A near Minimum Iterative Analytical Procedure for Obtaining a Robot-Manipulator Dynamic Model

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Summary

The dynamic control synthesis of robot manipulators requires a great number of arithmetic operations, and it cannot be effected in real time unless this number is reduced. This paper presents a systematic analytical procedure for obtaining the dynamic model necessary for the dynamic control synthesis. This procedure which uses a Lagrangian formulation is applicable to all manipulators having a simple kinematic chain structure with revolute and/or prismatic joints.
An example shows that the calculation of the dynamic model requires 368 multiplications and 271 additions for a particular 6 revolute joint manipulator using the systematic procedure. The examination of the results shows that only a few simpliciations are a fortiori possible and proves that the procedure is near-minimum.

1. Introduction

The dynamic characteristics of robot manipulators are highly nonlinear. To control these systems, it is necessary to compute the forces and torques needed to drive all their joints accurately, and frequently at adequate sampling frequency. This necessitates the establishment of the so-called robot manipulator dynamic model, in other words the relationship between the generalized forces which have to be applied at each joint and the generalized coordinates, velocities, and accelerations q, $\dot{q}$ and $\ddot{q}$, such that :

$$\Gamma = \Gamma(q, \dot{q}, \ddot{q})$$

The numerical calculations necessary for the dynamic control synthesis cannot be carried out in real-time unless the number of arithmetic operations is reduced.

The aim of this paper is to present a method for computing the dynamic model which minimizes this number. To achieve this, it is demonstrated that analytical methods are more effective than numerical ones.

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In order to compare analytical and numerical methods and to arrive at identical results, the same data must be used, i.e.

i) the constant parameters of the manipulator such as angles, lengths, masses, inertias, etc., and every constant quantity which can be previously calculated,

ii) the generalized coordinates, velocities and accelerations.

The unknown quantities of the problem are the generalized forces which have to be applied at each joint.

To simplify the presentation, the worst case of robot manipulators having a simple kinematic chain structure with n joints of the revolute type is studied, as it is well known that the dynamic model is simpler when the robot manipulator possesses some joints of the prismatic type.

2. Formulations for obtaining the dynamic model of robot manipulators

The most important formulations for obtaining the dynamic model of robot manipulators are :

a) the Newton-Euler formulation, and

b) the Lagrange formulation.

The generalized forces numerically obtained using the first or the second formulation are, obviously, the same : thus it is possible to establish relationships between the two sets of equations obtained [12]. The real problem is not, in the author's opinion, to choose between an analytical or numerical method.

The Newton-Euler formulation is oriented towards the numerical method and the Lagrange formulation towards the analytical method. Indeed :

a) an analytical method of computation can be obtained using the Newton-Euler formulation, but there are great difficulties in the elimination of constraint forces and torques at the different joints of the manipulator. Nevertheless, this solution is used by several authors [7-6-16],

b) a numerical method of computation using the Lagrange formulation is difficult to imagine as the calculation of the partial derivatives poses problems.

This is the reason why the most frequently used methods are :

- the numerical Newton-Euler method,
- the analytical Lagrange method.