A DIRECT DETERMINATION OF MINIMUM INERTIAL PARAMETERS OF ROBOTS

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Abstract

The dynamic model of robots is obtained classically as function of 10 inertial parameters per link. However, some of these parameters have no effect on the dynamic model and some others may appear in linear combinations. The classical parameters affecting the dynamic model separetelly and the regrouped parameters constitute the set of minimum inertial parameters. The determination of this set of parameters contributes at reducing the computational cost of the dynamic models and increases the robustness of the identification of the inertial parameters.

This paper presents a direct method to determine the set of minimum number of inertial parameters of robots. The given method permits to determine most of the regrouped parameters by means of closed form relations function of the geometric parameters of the robot, we prove that the minimum number of inertial parameters is equal to or less than 7n–4, where n is the number of joints.

1. Introduction

In the last years many control schemes based on the dynamic model have been presented [1,2,3,4] . Two problems have to be solved for real time implementation of these control schemes:

1- The numerical values of the inertial parameters must be known. The solution of this problem has been investigated by the use of identification procedures based on a dynamic model linear in the inertial parameters [5,6,7,8,9]. However, some of these parameters do not affect the dynamic model and some others appear in linear combination. The classification and the determination of the identifiable parameters increases the robustness of the identification process. This set of identifiable parameters constitutes what we call in this paper the set of minimum number of inertial parameters.

2- The computational cost of the dynamic model must be reduced. The solution of this problem has been carried out by the use of the customized Newton-Euler method [10,11] . This method takes into account the particular values of the inertial and geometric parameters of the robot. To increase the number of parameters which are equal to zero and consequently reducing the number of operations of the dynamic model, once more the use of the set of minimum number of inertial parameters and a dynamic model linear in the inertial parameters have been proposed [12,13].

The definition and the determination of the minimum number of inertial parameters have been investigated in [12, 13, 14, 15]:

- Khosla [14] has used a symbolic Newton-Euler algorithm. Some results concerning some parameters at the case of robots whose successive axes are either parallel or perpendicular are given. The detection of most of the combined parameters needs a long examination of the symbolic Newton Euler model on a case by case basis.

- Khalil and al.[12, 13, 15] , have determined the set of minimum parameters by the examination of the symbolic expressions of the inertia matrix and the gravity forces coefficients of a lagrangian dynamic model. Although that these expressions are generated automatically by computer [16], the detection of the combined inertial parameters may need several hours .

The aim of this paper is to present a general and direct method to determine the set of minimum parameters. In fact , most of these parameters , or in many cases all of them, will be determined by a recursive and closed form solution. A very little number of parameters concerning the first links may need more detailed study, which can be carried out in few minutes.

2. Description of the robot

The system to be considered is an open loop mechanism.The description of the system will be carried out by the use of the modified Denavit - Hartenberg notation [17] . The coordinate frame $\mathbf{R}_i$ (x_i,y_i,z_i) is assigned fixed with respect to link i. The $\mathbf{Z}_i$ axis is along the axis of joint i, the $\mathbf{X}_i$ axis is along the common perpendicular of $\mathbf{Z}_i$ and $\mathbf{Z}_{i+1}$. The frame i coordinate will be defined with respect to the frame i-1 by the matrix ${}^{i-1}\mathbf{T}_i$ which is function of the parameters (α_i, d_j, θ_j, r_j) , Figure (1), such that :

$$ {}^{i-1}\mathbf{T}_i = \begin{bmatrix} {}^{i-1}\mathbf{A}_i & {}^{i-1}\mathbf{P}_i \\ 0\,0\,0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & d_i \\ S\theta_i C\alpha_i & C\theta_i C\alpha_i & -S\alpha_i & -r_i S\alpha_i \\ S\theta_i S\alpha_i & C\theta_i S\alpha_i & C\alpha_i & r_i C\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1) $$

where:
${}^{i-1}\mathbf{A}_i$ defines the orientation of frame i with respect to frame i-1,
${}^{i-1}\mathbf{P}_i$ defines the position of the origin of frame i with respect to frame i-1.

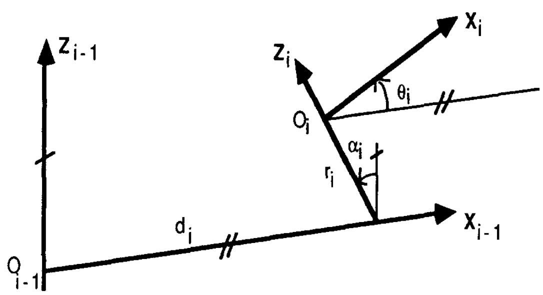

Figure 1

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The joint variable i will be denoted by :

$$q_i = \bar{\sigma}_i \, \theta_i + \sigma_i r_i \quad (2)$$

where $\sigma_i = 0$ for i rotational , $\sigma_i = 1$ for i translational, and $\bar{\sigma}_i = (1-\sigma_i)$.

3- Definition of the inertial parameters

Let us denote:

m_j the mass of link j

${}^j mS_j$ the first moment of link j about the origin of frame j , referred to frame j= $[\, mX_j \;\; mY_j \;\; mZ_j]^T$

${}^j J_j$ the inertia tensor of link j about the origin of frame j, it will be denoted by:

$${}^j\mathbf{J}_j = \begin{bmatrix} XX_j & XY_j & XZ_j \\ XY_j & YY_j & YZ_j \\ XZ_j & YZ_j & ZZ_j \end{bmatrix}$$

The inertial parameters of link j will be represented by the vector $\mathbf{X}^j$, denoted by:

$$\mathbf{X}^j = \begin{bmatrix} XX_j & XY_j & XZ_j & YY_j & YZ_j & ZZ_j & mX_j & mY_j & mZ_j & m_j \end{bmatrix}^T \quad (3)$$

The inertial parameters of the robot will be represented by the (10nx1) vector $\mathbf{X}$, denoted by:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}^{1^T} & \mathbf{X}^{2^T} & \cdots & \mathbf{X}^{n^T} \end{bmatrix}^T \quad (4)$$

4. The lagrangian dynamic model of a robot

The Lagrangian equation is given by:

$$\Gamma_j = \frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_j}\right) - \frac{\partial E}{\partial q_j} + \frac{\partial U}{\partial q_j} \qquad j= 1\ldots n \quad (5)$$

where:

Γ is nx1 vector representing the actuators torques (or forces),

E is the total kinetic energy, and U is the total potential energy of the robot.

4-1 Calculation of the kinetic energy

The kinetic energy of the robot can be calculated as :

$$E = \sum_{j=1}^{n} \frac{1}{2}\left({}^j\omega_j^T \, {}^j\mathbf{J}_j \, {}^j\omega_j + m_j \, {}^j\mathbf{V}_j^T \, {}^j\mathbf{V}_j + 2\, {}^j\mathbf{V}_j^T \left({}^j\omega_j \times {}^j\mathbf{mS}_j \right) \right) \quad (6)$$

where:

${}^j\mathbf{V}_j$ the velocity of the origin of the link j fixed frame, referred to frame j,

${}^j\omega_j$ the angular velocity of link j, referred to frame j.

${}^j\omega_j$ and ${}^j\mathbf{V}_j$ can be calculated by the following recursive equations [13, 18]:

$${}^j\omega_j = {}^j\mathbf{A}_{j-1} \, {}^{j-1}\omega_{j-1} + \bar{\sigma}_j \mathbf{Z}_0 \, \dot{q}_j \quad (7)$$

$${}^j\mathbf{V}_j = {}^j\mathbf{A}_{j-1} \, {}^{j-1}\mathbf{V}_{j-1} + {}^j\mathbf{A}_{j-1} \left({}^{j-1}\omega_{j-1} \times {}^{j-1}\mathbf{P}_j \right) + \sigma_j \mathbf{Z}_0 \, \dot{q}_j \quad (8)$$

where: $\mathbf{Z}_0 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

4-2 Calculation of the potential energy

The potential energy is calculated as [13, 18]:

$$U = -\sum_{j=1}^{n} \left(\mathbf{g}^T \left(m_j \, {}^0\mathbf{P}_j + {}^0\mathbf{A}_j \, {}^j\mathbf{mS}_j \right) \right) \quad (9)$$

where $\mathbf{g}$ is the acceleration of gravity.

4-3 The linearity of the dynamic model in the inertial parameters

From equations (6,...,9) we deduce that E and U are linear in the inertial parameters. Thus :

$$E = \sum_{i=1}^{m} \frac{\partial E}{\partial X_i} X_i = \sum_{i=1}^{m} DE_i \, X_i \quad (10)$$

$$U = \sum_{i=1}^{m} \frac{\partial U}{\partial X_i} X_i = \sum_{i=1}^{m} DU_i \, X_i \quad (11)$$

where X_i is an inertial parameter, DE_i is function of $\mathbf{q}$, $\dot{\mathbf{q}}$, and the geometric parameters of the robot, and DU_i is function of $\mathbf{q}$ and the geometric parameters.

From (10) and (11) we obtain that the dynamic model is also linear in the inertial parameters. It can be given as:

$$\Gamma = \mathbf{H} \; \mathbf{X} \quad (12)$$

where:

$\mathbf{H}$ is a nxm matrix, function of $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ and the constant geometric parameters.

5. Determination of the parameters having no effect on the dynamic model

In terms of energy, and by the use of (5),(10), (11), we see that if:

$$DE_i = 0 \qquad \text{and} \qquad DU_i = \text{constant} \quad (13)$$

then the parameter X_i has no effect on Γ. Thus X_i is not an element of the minimum inertial parameters, it can be put equal to zero in (12).

6- Conditions of regrouping the inertial parameters

The regrouping of an inertial parameter X_i , to some other parameters $X_{i1},\ldots,X_{ir}$ can be carried out by detecting the linear combination of DE_i and DU_i.

In fact if :

$$DE_i = \alpha_{i1} \, DE_{i1} + \ldots + \alpha_{ir} \, DE_{ir} = \sum_{k=1}^{r} \alpha_{ik} \, DE_{ik} \quad (14)$$

and

$$DU_i = \alpha_{i1} \, DU_{i1} + \ldots + \alpha_{ir} \, DU_{ir} = \sum_{k=1}^{r} \alpha_{ik} \, DU_{ik} \quad (15)$$

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with α_{ik} = constant, then we get the same value of E and U, (and consequently Γ), if we use the parameters $X_i, X_{i1},...,X_{ir}$, or if we put X_i equal to zero and the parameters X_{ik} equal to XR_{ik} where:

$$XR_{ik} = X_{ik} + \alpha_{ik} X_i$$

We choose to eliminate the parameter belonging to the farthest link from the base.

7. Closed form solution of regrouping the inertial parameters

Let X^j_i be the i^{th} element of (3) defining the i^{th} inertial parameter of link j, with i=1, ..., 10.

Denoting $\mathbf{DE}^j$ the vector of the components $\partial E/\partial X_i^j$, and $\mathbf{DU}^j$ the vector of the components $\partial U/\partial X_i^j$, i = 1, ...,10, thus :

$$\mathbf{DE}^j=\left[\frac{\partial E}{\partial X_1^j} \quad \frac{\partial E}{\partial X_2^j} \quad \cdots \quad \frac{\partial E}{\partial X_{10}^j}\right]^T, \mathbf{DU}^j=\left[\frac{\partial U}{\partial X_1^j} \quad \cdots \quad \frac{\partial U}{\partial X_{10}^j}\right]^T \quad (16)$$

The main idea is to calculate $\mathbf{DE}^j$ and $\mathbf{DU}^j$ as function of $\mathbf{DE}^{j-1}$ and $\mathbf{DU}^{j-1}$ respectively.

7 -1 Recursives relations of $\mathbf{DE}^j$ and $\mathbf{DU}^j$

The recursives relations of $\mathbf{DE}^j$ and $\mathbf{DU}^j$ can be written as:

$$\frac{\partial E}{\partial X_i^j} = \sum_{k=1}^{10} \lambda^j_{i,k} \frac{\partial E}{\partial X_k^{j-1}} + f_i^j(\mathbf{q},\dot{\mathbf{q}}) \quad (17)$$

$$\frac{\partial U}{\partial X_i^j} = \sum_{k=1}^{10} \beta^j_{i,k} \frac{\partial U}{\partial X_k^{j-1}} \quad (18)$$

As $\partial U/\partial X_k^{j-1} = 0$ for k = 1,...,6, and taking into account that (see appendix):

$\beta_{i,k}^{j-1} = \lambda_{i,k}^{j-1}$, (k = 7 ,..., 10) , then eq. (18) can be written as:

$$\frac{\partial U}{\partial X_i^j} = \sum_{k=7}^{10} \lambda^j_{i,k} \frac{\partial U}{\partial X_k^{j-1}} = \sum_{k=1}^{10} \lambda^j_{i,k} \frac{\partial U}{\partial X_k^{j-1}} \quad (19)$$

The expressions of λ and $\mathbf{f}$ are given in the appendix.

7-2 General regrouping relations of inertial parameters

The necessary conditions have been given by (14) and (15). By the use of the recursive relations of energy given in appendix , we consider the following two cases:

a- If joint j is rotational, σ_j=0, we see that:

i- The coefficients $\lambda_{9,k}^j$ and $\lambda_{10,k}^j$ are constant, for k=1,...,10, while the elements $f_9^j(\mathbf{q},\dot{\mathbf{q}}) = 0$ and $f_{10}^j(\mathbf{q},\dot{\mathbf{q}}) = 0$, i.e DE_9^j and DE_{10}^j verify the condition (14). Taking into account the relation (19) , then DU_9^j and DU_{10}^j verify also the condition (15). Thus the parameters 9 and 10 , mZ_j and m_j , can be regrouped to the parameters of the link j–1.

ii- The sum of DE_1^j and DE_4^j, corresponding to the elements XX_j and YY_j respectively, can be expressed through constant coefficients as function of DE_i^{j-1} . Thus XX_j , or YY_j , can be regrouped to the parameters of link j-1 and YY_j , or XX_j . We choose to regroup YY_j.

As a conclusion, the parameters (YY_j, mZ_j , m_j) can be eliminated while the combined parameters of links j and j-1 are given by :

$$\begin{aligned}
XXR_j &= XX_j - YY_j\\
XXR_{j-1} &= XX_{j-1} + YY_j + 2\, r_j\, mZ_j + r_j^2\, m_j\\
YYR_{j-1} &= YY_{j-1} + C\alpha_j^{\,2}\, YY_j + 2\, r_j\, C\alpha_j^{\,2}\, mZ_j + (d_j^2 + r_j^2\, C\alpha_j^2)\, m_j\\
ZZR_{j-1} &= ZZ_{j-1} + S\alpha_j^2\, YY_j + 2\, r_j S\alpha_j^2\, mZ_j + (d_j^2 + r_j^2\, S\alpha_j^2)\, m_j\\
XYR_{j-1} &= XY_{j-1} + d_j\, S\alpha_j\, mZ_j + d_j r_j S\alpha_j\, m_j\\
XZR_{j-1} &= XZ_{j-1} - d_j\, C\alpha_j\, mZ_j - d_j\, r_j\, C\alpha_j\, m_j\\
YZR_{j-1} &= YZ_{j-1} + C\alpha_j\, S\alpha_j\, YY_j + 2\, r_j\, C\alpha_j S\alpha_j\, mZ_j + r_j^2\, C\alpha_j S\alpha_j\, m_j\\
mXR_{j-1} &= mX_{j-1} + d_j\, m_j\\
mYR_{j-1} &= mY_{j-1} - S\alpha_j\, mZ_j - r_j\, S\alpha_j\, m_j\\
mZR_{j-1} &= mZ_{j-1} + C\alpha_j\, mZ_j + r_j\, C\alpha_j\, m_j\\
mR_{j-1} &= m_{j-1} + m_j
\end{aligned} \quad (20)$$

b- if joint j is translational, $\sigma_j = 1$, we see that:

$\lambda_{i,k}$ are constant while $f_i(\mathbf{q},\dot{\mathbf{q}}) = 0$, for i= 1,...,6 and k=1,...,10. Thus the inertial parameters 1,...,6 of link j can be regrouped to the inertial parameters of link j-1.

This means that all the parameters of $\mathbf{J}_j$ will be combined to the parameters of $\mathbf{J}_{j-1}$, by the use of the following relations:

$$\begin{aligned}
XXR_{j-1} &= XX_{j-1} + C\theta_j^2\, XX_j + S\theta_j^2\, YY_j - 2\, C\theta_j S\theta_j\, XY_j\\
YYR_{j-1} &= YY_{j-1} + S\theta_j^2 C\alpha_j^2 XX_j + C\theta_j^2 C\alpha_j^2 YY_j + S\alpha_j^2\, ZZ_j +\\
&\quad 2C\theta_j S\theta_j C\alpha_j^2\, XY_j - 2S\theta_j C\alpha_j S\alpha_j XZ_j - 2C\theta_j C\alpha_j S\alpha_j YZ_j\\
ZZR_{j-1} &= ZZ_{j-1} + S\theta_j^2 S\alpha_j^2 XX_j + C\theta_j^2 S\alpha_j^2 YY_j + C\alpha_j^2\, ZZ_j +\\
&\quad 2C\theta_j S\theta_j S\alpha_j^2\, XY_j + 2S\theta_j C\alpha_j S\alpha_j XZ_j + 2C\theta_j C\alpha_j S\alpha_j YZ_j\\
XYR_{j-1} &= XY_{j-1} + C\theta_j\, S\theta_j S\alpha_j XX_j - C\theta_j S\theta_j C\alpha_j YY_j + (C\theta_j^2 - S\theta_j^2)\, C\alpha_j\, XY_j\\
&\quad - C\theta_j S\alpha_j XZ_j + S\theta_j S\alpha_j YZ_j\\
XZR_{j-1} &= XZ_{j-1} - C\theta_j\, S\theta_j S\alpha_j XX_j - C\theta_j S\theta_j S\alpha_j YY_j + (C\theta_j^2 - S\theta_j^2)\, S\alpha_j\, XY_j\\
&\quad + C\theta_j C\alpha_j XZ_j - S\theta_j C\alpha_j YZ_j\\
YZR_{j-1} &= YZ_{j-1} - S\theta_j^2 C\alpha_j S\alpha_j XX_j + C\theta_j^2 C\alpha_j S\alpha_j YY_j - C\alpha_j S\alpha_j ZZ_j +\\
&\quad 2C\theta_j\, S\theta_j C\alpha_j S\alpha_j\, XY_j + S\theta_j (C\alpha_j^2 - S\alpha_j^2)\, XZ_j + C\theta_j\, (C\alpha_j^2 - S\alpha_j^2)\, YZ_j
\end{aligned} \quad (21)$$

Remark:

We note that the relations (21) correspond to the following matrix equation :

$$\mathbf{JR}_{j-1} = \mathbf{J}_{j-1} + {}^{j-1}\mathbf{A}_j\, \mathbf{J}_j\, {}^{j}\mathbf{A}_{j-1} \quad (22)$$

This relation could be proved directly by noting that the angular speed of link j is the same as that of link j-1, Eq.(7).

8-Practical utilization of the regrouping relations

At first we give the following theorems , some of them will be given without demonstration owing to the limited number of pages :

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Theorem 1: If all the parameters of link j-1, X_i^{j-1} for i=1,...,10, have $DE_i^{j-1} \neq 0$ or DU_i^{j-1} not constant, then the relations (20) and (21) detect all the regrouping parameters for links j,...,n. Moreover, no elimination is possible using conditions (13).

Theorem 2: In order that a parameter of link j may be regrouped to one or some parameters of link j–2, this parameter must be combined at first to one or some parameters of link j-1, which will be combined afterwards to the parameters of link j-2.

Thereom 3: If joint 1 is rotational the parameters XX_1, XY_1, XZ_1, YY_1, YZ_1, mZ_1, m_1 have no effect on the dynamic model. Moreover, if the axis of the first joint is along the direction of the acceleration of gravity then the parameters mX_1, mY_1, have no effect also.

Theorem 4: if joint 1 is translational the parameters XX_1, XY_1, XZ_1, YY_1, YZ_1, ZZ_1, mZ_1, mX_1, mY_1 have no effect on the dynamic model.

Theorem 5: If joint j is rotational, then the parameters (YY_j, mZ_j, m_j) can be eliminated by the use of the relations (20).

Theorem 6: If joint j is translational then the parameters (XX_j, XY_j, XZ_j, YY_j, YZ_j, ZZ_j) can be eliminated by the use of the relations (21).

Theorem 7: From theorems 3,4,5,6, we can deduce that the number of minimum inertial parameters is equal or less than:

$$7n_r + 4n_t - 3 - \bar{\sigma}_1 \quad (23)$$

where: n_r = number of rotational joints = $\Sigma\, \bar{\sigma}_j$

n_t = number of translational joints = $\Sigma\, \sigma_j$

The proposed algorithm of the detection of the set of minimum parameters will be carried out as follows:

a-Calculate DE_i and DU_i for the parameters of links 1,...,k, where k is the first link satisfying the condition of thereom 1, and apply the relations (13) and theorem 3 to eliminate the parameters having no effect on the model. The recursive relations given in appendix are easy to use for this purpose, even by hand.

b-Apply the relations (20) and (21) in cascade from link n to link 1.

c-Apply the relations (14) and (15) from link 1 to k, to detect the existence of supplementary combination.

Steps a and b are calculated automatically by computer, and step c is easy to carry out by hand, using the results of step a.

9-Example

Find the minimum inertial parameters of the SCEMI robot, Figure 2.

Step a: the use of relations (13) and theorem 3 for links 1,2,3, gives that the parameters XX_1, XY_1, XZ_1, YY_1, YZ_1, mX_1, mY_1, mZ_1, m_1, mZ_2, m_2, have no effect on the dynamic model.

Step b: we apply the relations (20) for links 6, ..., 1.

link 6:

$XXR_6 = XX_6 - YY_6$

$XXR_5 = XX_5 + YY_6$

$ZZR_5 = ZZ_5 + YY_6$

$mYR_5 = mY_5 + mZ_6$

$mR_5 = m_5 + m_6$

The minimum parameters of link 6 are : XXR_6, XY_6, XZ_6, YZ_6, ZZ_6, mX_6, mY_6.

link 5:

$XXR_5 = XXR_5 - YY_5 = XX_5 + YY_6 - YY_5$

$XXR_4 = XX_4 + YY_5$

$ZZR_4 = ZZ_4 + YY_5$

$mYR_4 = mY_4 - mZ_5$

$mR_4 = m_4 + mR_5 = m_4 + m_5 + m_6$

The minimum parameters of link 5 are : XXR_5, XY_5, XZ_5, YZ_5, ZZR_5 mX_5, mYR_5

link 4:

$XXR_4 = XXR_4 - YY_4 = XX_4 + YY_5 - YY_4$

$XXR_3 = XX_3 + YY_4 + 2\, RL4\, mZ_4 + RL4^2\, (m_4 + m_5 + m_6)$

$ZZR_3 = ZZ_3 + YY_4 + 2\, RL4\, mZ_4 + RL4^2\, (m_4 + m_5 + m_6)$

$mYR_3 = mY_3 + mZ_4 + RL4\, (m_4 + m_5 + m_6)$

$mR_3 = m_3 + m_4 + m_5 + m_6$

The minimum parameters of link 4 are : XXR_4, XY_4, XZ_4, YZ_4, ZZR_4, mX_4, mYR_4

link 3:

$XXR_3 = XXR_3 - YY_3 = XX_3 + YY_4 + 2\, RL4\, mZ_4 + RL4^2\, (m_4 + m_5 + m_6) - YY_3$

$XXR_2 = XX_2 + YY_3$

$YYR_2 = YY_2 + D3^2\, (m_3 + m_4 + m_5 + m_6)$

$ZZR_2 = ZZ_2 + D3^2\, (m_3 + m_4 + m_5 + m_6)$

$XZR_2 = XZ_2 - D3\, mZ_3$

$mXR_2 = mX_2 + D3\, (m_3 + m_4 + m_5 + m_6)$

The minimum parameters of link 3 are : XXR_3, XY_3, XZ_3, YZ_3, ZZR_3, mX_3, mYR_3.

link 2:

$XXR_2 = XXR_2 - YYR_2 = XX_2 + YY_3 - YY_2 - D3^2\, (m_3 + m_4 + m_5 + m_6)$

$ZZR_1 = ZZ_1 + YY_2 + D3^2\, (m_3 + m_4 + m_5 + m_6)$

The minimum parameters of link 2 are : XXR_2, XY_2, XZR_2, YZ_2, ZZR_2, mXR_2, mY_2.

link 1:

The minimum parameter of link 1 is : ZZR_1.

Step c: we apply the conditions (14) and (15) to links 1,2,3, it gives no supplementary regrouping.

10- Application to the Identification

The set of minimum number of inertial parameters are the only identifiable parameters, and they are only needed to the calculation of the control law. They represent, at the case of the given example 36 parameters, i.e. 24 parameters less than the classical parameters. This will facilitate greatly the identification process. The regrouped relations may not be important in this case, only the knowledge of the parameters to be eliminated either by regrouping or because they have no effect is essential. The identification process would give the values of the identifiable parameters directly.

11- Application to the control

The application of the set of minimum number of inertial parameters in an algorithm of customized Newton-Euler will

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contribute greatly in reducing the cost of calculation of the dynamic model, see table 1 [13, 18]. If we know the numerical values of the inertial parameter by means of measuring or calculation procedure, then the regrouping relations are necessary to get the values of the regrouped inertial parameters.

		General robot		Stanford		Scemi	
		n	n=6	Gen*	Simp**	Gen*	Simp**
Classical parameters	Mult.	101n-127	477	291	193	294	201
	Add.	90n-118	422	284	149	283	152
Minimum parameters	Mult.	92n-127	425	232	142	253	159
	Add.	81n-117	369	218	99	238	113

General robot means that the geometric and inertial parameters can take any values.
* Gen(eral) means that no inertial parameter is taken equal to zero,
**Simp(lified) means that the links inertia tensors are supposed diagonal, the first moment vector contains only one element different than zero, motors inertias and terminal forces are taken into account.

Table 1.

12- Conclusion

This paper presents a direct method to determine the set of minimum number of inertial parameters of robots. The given method permits to determine most of the regrouped parameters by means of closed form relations function of the geometric parameters of the robot. We prove that the minimum number of inertial parameters is less than 7n–4, where n is the number of joints. The method is general whatever the values of the geometric parameters.

The method can be extended to tree structure robots and will be integrated to our software package SYMORO.

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APPENDIX

A.1-Calculation of the recursive relations of the kinetic energy

Owing to the limited number of pages, we give directly final results, [19].

Let:

$$\mathbf{DE}^j = \left[\frac{\partial E}{\partial XX} \quad \frac{\partial E}{\partial XY} \quad \frac{\partial E}{\partial XZ} \quad \frac{\partial E}{\partial YY} \quad \frac{\partial E}{\partial YZ} \quad \frac{\partial E}{\partial ZZ} \quad \frac{\partial E}{\partial mX} \quad \frac{\partial E}{\partial mY} \quad \frac{\partial E}{\partial mZ} \quad \frac{\partial E}{\partial m} \right]_j^T$$

$$\mathbf{DE}^j = \left[DE_1 \quad DE_2 \quad DE_3 \quad DE_4 \quad DE_5 \quad DE_6 \quad DE_7 \quad DE_8 \quad DE_9 \quad DE_{10} \right]_j^T$$

We find that:

$$DE_1 = \frac{1}{2}\omega_1^2,\ DE_2 = \omega_1\omega_2,\ DE_3 = \omega_1\omega_3,\ DE_4 = \frac{1}{2}\omega_2^2,\ DE_5 = \omega_2\omega_3,\ DE_6 = \frac{1}{2}\omega_3^2$$

$$DE_7 = \omega_3 V_2 - \omega_2 V_3,\ DE_8 = \omega_1 V_3 - \omega_3 V_1,\ DE_9 = \omega_2 V_1 - \omega_1 V_2,\ DE_{10} = \frac{1}{2} \mathbf{V}^T \mathbf{V}$$

Where:

$${}^j\omega_j = \left[\omega_1 \quad \omega_2 \quad \omega_3 \right]_j^T, \quad {}^j\mathbf{V}_j = \left[V_1 \quad V_2 \quad V_3 \right]_j^T$$

Using (7) and (8), and writting α_j, θ_j, r_j, d_j without the subscript, we get:

$$\mathbf{DE}^j = \lambda^j \ \mathbf{DE}^{j-1} + \mathbf{f}^j$$

where : λ^j is a 10x10 matrix , function of the geometric parameters of link j, defined as:

$$\lambda^j = \begin{bmatrix} \mathbf{DJ1} & \mathbf{0} \\ \mathbf{DS1} & {}^{j-1}\mathbf{T}_j^T \\ \mathbf{DM1} & \end{bmatrix}$$

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$\mathbf{f}^j$ is a vector defined as :

$$\mathbf{f}^j=\dot{q}_j\left[\begin{bmatrix}\bar{\sigma}_j\mathbf{DJ2}\\ \sigma_j\mathbf{DS2}+\bar{\sigma}_j\mathbf{DS3}\\ \sigma_j\mathbf{DM2}\end{bmatrix}{}^{j-1}\omega_{j-1}+\begin{bmatrix}\mathbf{0}\\ \bar{\sigma}_j\mathbf{DS4}\\ \sigma_j\mathbf{DM3}\end{bmatrix}{}^{j-1}\mathbf{V}_{j-1}\right]$$

$$+\frac{1}{2}\dot{q}_j^2\begin{bmatrix}0 & 0 & 0 & 0 & 0 & \bar{\sigma}_j & 0 & 0 & 0 & \sigma_j\end{bmatrix}^T$$

where:

DJ1 =

$$\begin{bmatrix}C^2\theta & CS\theta C\alpha & CS\theta S\alpha & S^2\theta C^2\alpha & S^2\theta CS\alpha & S^2\theta S^2\alpha\\ -2CS\theta & \left(C^2\theta-S^2\theta\right)C\alpha & \left(C^2\theta-S^2\theta\right)S\alpha & 2CS\theta C^2\alpha & 2CS\theta CS\alpha & 2CS\theta S^2\alpha\\ 0 & -C\theta S\alpha & C\theta C\alpha & -2S\theta CS\alpha & S\theta\left(C^2\alpha-S^2\alpha\right) & 2S\theta CS\alpha\\ S^2\theta & -CS\theta C\alpha & -CS\theta S\alpha & C^2\theta C^2\alpha & C^2\theta CS\alpha & C^2\theta S^2\alpha\\ 0 & S\theta S\alpha & -S\theta C\alpha & -2C\theta CS\alpha & C\theta\left(C^2\alpha-S^2\alpha\right) & 2C\theta CS\alpha\\ 0 & 0 & 0 & S^2\alpha & -CS\alpha & C^2\alpha\end{bmatrix}$$

Where CS(.) means Cos(.)Sin(.)

$$\mathbf{DS1}^T=\begin{bmatrix}0 & 0 & 2r\\ C\theta S\alpha r-S\theta C\alpha d & -C\theta C\alpha d-S\theta S\alpha r & S\alpha d\\ -C\theta C\alpha r-S\theta S\alpha d & -C\theta S\alpha d+S\theta C\alpha r & -C\alpha d\\ 2\left(C\theta d+S\theta CS\alpha r\right) & 2\left(C\theta CS\alpha r-S\theta d\right) & 2CC\alpha r\\ S\theta r\left(SS\alpha-CC\alpha\right) & C\theta r\left(SS\alpha-CC\alpha\right) & 2CS\alpha r\\ 2\left(C\theta d-S\theta CS\alpha r\right) & 2\left(-C\theta CS\alpha r-S\theta d\right) & 2SS\alpha r\end{bmatrix}$$

$$\mathbf{DM1}=\begin{bmatrix}r^2 & S\alpha dr & -C\alpha dr & C^2\alpha r^2+d^2 & CS\alpha r^2 & S^2\alpha r^2+d^2\end{bmatrix}$$

$$\mathbf{DJ2}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ C\theta & S\theta C\alpha & S\theta S\alpha\\ 0 & 0 & 0\\ -S\theta & C\theta C\alpha & C\theta S\alpha\\ 0 & -S\alpha & C\alpha\end{bmatrix},\ \mathbf{DS2}=\begin{bmatrix}S\theta & -C\theta C\alpha & -C\theta S\alpha\\ C\theta & S\theta C\alpha & S\theta S\alpha\\ 0 & 0 & 0\end{bmatrix}$$

$$\mathbf{DS3}=\begin{bmatrix}-C\theta r & -C\theta S\alpha d-S\theta C\alpha r & C\theta C\alpha d-S\theta S\alpha r\\ S\theta r & -C\theta C\alpha r+S\theta S\alpha d & -C\theta S\alpha r-S\theta C\alpha d\\ 0 & 0 & 0\end{bmatrix},\ \mathbf{DS4}=\begin{bmatrix}-S\theta & C\theta C\alpha & C\theta S\alpha\\ -C\theta & -S\theta C\alpha & -S\theta S\alpha\\ 0 & 0 & 0\end{bmatrix}$$

$$\mathbf{DM2}=\begin{bmatrix}0 & -C\alpha d & -S\alpha d\end{bmatrix},\ \mathbf{DM3}=\begin{bmatrix}0 & -S\alpha & C\alpha\end{bmatrix}$$

Initialization of the recursive relations of DE

The recursive relations begin by link 0. Since link 0 is fixed then:

$^0\omega_0 = \mathbf{0}$, $^0\mathbf{V}_0 = \mathbf{0}$, $E^0 = 0$, $\mathbf{DE}^0 = \partial E/\partial \mathbf{X}^0 = \mathbf{0}$

Thus we initialize the calculation by :

$$\mathbf{DE}^0=\begin{bmatrix}0 & 0 & \ldots & 0\end{bmatrix}^T$$

A-2-Calculation of the recursive relations of the potential energy

Let U^j denote the potential energy of link j.

It is evident that $DU_i^j = 0$, i = 1 ,...,6 , and we prove that:

$$\begin{bmatrix}DU_7\\ DU_8\\ DU_9\\ DU_{10}\end{bmatrix}_j=\begin{bmatrix}{}^{j-1}\mathbf{A}_j^T & \begin{matrix}0\\0\\0\end{matrix}\\ {}^{j-1}\mathbf{P}_j^T & 1\end{bmatrix}\begin{bmatrix}DU_7\\ DU_8\\ DU_9\\ DU_{10}\end{bmatrix}_{j-1}={}^{j-1}\mathbf{T}_j^T\begin{bmatrix}DU_7\\ DU_8\\ DU_9\\ DU_{10}\end{bmatrix}_{j-1}$$

Initialization of the recursive relations of DU

$U^0 = -\mathbf{g}^T\,{}^0\mathbf{P}_0\, m_0 - \mathbf{g}^T\,{}^0\mathbf{A}_0\,{}^0\mathbf{mS}_0$

$\mathbf{g}^T = [g_1\ g_2\ g_3]$

${}^0\mathbf{P}_0 = 0$, ${}^0\mathbf{A}_0 = \mathbb{1}$, the identity matrix.

Then:

$$\mathbf{DU}^0=\begin{bmatrix}0 & \ldots & 0 & -g_1 & -g_2 & -g_3 & 0\end{bmatrix}^T$$

j	σ_j	α_j	d_j	θ_j	r_j
1	0	0	0	θ_1	0
2	0	90	0	θ_2	0
3	0	0	D3	θ_3	0
4	0	−90	0	θ_4	RL4
5	0	90	0	θ_5	0
6	0	−90	0	θ_6	0

The geometric parameters

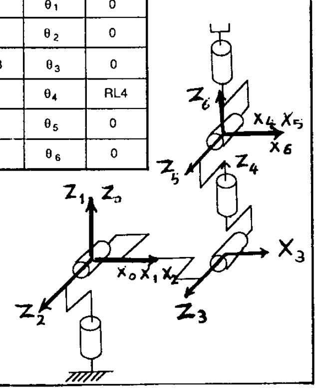

Figure 2.

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