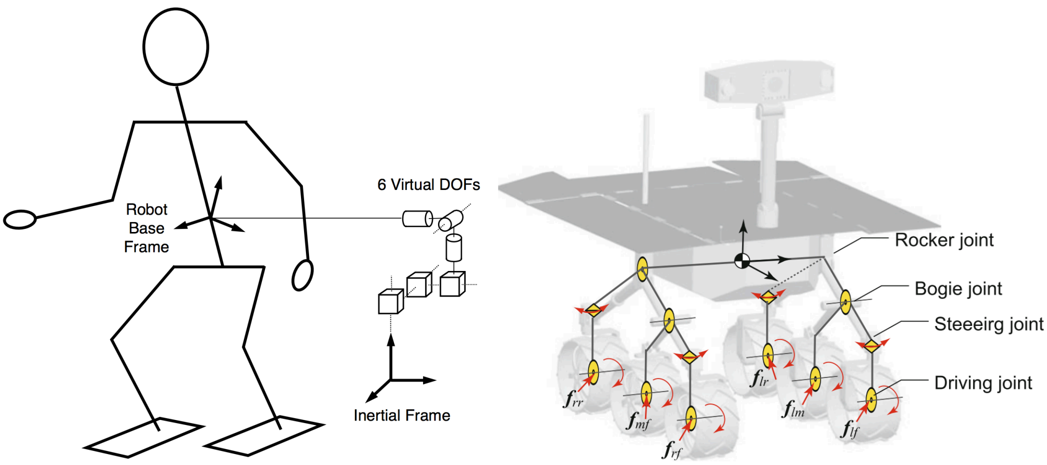
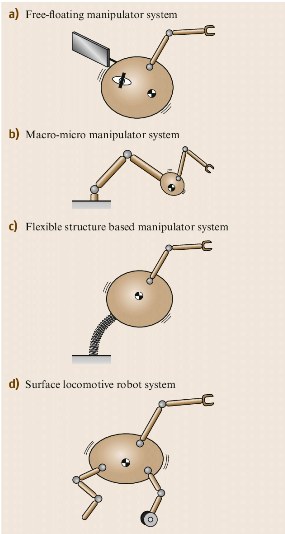
Moving base robots

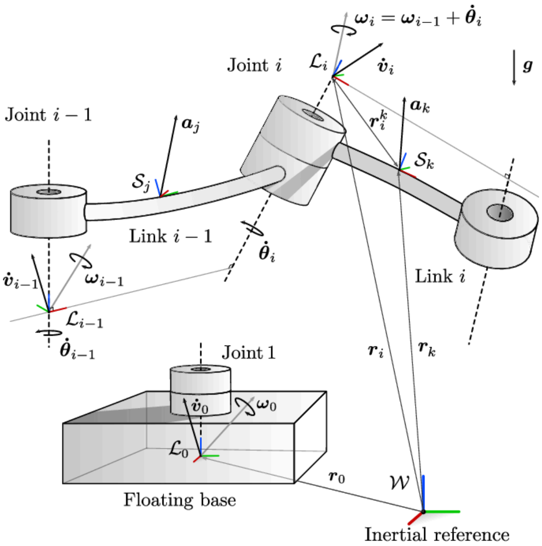
A new paradigm of moving base robotics, for a class of robots which have manipulator systems on a moving base. In some sense we can see at this case as a generalisations of classic robot models, where base frame connected to first link is assumed to be inertial and rigid, able to withstand any reaction. This is not actually the case even for some common applicative case. For such robots, kinematic and dynamic coupling between the manipulator arm and the base degrades positioning accuracy and operational dexterity. Moving base robots are classified into the following four categories:

- Free Floating Manipulator System,
- Flexible Structure mounted Manipulator System,
- Macro-Mini Manipulator System, and
- Mobile Vehicle mounted Manipulator System. (wheeled robots)
- Surface locomotive systems (walking robots)

Base motion imply significative contributes for dynamics and kinematics especially in some applications where base motion is common and not negligible.
Some classic examples include spacecraft mounted robots, movable base robots, humanoid and legged robots, walking robots in general, wheeled robots etc. Typically considered configurations are reported in figure.







For such a system, the equation of motion is expressed as follows. Case equation of motion taken form [Siciliano chap 54 Space robotics]:

$$\begin{bmatrix} H_{bb} & H_{bm} \\ H_{bm}^T & H_{mm} \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \begin{pmatrix} c_b \\ c_m \end{pmatrix} = \begin{pmatrix} F_b \\ \tau_m \end{pmatrix} + \begin{pmatrix} J_b^T \\ J_m^T \end{pmatrix} F_e$$

Free base motion imply acting external forces at the base F_b should be null. But equations allow to manage general case of forced base motion.
Remark that essentially structure of equations do not changes significantly. Indeed one way to see the problem is as robot with an additional root link which motion could be imposed, free or subject to deterministic forces. Results shall be in the end equivalent, provided assigned boundary conditions are respected.

The kinematic relationship among x_h, x_b, and ϕ is expressed using Jacobian matrices as

$$\dot{x}_h = J_m \dot{\phi} + J_b \dot{x}_b$$
$$\ddot{x}_h = J_m \ddot{\phi} + \dot{J}_m \dot{\phi} + J_b \ddot{x}_b + \dot{J}_b \dot{x}_b$$

and H_b, H_m, and H_{bm} are inertia matrices for the base body, manipulator arm, and the coupling between the base and the arm, respectively, c_p and c_q are nonlinear Coriolis and centrifugal forces, respectively. For a free-floating manipulator in orbit, the gravity forces exerted on the system can be *neglected*, and so the nonlinear term becomes

$$c_b = \dot{H}_b \dot{x}_b + \dot{H}_{bm} \dot{\phi}_b$$

Integrating the upper set of equation in (55.1) with respect to time, we obtain the total momentum of the system as

Free floating base equations

Let us first consider a free-floating system with a single or multiple manipulator arm(s) mounted on a mobile base. The pioneering work in the mathematical models of this type of space manipulator systems were conducted in late 1980s and early 1990s and collected in the book [55.48], published in 1993. We introduce models that are widely accepted today. The base body, termed link 0, is floating in inertial space without any external forces or moments (free motion). At the end point of the arm(s), external forces/moments F_e may apply.

$$\begin{bmatrix} H_{bb} & H_{bm} \\ H_{bm}^T & H_{mm} \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \begin{pmatrix} c_b \\ c_m \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix} + \begin{pmatrix} J_b^T \\ J_m^T \end{pmatrix} F_e$$

Typical applicative examples comes from space dynamics where arms or generic mechanical systems are mounted on a free floating base (expected for perturbative forces, like drag, solar pressure, propulsion or other perturbations and respective gradients).
Inclusion of flexibility contributions is also possible in this case, setting properly forcing contributes

$$\begin{bmatrix} H_{bb} & H_{bm} \\ H_{bm}^T & H_{mm} \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \begin{pmatrix} c_b \\ c_m \end{pmatrix} + \begin{bmatrix} D_b & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \begin{bmatrix} K_b & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta x_b \\ \Delta q \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{pmatrix} J_b^T \\ J_m^T \end{pmatrix} F_e$$

For the case in which reaction wheels are mounted on the base body, they are included as additional manipulator arms.

Imposed, forced and flexible base motion,

In case base motion is not free but imposed, substantially there are not significative differences. Structure of equations remain the same, the only difference is that (like in inverse dynamic case) we assume motion of base to be provided and imposed, leading to equations simplifications. Also the case of forced and flexible base could be treated generally in same framework using proper contributes into right side of equations, including possible flexibility contributes from the base, and eventually of links.

$$\begin{bmatrix} H_{bb} & H_{bm} \\ H_{bm}^T & H_{mm} \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \begin{pmatrix} c_b \\ c_m \end{pmatrix} + \begin{bmatrix} D_b & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{x}_b \\ \dot{q} \end{pmatrix} + \begin{bmatrix} K_b & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta x_b \\ \Delta q \end{pmatrix} = \begin{pmatrix} F_b \\ \tau_m \end{pmatrix} + \begin{pmatrix} J_b^T \\ J_m^T \end{pmatrix} F_e$$

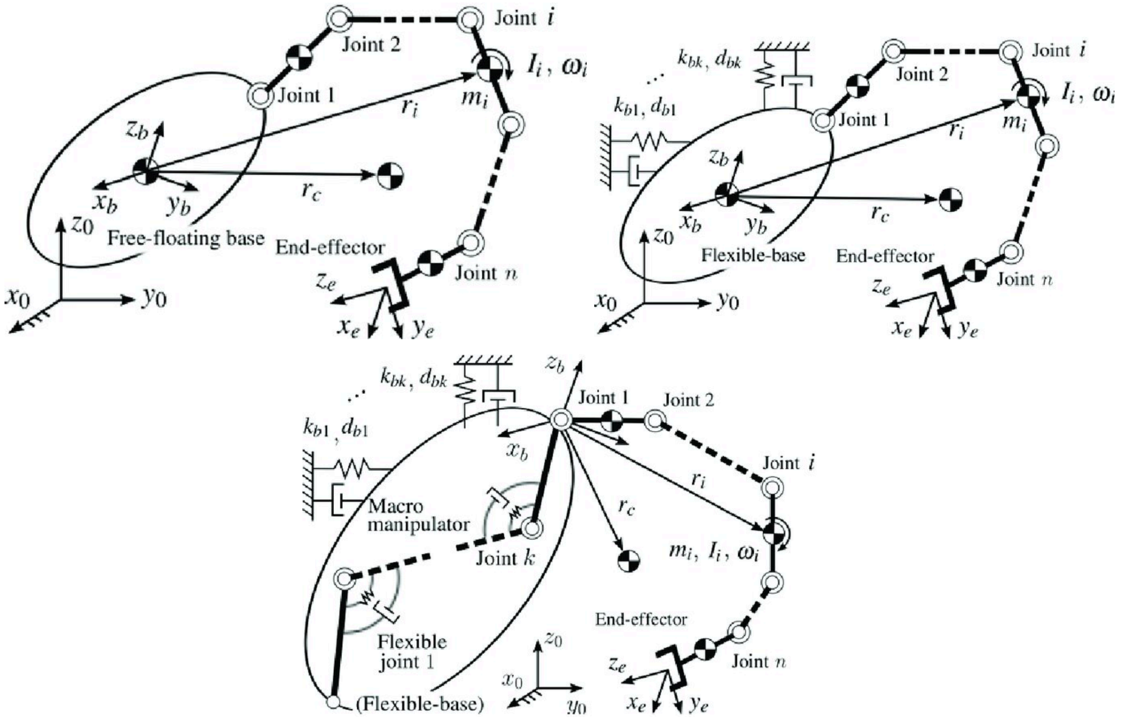
Derivation of dynamic equation for a imposed motion base in symbolic form is straightforward using RNEA method, allowing model derivation as in fixed base case. Nonetheless complexity of equations increases significantly and when possible simplification assumptions are made.

It is worthy to note also multi arms case is treatable in same analytical model. Extension to Multi-Arm robots for free-flying robot and not, has i manipulator arms mounted on a base, the manipulators comprise a tree-like structure. Each manipulator arm has n_k joints, $k = 1..l$ resulting in the total number of joints of $n = \sum_{k=1}^{l} n_k$. External forces may act on the base as well as on one or more of the end-links. The dynamic equation set then becomes as above as general case.

Introduction of quasi coordinates is also possible leading to equations decoupling

$$\begin{bmatrix} M_C & 0 \\ 0 & M_m \end{bmatrix} \begin{pmatrix} \ddot{x}_c \\ \ddot{q} \end{pmatrix} + \begin{pmatrix} C_c \\ c_m \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{pmatrix} J_{ec}^T \\ J^T \end{pmatrix} F_e$$

Other cases where base motion derive from dynamic interaction with base body including flexibility effects is more complex and specific methods shall be applied.
Some simple schematics of moving base robotic on a free floating and flexible base with mounted arms are given in following pictures:



$\mathbf{M}_m$	$\in \Re^{n\times n}$	: fixed-base manipulator link inertia matrix
$\mathbf{M}_b$	$\in \Re^{6\times 6}$	: system articulated body inertia matrix
$\mathbf{M}_{bm}$	$\in \Re^{6\times n}$	: coupling inertia matrix
$\mathbf{c}_m$	$\in \Re^{n}$	: fixed-base manipulator link Coriolis and centrifugal forces
C_b	$\in \Re^{6}$	: Coriolis and centrifugal forces on the system articulated body
τ	$\in \Re^{n}$	: manipulator joint torque vector
$\mathcal{V}_b$	$\in \Re^{6}$	: spatial velocity of the base
$\mathcal{F}_b,\ \mathcal{F}_e$	$\in \Re^{6}$	: spatial forces on the base and the end-link respectively
${}^b\mathbf{T}_e$	$\in \Re^{6\times 6}$	: spatial coordinate transform
$\mathbf{J}_m$	$\in \Re^{6\times n}$	: fixed-base manipulator Jacobian matrix

For purpose of study, maybe in some case unpractical deriving a complete analytical model of a system with base motion. Conversely the design of a multibody model in Simscape (as well as other multibody environments) is rather easy and little time taking.
That's why the use of a digital twin under a purposed tool may be a suitable and easy way to reproduce a real experiment in virtual environment and get some insights in expected response.

For instance we could desire to design an identification experiment but avoiding real physical setup could be expensive and time taking, in order to see what could be obtained using different trajectories or even a floating base motion or even combinations of, making our digital twin a sort of experiment design tools able to save a lot of time and efforts.

Set of equations can be used in same way of classic dynamics equations for identification purposes. None the less some considerations lead to prefer a different approach, for instance in humanoid robot identification.

.For more details see

Yoshida, K., Nenchev, D., Ishigami, G., Tsumaki, Y. (2014). Space Robotics. In: Macdonald, M., Badescu, V. (eds) The International Handbook of Space Technology. Springer Praxis Books(). Springer, Berlin, Heidelberg.

Yoshida, K., Nenchev, D., Ishigami, G., Tsumaki, Y. (2014). Space Robotics. In: Macdonald, M., Badescu, V. (eds) The International Handbook of Space Technology. https://doi.org/10.1007/978-3-642-41101-4_19

Base-lnk equations and base parameters

First equation of

$$\begin{bmatrix} H_{bb} & H_{bm} \\ H_{bm}^T & H_{mm} \end{bmatrix} \begin{pmatrix} \ddot{x}_b \\ \ddot{q} \end{pmatrix} + \begin{pmatrix} c_b \\ c_m \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix} + \begin{pmatrix} J_b^T \\ J_m^T \end{pmatrix} F_e$$

$$H_{bb}\ddot{x}_b + H_{bm}\ddot{q} + c_b = J_b^T F_e$$

describes dynamic of base-link as seen afore. Rewriting it as function of inertial parameters as (following conventions of [Identification of Human Mass Properties From Motion])

$$Y_B \phi_B = \begin{bmatrix} Y_{B1} \\ Y_{B2} \end{bmatrix} \phi_B = \begin{bmatrix} 0 \\ \tau \end{bmatrix} + \sum_{k=1}^{N_c} \begin{bmatrix} K_{k1} \\ K_{k2} \end{bmatrix} F_k$$

first equation do not depend on joints actuation but only on contact forces

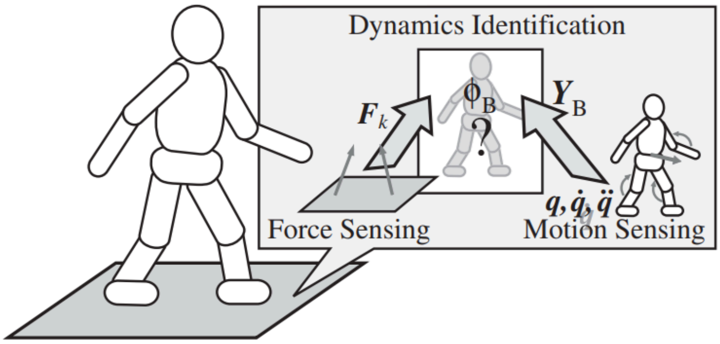
$$Y_{B1} \phi_B = \sum_{k=1}^{N_c} K_{k1} F_k$$

The model given by full set or the identification model given by first one leads to similarly identify the base parameters. However when using full set or second set it is necessary to have a measurement of the base-link position and orientation, the joint coordinates for each joint, the contact forces and the joint torque for each joint. It is also necessary to take into account the visco-elastic properties of the joints if there are some Khalil and Dombre [2002], Venture et al. [2006]. And eventually to use a model of the actuation to obtain the joint torques: friction model for humanoids, muscle model for humans, which are still open problems. In conventional methods the identified inertial parameters are thus contaminated by the inaccuracies in the respective models. These issues are eliminated when using the base-link approach given in first set, as it is neither a function of the joint torque τ nor the visco-elasticities as shown in figure. The inertial parameters are then estimated straightforwardly. Nevertheless, as there is no local measurements of the joint torque a drawback of the base-link approach resides in the design of exciting motions. Typical used dynamic excitation is using imitation of human motion, based on conductor-like motions. These motions are performed without using the joint full range of motion. Use the full-range of motion is also possible. The use of combination of multiple motions s commonly used for the identification, some motion being used for cross-validations.

Motion acquisition can be obtained with different methods, one being capture motion by multiple camera and markers, gyros and other onboard sensors and instrumentations. The contact forces are measured by means of force-plates. Notably authors remarket similar methods are applicable also to living beings not equipped with actuator sensing or to any similar kinematic tree structured system, robotic, artificial,natural or what else.

Nonetheless the method is applicable also to traditional industrial manipulators but since requiring normally not available and somewhat expensive measurements equipment, is usually apple only on specific high value systems justifying costs and efforts.

Once more the use of digital twin approach and virtual experimentation make it possible a cost effective experiment design and study/analysis.



Principle of the identification of bipeds from base-link dynamics

Minimum inertial parameters for humanoid robots

Different approach used in base-link identification method allows for application of alternative methods of base parameters vector.

$$\phi_B = \begin{bmatrix}\phi_{B0}^T & \phi_{B1}^T & \dots & \phi_{B30}^T\end{bmatrix}^T \quad (5)$$

such that:

$$\phi_{Bi} = \begin{cases} [\ M_i\ MS_{i,x}\ MS_{i,y}\ MS_{i,z}\ J_{i,xx} \\ \quad J_{i,yy}\quad J_{i,zz}\quad J_{i,yz}\quad J_{i,zx}\ J_{i,xy}\ ]^T & (i=0) \\ [\ MS_{i,x}\ MS_{i,y}\ J_{i,xx}-J_{i,yy}\ J_{i,zz} \\ \quad J_{i,yz}\quad J_{i,zx}\quad J_{i,xy}\ ]^T & (i>0) \end{cases}$$

where:

- M_i is the base parameter of link i representing the sum of the masses of links that are lower in the chain: $M_{i-1} = m_{i-1} + M_i$,
- i from 30 to 1.
- MS_i is the base parameter of link i representing the sum of the first moment of inertia, for i from 30 to 1: $\boldsymbol{MS}_{i-1} = \boldsymbol{ms}_{i-1} + M_i{}^{i-1}\boldsymbol{p}_i + {}^{i-1}\boldsymbol{r}_i$,
- J_i is the base parameter of link i representing the inertia, for i from 30 to 1:

$$\begin{aligned}\boldsymbol{J}_{i-1} &= \boldsymbol{I}_{i-1} + M_i[{}^{i-1}\boldsymbol{p}_i\times]^T[{}^{i-1}\boldsymbol{p}_i\times] \\ &+ [{}^{i-1}\boldsymbol{p}_i\times]^T[{}^{i-1}\boldsymbol{r}_i\times] + [{}^{i-1}\boldsymbol{r}_i\times]^T[{}^{i-1}\boldsymbol{p}_i\times] \\ &+ J_{i,yy}{}^{i-1}\boldsymbol{R}_i\boldsymbol{U}^{i-1}\boldsymbol{R}_i^T\end{aligned}$$

- ${}^{i-1}\boldsymbol{R}_i$ is the rotation matrix from the frame attached to link $i-1$ to the frame attached to link i,
- ${}^{i-1}\boldsymbol{p}_i$ is the translational vector from the frame attached to link $i-1$ to the frame attached to link i,
- ${}^{i-1}\boldsymbol{r}_i$ and $\boldsymbol{U}$ defined as follow:

$${}^{i-1}\boldsymbol{r}_i = {}^{i-1}\boldsymbol{R}_i \begin{bmatrix} 0 \\ 0 \\ MS_{i-1,z}\end{bmatrix}, \boldsymbol{U} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0\end{bmatrix}$$

The links are numbered as follow: B_0 for the base-link, B_1 to B_3 for the head, B_4 to B_{10} for the left arm, B_{11} to B_{17} for the right arm, B_{18} for the waist, B_{19} to B_{24} for the left leg, B_{25} to B_{30} for the right leg. To simplify the computation it

Reaction Null-Space (RNS)

From a practical point of view, any change in the base attitude is undesirable. As such, manipulator motion planning methods that minimize the base attitude disturbance have been investigated extensively. Analysis of the angular momentum equation reveals that the ultimate goal of achieving zero disturbance is possible. The following is the angular momentum equation with zero initial angular momentum $L = 0$ and $\omega_b = 0$the zero attitude disturbance given in (55.14)

$$\bar{H}_{bm}\dot{\phi} = 0$$

HQ bmP D 0 : (55.16)

This equation yields the following null-space solution

$$\bar{H}_{bm}\dot{\phi} = 0$$

$$\dot{\phi} = \left(I - \tilde{H}_{bm}^{+}\tilde{H}_{bm}\right)\dot{\zeta}$$

P D .I HQ C bmHQ bm/ P : (55.17)

The joint motion given by (55.17) is guaranteed not to disturb the base attitude. Here, the vector $\dot{\zeta} \in \Re^{n}$ is arbitrary and the null-space of the inertia matrix $\widetilde{H}_{bm} \in \Re^{3xn}$ is called the *reaction null-space* (RNS) [55.61].