QUASI-MINIMAL COMPUTATION OF THE DYNAMIC MODEL OF A ROBOT MANIPULATOR UTILIZING THE NEWTON-EULER FORMALISM AND THE NOTION OF AUGMENTED BODY

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ABSTRACT

Real-time dynamic control of robot manipulators requires on-line computation of the dynamic model that expresses the generalized forces to be applied to their joints, as a function of their generalized coordinates, velocities and accelerations. To do this, this paper presents a method of computation of this model that uses a quasi-minimal number of elementary arithmetical operations and that can be applied systematically to robot manipulators with a simple kinematic chain structure and revolute and/or prismatic joints. To reach this quasi-minimal number, use if primarily made of the following :

* a computation that is intrinsic rather than extrinsic, analytical rather than numerical and iterative rather than developed ;
* the Newton-Euler formalism rather than the Lagrangian one and
* the notion of augmented body, generalized to this type of structure.

An example demonstrates that the computation of the dynamic model of an industrial robot manipulator with six revolute joints (the most complicated case in practice) can be effected with less than 300 arithmetical operations (adds and multiplies).

INTRODUCTION

When a robot manipulator has to be controlled in real-time and when the desired evolution has to be rapid and precise, the dynamic variations must be taken into account. An adequate control may be obtained if it is possible to ensure on-line computation of the relation -called dynamic model- between the generalized forces to be applied to joints Γ and the generalized coordinates q, velocities $\dot{q}$ and acceleration $\ddot{q}$, they produce :

$$\Gamma = f(q, \dot{q}, \ddot{q})$$

The aim of this paper is to present a method of computation of this model that utilizes a quasi-minimal number of arithmetical-operations (additions/substractions, multiplications) and that can be applied systematically to robot manipulators with a simple kinematic chain structure and arbitrarily distributed revolute and/or prismatic joints.

Examination of the principal methods presented in the litterature[13] makes it possible to classify them roughly as follows :

* the methods based on the Newton-Euler formalism leading to an expression of the dynamic model in which the generalized coordinates q, velocities $\dot{q}$, and accelerations $\ddot{q}$ occur non-separately between themselves (referred to here as the implicit model)[9,16,15,6]
* Those that utilize the Lagrange formalism leading to an expression of the dynamic model in which the generalized coordinates, q, velocities $\dot{q}$ and accelerations $\ddot{q}$ occur separately between themselves (referred to as the explicit model)[14,5,8,10,11,12].

Because the implicit computation is naturally simpler than the explicit one, the methods pertaining to the first group allow the model to be obtained more easily than those of the second group. However, the methods of the first group and most methods of the second group suprisingly disregard the simplifying notion of augmented body introduced in 1906 by O. Fischer[3]. This is all the more surprising as this notion has been widely used by various authors[4,17] who utilized the Newton-Euler formalism but erroneously tried to carry out the explicit computation of the model with this formalism.

These reasons led us to propose in this article a method of computation of the model using the Newton-Euler formalism and the notion of augmented body that, however, was generalized to take account of possible prismatic joints. This method primarily relies on a computation that is intrinsic rather than extrinsic, analytical rather than numerical and iterative rather than developed.

The intrinsic computation requires first to use such intrinsic elements as vectors and tensors to obtain the model under a general theoretical form and then to project them on suitable frames in order to obtain the model under as simple a specific form as possible. The analytical computation consists of furnishing a literal expression of the model that allows deletion of redundant calculations (add with 0, multiply by 0 or 1). The iterative computation relies on naming any new quantity occurring during the development and involving at least one add or multiply, and on utilizing this name in the subsequent development.

DEFINITION OF OPERATIONS INVOLVING VECTORS AND TENSORS

Intrinsic computations utilize operations between vectors and tensors. These operations are defined as follows : vectors and tensors, of order 2 only, of the usual 3 dimensional Euclidian vectorial space are used. In accordance with Gibb's notation vectors are underlined once (e.g. $\underline{a}$) and tensors twice (e.g. $\underline{\underline{t}}$). If $(\underline{x},\underline{y},\underline{z})$ represents an orthonormal frame of this space, it is thus possible to write :

$\underline{a}=a_x\underline{x}+a_y\underline{y}+a_z\underline{z}$ and $\underline{\underline{t}}=t_{xx}\underline{x}\otimes\underline{x}+t_{xy}\underline{x}\otimes\underline{y}+\ldots+t_{zz}\underline{z}\otimes\underline{z}$

where (a_x,a_y,a_z) and $(t_{xx},t_{xy},\ldots,t_{zz})$ are the vector $\underline{a}$ and tensor $\underline{\underline{t}}$ components, respectively, with regard to the frame $(\underline{x},\underline{y},\underline{z})$. ($\otimes$ refers to the tensor product defined hereafter).

The following operations are used :

(a) outer product (designated by "symbol-free"):

$$\lambda\underline{a} = (\lambda a_x)\underline{x}+(\lambda a_y)\underline{y}+(\lambda a_z)\underline{z}$$

$$\lambda\underline{\underline{t}} = (\lambda t_{xx})\underline{x}\otimes\underline{x}+(\lambda t_{xy})\underline{x}\otimes\underline{y}+\ldots+(\lambda t_{zz})\underline{z}\otimes\underline{z}$$

(b) inner product (designated by .)

$$\underline{a}.\underline{b}=a_xb_x + a_yb_y + a_zb_z$$

$$\underline{\underline{t}}.\underline{a}=(t_{xx}a_x+t_{xy}a_y+t_{xz}a_z)\underline{x}+\ldots+(t_{zx}a_x+t_{zy}a_y+t_{zz}a_z)\underline{z}$$

$$\underline{\underline{t}}.\underline{\underline{u}}=(t_{xx}u_{xx}+t_{xy}u_{yx}+t_{xz}u_{zx})\underline{x}\otimes\underline{x}+\ldots+(t_{zx}u_{xz}+t_{zy}u_{yz}+t_{zz}u_{zz})\underline{z}\otimes\underline{z}$$

(c) cross product (designated by x) :

$$\underline{a}\times\underline{b}=(a_yb_z-a_zb_y)\underline{x}+(a_zb_x-a_xb_z)\underline{y}+(a_xb_y-a_yb_x)\underline{z}$$

If the skew-symmetric tensor $\underline{\underline{\hat{a}}}$, corresponding to vector $\underline{a}$ is defined such that :

$$\underline{\underline{\hat{a}}} = -a_z\underline{x}\otimes\underline{y}+a_y\underline{x}\otimes\underline{z}+a_z\underline{y}\otimes\underline{x}-a_x\underline{y}\otimes\underline{z}-a_y\underline{z}\otimes\underline{x}+a_x\underline{z}\otimes\underline{y}$$

it can be proved that :

$$\underline{a}\times\underline{b} = \underline{\underline{\hat{a}}}\,.\,\underline{b}$$

(d) tensor product (designated by $\otimes$) :

$$\underline{a}\otimes\underline{b}=(a_xb_x)\underline{x}\otimes\underline{x}+(a_xb_y)\underline{x}\otimes\underline{y}+\ldots+(a_zb_z)\underline{z}\otimes\underline{z}$$

INTRINSIC CALCULATION OF THE DYNAMIC MODEL

Definition of the kinematic chain of the robot-manipulator

Robot manipulators consist of n-solid bodies interconnected, in accordance with a simple kinematic chain structure, by revolute (R) and/or prismatic (P) joints.

The first body of the chain is joined around a fixed mount and the last body constitutes the end-effector. The bodies and joints are numbered in ascending order from 1 to n starting from the mount (body 0) (see figure 1).

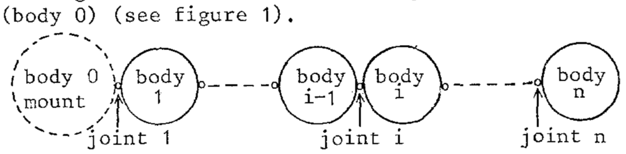


Figure 1. Numbering of the bodies and joints of the robot manipulator

The binary coefficient σ_i and its conjugate $\bar{\sigma}_i$ identify the type of ith joint :

$$\sigma_i = \begin{cases} 0 & \text{if the joint is revolute R} \\ 1 & \text{if the joint is prismatic P} \end{cases}$$

Definition of points and vectors typical of body i

Body-dependant quantities are denoted by a symbol with a right hand subscript/superscript corresponding to the number of the associated bodies. A subscript is used if the associated quantity does not involve mass in its dimensional equation and a superscrip is used in the opposite case. The computation of the dynamic model is obtained relative to an affine orthonormal frame attachedto the mount :

$$R_0 = (O_0,\ \underline{x}_0,\ \underline{y}_0,\ \underline{z}_0)$$

$\underline{z}_0$ being directed according to the vertical ascendent.

Let O_{i-1} the intersecting point of the perpendicular common to the joint axes i-1 and i located on the joint axis i-1 and G_i the centre of mass of body i (see fig. 2). The axis of ith joint (R or P) is defined by the arbitrarily oriented vector $\underline{z}_i$ (see Fig. 2). The motion around or along this axis is defined by the ith generalized coordinate q_i.

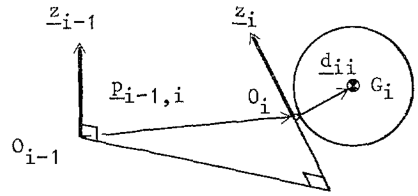


Figure 2. Points and vectors typical of body i

We define respectively by (see Fig. 2) :

$\underline{p}_{ij}$ the vector of origin O_i and extremity O_j and
$\underline{d}_{ij}$ the vector of origin O_i and extremity G_j.

Definition of the moments of body i at point O_i

If m^i and $\underline{\underline{\varphi}}^i$ denote respectively the mass and inertia tensor at point G_i of body i, the moments of order 0, 1 and 2 of body i at point O_i are respectively :

$$m^i$$

$$\underline{u}^i = m^i\,\underline{d}_{ii} \quad \text{and}$$

$$\underline{\underline{k}}^i = \underline{\underline{\varphi}}^i - m^i\,\underline{\underline{\hat{d}}}_{ii}\,.\,\underline{\underline{\hat{d}}}_{ii} \quad \text{(Huygens' theorem)}$$

Definition of augmented body i

The augmented body i is defined as the fictitious body consisting of the particles of body i itself and of a mass, equal to that of bodies i+1, i+2,...,n, attached to O_{i+1} (see fig. 3). This definition which can be applied regardless of the type of joint (i.e., R or P) generalizes O.Fischer's notion[3]. It differs slightly from the one presented in 1965 by W.W. Hooker and G. Margulies[4,17] in that the mass of the augmented body is not the total mass of the robot manipulator. Note that the augmented body i is solid if and only if $\sigma_{i+1} = 0$

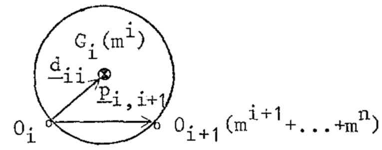


Figure 3. Augmented body i

Typical augmented body quantities are denoted by a symbol with a right hand side superscript (*). Moments of order 0, 1, and 2 of augmented body i at point O_i are respectively :

$$m^{*i} = \sum_{j=i}^{n} m^j$$
$$\underline{u}^{*i} = \underline{u}^i + m^{*(i+1)} \underline{p}_{i,i+1}$$
$$\underline{\underline{k}}^{*i} = \underline{\underline{k}}^i - m^{*(i+1)} \hat{\underline{\underline{p}}}_{i,i+1} \cdot \hat{\underline{\underline{p}}}_{i,i+1} \quad \text{(Huygens' theorem)}$$

Definition of the translational and rotational velocities and linear and angular momenta of body i

Let $\underline{v}_i$, the translational velocity of point G_i of body i and $\underline{\omega}_i$ the instantaneous rotational velocity of body i with respect to frame R_0.

To simplify the notation, let us write the translational and rotational accelerations as follows:

$$\underline{\gamma}_i = \dot{\underline{v}}_i$$
$$\underline{a}_i = \dot{\underline{\omega}}_i$$

In these conditions :

$\underline{p}^i = m^i \underline{v}_i$ is the linear momentum of body i,

$\underline{h}^i = \underline{\underline{\varphi}}^i \cdot \underline{\omega}_i$ is the angular momentum of body i.

The derivatives, with respect to time, of the linear and angular momenta can be written as :

$$\dot{\underline{p}}^i = m^i \underline{\gamma}_i \quad ; \quad \dot{\underline{h}}^i = \underline{\underline{\varphi}}^i \cdot \underline{a}_i + \underline{\omega}_i \times (\underline{\underline{\varphi}}^i \cdot \underline{\omega}_i)$$

Definition of the forces and torques acting on body i

Let :

$\underline{f}^i$ the joint force exerted at point O_i by body i-1 on body i,

$\underline{n}^i$ the joint torque exerted at point O_i by body i-1 on body i.

If it is assumed that body i is only subjected to joint forces and torques (at joints i and i+1) and to the force due to gravity, it is possible to compute the external total force $\underline{F}^i$ and the external total torque $\underline{N}^i$ applied onto body i (see Fig. 4):

$$\underline{F}^i = \underline{f}^i - \underline{f}^{i+1} - m^i g \, \underline{z}_0$$
$$\underline{N}^i = \underline{n}^i - \underline{n}^{i+1} - \underline{d}_{ii} \times \underline{f}^i + \underline{d}_{i+1,i} \times \underline{f}^{i+1}$$

g being the acceleration due to gravity.

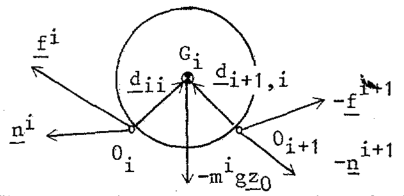


Figure 4. Forces and torques exerted on body i.

Note that when joint i is revolute, the joint torque $\underline{n}^i$ can be divided into a control torque $\underline{\Gamma}^i$ carried by the joint axis and a constraint torque at right angles to this axis whereas when joint i is prismatic the joint force $\underline{f}^i$ can be divided into a control force $\underline{\Gamma}^i$ carried by the joint axis and a constraint force at right angles to this axis. In all cases, the control force or torque, can be written as :

$$\underline{\Gamma}^i = \Gamma^i \underline{z}_i \quad \text{with :}$$
$$\Gamma^i = \bar{\sigma}_i (\underline{z}_i \cdot \underline{n}^i) + \sigma_i (\underline{z}_i \cdot \underline{f}^i)$$

Γ^i being the generalized force.

Classical iterative calculation

To obtain the dynamic model we use the linear momenta theorem (i.e., Newton's theorem) as well as the angular momenta theorem (Euler's theorem) for each one of the bodies of the robot manipulator. Applied to body i, these theorems are expressed as:

$$\underline{F}^i = \dot{\underline{p}}^i \quad ; \quad \underline{N}^i = \dot{\underline{h}}^i$$

or, under a more explicit form based on the preceding definitions :

$$m^i (\underline{\gamma}_i + g \underline{z}_0) = \underline{f}^i - \underline{f}^{i+1}$$
$$\underline{\underline{\varphi}}^i \cdot \underline{a}_i + \underline{\omega}_i \times (\underline{\underline{\varphi}}^i \cdot \underline{\omega}_i) = \underline{n}^i - \underline{n}^{i+1} - \underline{d}_{ii} \times \underline{f}^i + \underline{d}_{i+1,i} \times \underline{f}^{i+1}$$

Calculation of the dynamic model consists of a forward iterative computation of the body velocities and accelerations according to the velocities and accelerations of the precedings bodies and of a backward iterative computation of the joint forces and torques of the bodies according to the joint forces and torques of the following bodies.

If we set by definition : $\underline{\alpha}_i = \ddot{\underline{p}}_{0i} + g \, \underline{z}_0$,

and if we note that : $\ddot{\underline{p}}_{0i} = \ddot{\underline{p}}_{0,i-1} + \ddot{\underline{p}}_{i-1,i}$

$$\ddot{\underline{p}}_{i-1,i} = \underline{a}_{i-1} \times \underline{p}_{i-1,i} + \underline{\omega}_{i-1} \times (\underline{\omega}_{i-1} \times \underline{p}_{i-1,i}) + \sigma_i (2\dot{q}_i \underline{\omega}_{i-1} \times \underline{z}_i + \ddot{q}_i \underline{z}_i)$$

and that : $\underline{\gamma}_i = \dot{\underline{v}}_i = \ddot{\underline{d}}_{0i} = \ddot{\underline{p}}_{0i} + \ddot{\underline{d}}_{ii}$ with :

$$\ddot{\underline{d}}_{ii} = \underline{a}_i \times \underline{d}_{ii} + \underline{\omega}_i \times (\underline{\omega}_i \times \underline{d}_{ii})$$

forward iterative computation utilizing the initial conditions : $\underline{\omega}_0 = \underline{a}_0 = \underline{0}$ and $\underline{\alpha}_0 = g \, \underline{z}_0$

is as follows :

Do i=1,2,...,n

$$\underline{\omega}_i = \underline{\omega}_{i-1} + \bar{\sigma}_i \dot{q}_i \underline{z}_i$$
$$\underline{a}_i = \underline{a}_{i-1} + \bar{\sigma}_i (\dot{q}_i \underline{\omega}_{i-1} \times \underline{z}_i + \ddot{q}_i \underline{z}_i)$$
$$\underline{\underline{b}}_i = \hat{\underline{\underline{a}}}_i + \hat{\underline{\underline{\omega}}}_i \cdot \hat{\underline{\underline{\omega}}}_i$$
$$\sigma_i \underline{t}_i = \sigma_i (2\dot{q}_i \underline{\omega}_{i-1} \times \underline{z}_i + \ddot{q}_i \underline{z}_i)$$
$$\underline{r}_{i-1} = \underline{\alpha}_{i-1} + \underline{\underline{b}}_{i-1} \cdot \underline{p}_{i-1,i}$$
$$\underline{\alpha}_i = \sigma_i \underline{t}_i + \underline{r}_{i-1}$$

End do.

Backward iterative computation utilizing the initial conditions :

$\underline{f}^{n+1}$: force exerted by the robot manipulator end-effector on the environment,

$\underline{n}^{n+1}$: torque exerted by the robot manipulator end-effector on the environment.

is as follows :

Do i = n, n-1,..., 1

$$\underline{f}^i = \underline{f}^{i+1} + m^i (\underline{\alpha}_i + \underline{\underline{b}}_i \cdot \underline{d}_{ii})$$
$$\underline{n}^i = \underline{n}^{i+1} + \underline{\underline{\varphi}}^i \cdot \underline{a}_i + \underline{\omega}_i \times (\underline{\underline{\varphi}}^i \cdot \underline{\omega}_i) + \underline{d}_{ii} \times \underline{f}^i - \underline{d}_{i+1,i} \times \underline{f}^{i+1}$$
$$\Gamma^i = \underline{z}_i \cdot (\bar{\sigma}_i \underline{n}^i + \sigma_i \underline{f}^i)$$

End do

Simplified iterative calculation

The notion of augmented body allows the preceding backward iterative computation to be slightly simplified. Indeed if we note that :

$$\underline{\alpha}_j=\underline{\alpha}_i+\sum_{k=i}^{j-1}(\underline{\underline{b}}_k\cdot\underline{p}_{k,k+1}+\sigma_{k+1}\underline{t}_{k+1})$$

it can be stated :

$$\underline{f}^j=\underline{f}^{j+1}+m^j(\underline{\alpha}_i+\sum_{k=i}^{j-1}(\underline{\underline{b}}_k\cdot\underline{p}_{k,k+1}+\sigma_{k+1}\underline{t}_{k+1}))+\underline{\underline{b}}_j\cdot\underline{u}^j$$

$j=i,\ i+1,\dots,n$

The sum of the preceding equations leads to :

$$\underline{f}^i=\underline{f}^{n+1}+m^{*i}\underline{\alpha}_i+\sum_{j=i}^{n}\underline{\underline{b}}_j\cdot\underline{u}^j+\sum_{j=i+1}^{n}m^j\sum_{k=i}^{j-1}(\underline{\underline{b}}_k\cdot\underline{p}_{k,k+1}+\sigma_{k+1}\underline{t}_{k+1})$$

By inverting j and k and by transposing both symbols $\sum$ of the last right hand term of the preceding equation we can write :

$$\underline{f}^i=\underline{f}^{n+1}+m^{*i}\underline{\alpha}_i+\sum_{j=i}^{n}\underline{\underline{b}}_j\cdot\underline{u}^j+\sum_{j=i}^{n}m^{*(j+1)}(\underline{\underline{b}}_j\cdot\underline{p}_{j,j+1}+\sigma_{j+1}\underline{t}_{j+1})$$

and finally :

$$\underline{f}^i=\underline{e}^i+m^{*i}\underline{r}_{i-1}\quad\text{with :}$$

$$\underline{e}^i=\underline{f}^{n+1}+\sigma_i m^{*i}\underline{t}_i+\sum_{j=i}^{n}(\underline{\underline{b}}_j\cdot\underline{u}^{*j}+\sigma_{j+1}m^{*(j+1)}\underline{t}_{j+1})$$

In addition, it can be stated :

$$\underline{\underline{\varphi}}^i\cdot\underline{a}_i+\underline{\omega}_i\times(\underline{\underline{\varphi}}^i\cdot\underline{\omega}_i)=\underline{n}^i-\underline{n}^{i+1}-\underline{d}_{ii}\times(\underline{f}^i-\underline{f}^{i+1})-\underline{p}_{i,i+1}\times\underline{f}^{i+1}$$

let :

$$\underline{\underline{\varphi}}^i\cdot\underline{a}_i+\underline{\omega}_i\times(\underline{\underline{\varphi}}^i\cdot\underline{\omega}_i)=\underline{n}^i-\underline{n}^{i+1}-m^i\underline{d}_{ii}\times(\underline{\alpha}_i+\underline{\underline{b}}_i\cdot\underline{d}_{ii})$$

$$-\underline{p}_{i,i+1}\times(\underline{f}^{n+1}+m^{*(i+1)}(\underline{\alpha}_i+\underline{\underline{b}}_i\cdot\underline{p}_{i,i+1}+\sigma_{i+1}\underline{t}_{i+1})$$

$$+\sum_{j=i+1}^{n}(\underline{\underline{b}}_j\cdot\underline{u}^{*j}+\sigma_{j+1}m^{*(j+1)}\underline{t}_{j+1}))$$

This equation can be written as :

$$\underline{\underline{k}}^{*i}\cdot\underline{a}_i+\underline{\omega}_i\times(\underline{\underline{k}}^{*i}\cdot\underline{\omega}_i)=\underline{n}^i-\underline{n}^{i+1}-\underline{u}^{*i}\times\underline{\alpha}_i-\underline{p}_{i,i+1}\times\underline{e}^{i+1}$$

which involves the augmented body i quantities.

In these conditions the preceding backward iterative computation can be replaced by the following one :

Initial conditions :

$\underline{e}^{n+1}=\underline{f}^{n+1}$ see before

$\underline{n}^{n+1}$ see before

Do $i=n,n-1,\dots,1$

$$\underline{e}^i=\underline{e}^{i+1}+\underline{\underline{b}}_i\cdot\underline{u}^{*i}+\sigma_i m^{*i}\underline{t}_{i(i)}$$

$$\sigma_i\underline{f}^i=\sigma_i(\underline{e}^i+m^{*i}\underline{r}_{i-1})$$

$$\underline{n}^i=\underline{n}^{i+1}+(\underline{\underline{k}}^{*i}\cdot\underline{a}_i+\underline{\omega}_i\times(\underline{\underline{k}}^{*i}\cdot\underline{\omega}_i))+\underline{u}^{*i}\times\underline{\alpha}_i+\underline{p}_{i,i+1}\times\underline{e}^{i+1}$$

$$\Gamma^i=\underline{z}_i\cdot(\overline{\sigma}_i\underline{n}^i+\sigma_i\underline{f}^i)$$

End do

EXTRINSIC CALCULATION OF THE DYNAMIC MODEL

The extrinsic computation of the dynamic model is based on a systematic procedure. It consists of associating first a direct orthonormal frame to each one of the robot manipulator bodies. These frames differ slightly from those used by J. Denavit and R.S. Hartenberg[2] as we intend to link the affine orthonormal frame R_i to body i and to use vector $\underline{z}_i$ as the unit vector of joint i. Recently these new frames have been concurrently introduced by several authors[7,1]. They are represented in Fig. 5 together with the four parameters defining the position and orientation (termed "situation" here) of frame R_i relative to frame R_{i-1}. Among such parameters (α_{i-1}, a_{i-1}, θ_i, r_i) three are constants while the fourth one corresponds to the generalized coordinate q_i. It is :

θ_i if $\sigma_i=0$ and r_i if $\sigma_i=1$

Hence : $q_i=\overline{\sigma}_i\theta_i+\sigma_i r_i$

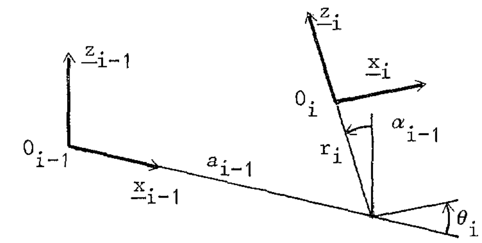


Figure 5. Frame R_i connected with body i and associated parameters

The homogeneous transfer matrix $T_{i-1,i}$ from frame R_{i-1} to frame R_i is expressed as :

$$T_{i-1,i}=\begin{bmatrix}\cos\theta_i & -\sin\theta_i & 0 & a_{i-1}\\ \cos\alpha_{i-1}\sin\theta_i & \cos\alpha_{i-1}\cos\theta_i & -\sin\alpha_{i-1} & -r_i\sin\alpha_{i-1}\\ \sin\alpha_{i-1}\sin\theta_i & \sin\alpha_{i-1}\cos\theta_i & \cos\alpha_{i-1} & r_i\cos\alpha_{i-1}\\ 0 & 0 & 0 & 1\end{bmatrix}$$

This matrix is of the form :

$$T_{i-1,i}=\left[\begin{array}{c|c}R_{i-1,i} & P_{i-1,i}\\ \hline 0 & 1\end{array}\right]$$

where $R_{i-1,i}$ is the classical transfer matrix from frame R_{i-1} to frame R_i. We will designate respectively by $\underline{v}_{(i)}$ and $\underline{\underline{t}}_{(i)}$ the column matrix 3 x 1 and the square matrix 3 x 3 of the components of vector $\underline{v}$ and tensor $\underline{\underline{t}}$, respectively, relative to frame R_i.

In addition, note :

$$\underline{u}^i_{(i)}=\begin{bmatrix}x^i\\ y^i\\ z^i\end{bmatrix}\quad\text{and}\quad\underline{\underline{k}}^i_{(i)}=\begin{bmatrix}a^i & -f^i & -e^i\\ -f^i & b^i & -d^i\\ -e^i & -d^i & c^i\end{bmatrix}$$

$$\underline{u}^{*i}_{(i)}=\begin{bmatrix}x^{*i}\\ y^{*i}\\ z^{*i}\end{bmatrix}\quad\text{and}\quad\underline{\underline{k}}^{*i}_{(i)}=\begin{bmatrix}a^{*i} & -f^{*i} & -e^{*i}\\ -f^{*i} & b^{*i} & -d^{*i}\\ -e^{*i} & -d^{*i} & c^{*i}\end{bmatrix}$$

To effect the extrinsic computation of the dynamic model it suffices to project adequately the intrinsic iterative relations obtained in part 3.

Do $i=1,2,\dots,n$

(0a) $\sigma_{i+1}\underline{u}^{*i}_{(i)}=\sigma_{i+1}(\underline{u}^i_{(i)}+m^{*(i+1)}\underline{p}_{i,i+1(i+1)})$

(0b) $\sigma_{i+1}\underline{\underline{k}}^{*i}_{(i)}=\sigma_{i+1}(\underline{\underline{k}}^i_{(i)}-m^{*(i+1)}\hat{\underline{\underline{p}}}_{i,i+1(i)}\hat{\underline{\underline{p}}}_{i,i+1(i)})$

Forward iterative calculation

Initial conditions (*)

$\underline{\alpha}_{0(0)}=\underline{a}_{0(0)}=(0\ 0\ 0)^t$; $\underline{\alpha}_{0(0)}=gZ$

with $Z=(0\ 0\ 1)^t$

Do $i=1,2,\dots,n$

(1a) $\underline{\omega}_{i-1(i)}=R_{i,i-1}\underline{\omega}_{i-1(i-1)}$

(1b) $\underline{\omega}_{i(i)}=\underline{\omega}_{i-1(i)}+\overline{\sigma}_i\dot{q}_iZ$

(2) $\underline{\underline{\delta}}_{i(i)}=(\underline{\omega}_i\boxtimes\underline{\omega}_i)_{(i)}$

(*) t denotes the transposed of a matrix

(3a) $\underline{a}_{i-1(i)} = R_{i,i-1}\underline{a}_{i-1(i-1)}$

(3b) $\underline{a}_{i(i)} = \underline{a}_{i-1(i)} + \bar{\sigma}_i(\dot{q}_i\underline{\underline{\hat{\omega}}}_{i-1(i)}Z + \ddot{q}_i Z)$

(4) $\underline{\underline{b}}_{i(i)} = \underline{\underline{\hat{a}}}_{i(i)} + \underline{\underline{\hat{\omega}}}_{i(i)}\underline{\underline{\hat{\omega}}}_{i(i)}$ expressed as a function of $\underline{a}_{i(i)}$ and $\underline{\underline{\delta}}_{i(i)}$ components

(5) $\sigma_i\underline{t}_{i(i)} = \sigma_i(2\dot{q}_i\underline{\underline{\hat{\omega}}}_{i-1(i)}Z + \ddot{q}_i Z)$

(6a) $\underline{r}_{i-1(i-1)} = \underline{\alpha}_{i-1(i-1)} + \underline{\underline{b}}_{i-1(i-1)}\underline{p}_{i-1,i(i-1)}$

(6b) $\underline{r}_{i-1(i)} = R_{i,i-1}\underline{r}_{i-1(i-1)}$

(6c) $\underline{\alpha}_{i(i)} = \sigma_i\underline{t}_{i(i)} + \underline{r}_{i-1(i)}$

End do

<u>Backward iterative calculation</u>

Initial conditions

$\left.\begin{array}{l} \underline{e}^{n+1}_{(n)} = \underline{f}^{n+1}_{(n)} \quad \text{and} \\ \underline{n}^{n+1}_{(n)} \end{array}\right\}$ unspecified given values

Do i=n, n-1,...,1

(7a) $\underline{e}^{i}_{(i)} = \underline{e}^{i+1}_{(i)} + \underline{\underline{b}}_{i(i)}\cdot\underline{u}^{*i}_{(i)} + \sigma_i m^{*i}\underline{t}_{i(i)}$

(7b) $\underline{e}^{i}_{(i-1)} = R_{i-1,i}\,\underline{e}^{i}_{(i)}$

(8) $\sigma_i\underline{f}^{i}_{(i)} = \sigma_i(\underline{e}^{i}_{(i)} + m^{*i}\underline{r}_{i-1(i)})$

(9a) $\underline{n}^{i}_{(i)} = \underline{n}^{i+1}_{(i)} + \underline{\underline{k}}^{*i}_{(i)}\underline{a}_{i(i)} + \underline{\underline{\hat{\omega}}}_{i(i)}\underline{\underline{k}}^{*i}_{(i)}\underline{\omega}_{i(i)}$
$\qquad + \underline{\underline{\hat{u}}}^{*i}_{(i)}\underline{\alpha}_{i(i)} + \underline{\underline{\hat{p}}}_{i,i+1(i)}\,\underline{e}^{i+1}_{(i)}$

(9b) $\underline{n}^{i}_{(i-1)} = R_{i-1,i}\underline{n}^{i}_{(i)}$

(10) $\Gamma^{i} = Z^{t}(\bar{\sigma}_i\underline{n}^{i}_{(i)} + \sigma_i\underline{f}^{i}_{(i)})$

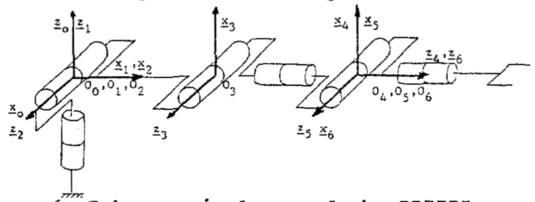
End do

Relations (0) through (10) furnish the dynamic model. The iterative numerical computation can be effected by following these eleven steps; however, it is preferable to effect the calculation under an analytical form first so as to reduce the number of arithmetical operations. As a result it is no longer necessary to use adds with 0 or multiplies by 0 or 1. The model is obtained under the form of auxiliary equations occurring at the various steps in the calculation whose first term is one of the previously introduced notations and the second includes at least one add or multiply.

The notations thus introduced are used as such up to the end of the calculation without being ever developed as a function of the previously introduced notations.

EXAMPLE

Consider the robot manipulator with six revolute joints (6R) represented in Fig. 6.



<u>Figure 6</u>. Robot manipulator of the RRRRRR type

This robot manipulator is characterized by the following parameters

Par. \ i	1	2	3	4	5	6
σ_i	0	0	0	0	0	0
α_{i-1}	0	$\pi/2$	0	$\pi/2$	$-\pi/2$	$\pi/2$
a_{i-1}	0	0	a_2	0	0	0
θ_i	q_1	q_2	q_3	q_4	q_5	q_6
r_i	0	0	0	r_4	0	0
m^i	m^1	m^2	m^3	m^4	m^5	m^6
x^{*i}	0	$x^2+m^{*3}a_2$	0	0	0	0
y^{*i}	0	0	$y^3-m^{*4}r_4$	0	y^5	0
z^{*i}	z^1	0	0	z^4	0	z^6
a^{*i}	a^1	a^2	$a^3+m^{*4}(r_4)^2$	a^4	a^5	a^6
b^{*i}	b^1	$b^2+m^{*3}(a_2)^2$	b^3	b^4	b^5	b^6
c^{*i}	c^1	$c^2+m^{*3}(a_2)^2$	$c^3+m^{*4}(r_4)^2$	c^4	c^5	c^6

<u>Remark</u> : It is assumed that the centres of mass of the bodies are located on one of the axes of the associated frames and that the inertia products, relative to such frames, are zero. (Since it gives a good idea of reality with respect to industrial robot manipulators).

The calculation of the dynamic model leads to the following result :

Step n°	Number of multiplies	Number of adds
0	Not required	
1a and 1b	18	12
2	25	0
3a and 3b	28	22
4	0	15
5	Not required	
6a,6b and 6c	22	12
7a and 7b	26	16
8	Not required	
9a and 9b	62	50
10	Not required ; included in 9a and 9b	
TOTAL	181	127

This example demonstrates the efficiency of our method compared to the previous ones[9]. However, the result obtained is not minimal. Indeed, it can be noted that y^{*5} and z^{*6} always occur coupled under the form of their difference. This leads to a slight simplification of the computation. A quick examination of the model obtained allows to bring the computations down to 164 multiplies and 115 adds, i.e., less than 300 arithmetical operations.

CONCLUSION

In this paper we have presented a method of computation of the dynamic model of a robot-manipulator -with a simple kinematic chain structure and revolute and/or prismatic joints- using a quasi-minimal number of arithmetical operations. It has been shown why the Newton-Euler formalism which leads to the implicit model in more suited than the Lagrange formalism leading to the explicit model. However, it must be recalled that the latter formalism is particularly useful if the inverse model is equally searched for (to compute the generalized accelerations as a function of the generalized positions and velocities on the one hand and the generalized forces on the other).

The advantage of our method lies in the use of the notions of intrinsic, analytical, iterative calculation and of augmented body that had never been generalized to this type of structure altough it had been known for a long time. Indeed, this method allows a-priori gathering together of most parameters occuring together in the model. However, note that, as shown in the example, some gatherings together cannot be planned with this notion.

This leads us to focus our research work on a new notion that would ensure an exhaustive gathering together of all the parameters occuring together in the model. This offers a two fold advantage : firstly it must lead to a new simplification of the computation of the model and secondly it is needed when an identification procedure is required to regroup all the parameters involved in the model.

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