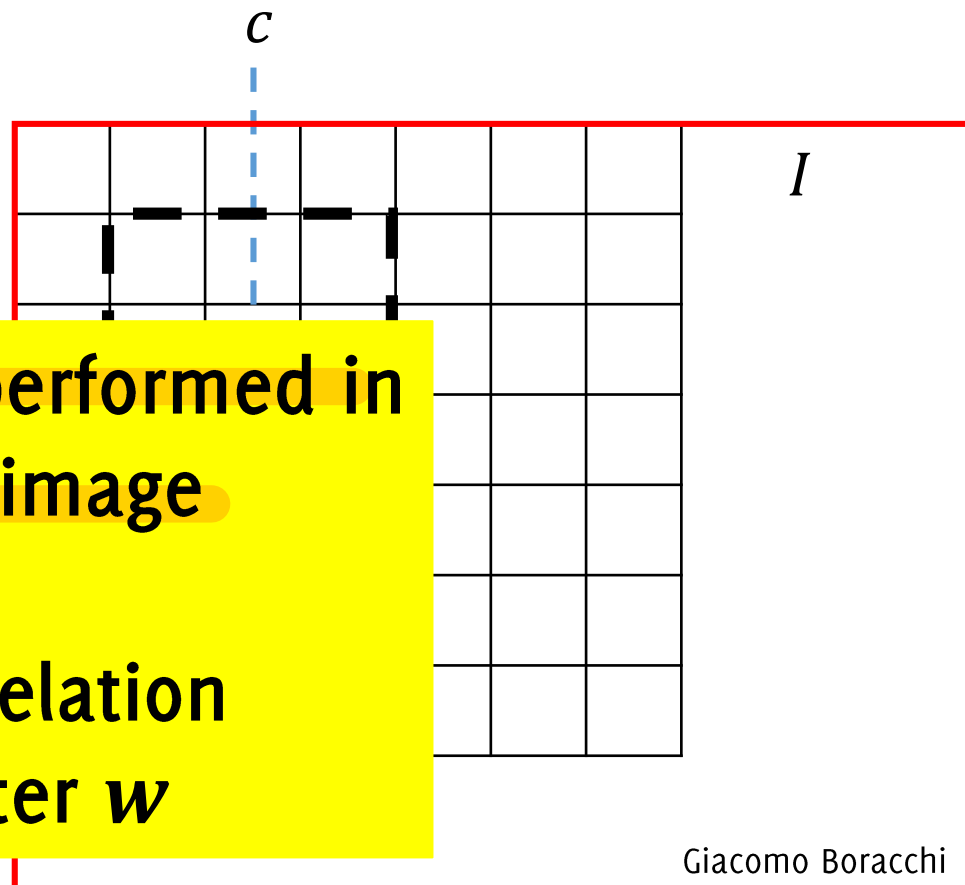
Convolution is a linear transformation. Linearity implies that

$$(I \circledast w)(r,c) = \sum_{(u,v)\in U} w(u,v) * I(r-u, c-v)$$

w

$w(1,1)$	$w(1,0)$	$w(1,-1)$
$w(0,1)$	$w(0,0)$	$w(0,-1)$
$w(-1,1)$	$w(-1,0)$	

Rmk: indexes have been shifted in the filter w



The same operation is being performed in each pixel of the input image

It is equivalent to 2D Correlation up to a «flip» in the filter w

We can con

The filter h e

Convolution op

2D Convolution

Convolution is a linear transformation. Linearity implies that

$$(I \circledast w)(r,c) = \sum_{(u,v)\in U} w(u,v) * I(r-u, c-v)$$

Convolution is defined up to the "filter flip" for the Fourier Theorem to apply. Filter flip has to be taken into account when computing convolution in Fourier domain and when designing filters.

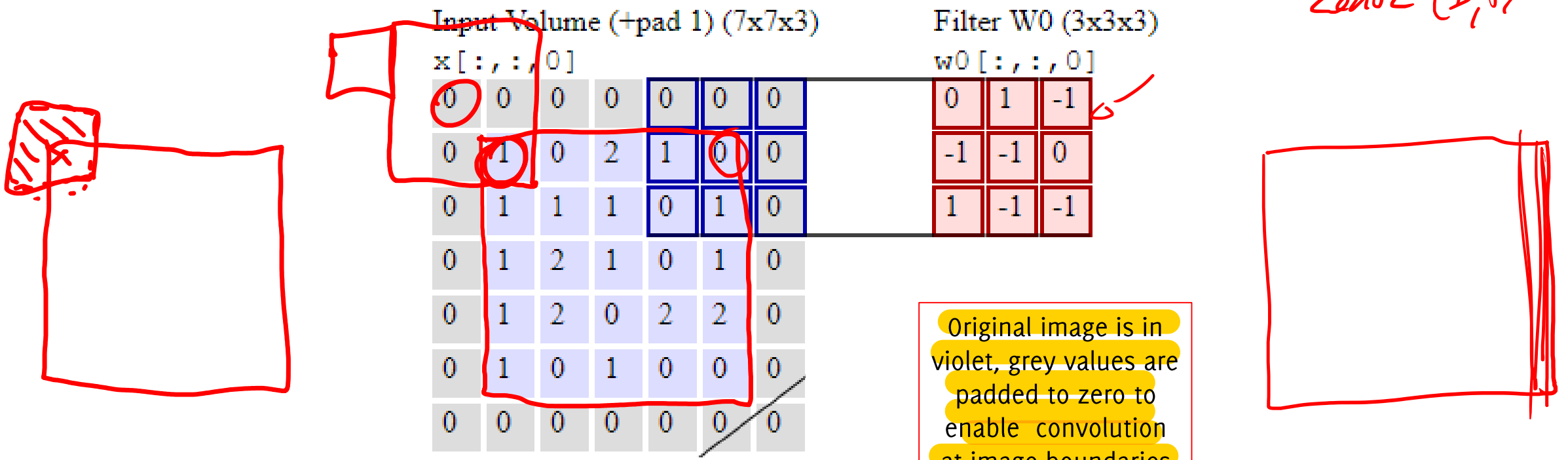
However, in CNN, convolutional filters are being learned from data, thus it is only important to use these in a consistent way.

In practice, **in CNN arithmetic there is no flip!**

Convolution: Padding

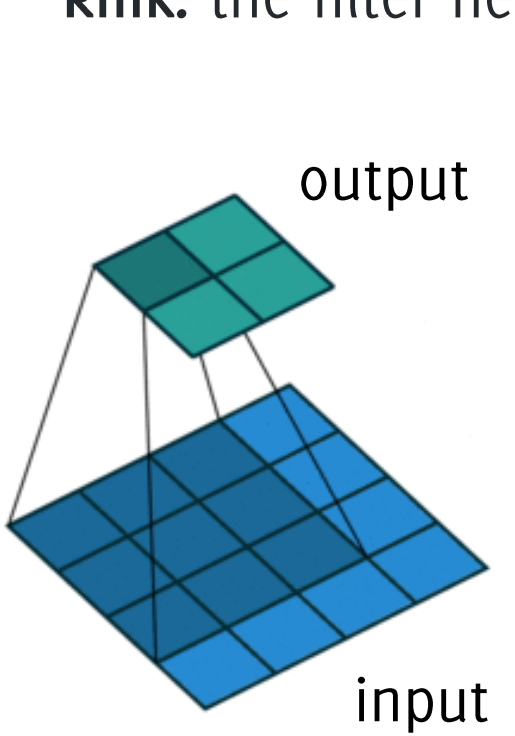
How to define convolution output close to image boundaries?

Padding with zero is the most frequent option, as this does not change the output size. However, no padding or symmetric padding are also viable options

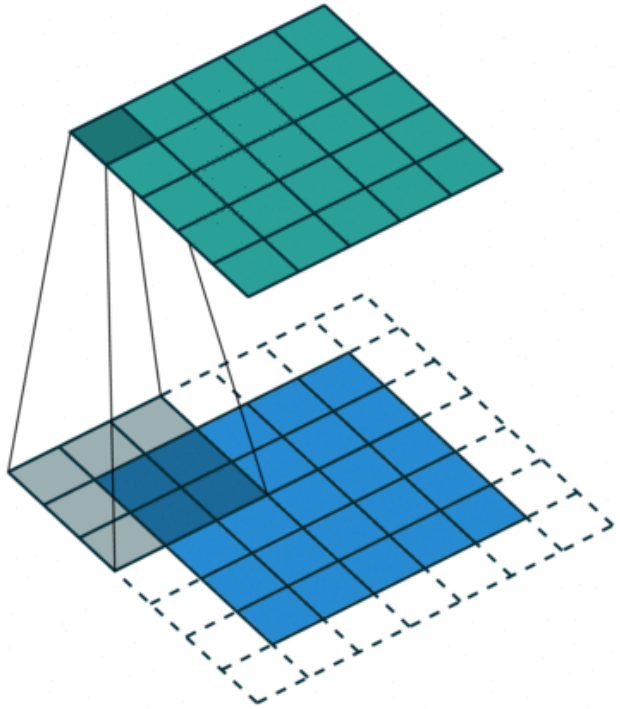


Padding Options in Convolution Animation

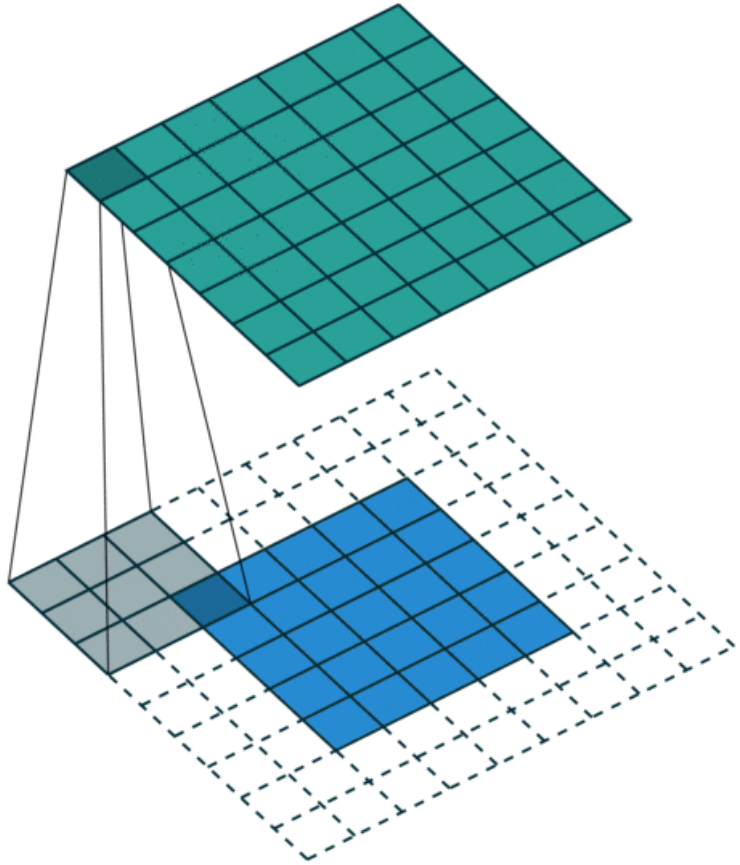
Rmk: Blue maps are inputs, and cyan maps the outputs.
Rmk: the filter here is 3×3



No padding
«valid»



Half padding
«same»



full padding
«full»

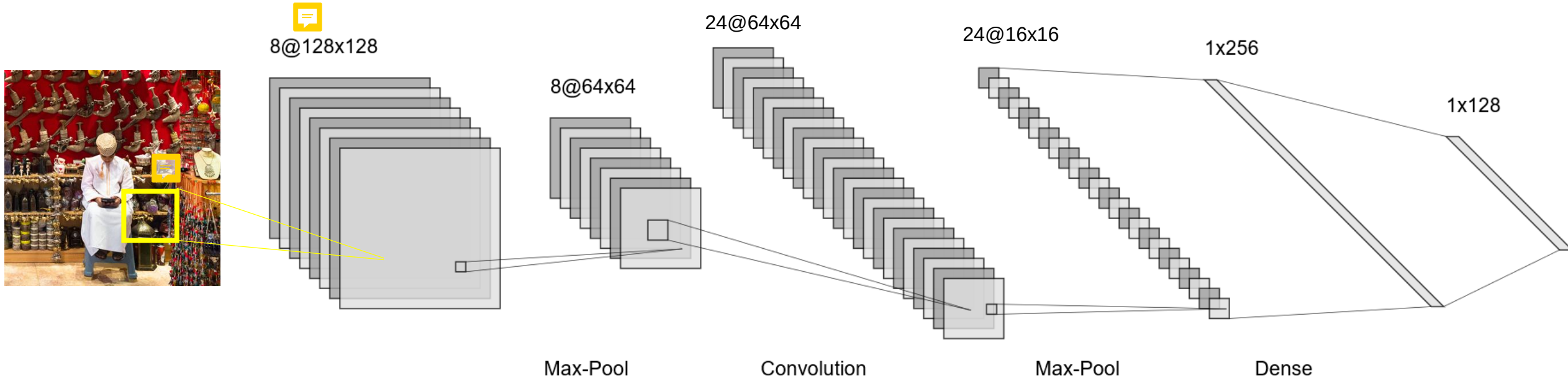
Vincent Dumoulin, Francesco Visin - A guide to convolution arithmetic for deep learning

Convolutional Neural Networks

CNNs

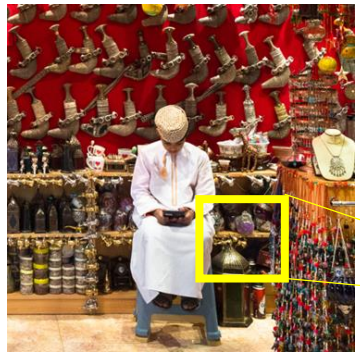
Giacomo Boracchi

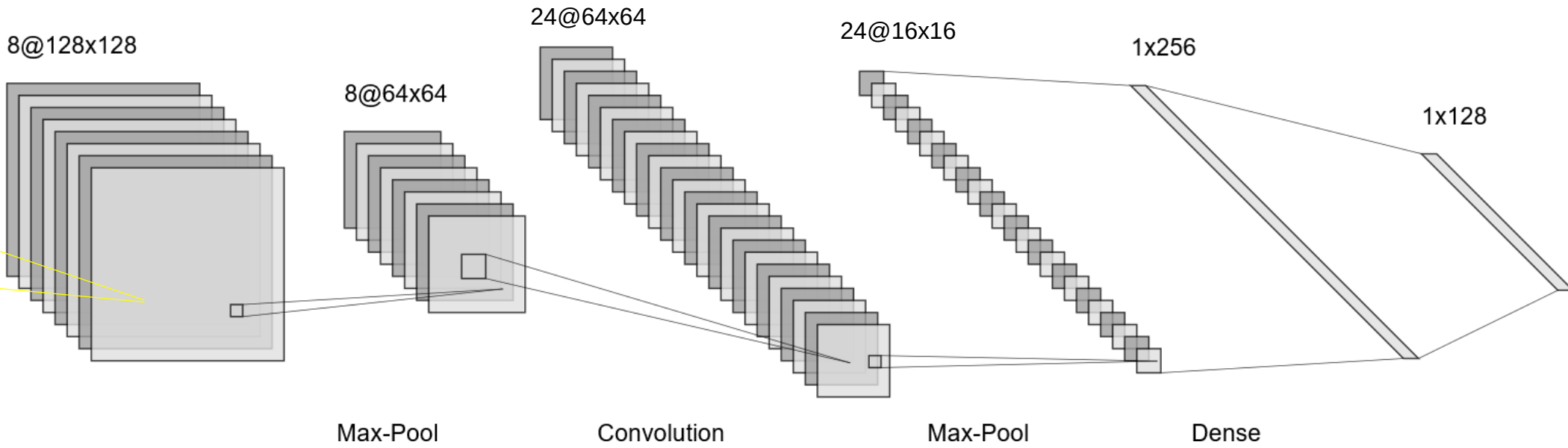
The typical architecture of a CNN



The typical architecture of a CNN

The input of a CNN is an entire image

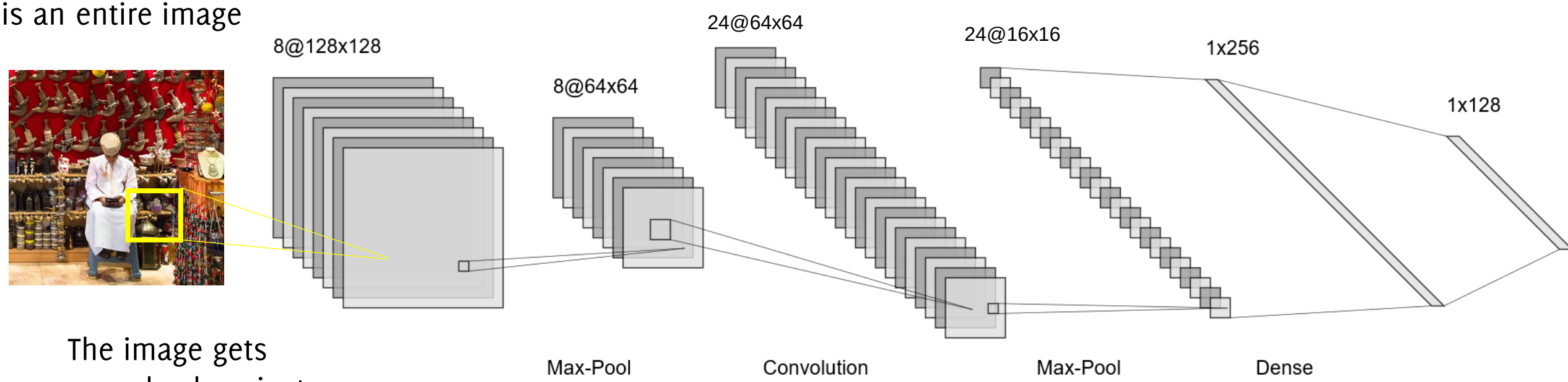




The image gets convolved against many filters

The typical architecture of a CNN

The input of a CNN is an entire image

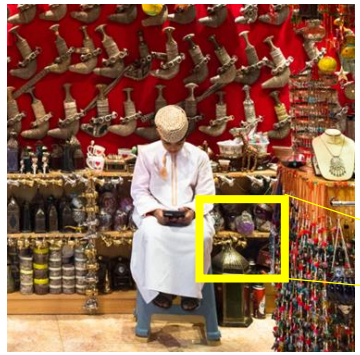


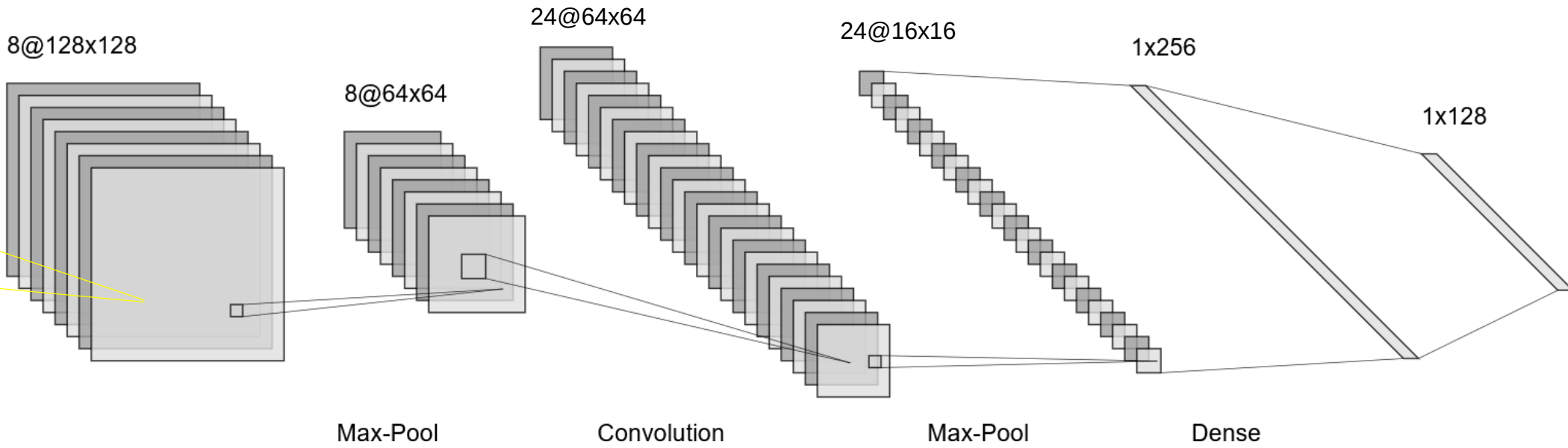
The image gets convolved against many filters

When progressing along the network, the «number of images» or the «number of channels in the images» increases, while the image size decreases

The typical architecture of a CNN

The input of a CNN is an entire image

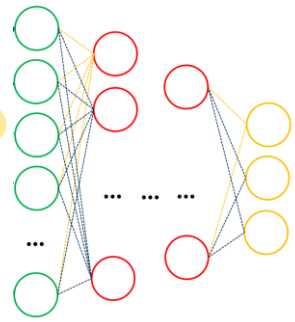




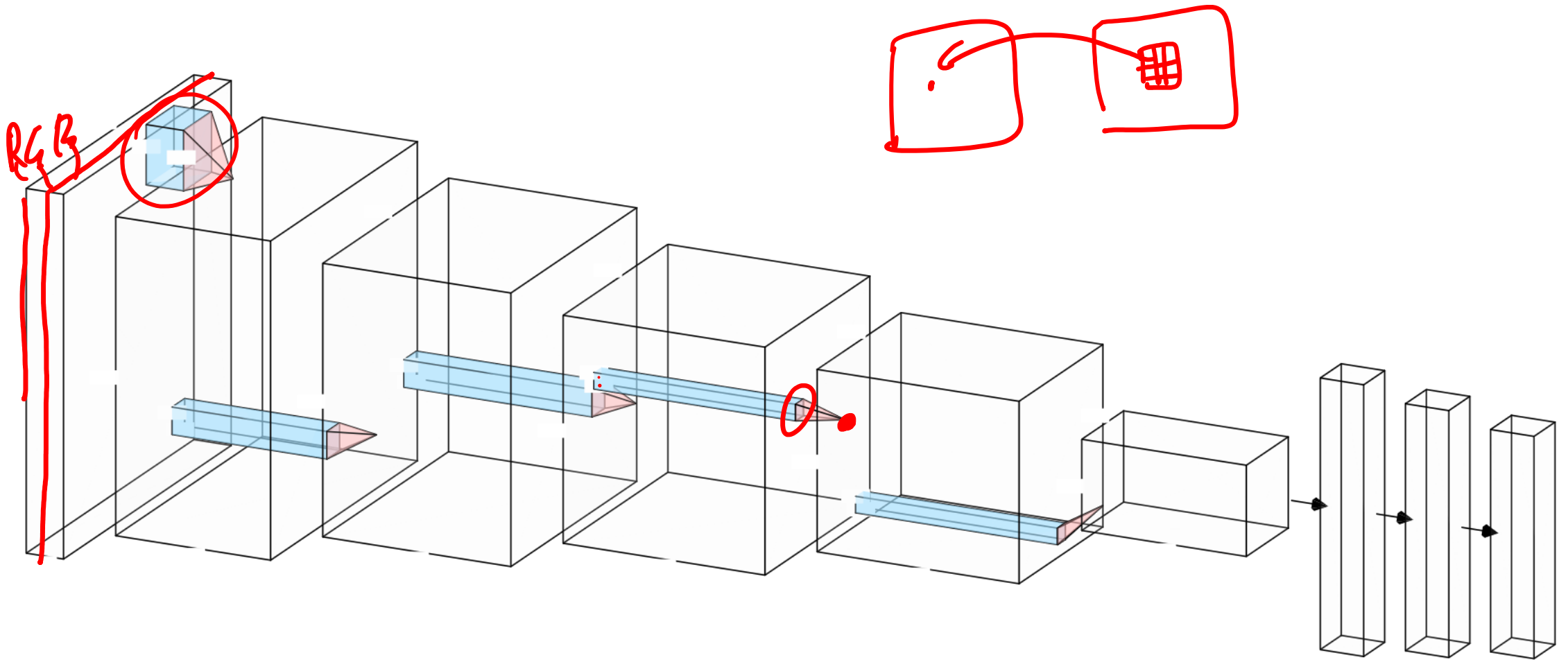
The image gets convolved against many filters

When progressing along the network, the «number of images» or the «number of channels in the images» increases, while the image size decreases

Once the image gets to a vector, this is fed to a traditional neural network



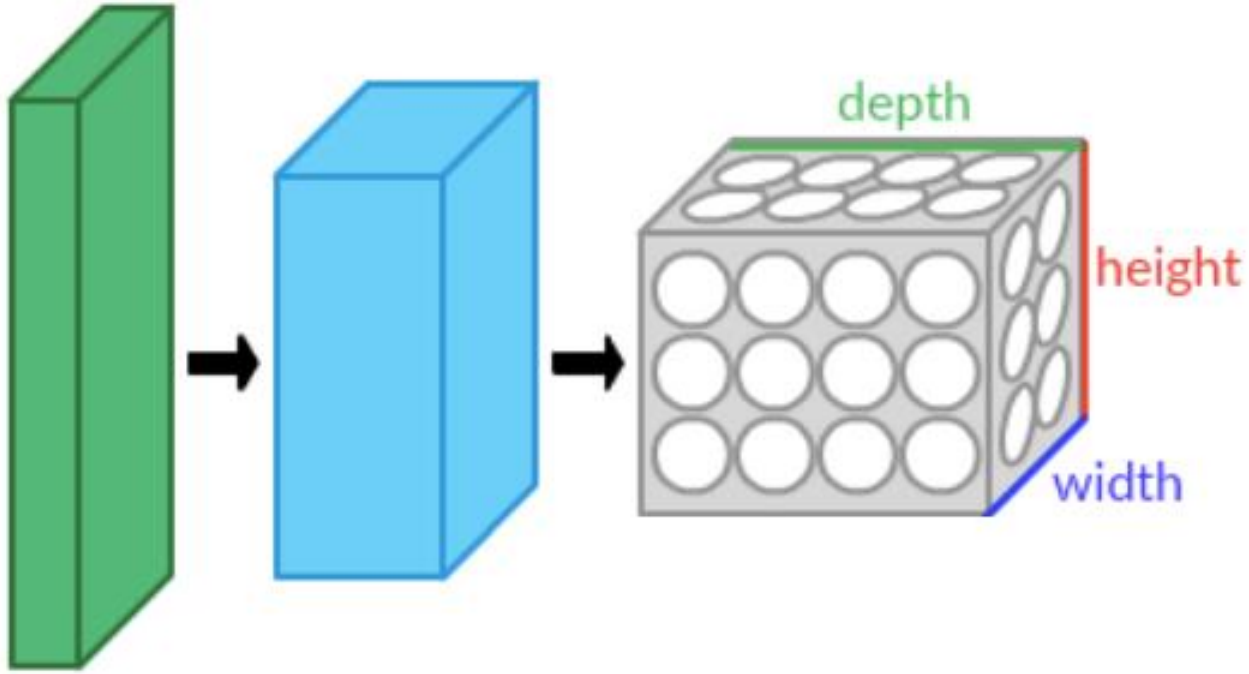
The typical architecture of a CNN



Convolutional Neural Networks (CNN)

CNN are typically made of blocks that include:

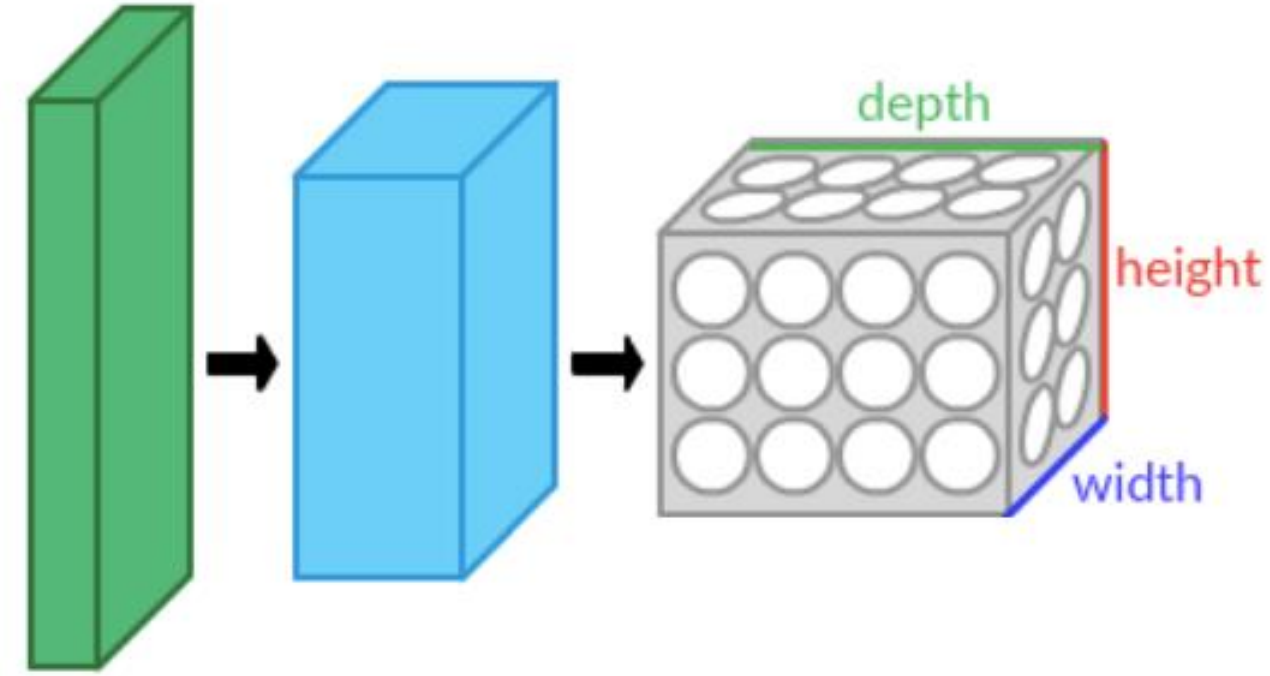
- Convolutional layers
- Nonlinearities (activation functions)
- Pooling Layers (Subsampling / maxpooling



By Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45661858

Convolutional Neural Networks (CNN)

- An image passing through a CNN is transformed in a sequence of volumes.
- As the depth increases, the height and width of the volume decreases
- Each layer takes as input and returns a volume



By Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45661858

Convolutional Layers

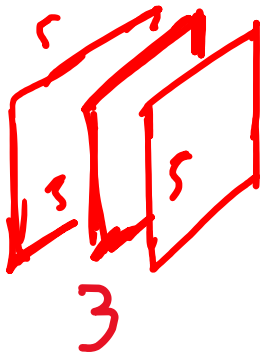
Giacomo Boracchi

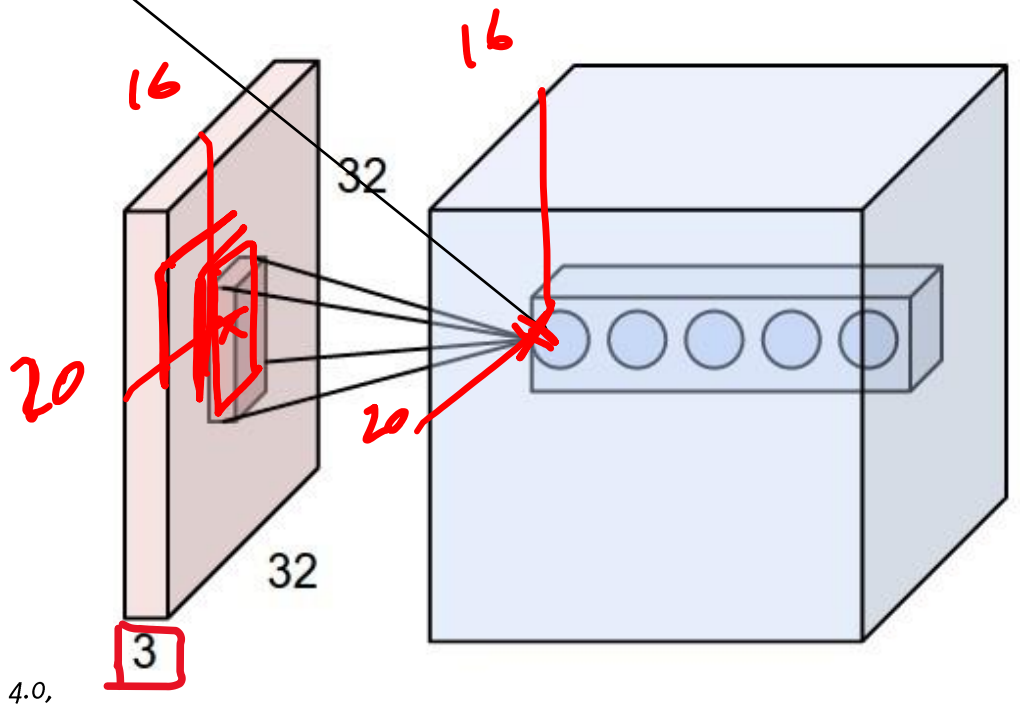
Convolutional Layers

Convolutional layers "mix" all the input components

The output is a linear combination of **all the values in a region of the input, considering all the channels**

$$a^1 = \sum_{i,j} w^1_{i,j}\, x_{i,j} + b^1$$





Filters need to have **the same number of channels** as the input, to process all the values from the input layer

By Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45659236

Convolutional Layers

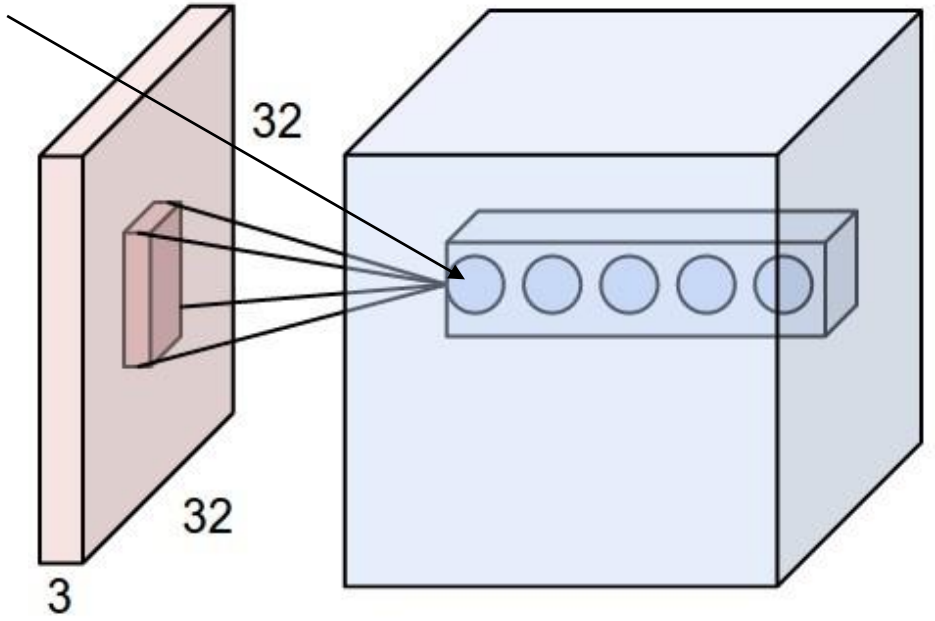
Convolutional layers "mix" all the input components

The output is a linear combination of **all the values in a region of the input, considering all the channels**

$$a^1(r,c) = \sum_{i,j} w^1_{i,j}\, x(r+i, c+j) + b^1$$

The parameters of this layer are **called filters**.

The same filter is used through the whole spatial extent of the input

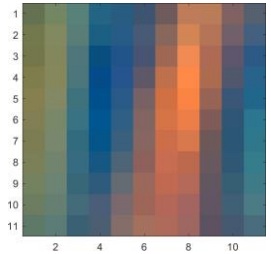


By Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45659236

Convolutional Layers

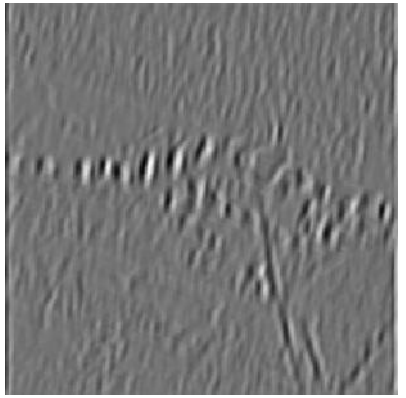
I

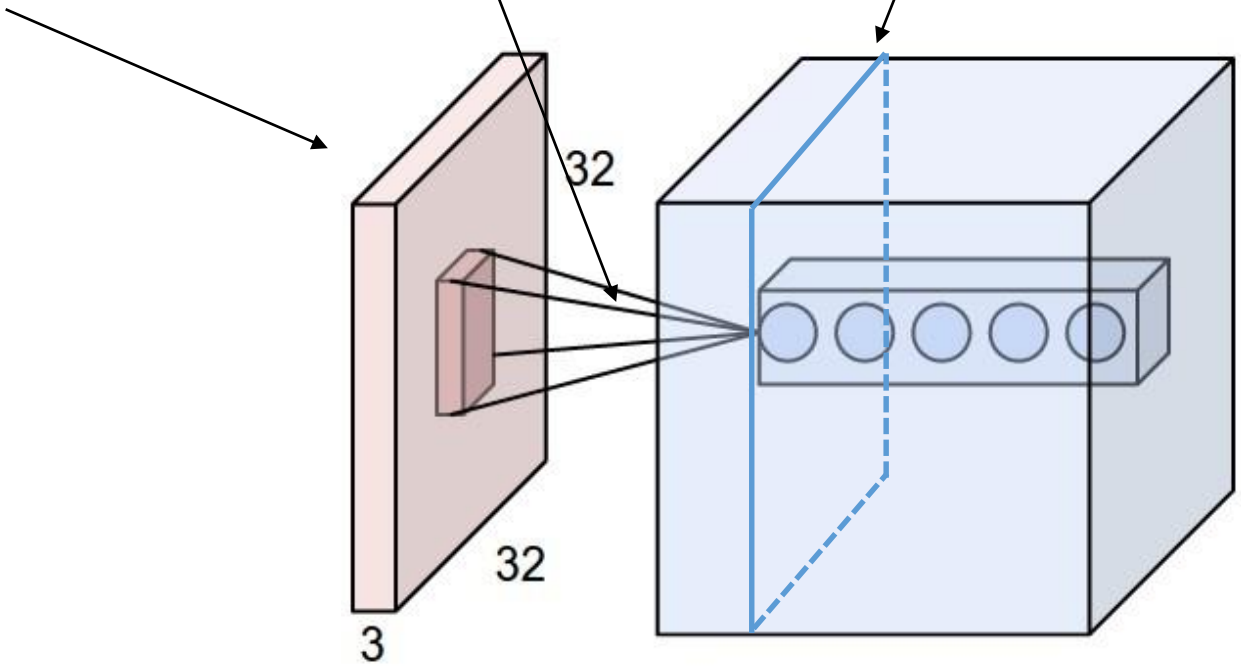
w^1



b^1

a^1





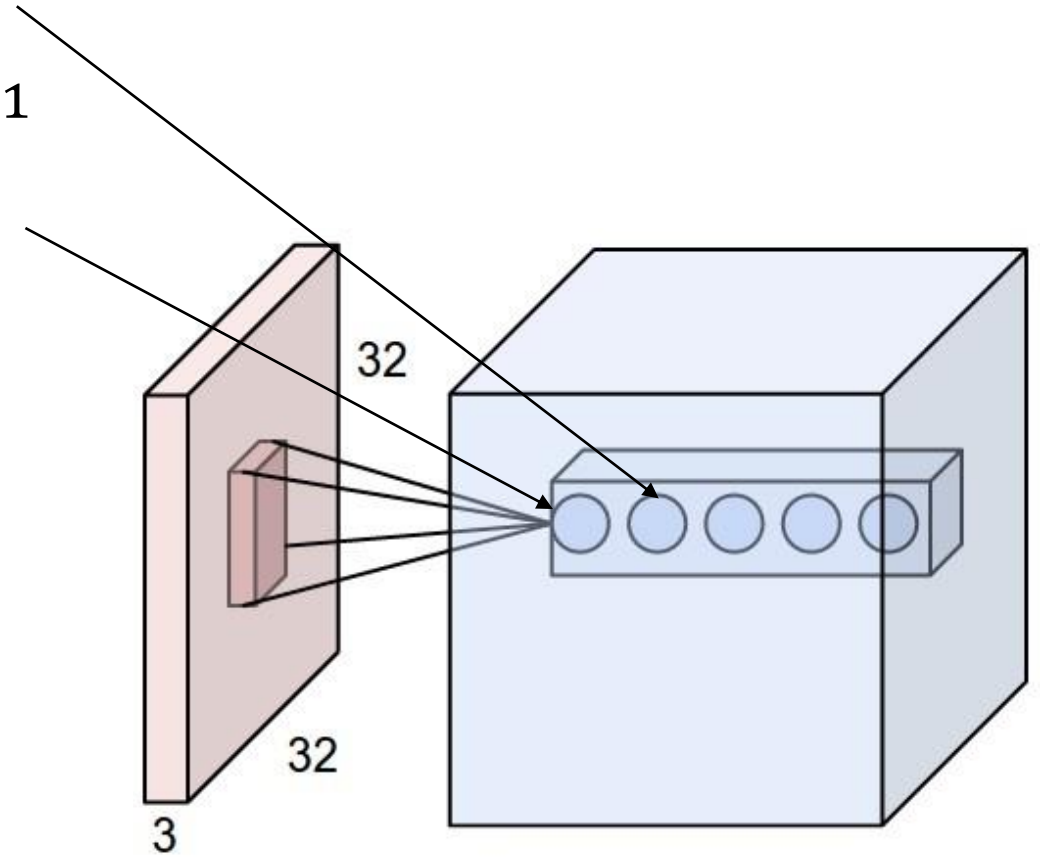
By Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45659236

Convolutional Layers

Different filters yield different layers in the output

$$\sum_{i,j} w^2_{i,j}\, x_{i,j} + b^2$$

$$\sum_{i,j} w^1_{i,j}\, x_{i,j} + b^1$$

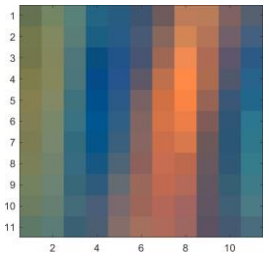


By Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45659236

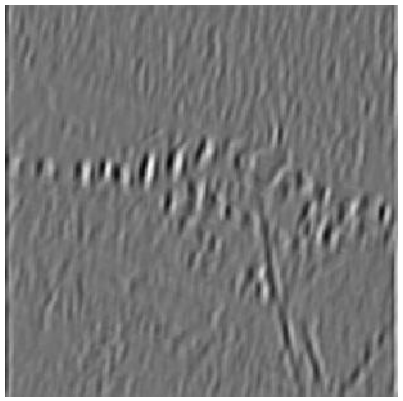
Convolutional Layers

I

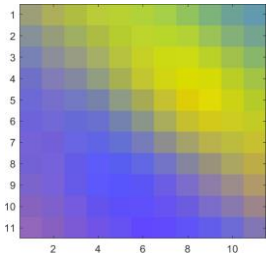
w^1, b^1



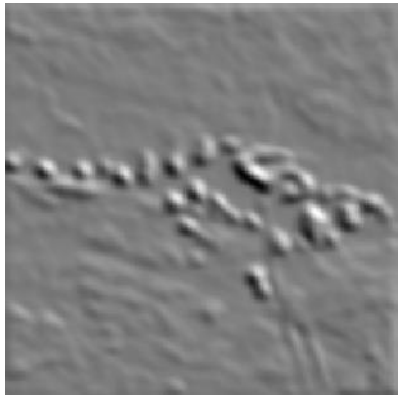
a^1

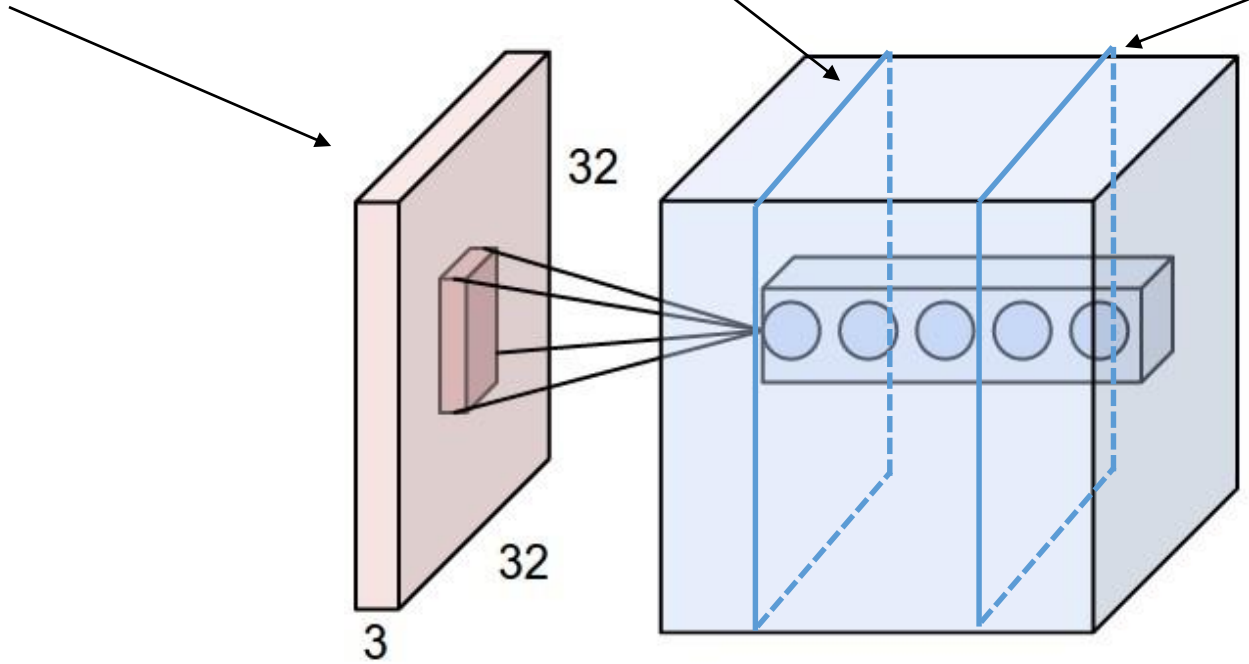


w^n, b^n



a^n



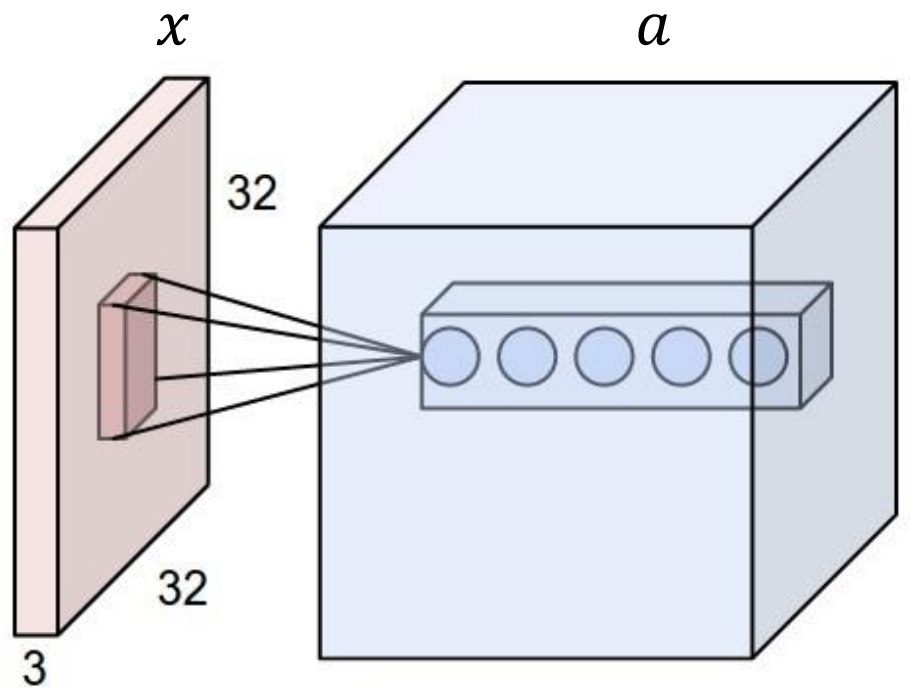


By Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45659236

Recap: Convolutional Layers

Convolutional layers "mix" all the input components

- The **output** is also called **volume** or **activation maps**
- Each **filter yields a different slice** of the output volume
- **Each filter has depth equal to the depth of the input volume**

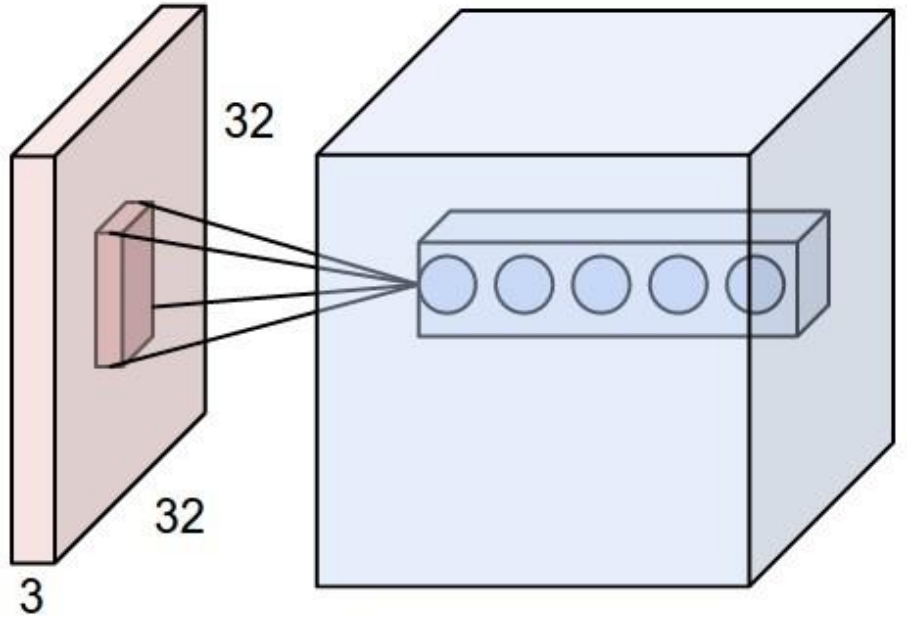


By Aphex34 - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=45659236

Convolutional Layers, remarks:

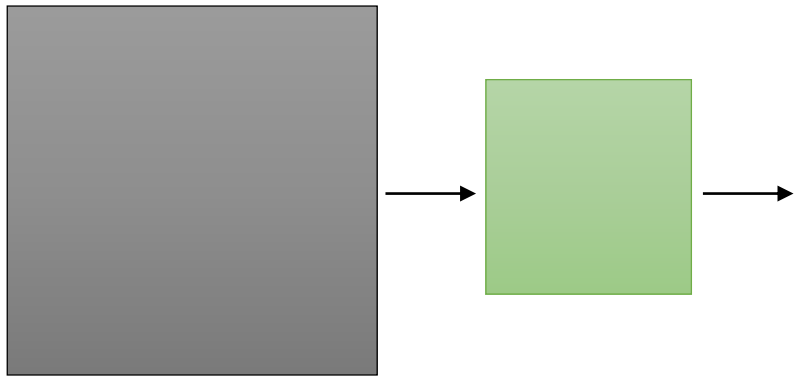
- Convolutional Layers are described by a set of **filters**
- **Filters represent the weights of these linear combination.**
- Filters have **very small spatial extent and large depth extent,**
- The **filter depth** is typically not specified, as it corresponds to the **number of layers of the input volume**

The output of the convolution
against a filter becomes a slice
in the volume feed to the next layer



By Aphex34 - Own work, CC BY-SA 4.0,
https://commons.wikimedia.org/w/index.php?curid=45659236

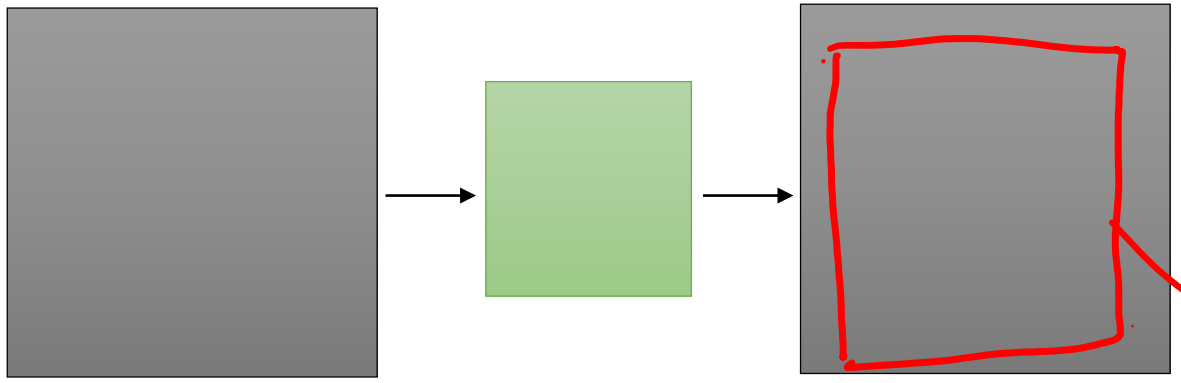
CNN Arithmetic



1 input map

1 filter 5x5

CNN Arithmetic



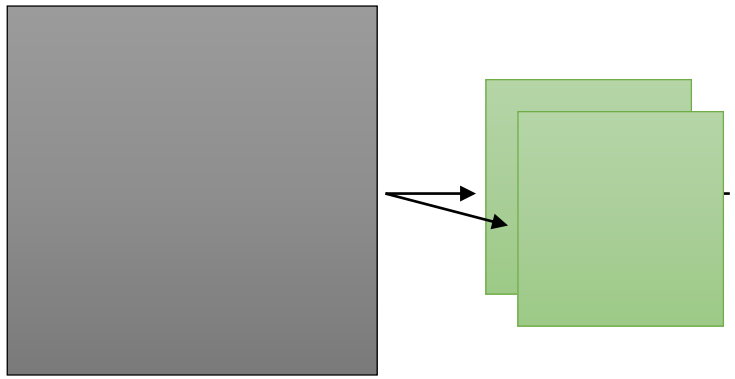
1 input map

1 filter 5x5

1 output map

In case of padding "valid"

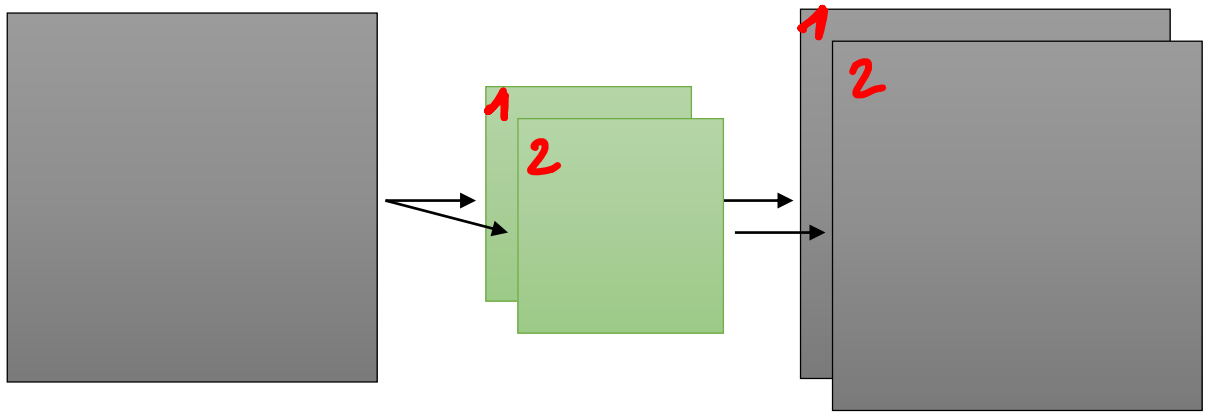
CNN Arithmetic



1 input map

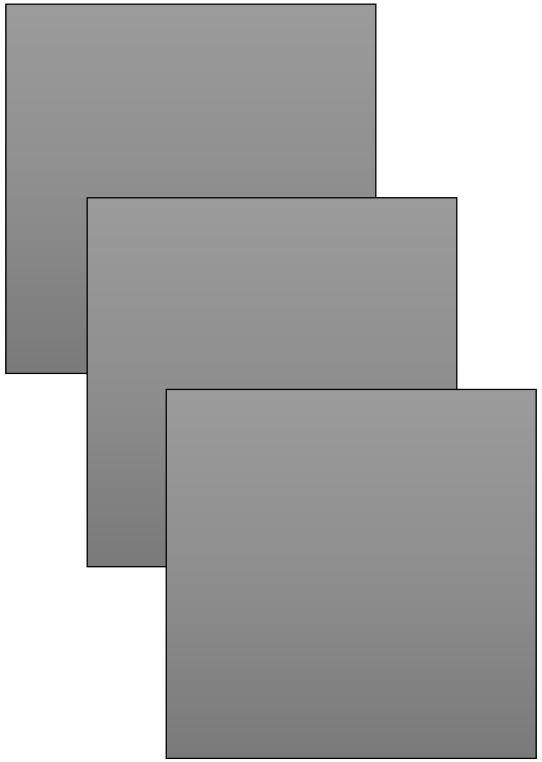
2 filter 5x5

CNN Arithmetic



1 input map 2 filter 5x5 2 output maps

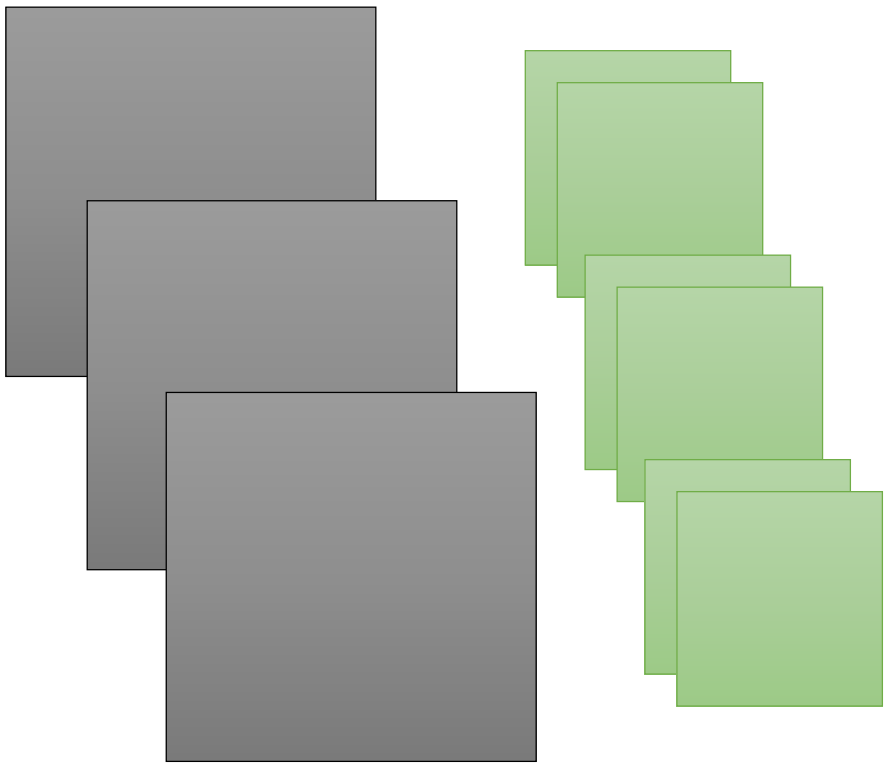
CNN Arithmetic



3 input maps

2 filters 5x5

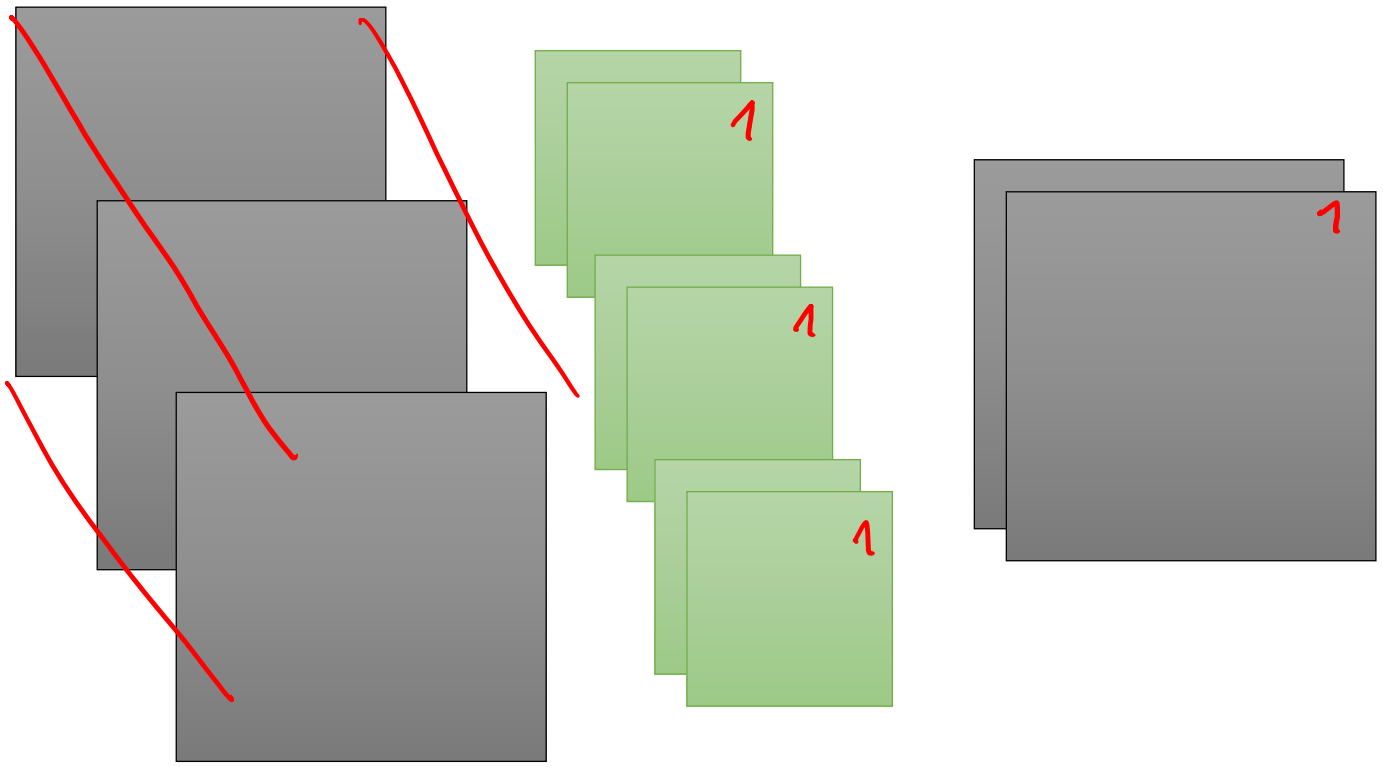
CNN Arithmetic



3 input maps

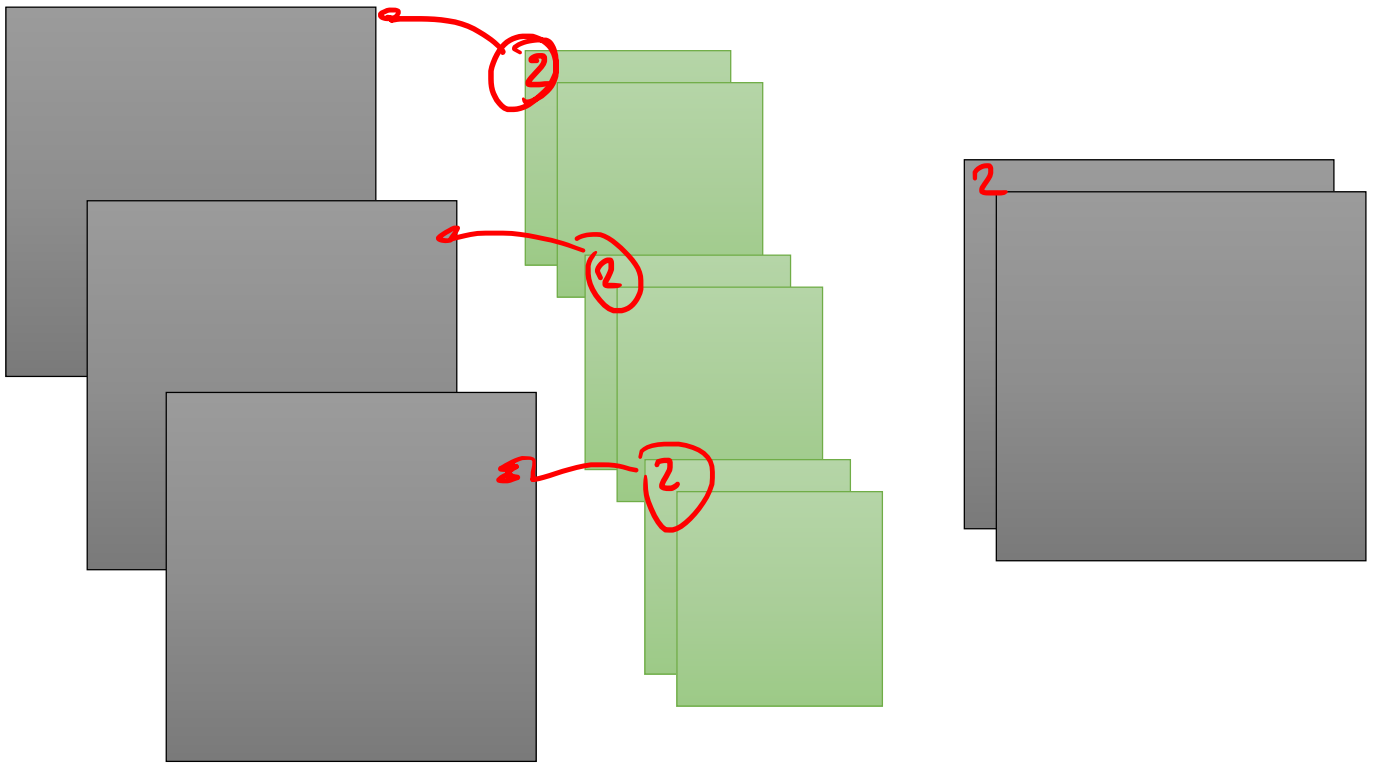
2 filters 5x5 (each filter has depth = 3)

CNN Arithmetic



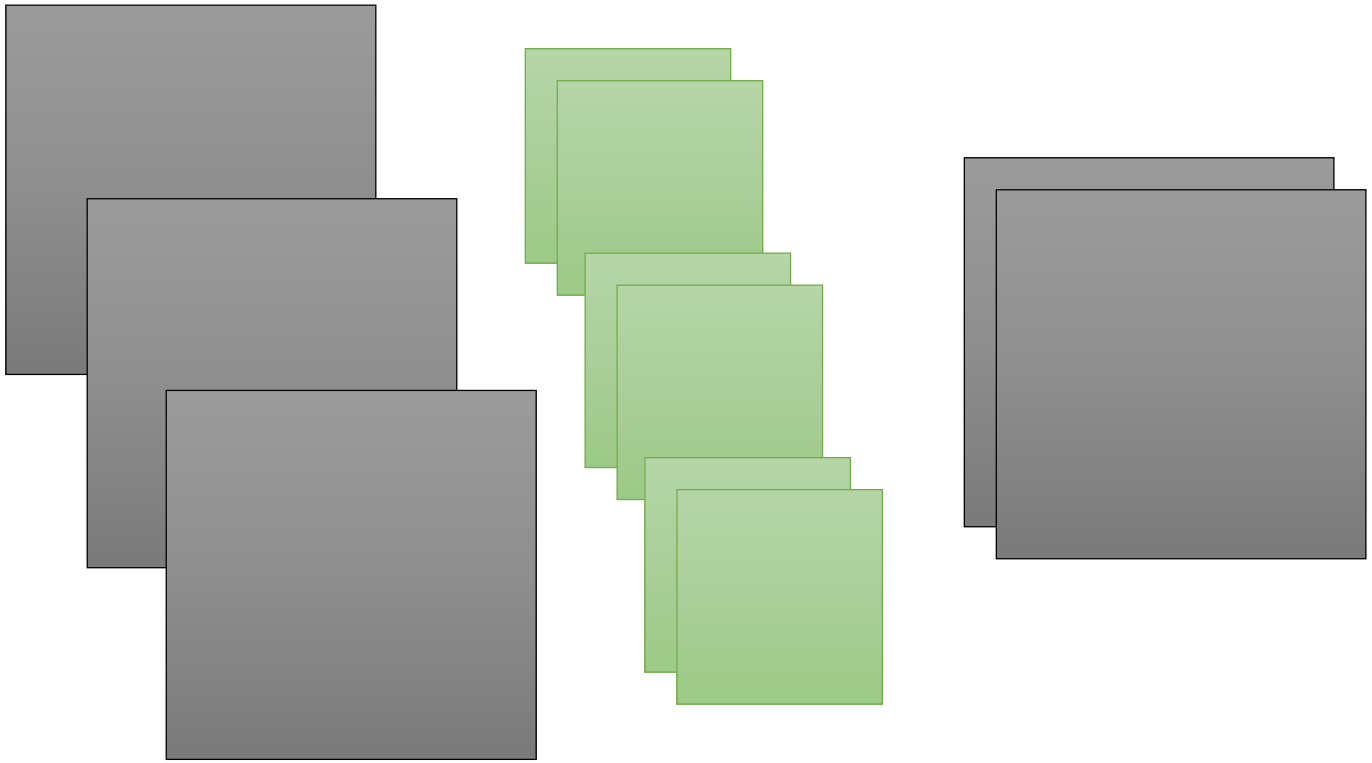
3 input maps 2 filters 5x5 2 output maps

CNN Arithmetic



3 input maps 2 filters 5x5 2 output maps

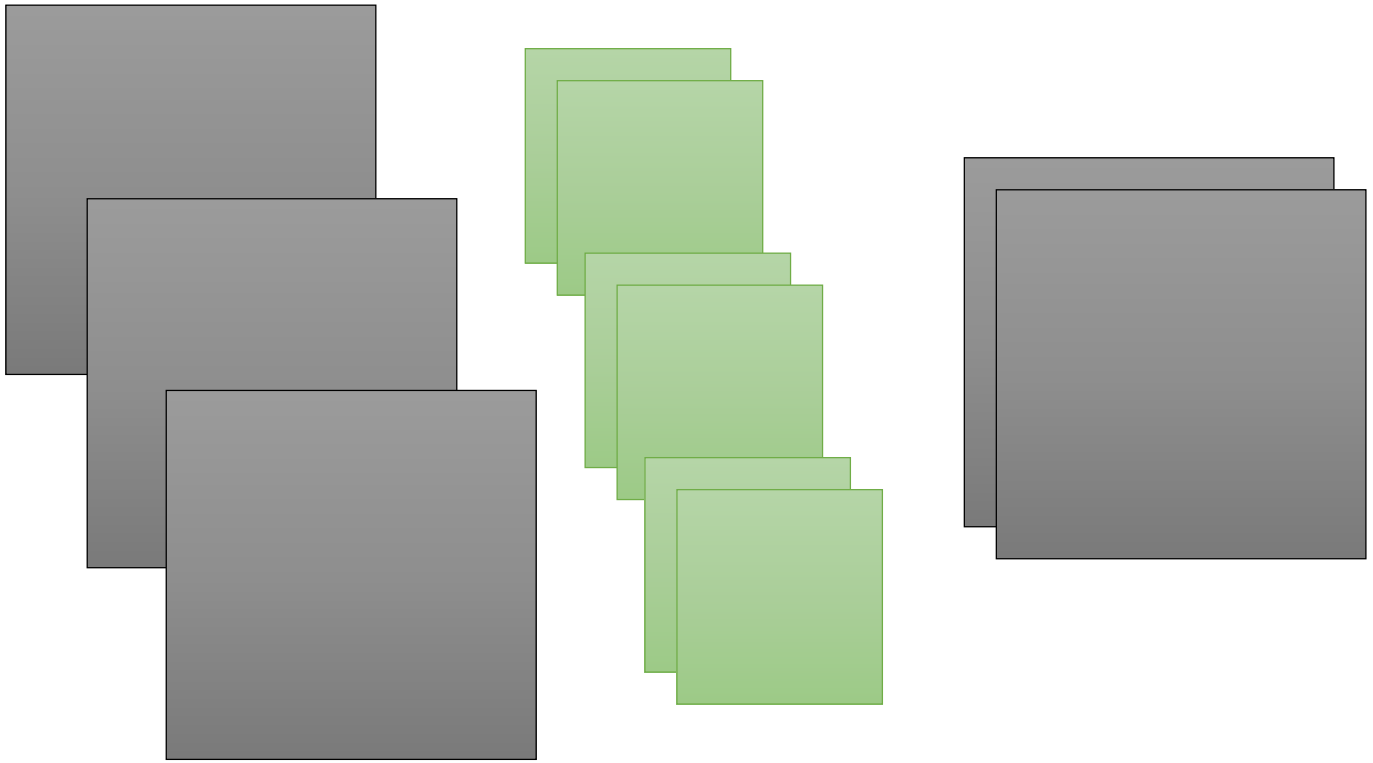
CNN Arithmetic



3 input maps 2 filters 5x5 2 output maps

Quiz: how many parameters
does this layer have?

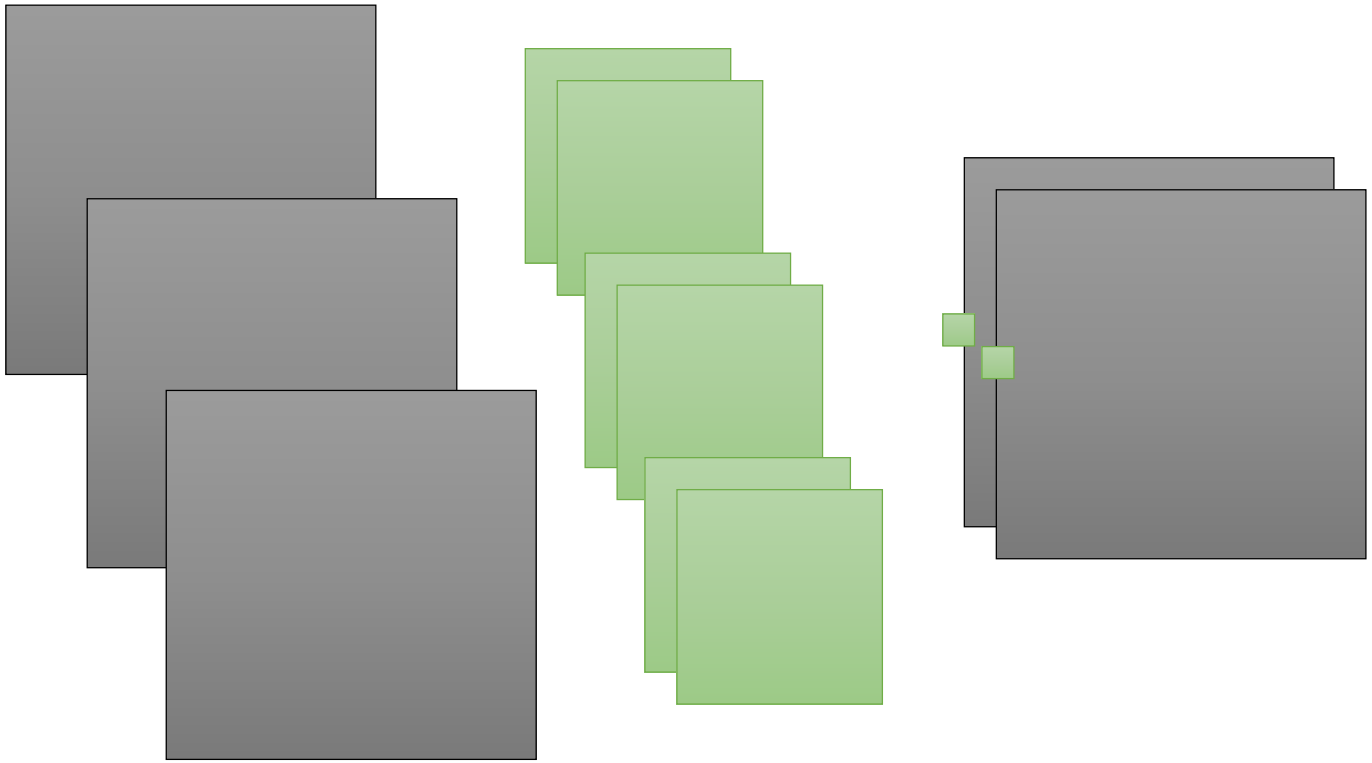
CNN Arithmetic



3 input maps 2 filters 5x5 2 output maps

= 150 parameters in the filters

CNN Arithmetic

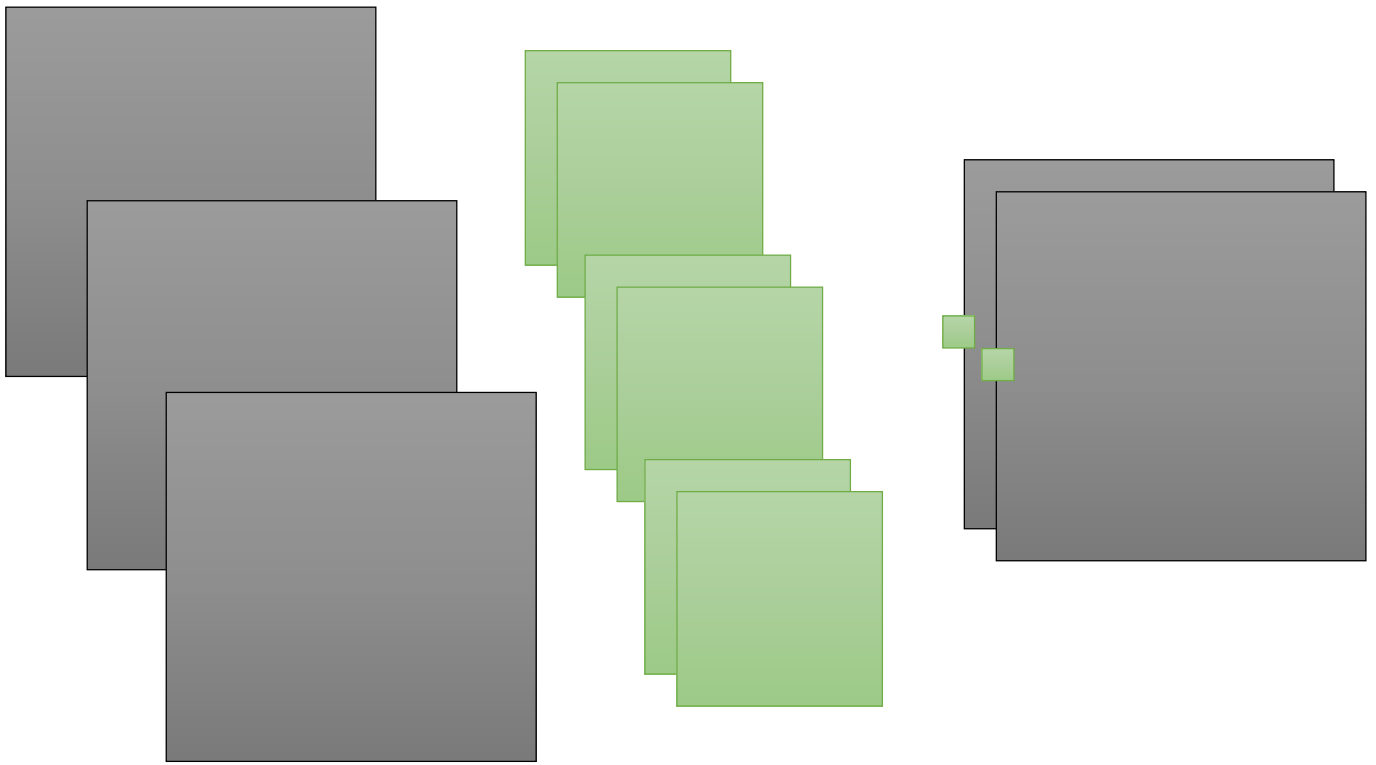


3 input maps

2 filters 5x5
= 150 ...

2 output maps
+ 2 biases

CNN Arithmetic



output maps = # of filters in the layer

3 input maps

2 filters 5x5

= 150 ...

2 output maps

+ 2 biases

= 152 trainable parameters (weights)

To Recap...

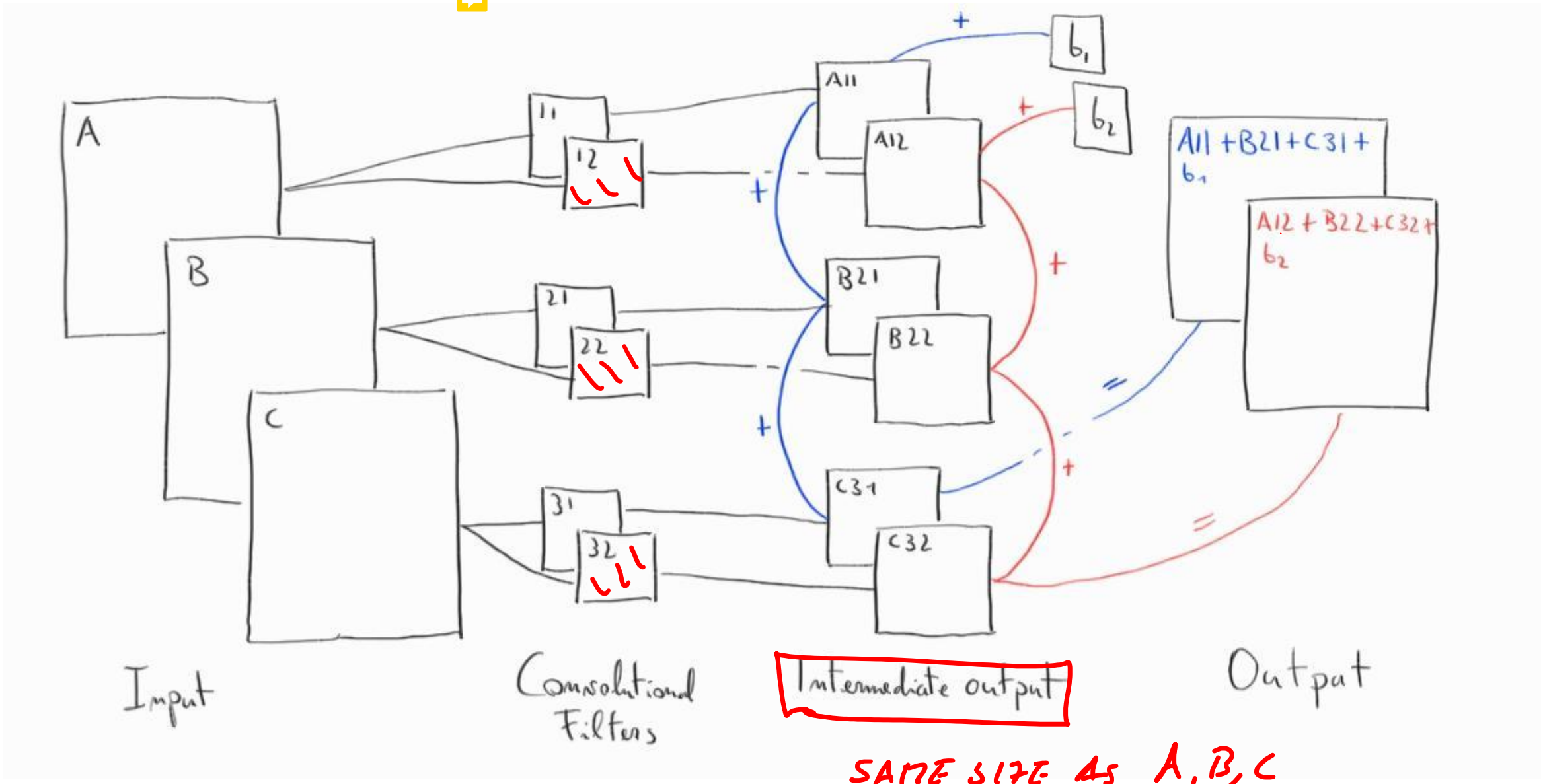


Image credits Nicolò Ghielmetti

Other Layers

Activation and Pooling

Giacomo Boracchi

Activation Layers

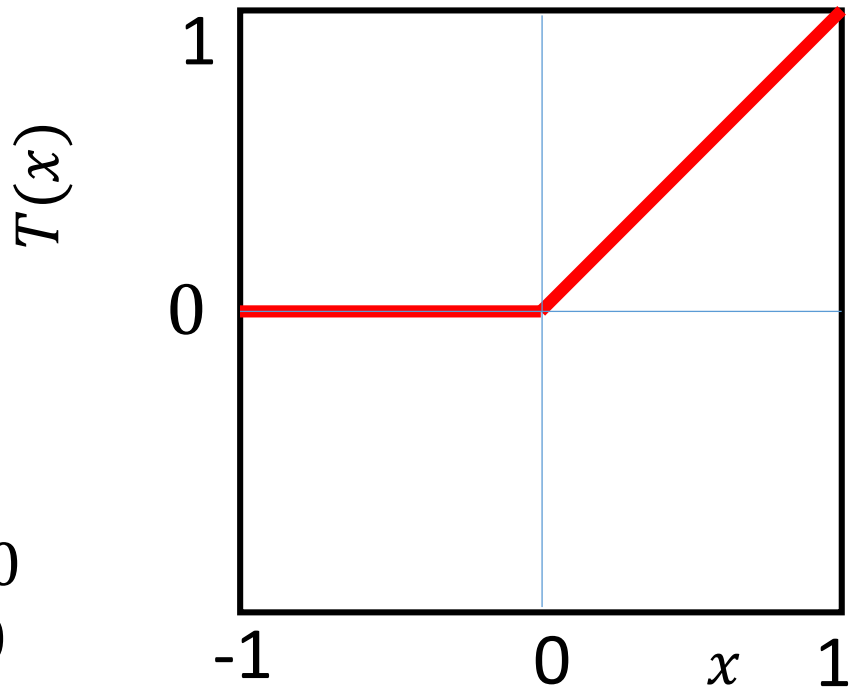
Introduce nonlinearities in the network, otherwise the CNN might be equivalent to a linear classifier…

Activation functions are scalar functions, namely they operate on each single value of the volume. **Activations don't change volume size**

RELU (Rectifier Linear Units): it's a thresholding on the feature maps, i.e., a $\max(0,\cdot)$ operator.

- By far the most popular activation function in deep NN (since when it has been used in AlexNet)
- Dying neuron problem: a few neurons become insensitive to the input (vanishing grandient problem)

$$T(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

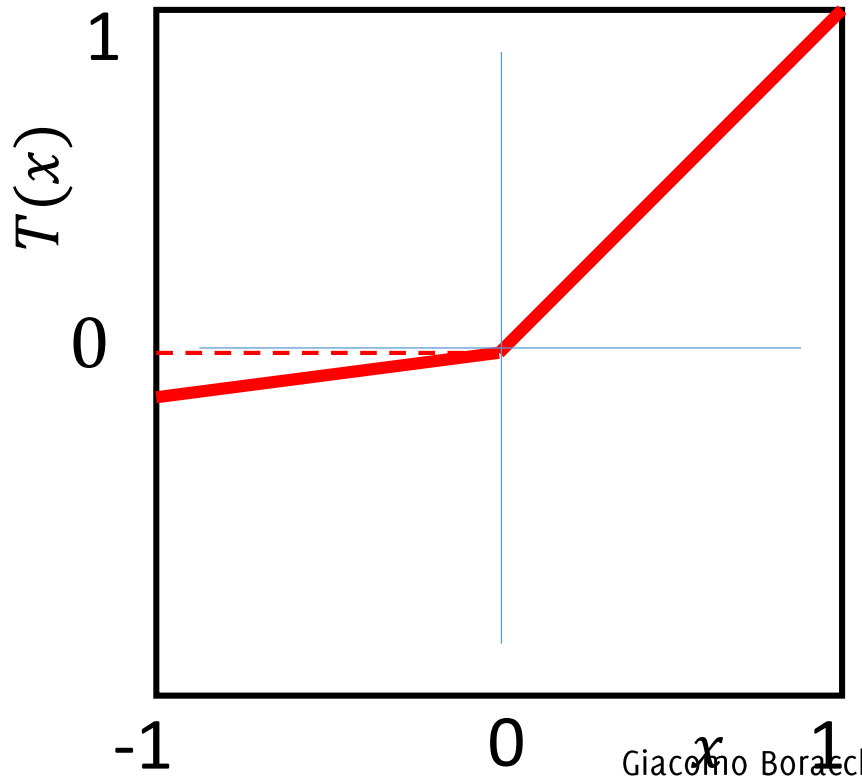


Activation Layers

Introduce nonlinearities in the network, otherwise the CNN might be equivalent to a linear classifier…

LEAKY RELU: like the relu but include a small slope for negative values

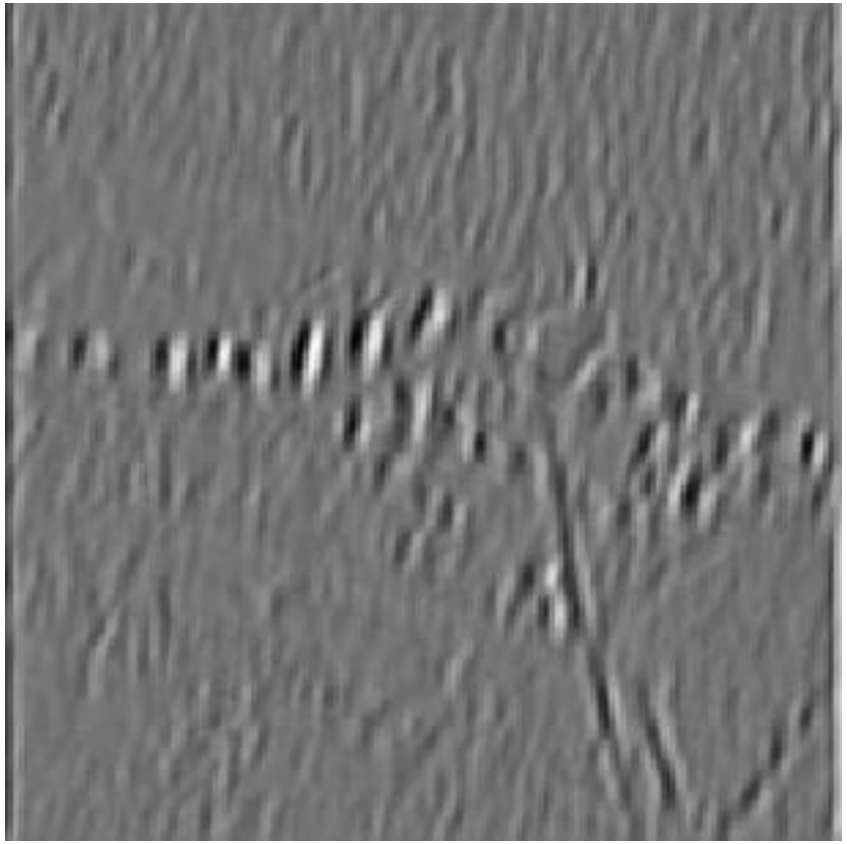
$$T(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 0.01 * x & \text{if } x < 0 \end{cases}$$

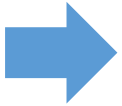


ReLu

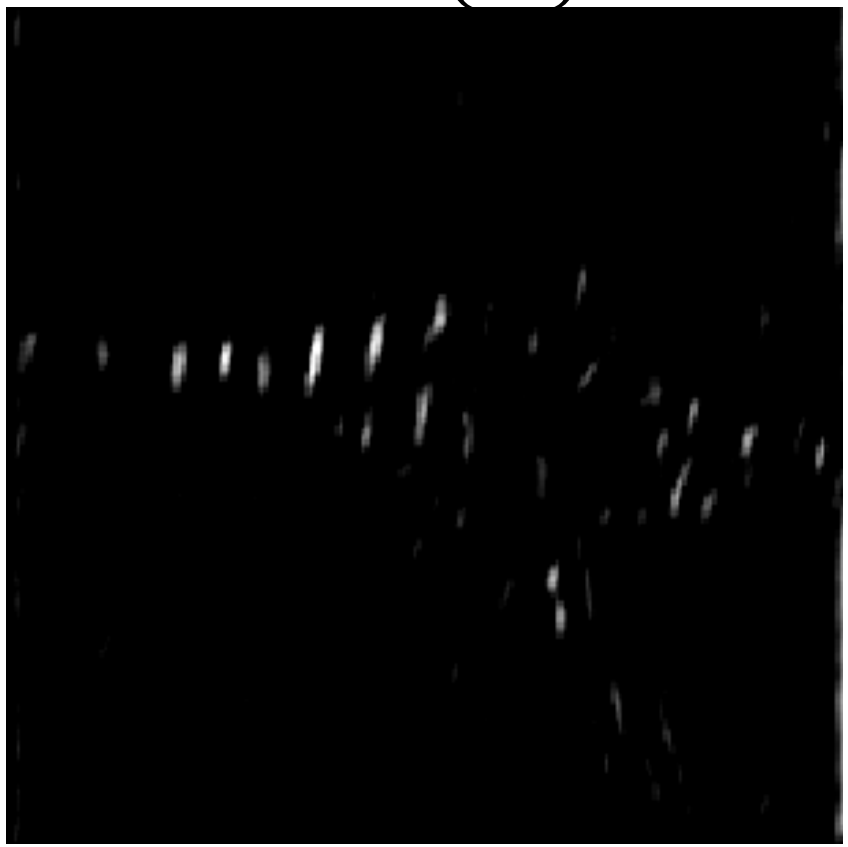
Acts separately on each layer

a^1





$ReLU(a^1)$

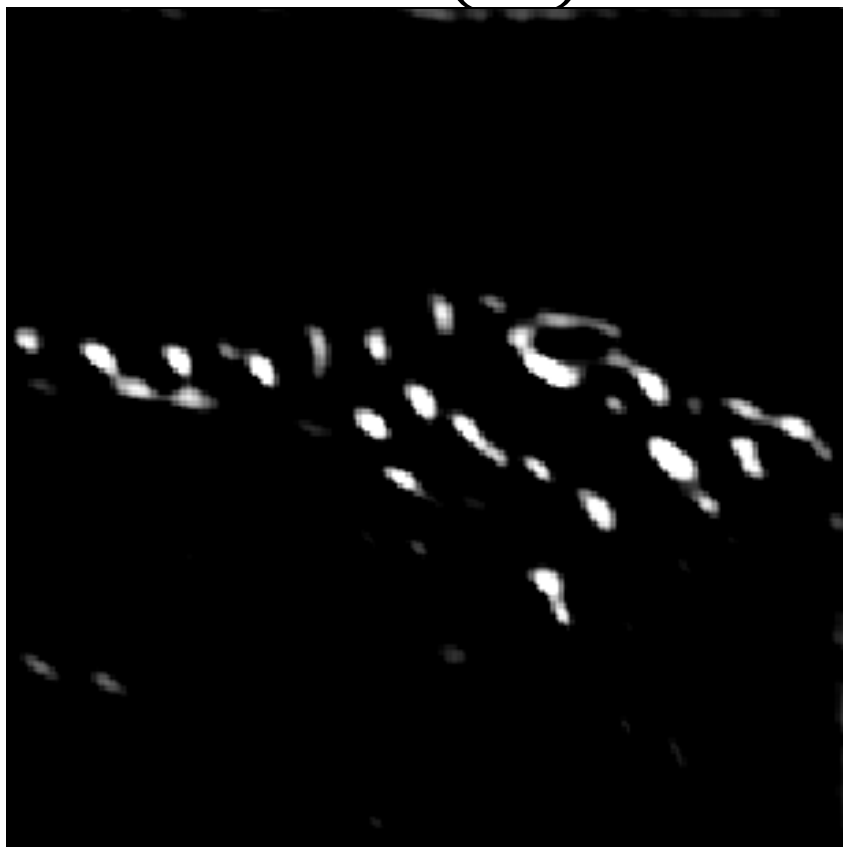


ReLu

Acts separately on each layer

a^2

$ReLU(a^2)$



Activation Layers

Introduce nonlinearities in the network, otherwise the CNN might be equivalent to a linear classifier…

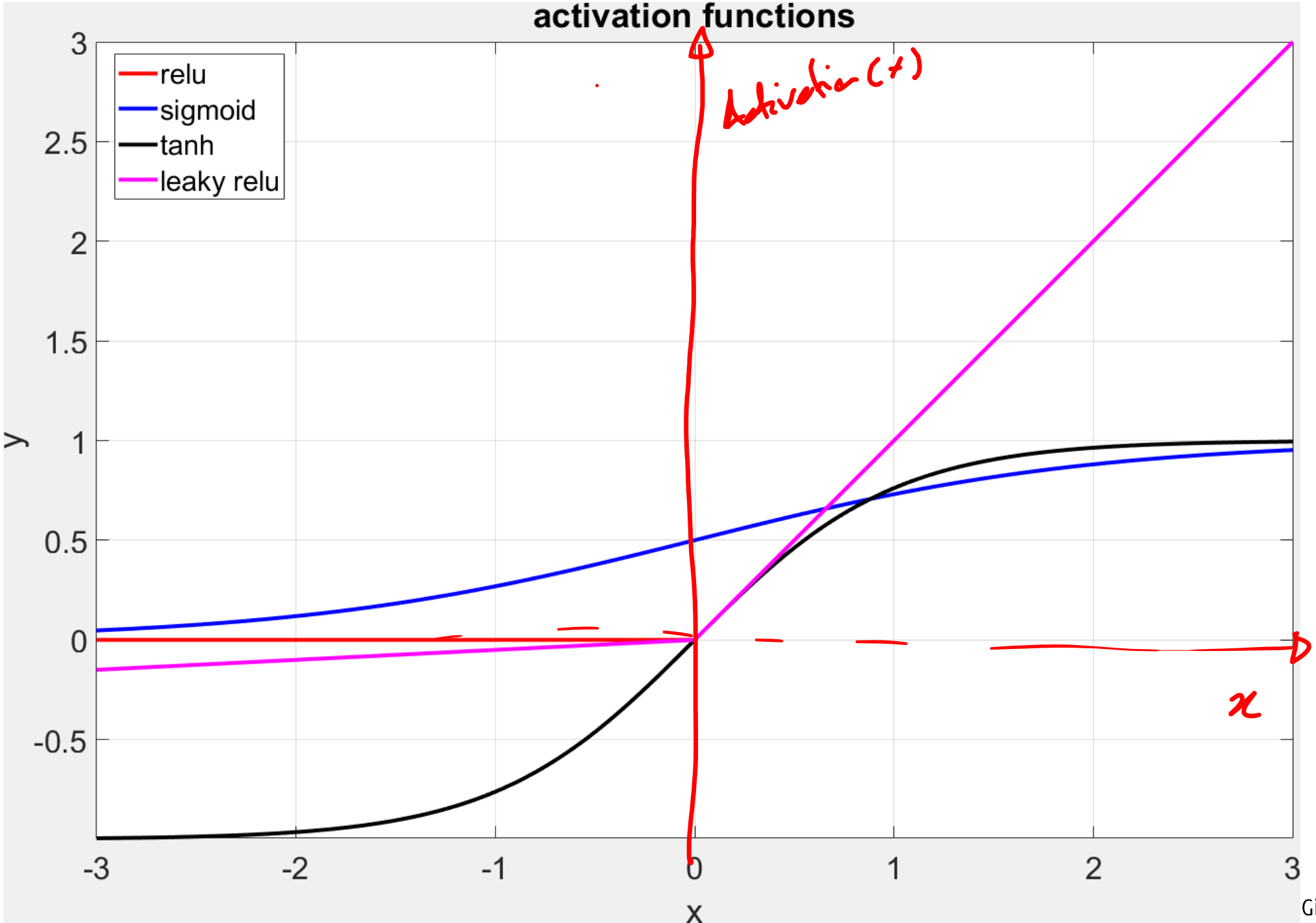
TANH (hyperbolic Tangent): has a range (-1,1), continuous and differentiable

$$T(x) = \frac{2}{1 + e^{-2x}} - 1$$

SIGMOID: has a range (0,1), continuous and differentiable

$$S(x) = \frac{1}{1 + e^{-2x}}$$

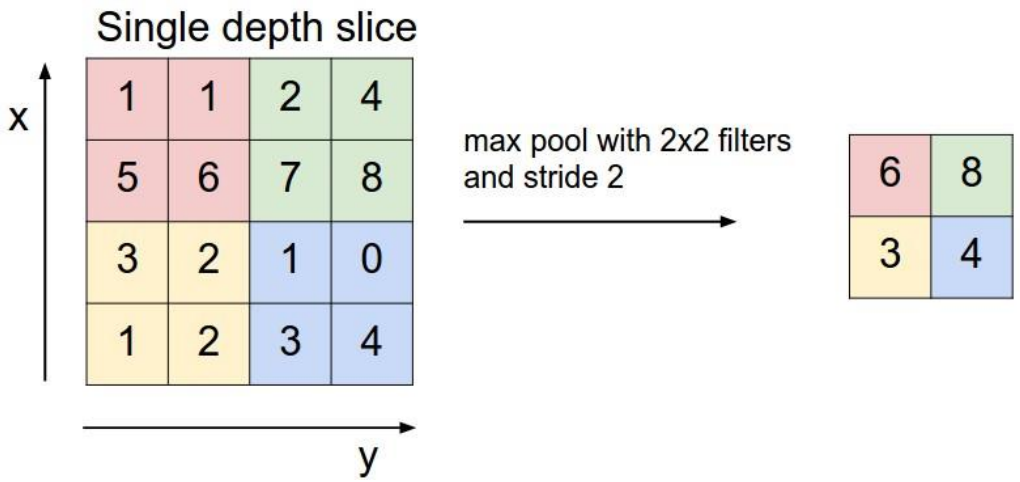
These activation functions are mostly popular **in MLP architectures**

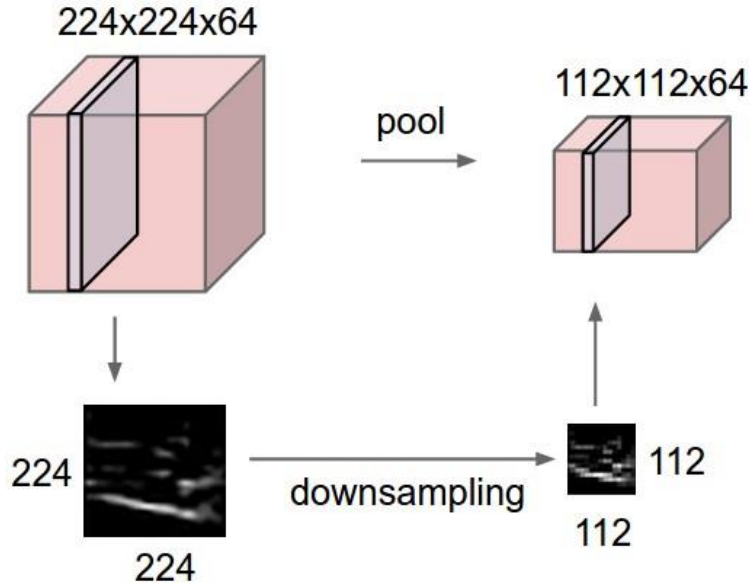

activation functions
relu
sigmoid
tanh
leaky relu
activation(x)
x
y
3
2.5
2
1.5
1
0.5
0
-0.5
-3
-2
-1
0
1
2
3

Pooling Layers

Pooling Layers reduce the **spatial** size of the volume.

The Pooling Layer operates **independently on every depth slice** of the input and **resizes it spatially**, **often using the MAX operation**.



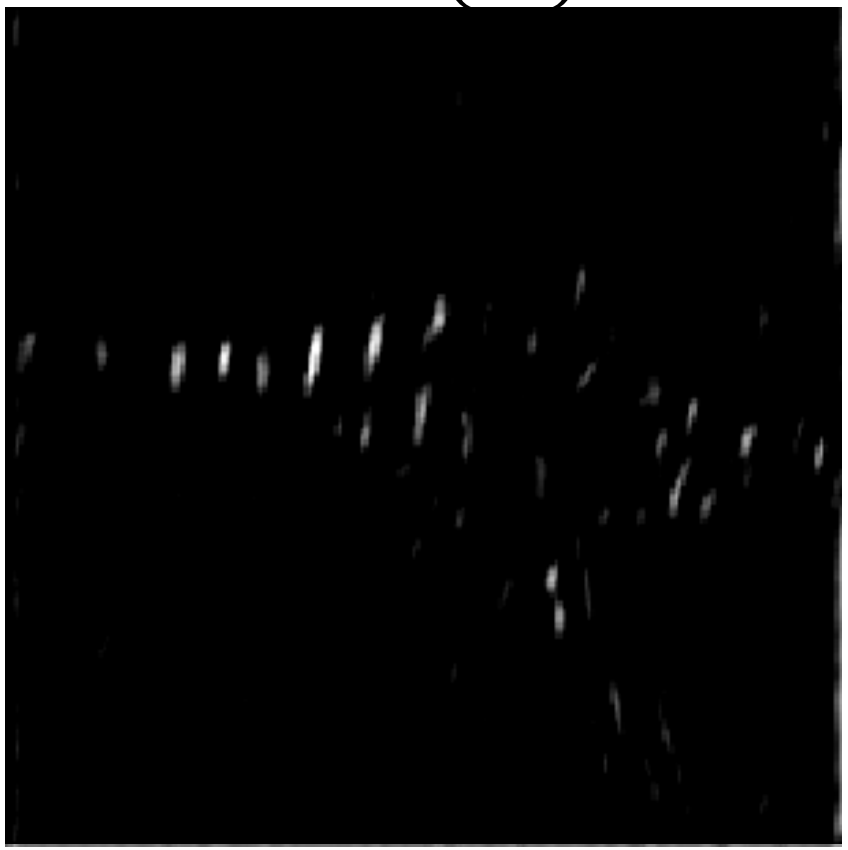


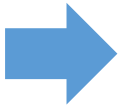
In a 2x2 support it discards 75% of samples in a volume

Max-Pooling (MP)

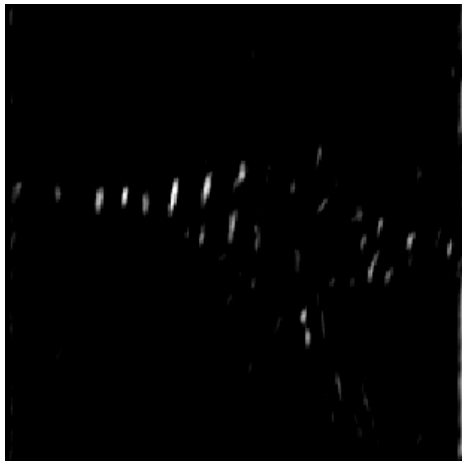
Acts separately on each layer

$ReLU(a^1)$





$\mathrm{MP}(ReLU(a^1))$

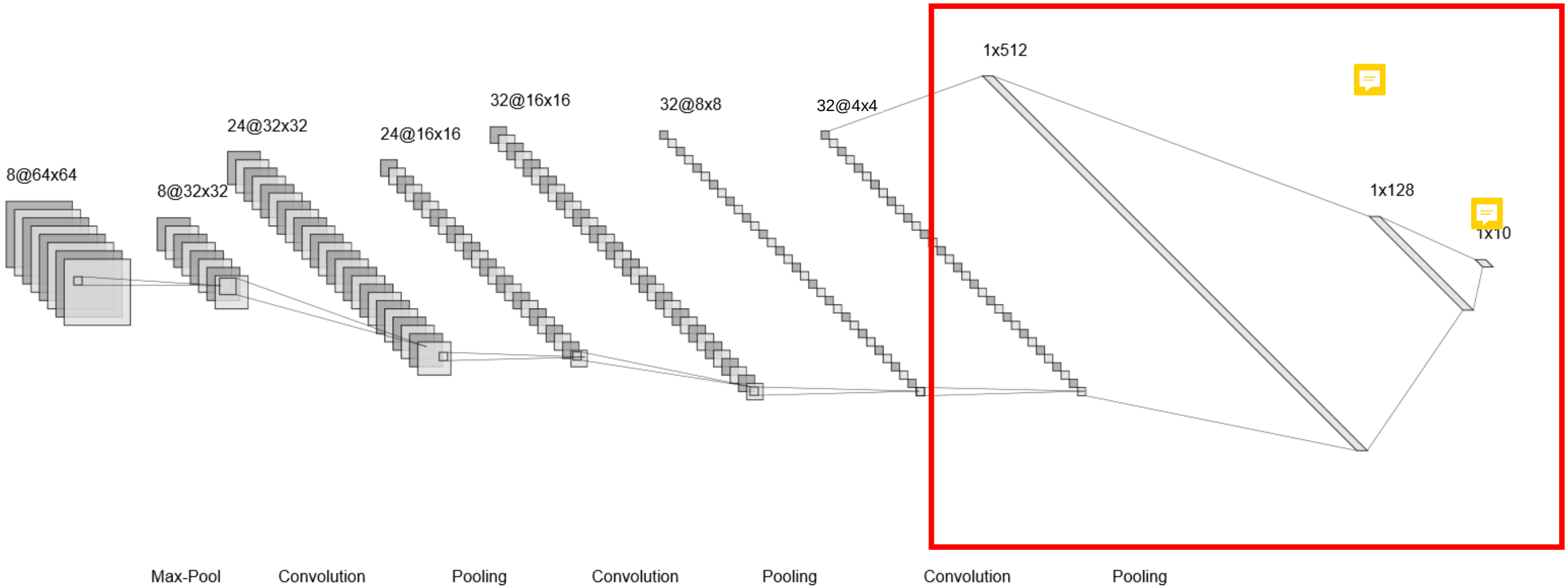


Dense Layers

As in feed-forward NN

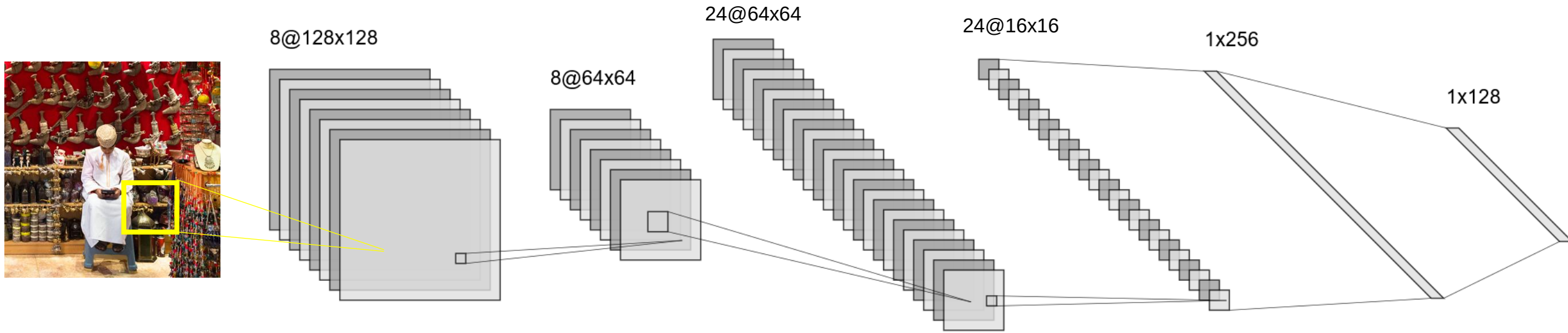
The Dense Layers

Here the spatial dimension is lost, the CNN stacks hidden layers from a MLP NN.
It is called Dense as each output neuron is connected to each input neuron



Convolutional Neural Networks (CNN)

The typical architecture of a convolutional neural network



Assume zero padding

Convolution: There are 8 filters

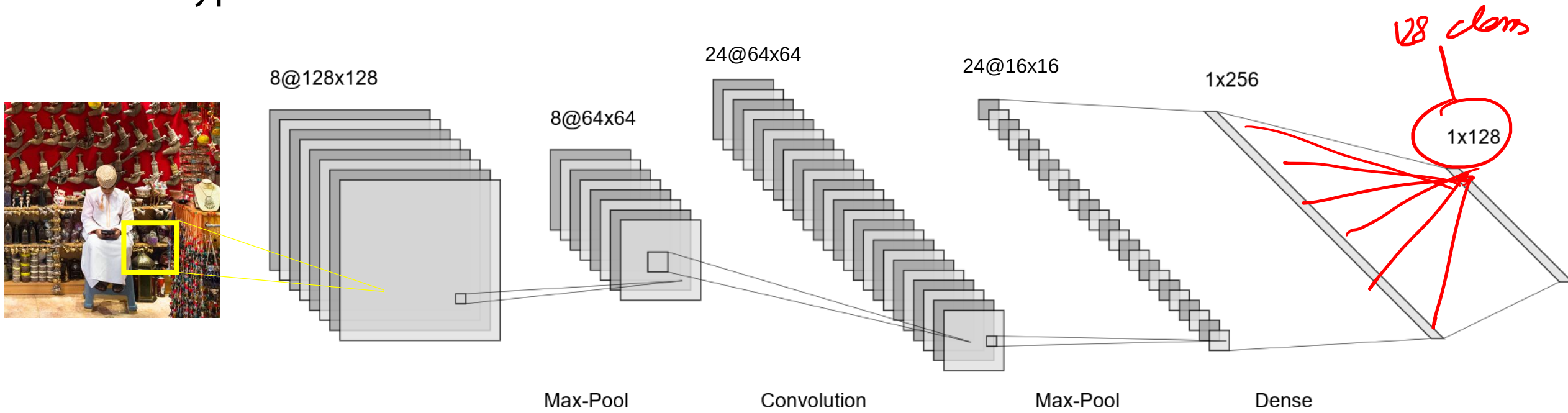
Max-pooling reduces spatial dimensions by 2

Convolution: There are 24 filters

LeCun, Y., Bottou, L., Bengio, Y., Haffner, P. *"Gradient-based learning applied to document recognition"* Proceedings of the IEEE, 1998 86(11), 2278-2324.

Convolutional Neural Networks (CNN)

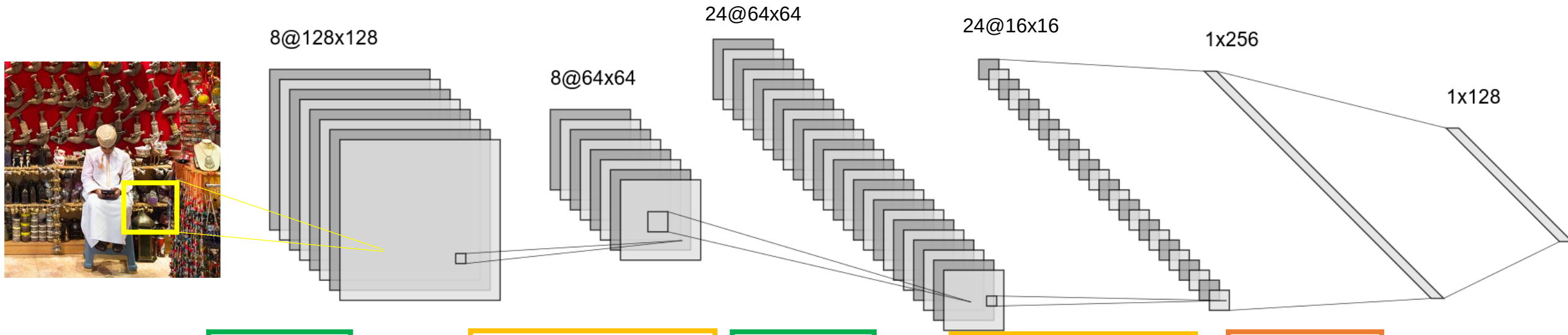
The typical architecture of a convolutional neural network



The output of the **fully connected (FC) layer** has the same size as the **number of classes**, and **provides a score** for the input image to belong to each class

LeCun, Y., Bottou, L., Bengio, Y., Haffner, P. *"Gradient-based learning applied to document recognition"* Proceedings of the IEEE, 1998 86(11), 2278-2324.

Convolutional Neural Networks (CNN)



Conv

Subsampling

Conv.

Subsampling

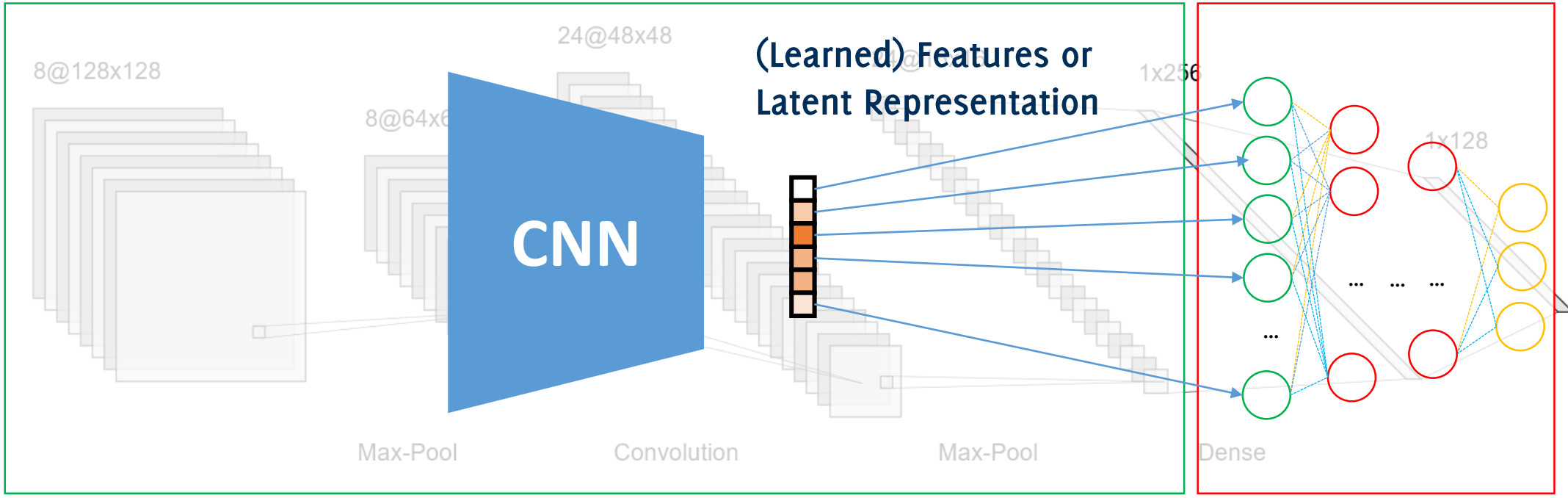
Dense

Convolution filters are learned for the classification task at hand

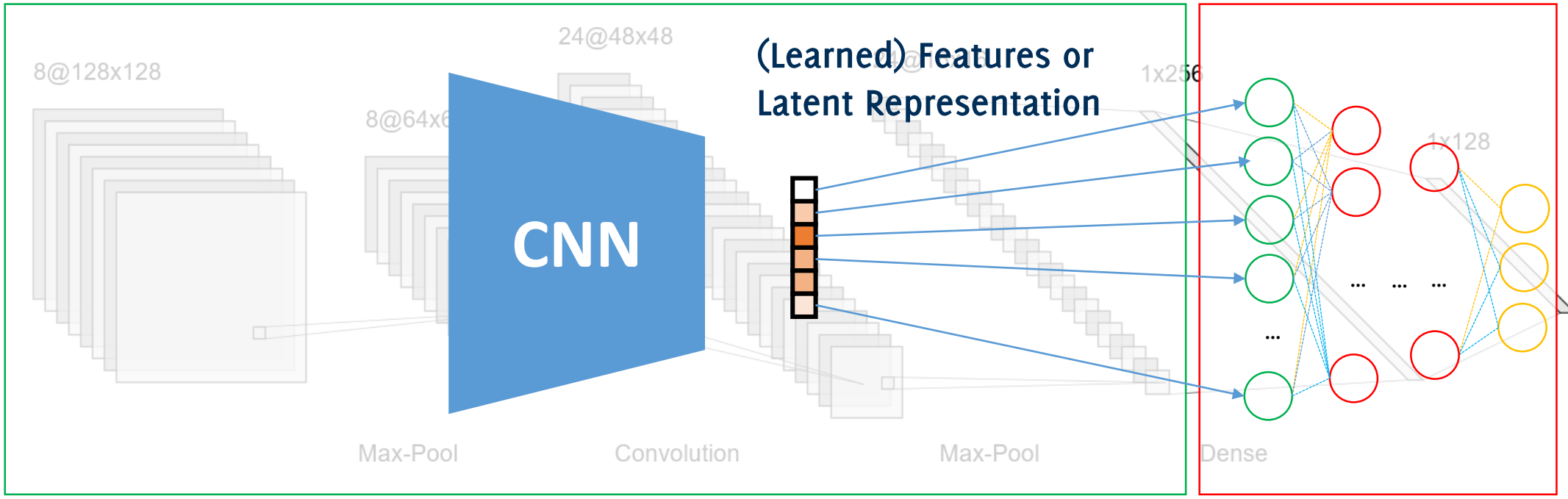
Thresholding + Downsampling (ReLu + Maxpooling)

Fully Connected Neural Network providing as output class probabilities

The typical architecture of a CNN



The typical architecture of a CNN



Data-Driven Feature extraction

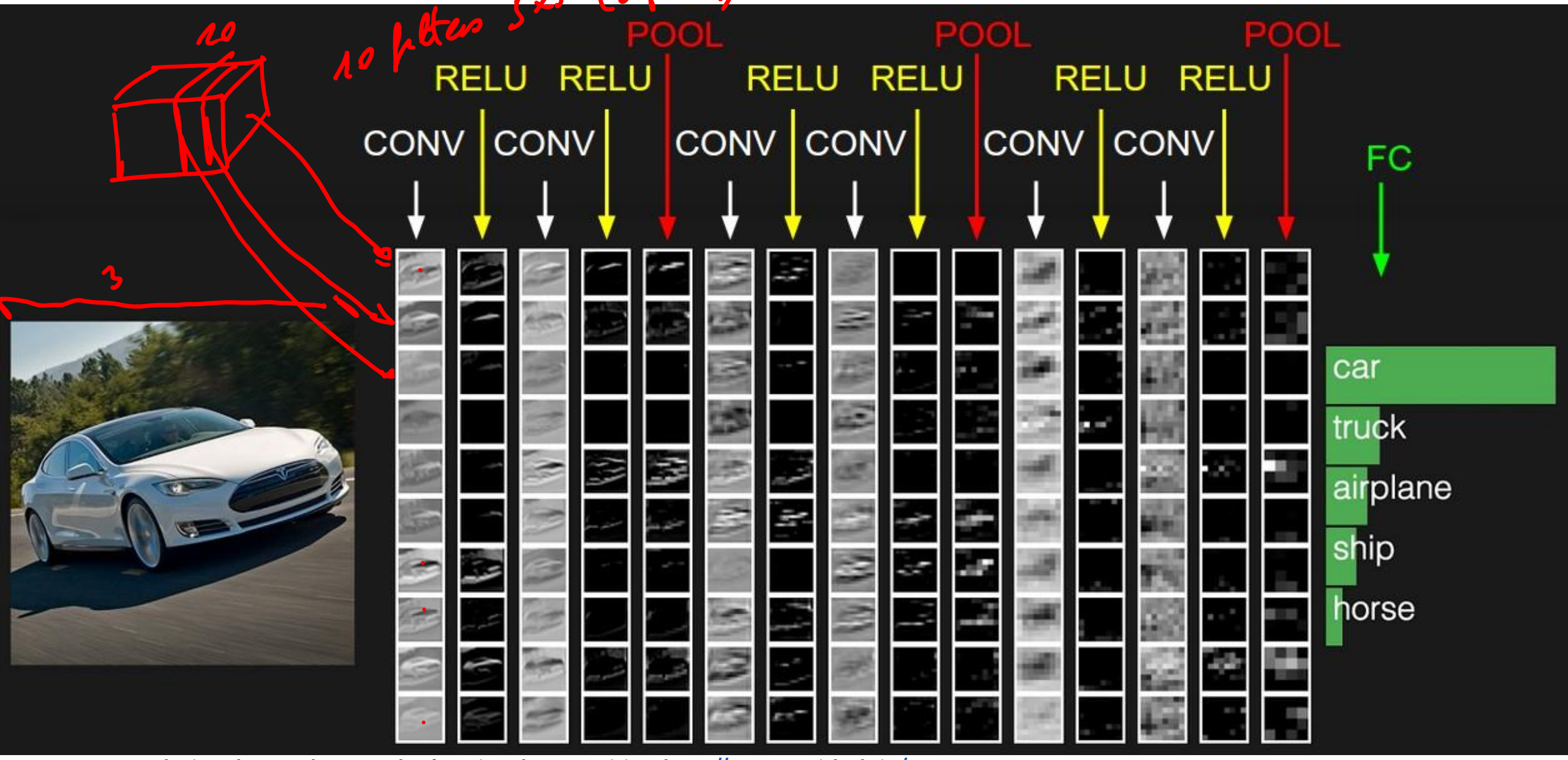
Feature Classification

Typically, to learn meaningful representations, many layers are required

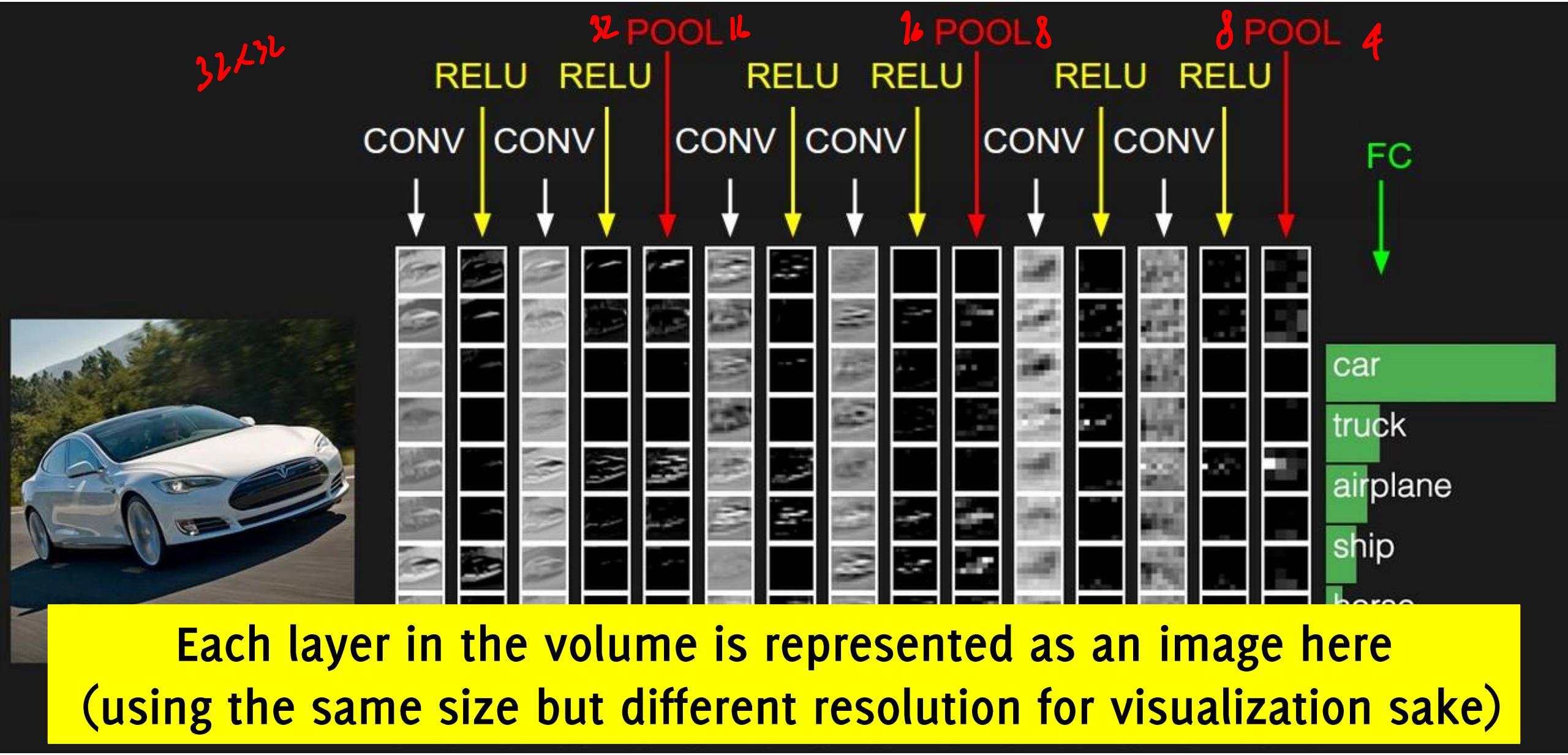
The network becomes deep

CNN «in action»

Activations in a convolutional network

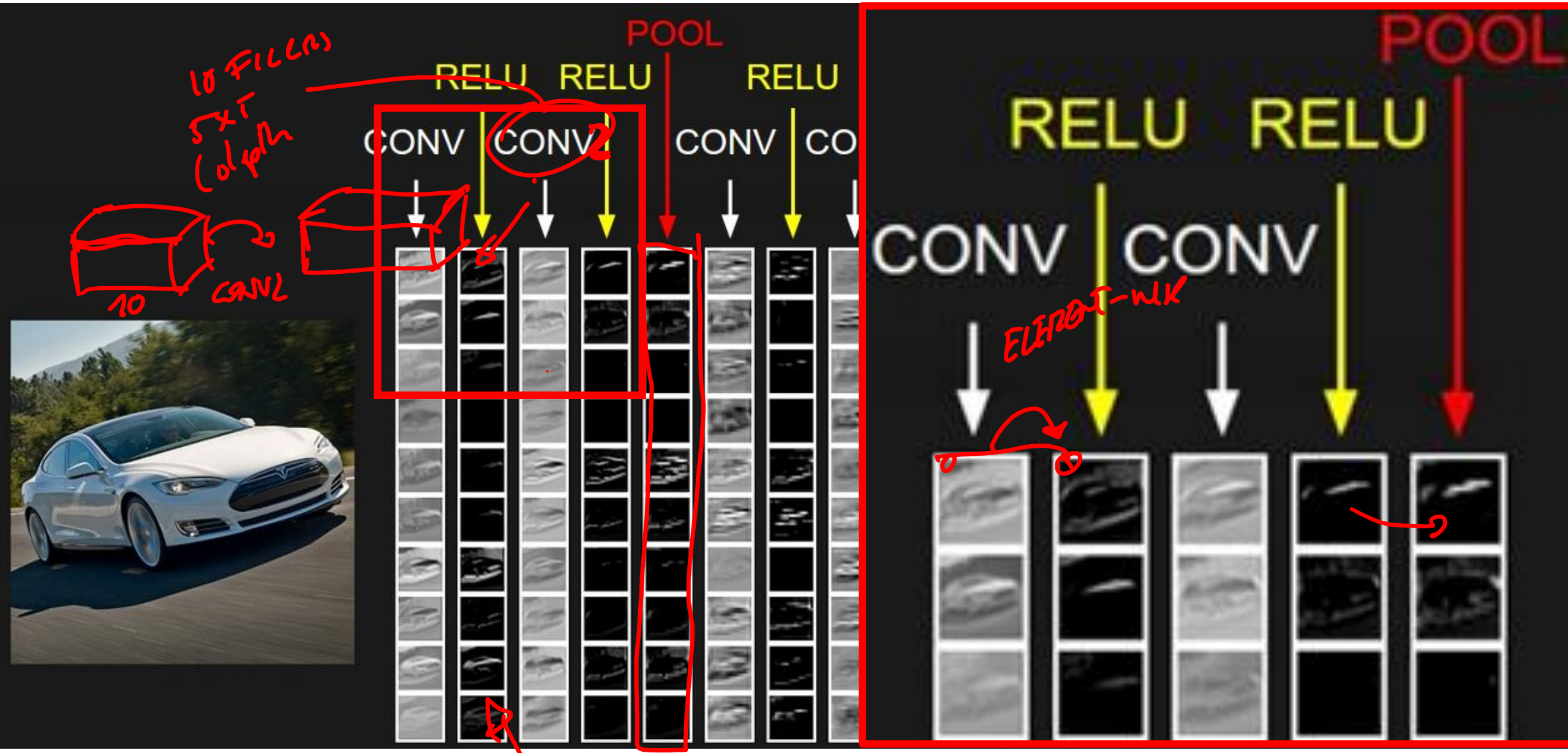


Activations in a convolutional network

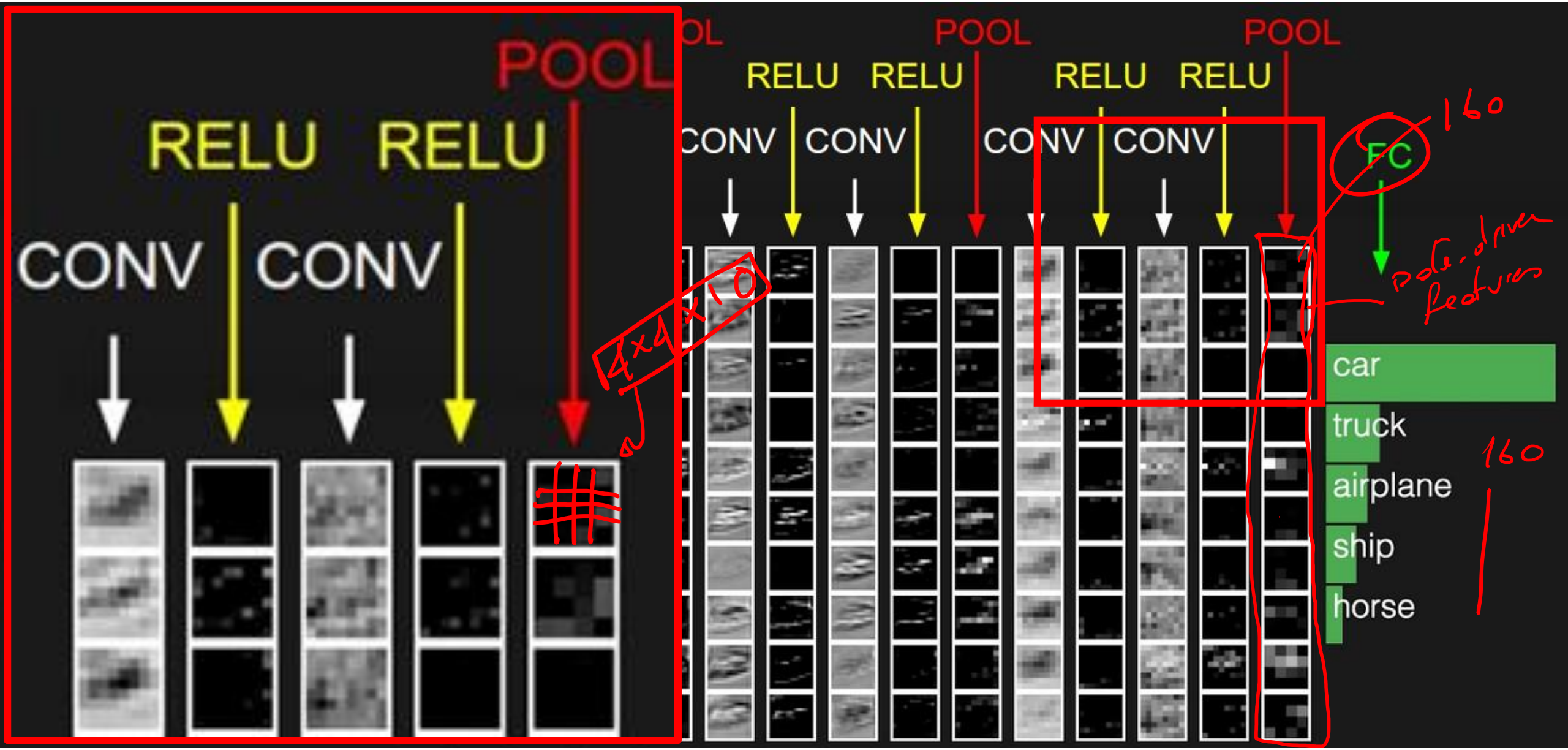


Each layer in the volume is represented as an image here (using the same size but different resolution for visualization sake)

Activations in a convolutional network

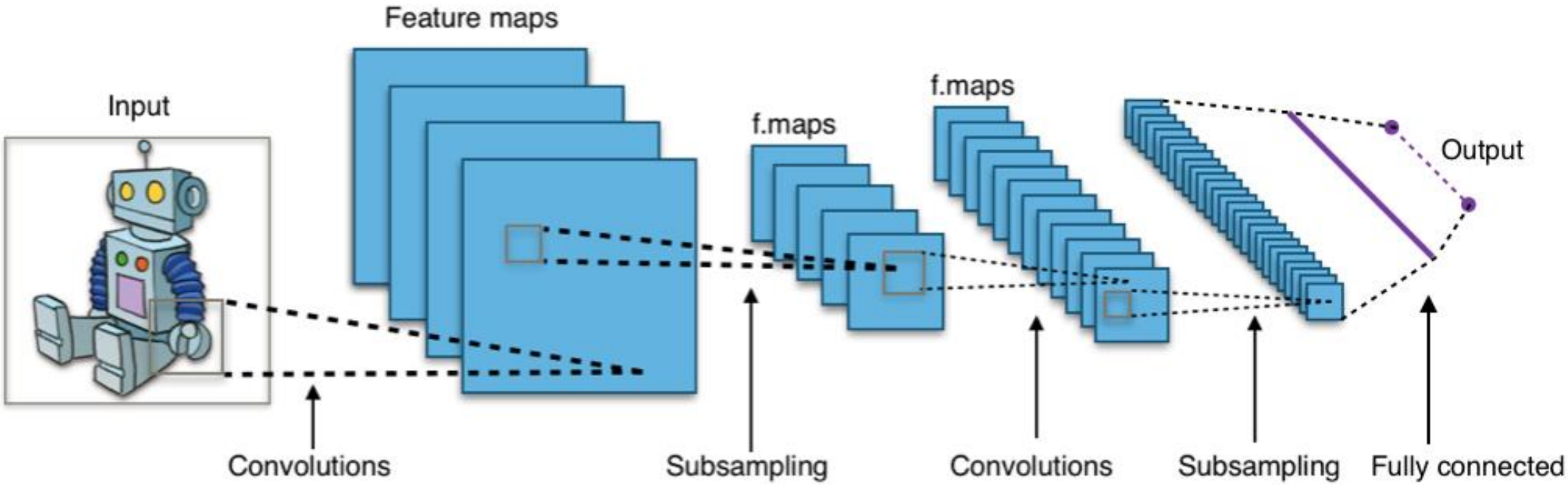


Activations in a convolutional network



Convolutional Neural Networks (CNN)

Btw, this figure contains an error.
If you are CNN-Pro, you should spot it!



By Aphex34 - Own work, CC BY-SA 4.0,
https://commons.wikimedia.org/w/index.php?curid=45679374

The First CNN

Giacomo Boracchi

Gradient-Based Learning Applied to Document Recognition

Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner

Abstract—
Multilayer Neural Networks trained with the backpropagation algorithm constitute the best example of a successful Gradient-Based Learning technique. Given an appropriate network architecture, Gradient-Based Learning algorithms can be used to synthesize a complex decision surface that can classify high-dimensional patterns such as handwritten characters, with minimal preprocessing. This paper reviews various methods applied to handwritten character recognition and compares them on a standard handwritten digit recognition task. Convolutional Neural Networks, that are specifically designed to deal with the variability of 2D shapes, are shown to outperform all other techniques.

I. Introduction

Over the last several years, machine learning techniques, particularly when applied to neural networks, have played an increasingly important role in the design of pattern recognition systems. In fact, it could be argued that the availability of learning techniques has been a crucial factor in the recent success of pattern recognition applications such as continuous speech recognition and handwriting recognition.

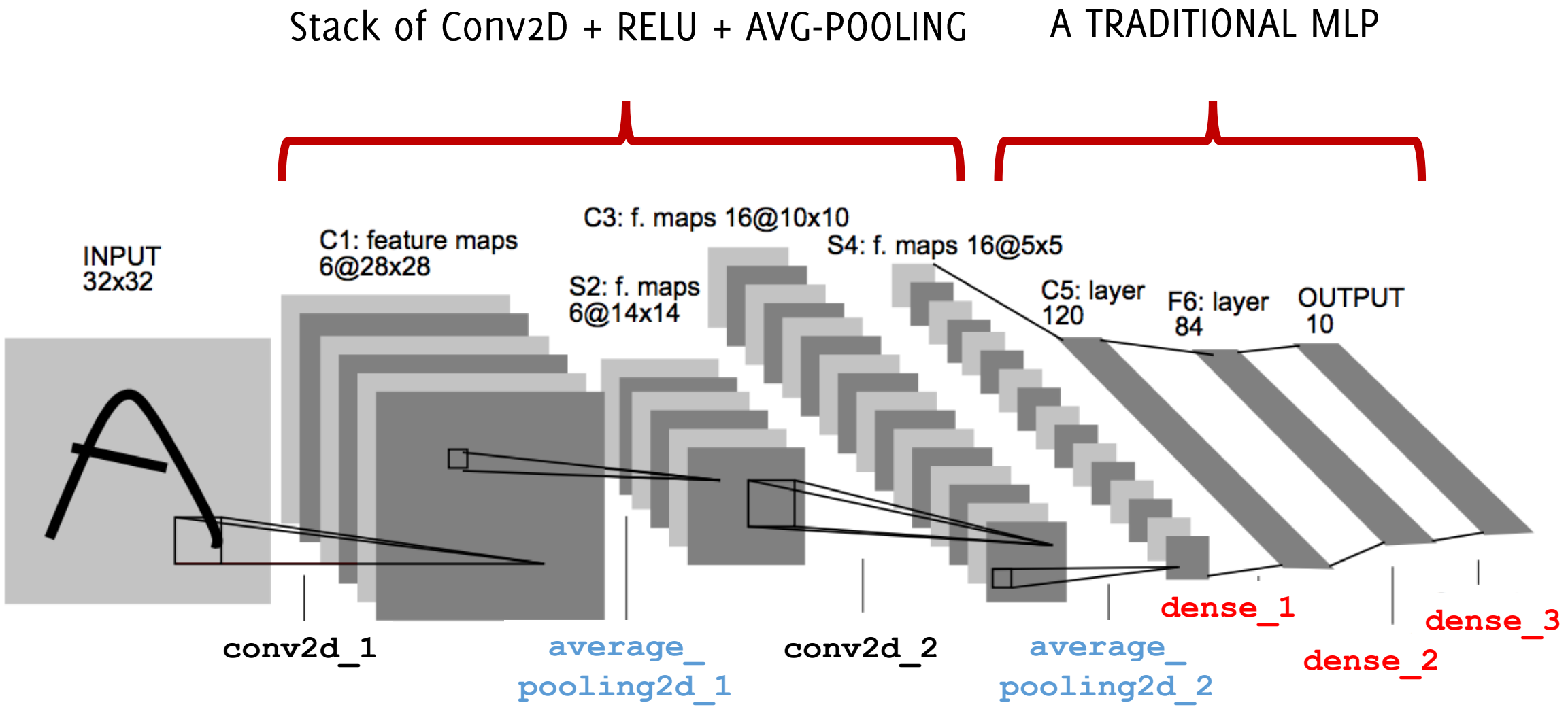
LeCun, Yann, et al. "Gradient-based learning applied to document recognition." *Proceedings of the IEEE* 86.11 (1998)

Fathers of the Deep Learning Revolution Receive ACM A.M. Turing Award

Bengio, Hinton and LeCun Ushered in Major Breakthroughs in Artificial Intelligence

https://awards.acm.org/about/2018-turing

LeNet-5 (1998)



LeCun, Yann, et al. "Gradient-based learning applied to document recognition." *Proceedings of the IEEE* 86.11 (1998)

The First CNN

Do not use each pixel as a separate input of a large MLP, because:

- images are highly spatially correlated,
- using individual pixel of the image as separate input features would not take advantage of these correlations.

The first convolutional layer: 6 filters 5x5

The second convolutional layer: 16 filters 5x5

LeNet-5 in Keras

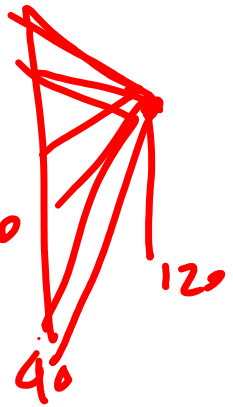
```
from keras.models import Sequential
from keras.layers import Dense, Flatten, Conv2D, AveragePooling2D

num_classes = 10;
input_shape=(32, 32, 1);

model = Sequential()
model.add(Conv2D(filters = 6, kernel_size = (5, 5), activation='tanh', input_shape=inp
ut_shape, padding = 'valid'))
model.add(AveragePooling2D(pool_size=(2, 2)))
model.add(Conv2D(filters = 16, kernel_size = (5, 5), activation='tanh',
padding = 'valid'))
model.add(AveragePooling2D(pool_size=(2, 2)))
model.add(Flatten())
model.add(Dense(120, activation='relu'))
model.add(Dense(84, activation='relu'))
model.add(Dense(num_classes, activation='softmax'))
```

model.summary()

```
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_1 (Conv2D)            (None, 28, 28, 6)         ...
_________________________________________________________________
average_pooling2d_1 (Average (None, 14, 14, 6)         ...
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 10, 10, 16)        ...
_________________________________________________________________
average_pooling2d_2 (Average (None, 5, 5, 16)          ...
_________________________________________________________________
flatten_1 (Flatten)          (None, 400)               ...
_________________________________________________________________
dense_1 (Dense)              (None, 120)               ...
_________________________________________________________________
dense_2 (Dense)              (None, 84)                ...
_________________________________________________________________
dense_3 (Dense)              (None, 10)                ...
=================================================================
Total params: 61,706
Trainable params: 61,706
Non-trainable params: 0
_________________________________________________________________
```

model.summary()

```
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_1 (Conv2D)            (None, 28, 28, 6)         156
_________________________________________________________________
average_pooling2d_1 (Average (None, 14, 14, 6)         0
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 10, 10, 16)        2416
_________________________________________________________________
average_pooling2d_2 (Average (None, 5, 5, 16)          0
_________________________________________________________________
flatten_1 (Flatten)          (None, 400)               0
_________________________________________________________________
dense_1 (Dense)              (None, 120)               48120
_________________________________________________________________
dense_2 (Dense)              (None, 84)                10164
_________________________________________________________________
dense_3 (Dense)              (None, 10)                850
=================================================================
Total params: 61,706
Trainable params: 61,706
Non-trainable params: 0
_________________________________________________________________
```



model.summary()

```
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_1 (Conv2D)            (None, 28, 28, 6)         156 (6 x 5 x 5 + 6)
_________________________________________________________________
average_pooling2d_1 (Average (None, 14, 14, 6)         0
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 10, 10, 16)        2416 (16 x 5 x 5 x 6 + 16)
_________________________________________________________________
average_pooling2d_2 (Average (None, 5, 5, 16)          0
_________________________________________________________________
flatten_1 (Flatten)          (None, 400)               0
_________________________________________________________________
dense_1 (Dense)              (None, 120)               48120
_________________________________________________________________
dense_2 (Dense)              (None, 84)                10164
_________________________________________________________________
dense_3 (Dense)              (None, 10)                850
=================================================================
Total params: 61,706
Trainable params: 61,706
Non-trainable params: 0
_________________________________________________________________
```

Input is a grayscale image

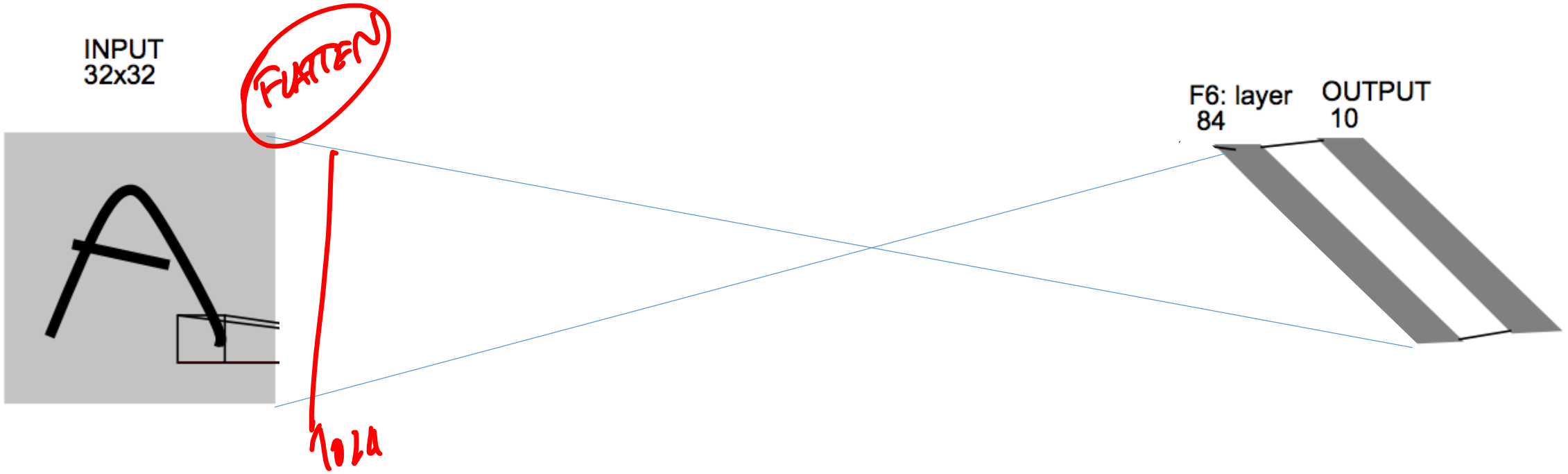
The input is a volume having depth = 6

Most parameters are still in the MLP

Most of parameters are in MLP

What about a MLP taking as input the whole image?

Input 32 x 32 = 1024 pixels, fed to a 84 neurons (the last FC layers of the network) -> 86950 parameters: 1024 * 84 + 84 + 84 * 10 + 10



Most of parameters are in MLP

What about a MLP taking as input the whole image?

Input 32 x 32 = 1024 pixels, fed to a 84 neurons (the last FC layers of the network) -> 86950 parameters

But.. If you take an RGB input: 32 x 32 x 3,

CNN: only the nr. of parameters in the filters at the first layer increases

$$156 + 61550 \rightarrow 456 + 61550$$
$$(6 \times 5 \times 5) \rightarrow (6 \times 5 \times 5 \times 3)$$

MLP: everything increases by a factor 3

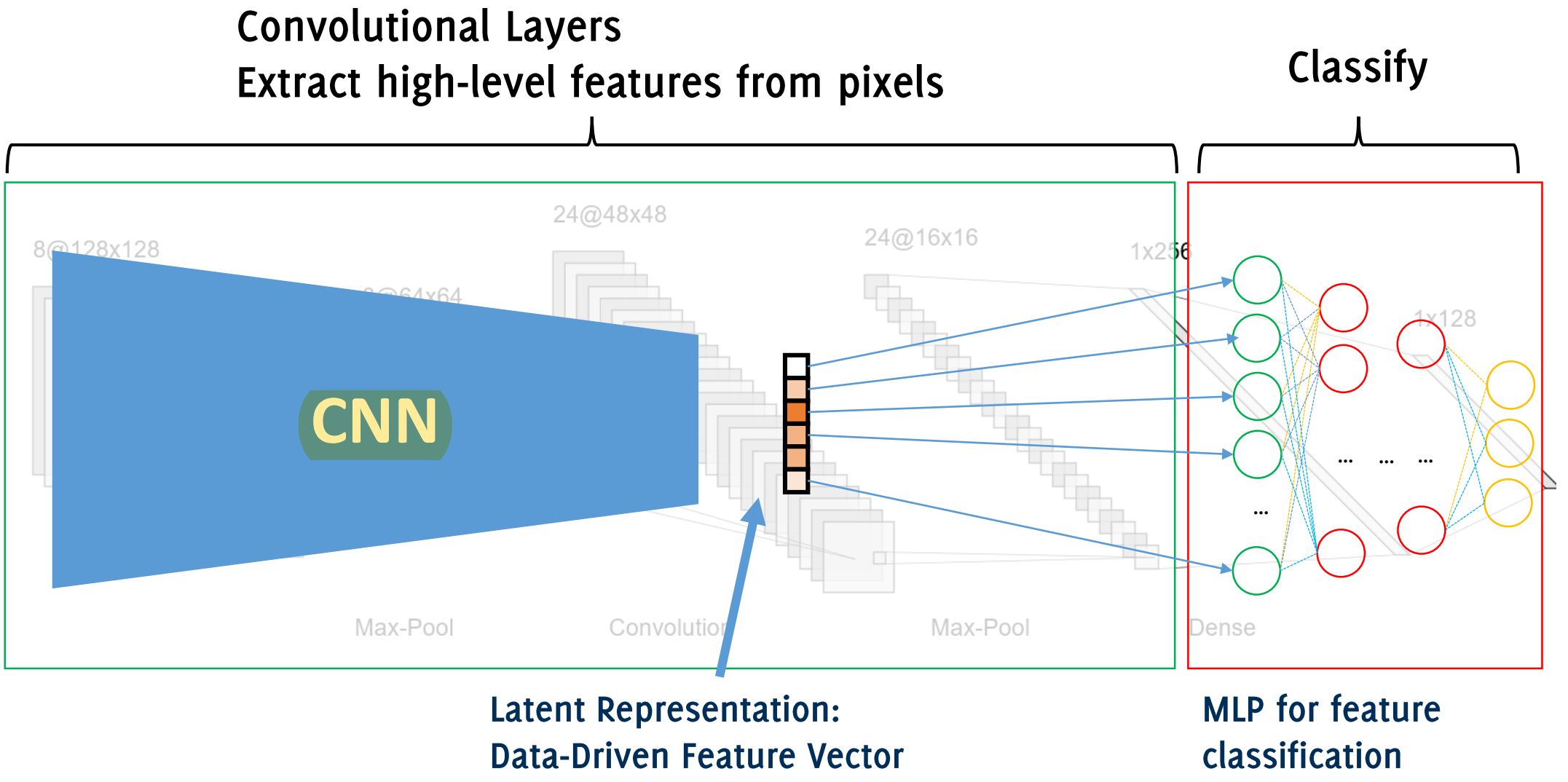
$$86950 \rightarrow 86950 \times 3$$

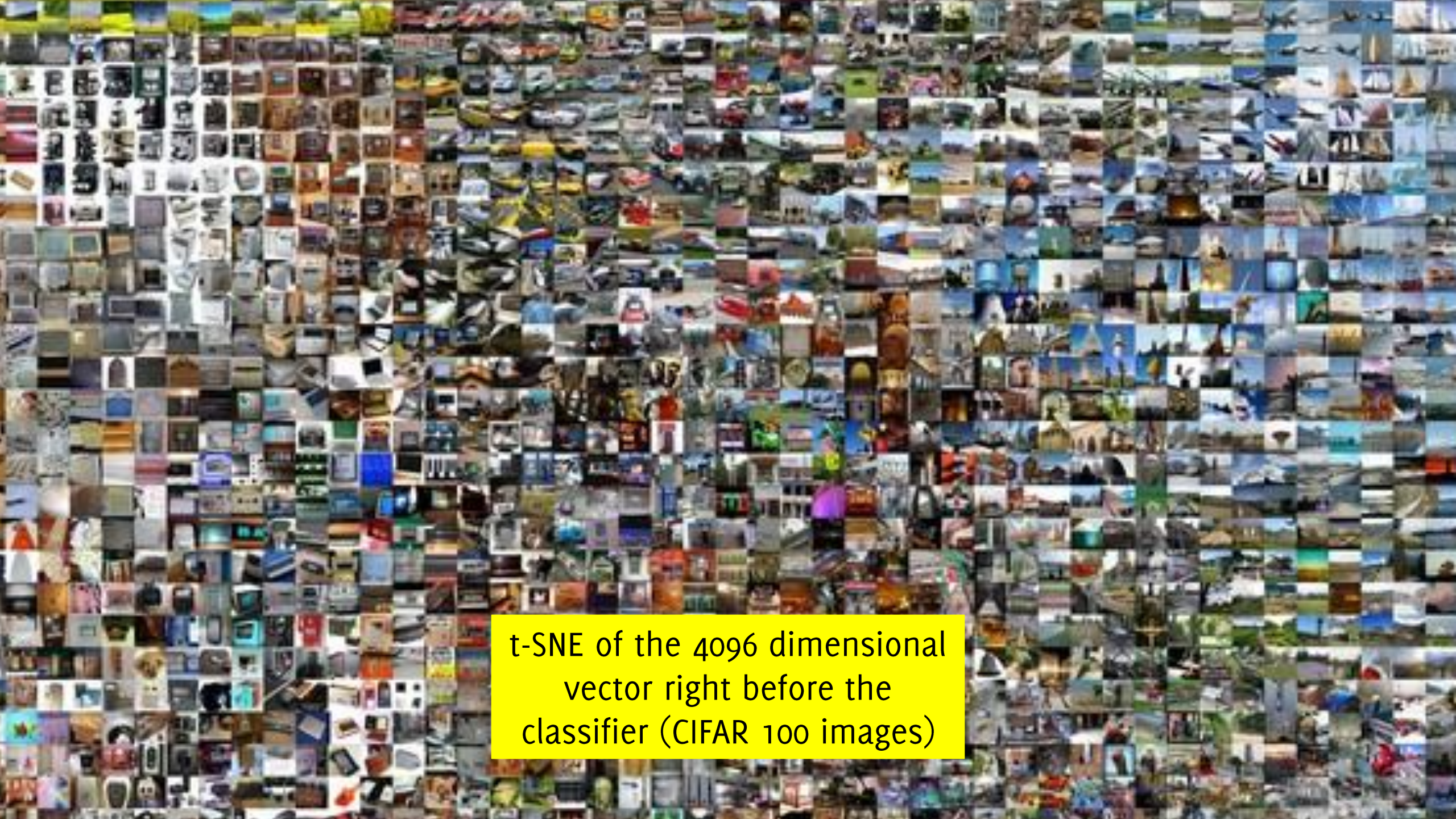
Latent representation in CNNs

Repeat the «t-SNE experiment» on the CIFAR dataset, using the last layer of the CNN as vectors

Giacomo Boracchi

The typical architecture of a CNN




t-SNE of the 4096 dimensional vector right before the classifier (CIFAR 100 images)


Distances in the latent representation of a CNN are much more meaningful than data itself

CNNs in Keras

Giacomo Boracchi

What is Keras?

An open-source library providing **high-level building blocks** for developing deep-learning models in Python

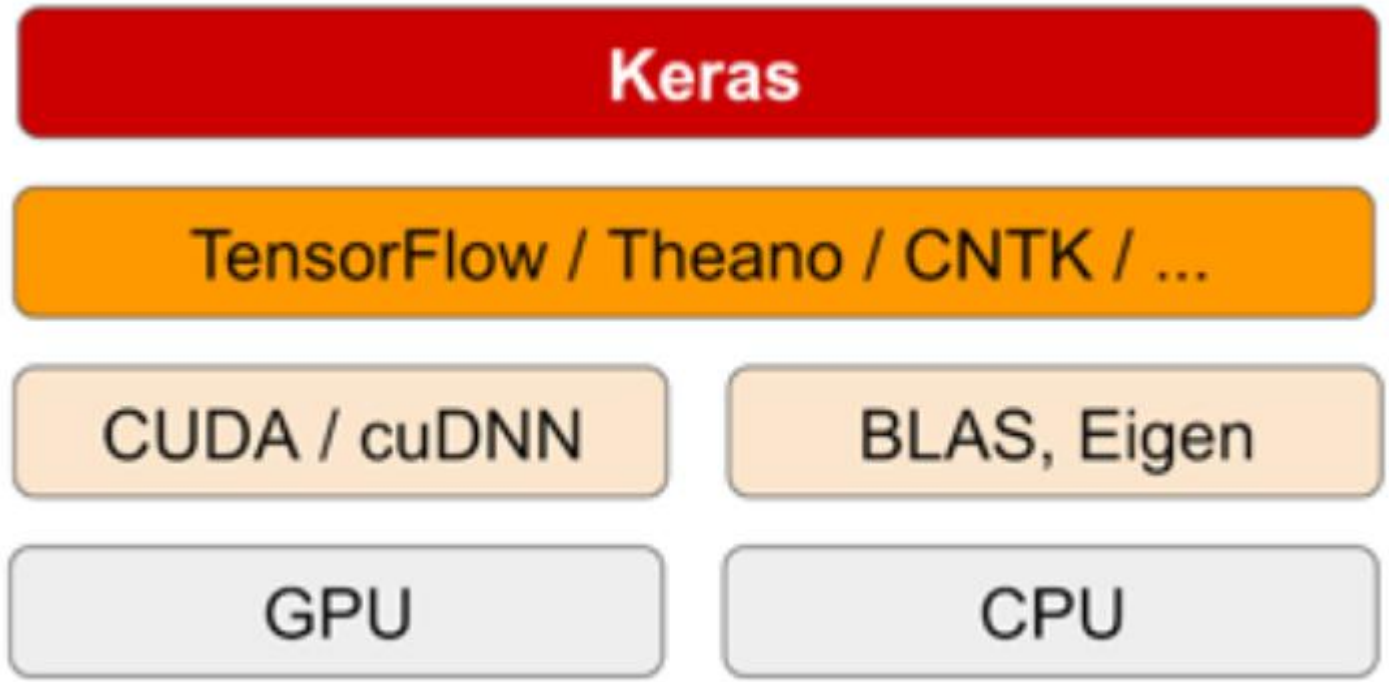
Designed to enable **fast experimentation with deep neural networks**, it focuses on being **user-friendly, modular, and extensible**

Doesn't handle low-level operations such as tensor manipulation and differentiation.

Relies on **backends** (TensorFlow, Microsoft Cognitive Toolkit, Theano, or PlaidML)

Enables **full access to the backend**

The software stack



Why Keras?

Pros:

Higher level → fewer lines of code

Modular backend → not tied to tensorflow

Way to go if you focus on applications

Cons:

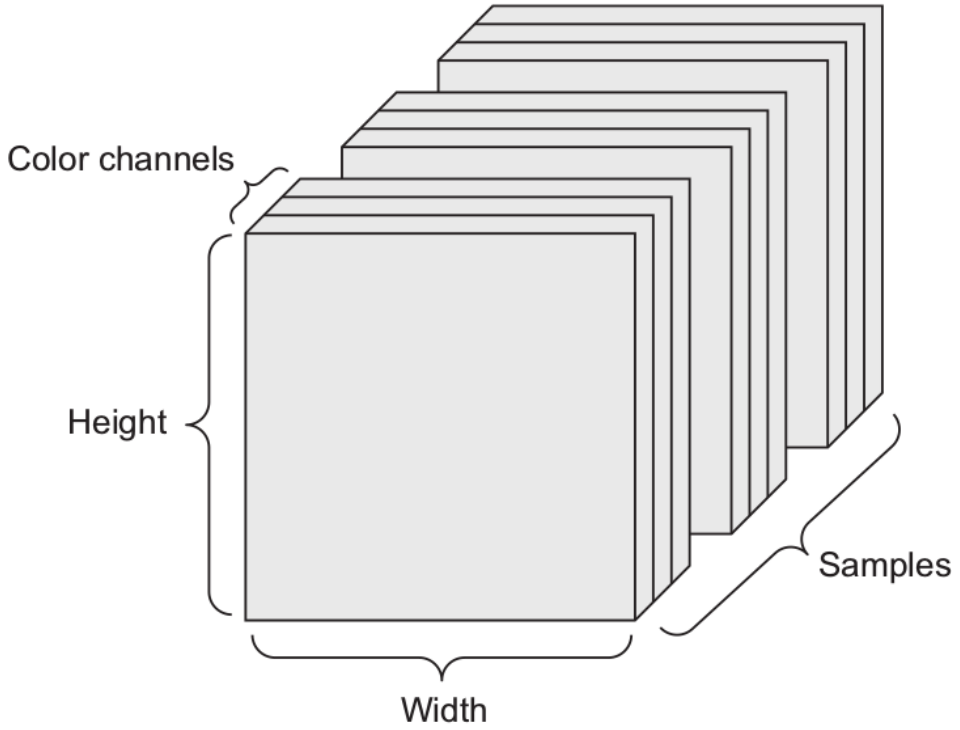
Not as flexible

Need more flexibility? Access the backend directly!

We will manipulate 4D tensors

Images are represented in 4D tensors:

Tensorflow convention: (samples, height, width, channels)



Building the Network

Convolutional Networks in Keras

```
# it is necessary to import some package
from keras.models import Sequential
from keras.layers import Dense, Flatten
from keras.layers import Conv2D, MaxPooling2D

# and initialize an object from Sequential()
model = Sequential()
```

A very simple CNN

```
# Network Layers are stacked by means of the
.add() method
model.add(Conv2D(filters = 64, kernel_size=3,
activation='relu', input_shape=(28,28,1)))
model.add(MaxPooling2D(pool_size=(2, 2)))
model.add(Flatten())
model.add(Dense(10, activation='softmax'))
```

Convolutional Layers

```
# Convolutional Layer
model.add(Conv2D(filters = 64, kernel_size=3,
activation='relu', input_shape=(28,28,1)))
# the input are meant to define:
# - The number of filters,
# - The spatial size of the filter (assumed
squared), while the depth depends on the network
structure
# - the activation layer (always include a
nonlinearity after the convolution)
# - the input size: (rows, cols, n_channels)
```

Convolutional Layers

```
# Convolutional Layer
model.add(Conv2D(filters = 64, kernel_size=3,
activation='relu', input_shape=(28,28,1)))
# This layer creates a convolution kernel that
is convolved with the layer input to produce a
tensor of outputs.
# When using this layer as the first layer in a
model, provide the keyword argument input_shape
(tuple of integers, does not include the batch
axis), e.g. input_shape=(128, 128, 3) for
128x128 RGB pictures in
data_format="channels_last".
```

Conv2D help

Arguments

filters: Integer, the dimensionality of the output space (i.e. the number of output filters in the convolution).

kernel_size: An integer or tuple/list of 2 integers, specifying the height and width of the 2D convolution window. Can be a single integer to specify the same value for all spatial dimensions.

strides: An integer or tuple/list of 2 integers, specifying the strides of the convolution along the height and width. Can be a single integer to specify the same value for all spatial dimensions. Specifying any stride value != 1 is incompatible with specifying any dilation_rate value != 1.

padding: one of "valid" or "same" (case-insensitive). Note that "same" is slightly inconsistent across backends with strides != 1, as described here

data_format: A string, one of "channels_last" or "channels_first". The ordering of the dimensions in the inputs. "channels_last" corresponds to inputs with shape (batch, height, width, channels) while "channels_first" corresponds to inputs with shape (batch, channels, height, width). It defaults to the image_data_format value found in your Keras config file at ~/.keras/keras.json. If you never set it, then it will be "channels_last".

Conv2D help

Arguments

dilation_rate: an integer or tuple/list of 2 integers, specifying the dilation rate to use for dilated convolution. Can be a single integer to specify the same value for all spatial dimensions. Currently, specifying any dilation_rate value != 1 is incompatible with specifying any stride value != 1.

activation: Activation function to use (see activations). If you don't specify anything, no activation is applied (ie. "linear" activation: a(x) = x).

use_bias: Boolean, whether the layer uses a bias vector.

kernel_initializer: Initializer for the kernel weights matrix (see initializers).

bias_initializer: Initializer for the bias vector (see initializers).

kernel_regularizer: Regularizer function applied to the kernel weights matrix (see regularizer).

bias_regularizer: Regularizer function applied to the bias vector (see regularizer).

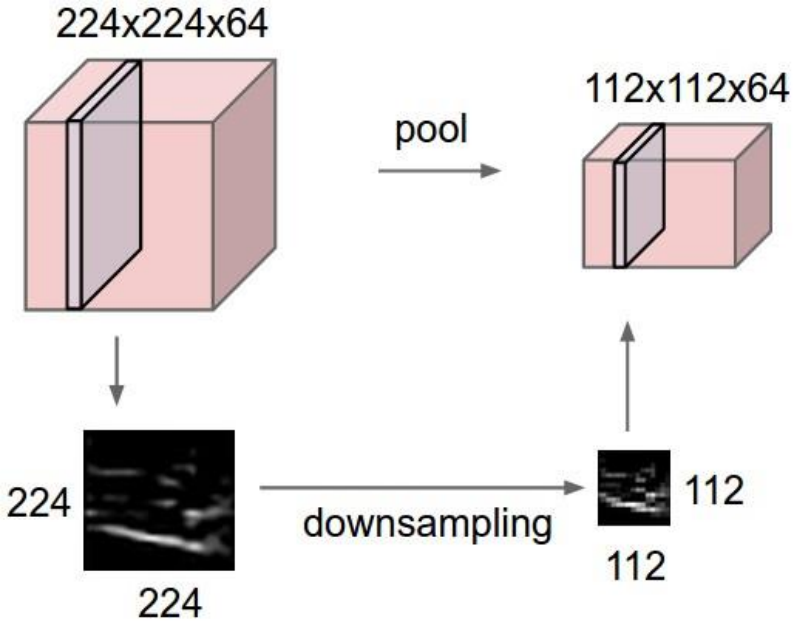
activity_regularizer: Regularizer function applied to the output of the layer (its "activation"). (see regularizer).

kernel_constraint: Constraint function applied to the kernel matrix (see constraints).

bias_constraint: Constraint function applied to the bias vector (see constraints).

MaxPooling Layers

```
# Maxpooling layer
model.add(MaxPooling2D(pool_size=(2, 2)))
# the only parameter here is the (spatial) size
to be reduced by the maximum operator
```



MaxPooling2D help

Arguments:

pool_size: integer or tuple of 2 integers, factors by which to downscale (vertical, horizontal). (2, 2) will halve the input in both spatial dimension. If only one integer is specified, the same window length will be used for both dimensions.

strides: Integer, tuple of 2 integers, or None. Strides values. If None, it will default to pool_size.

padding: One of "valid" or "same" (case-insensitive).

data_format: A string, one of channels_last (default) or channels_first. The ordering of the dimensions in the inputs. channels_last corresponds to inputs with shape (batch, height, width, channels) while channels_first corresponds to inputs with shape (batch, channels, height, width). It defaults to the image_data_format value found in your Keras config file at ~/.keras/keras.json. If you never set it, then it will be "channels_last".

MaxPooling2D help

Input shape:

If data_format='channels_last': 4D tensor with shape: (batch_size, rows, cols, channels)

If data_format='channels_first': 4D tensor with shape: (batch_size, channels, rows, cols)

Output shape:

If data_format='channels_last': 4D tensor with shape: (batch_size, pooled_rows, pooled_cols, channels)

If data_format='channels_first': 4D tensor with shape: (batch_size, channels, pooled_rows, pooled_cols)

Fully Connected Layers

```
# at the end the activation maps are "flattened" i.e.
they move from an image to a vector (just unrolling)
model.add(Flatten())
# Dense is a Fully Connected layer in a traditional
Neural Network.
model.add(Dense(units=10, activation='softmax'))
# Implements:
# output = activation(dot(input, kernel) + bias)
• activation is the element-wise activation function
  passed as the activation argument,
• kernel is a weights matrix created by the layer,
• bias is a bias vector created by the layer
# "Units" defines the number of neurons
```

Visualizing the model

```
# a nice output describing the model architecture
model.summary()
```

```
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_7 (Conv2D)            (None, 26, 26, 64)        640
_________________________________________________________________
flatten_3 (Flatten)          (None, 43264)             0
_________________________________________________________________
dense_4 (Dense)              (None, 10)                432650
=================================================================
Total params: 433,290
Trainable params: 433,290
Non-trainable params: 0
```

Training the Model

Compiling the model

Then we need to compile the model using the compile method and specifying:

- **optimizer** which controls the learning rate. Adam is generally a good option as it adjusts the learning rate throughout training.
- **loss function** the most common choice for classification is ‘categorical_crossentropy’ for our loss function. The lower the better.
- **Metric** to assess model performance, ‘accuracy’ is more interpretable

```
model.compile(optimizer='adam',
loss='categorical_crossentropy',
metrics=['accuracy'])
```

Training the model using

The **`fit()`** method of the model is used to train the model.

Specify at least the following inputs:

- training data (input images),
- target data (corresponding labels in categorical format),
- validation data (a pair of data, labels to be used only for computing validation performance)
- number of epochs (number of times the whole dataset is scanned during training)

```
model.fit(X_train, y_train,
validation_data=(X_test, y_test), epochs=3)
```

Training output

```
Epoch 22/100

18000/18000 [==============================] - 136s 8ms/step - loss: 0.7567
- acc: 0.6966 - val_loss: 1.9446 - val_acc: 0.4325

Epoch 23/100

18000/18000 [==============================] - 137s 8ms/step - loss: 0.7520
- acc: 0.6959 - val_loss: 1.9646 - val_acc: 0.4275

Epoch 24/100

18000/18000 [==============================] - 137s 8ms/step - loss: 0.7442
- acc: 0.7024 - val_loss: 1.9067 - val_acc: 0.4129
```

Advanced Training Options

Callbacks in Keras

A callback is a **set of functions** to be **applied at given stages** of the training procedure.

Callbacks give a **view on internal states** and statistics of the model **during training**.

You can pass **a list of callbacks** (as the keyword argument callbacks) to the .fit() method of the Sequential or Model classes.

The relevant methods of the callbacks will then be called at each stage of the training.

```
callback_list = [cb1,…,cbN]

model.fit(X_train, y_train,
validation_data=(X_test, y_test), epochs=3,
callbacks = callback_list)
```

Model Checkpoint

Training a network might take up to several hours

Checkpoints are snapshots of the state of the system to be saved in case of system failure.

When training a deep learning model, the checkpoint is the weights of the model. These weights can be used to make predictions as is, or used as the basis for ongoing training.

```
from keras.callbacks import ModelCheckpoint
```

[…]

```
cp = ModelCheckpoint(filepath,
monitor='val_loss', verbose=0,
save_best_only=False, save_weights_only=False,
mode='auto', period=1)
```

Early Stopping

The only stopping criteria when training a Deep Learning model is "reaching the required number of epochs."

However, it might be enough to train a model further, as sometimes the training error decreases but the validation error does not (overfitting)

Checkpoints are used to stop training when a monitored quantity has stopped improving.

```
from keras.callbacks import EarlyStopping
```

[…]

```
es = EarlyStopping(monitor='val_loss',
min_delta=0, patience=0, verbose=0, mode='auto',
baseline=None, restore_best_weights=False)
```

Testing the model

Predict() method

```
#returns the class probabilities for the input
image X_test
score = model.predict(X_test)
# select the class with the largest score
prediction_test = np.argmax(score, axis=1)
```

Tensorboard

When training a model it is important to monitor its progresses

Google has developed tensorboard a very useful tool for visualizing reports.

```
from keras.callbacks import TensorBoard
```

[…]

```
  tb = TensorBoard(log_dir="dirname")
```

… and add tb to the checkpoint list as well