Other methods for derivation of systems dynamics

Routhian mechanics

In classical mechanics, **Routh's procedure** or **Routhian mechanics** is a hybrid formulation of Lagrangian mechanics and Hamiltonian mechanics developed by Edward John Routh. Correspondingly, the **Routhian** is the function which replaces both the Lagrangian and Hamiltonian functions. Although Routhian mechanics is equivalent to Lagrangian mechanics and Hamiltonian mechanics, and introduces no new physics, it offers an alternative way to solve mechanical problems.
The Routhian is intermediate between L and H; some coordinates q_1, q_2, ..., q_n are chosen to have corresponding generalized momenta p_1, p_2, ..., p_n, the rest of the coordinates ζ_1, ζ_2, ..., ζ_s to have generalized velocities $d\zeta_1/dt$, $d\zeta_2/dt$, ..., $d\zeta_s/dt$, and time may appear explicitly. where again the generalized velocity dq_i/dt is to be expressed as a function of generalized momentum p_i via its defining relation. The choice of which n coordinates are to have corresponding momenta, out of the $n + s$ coordinates, is arbitrary.

The above is used by Landau and Lifshitz, and Goldstein. Some authors may define the Routhian to be the negative of the above definition.

For $n + s$ coordinates, with Routhian

$$R = \sum_{i=1}^{n} p_i \dot{q}_i \left(p_i\right) - L$$

$$\dot{p}_i = -\frac{\partial R}{\partial q_i} \dot{q}_i = \frac{\partial R}{\partial p_i}$$

$$\frac{d}{dt}\left(\frac{\partial R}{\partial \dot{\xi}_j}\right) = \frac{\partial R}{\partial \xi_j}$$

The total number of equations is $2n + s$, there are $2n$ Hamiltonian equations plus s Lagrange equations.

Gauss least constraint principle

The **principle of least constraint** is one variational formulation of classical mechanics enunciated by Carl Friedrich Gauss in 1829, equivalent to all other formulations of analytical mechanics. Intuitively, it says that the acceleration of a constrained physical system will be as similar as possible to that of the corresponding unconstrained system. The principle of

least constraint is a least squares principle stating that the true accelerations of a mechanical system of n masses is the minimum of the quantity

$$Z = \sum_{k=1} m_k \left| \ddot{r}_k - \frac{F_k}{m_k} \right|^2$$

where the *j*th particle has mass m_k, position vector r_k , and applied non-constraint force F_k acting on the mass. The corresponding accelerations $\ddot{r}_k$ satisfy the imposed constraints, which in general depends on the current state of the system, $\left(r_k(t), \dot{r}_k(t)\right)$

It is recalled the fact that due to active F_k and reactive (constraint) F_{cv} forces being applied, with resultant

$$R = \sum_{k}^{n} F_k + F_{ck}$$

a system will experience an acceleration

$$\ddot{r}_k = \sum_{k=1}^{n} \frac{F_k}{m_k} + \frac{F_{ck}}{m_k} = \sum_{k=1}^{n} a_k + a_{ck}$$

.

Gauss's principle is equivalent to D'Alembert's principle. The principle of least constraint is qualitatively similar to Hamilton's principle, which states that the true path taken by a mechanical system is an extremum of the action. However, Gauss's principle is a true (local) *minimal principle*, whereas the other is an *extremal principle*.

One can then think Gauss principle as a sort of father principle from which one can derive most of other methods.

Hertz's principle of least curvature is a special case of Gauss's principle, restricted by the three conditions that there are no externally applied forces, no interactions (which can usually be expressed as a potential energy), and all masses are equal. Without loss of generality, the masses may be set equal to one. Under these conditions, Gauss's minimized quantity can be written

$$ds^2 = \sum_{k=1}^{n} [dr_k]^2$$

Introducing kinetic energy T

$$\left(\frac{ds}{dt}\right)^2 = 2T$$

diving Z by 2T

$$K = \sum_{k=1}^{n} \left| \frac{d^2 r_k}{ds^2} \right|^2$$

Since $\sqrt{K}$ is the local curvature of the trajectory in the 3n-dimensional space of the coordinates, minimization of K is equivalent to finding the trajectory of least curvature (a geodesic) that is consistent with the constraints. Hertz's principle is also a special case of Jacobi's formulation of the least-action principle.

Gibbs-Appel equations

Starting from D'Alemebert principle we can derive another well known kind of equations of motions known as Gibbs or Appel dynamics. Also in this case we use assumption of holonomic or integrable constraint. Gibbs–Appel equations were introduced by Gibbs in 1879 and were subsequently studied and formalized by Appel in 1900. It is our intention in this section, to introduce this method and correlate its formulation to the methods developed previously by Kane and Lagrange. Let us introduce the Gibbs–Appel function. Consider a system S constituted of P particles with mass mi (i = 1,... , p), and let R represent the inertial fixed frame with center O. The Gibbs–Appel function is defined as

$$G = \frac{1}{2} \sum_{k=1}^{N} m_k a_k \cdot a_k$$

and the equations of motion are obtained by

$$\frac{\partial G}{\partial \ddot{q}_k} = Q_k$$

where f denotes the generalized active forces and x the generalized coordinates. Equations for a rigid body can be expressed as

$$G = \frac{1}{2} M a_k^2 + \frac{1}{2} \alpha_k^T I_k \alpha_k + \alpha_k \cdot \omega_k \wedge I_k \omega_k$$

where M_k is the mass of body k, a_k its center of mass acceleration, α_k the angular velocity, and I the dyadic of body k. For a multibody system, the function G can easily be formulated by substituting into it the explicit form of a_k, ω_k, and α_k, as developed from kinematic propagations. From a simple observation of equations one can see that the generalized

inertia forces are determined by $\frac{\partial G}{\partial \ddot{q}_k}$. Making use of Gibbs equations, we can show that this method also yields the numerical representation of the equations of motion given by equation of other methods.

Kane's equations

In alternative to classic Euler lagrange formulation, Kane's equation has been applied time by time in robotics as alternative method with some advantages. Kane's method is from one side intuitive like D'Alembert principle but also more general leading also to applications for non-holonomic systems, like wheeled robots, where quasi-velocities are applied in place of virtual displacements.
It also is quite suitable for
Kane's equation of motions are commonly used as way to manage multibody problems because its implementation adapts easily to arbitrary number of bodies and of joints configurations.

$$\sum_{i=1}^{N_B} m_i \vec{a}_{G_i} \frac{\partial \vec{v}_{G_i}}{\partial u_k} + \sum_{i=1}^{N_B} \left[\left(\underline{\underline{I}}_{G_i} \vec{\alpha}_i \right) + \vec{\omega} \times \vec{H}_{G_i} \right] \frac{\partial \vec{\omega_{B_i}}}{\partial u_k} = F_{u_k}$$

$$F_{u_k} = \sum_j \left(\vec{F}_j \cdot \frac{\partial \vec{v}_{P_j}}{\partial u_k} \right) + \sum_j \vec{M}_j \cdot \frac{\partial \vec{\omega}_{B_j}}{\partial u_k}$$

Essentially Kane's method has many advantages, one of being the use, in a similar way to D'Alembert principle of scalar product along generalized speed direction reducting number of equations involved to those producing or absorbing work. it is then an energy oriented principle. Differently from D'Alembert principe we do not need to constraint to integrable non holonomic constraints, because virtual speeds (quasi velocities) are directly used.in place of virtual displacements.

Relationship Between Kane's and Lagrange Equations

The equations of motion of a multibody system are *unique*; therefore irrelevant of the method they should be the same. In what follows is a proof that in essence Lagrange equations must be equivalent to Kane's method. It follows from equation above that the generalized inertia forces are

$$Q_k = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k}$$

$$Q_k = \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k}$$

where n denotes the number of degrees of freedom of the system. The generalized inertia forces are also given through Kane's equations as

$$Q_k^\star = F_k^\star \cdot \frac{\partial v_k}{\partial \dot{q}_k} + M_k^\star \cdot \frac{\partial \omega_k}{\partial \dot{q}_k}$$

where $F_k^\star$ is the inertia force and $M_k^\star$ the inertia torque of body k. Let us denote by $F_I^\star$ the contribution of the inertia forces and by $M_I^\star$ the corresponding inertia torques to the generalized inertia forces. Then, equation above becomes

$$Q_k^\star = F_I^\star + M_I^\star$$

$$Q_k^\star = F_k^\star \cdot \frac{\partial v_k}{\partial \dot{q}_k} = -m_k a_k \cdot \frac{\partial v_k}{\partial \dot{q}_k} = -\left[\frac{d}{dt}\left(m_k v_k \cdot \frac{\partial v_k}{\partial \dot{q}_k}\right) - m_k v_k \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial v_k}{\partial \dot{q}_k}\right)\right]$$

$$Q_k^\star = -\left[\frac{d}{dt}\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2} m_k v_k \cdot v_k\right) - m_k v_k \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial r_k}{\partial q_k}\right)\right]$$

$$Q_k^\star = -\left[\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial}{\partial q_k}\left(\frac{1}{2} m_k v_k \cdot v_k\right)\right] = -\left[\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k}\right]$$

It follows from the summation that the generalized inertia forces obtained from Kane's and Lagrange's method are equivalent. Similarly, it can be shown that the generalized active forces from both methods are the same. Hence the equations of motion of a system can be found through either method

Hamilton equations

Probably least used in robotic literature, maybe because sharing many of features with EL method, like calculation of energy and potential functions as function of momenta and coordinates, Hamilton equations find extensive application in celestial mechanics, complex systems mechanics, chaos theory and quantum mechanics. On the other side involvement in kinetic energy expression of the inverse of inertia matrix could be considered both an advantage (since often is required in a later time), or a disadvantage. Using hamiltonian formalism diagonalization and decoupling of dynamics equations is easier, and indeed one can find expression of transformations for diagonalization involves generalized momenta like quantities.

Hamilton equations is a system of 2N first order differential equations (ODE):

$$\dot{p}_k = -\frac{\partial H}{\partial q_k}$$
$$\dot{q}_k = \frac{\partial H}{\partial p_k}$$

A final Hamilton equation relates time variation of Hamiltonian function with Lagrangian.

$$\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

Since Hamiltonian is often a motion constant, it is often neglected.
Conjugate momenta are defined as (in some sense second set of Hamilton's equations are inverse relation):

$$p_k = \frac{\partial L}{\partial \dot{q}_k}$$

which are related to

$\dot{p} = -\frac{\partial H}{\partial q}$
$\dot{q} = \frac{\partial H}{\partial p}$

Hamiltonian function is defined by means of Legendre transformation

$$H = \sum_{k=1}^{n} \dot{q}_k p_k - L$$

but since in most of case we deal with holonomic conservative systems, it corresponds to system total energy

$$H = T + U$$

$H = K + U$

$p = M\dot{q} \Rightarrow M^{-1}p = \dot{q}$

Formalism is less intuitive because requires introduction of quantities like momenta not always so straightforward. Kinetic energy then becomes dependant on inverse of inertia matrix:

$$K = \frac{1}{2} p^T M^{-1} p$$

The proof is easy, assuming generalized momenta expression (for instance considering potential free motion)

$$p = M\dot{q} \Rightarrow \dot{q} = M(q)^{-1} p$$

$$K = \frac{1}{2}\dot{q}^T M(q)\dot{q} = \frac{1}{2}\dot{q}^T M(q)^{-T} M(q) M(q)^{-1}\dot{q} = \frac{1}{2}p^{-T} M(q)^{-1} p$$

$$K = \frac{1}{2}\dot{q}^T M\dot{q} = \frac{1}{2}p^T \left(M^{-1}\right)^T M M^{-1} p = \frac{1}{2}p^T \left(M^{-1}\right)^T p = \frac{1}{2}p^T M^{-1} p$$

On the other hand hamilton equations are often used when canonical transformations are applied, allowing for significative simplifications and leading to the identification of conserved quantities.
Applicative purpose of these models lead to a lower interest on this subject because broken by the interaction with environment leading to non conservation of motion integrals, making it a secondary point. Nonetheless could be a subject of research when looking for most simple possible formulations of system dynamics, introducing conserved quantities and applying perturbative techniques. We don't' go in details of such a specialistic theme.

Because of intrinsic nature definite positive of mass matrix, (then its invertibility), Hamiltonian function is a Lyapunov function for the system that results to be exponentially and globally stable, or in other words a dissipative system.

Consequently we can say that introduction of conjugate momenta could lead to a simpler formalism for robot dynamics representation, introducing in a mechanically and mathematically natural and justified way the introduction of a lower order set of equations, set of additional variables, and potential reduction of complexity like natural definition of motion constant and invariants, that in other formalisms would be harder to find and clearly depict.
Hamilton equations are directly suitable for integration in their canonical form, where usually lagrange equations require some rearrangement being second order. But on the other side using partial derivatives on less complex expression ease the understanding and structure of motion equations.
Indeed many of coriolis and centrifugal terms

Relation between momenta and mass matrix lead to

$$p = M\dot{q}$$
$$\dot{q} = M^{-1}p$$

$$\dot{p} = M(q)\ddot{q} + \dot{M}(q)\dot{q}$$
$$\dot{p} = M(q)\ddot{q} + \sum_{k=1}^{n} \frac{\partial M}{\partial q_k}\dot{q}\dot{q}_k$$

$$\dot{p} = M\ddot{q} + \dot{M}\dot{q}$$
$$\dot{p} = M\ddot{q} + \frac{\partial M}{\partial q}\dot{q}\dot{q}$$

Ergo at least part of centrifugal and coriolis terms are included into momenta derivative in place of their explicit writing in term of generalized speeds. Indeed is easy to see how some relevant laws of motion like angular and linear momentum conservation, energy conservation and variable independence and consequent decoupling arise evidently in equations structure.

A second step is required to reconvert result into mor intuitive coordinates space, but is not such a hard task.

Maggi's equations

Maggi's equations were introduced by Gian Antonio Maggi in 1896 as an alternative approach to deal with constrained multibody systems (with either holonomic or linear non-holonomic constraints). As described in, Maggi's equation can be understood as an extension of Lagrangian formalism that, by means of a simple algorithm based on finding a linear operator onto the kernel of the constraint equations Jacobian, allows the elimination of the undetermined multipliers from the equations of motion of a constrained mechanical. Whenever redundant generalized coordinates are used in a given formulation (which is inevitable in the case of non-holonomic systems and might be convenient for holonomic ones), the requirement of using indeterminate multipliers in the conventional Lagrangian formalism arises, leading to a system of differential-algebraic equations of motion (DAEs) instead of simply ordinary differential equations (ODEs) .

Maggi's equations are then written in the form:

$$S_k^T \left[\frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{\mathrm{d}L}{\mathrm{d}\dot{q}_k} \right) - \frac{\partial L}{\partial q_k} - Q_k \right] = 0$$

essentially Maggi's equations are a set of constrained lagrangian equations, and are then more general than purely lagrangian equations, since there's no need to change main coordinates systems (used in differential equations) in case further of different additional constraints are set into the problem. Nevertheless even differential equations generalized variables can be freely chosen, and that's potentially a free choice (assured a minimal set suitable for dynamic description is given) making automatization process not as easy as in other methods. General form taken for this kind of formulation is, in linear or linearized form, for constrained Lagrangian

$$L = T - V + \lambda^T \phi \left(q \right)$$

$$\begin{pmatrix} M & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \lambda \end{pmatrix} = \begin{pmatrix} F \\ \gamma \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u$$

it has been shown that the application of Maggi's equations, combined with a proper choice of a redundant set of generalized coordinates, leads to a simpler derivation of the dynamic equations of motion, in a suitable form for numerical simulations. Moreover, it also has been

shown that, the procedure for adding new bodies and obtaining differential equations of motion without the terms related to constraints forces is simple. [On the use of Maggi's equation in the dynamic modeling of multibody systems].

Maggi's equation is a quasi velocity method. In general, there is no set procedure on selecting quasi velocities. For a mechanical system of n generalized coordinates with m nonholonomic constraints, n quasi velocities are selected such that m of them span the constraint space, and (n-m) are independent quasi velocities. During the selection, one keeps in mind that the matrix Ψ obtained as in (13) is invertible with an inverse Φ.

Once we apply the Kane's approach it seems easy to work out a computational procedure, which allows to model a system methodically. However, to apply the Kane's approach it is necessary to calculate the acceleration of the centre of mass of each body, which is a disadvantage when compared to the Lagrangian formulation, in which only the velocities of centers of mass and angular velocity expressions of each body are required. Maggi's formalism also requires the computation of partial derivatives of the Lagrangian of the system. However, it is possible to take advantage of the use of redundant coordinates to simplify the energy terms as much as possible, which can make the modeling procedure much simpler when compared to the conventional

Modified Newton-Euler-Lagrange method

In this work we propose an original method derived from Newton Euler algorithm and modified to make it easier for implementation, readability and use. We use RNEA algorithm for propagating kinematics quantities along a serial chain or a kinematic tree of bodies, calculating speeds, in link frame. Then calculation of associated kinetic energy contributes is sum up into overall kinetic energy expression. We suggest for matter of reusability and post processing to keep both single link energy contribute and total system energy. Used approach is based on symbolic derivation of quantities, and is less suitable for numerical use.Once full energy formulation is known, we can use Eulero-Lagrange equations for derivation of dynamic equations. Equations of motion are derived using a two steps lagrangian method, calculating first inertial terms using only kinetic energy, adding later contributes deriving from potential, or calculating directly full lagrangian and taking out full equations. Considering single terms of kinetic energy expressions allows for using every single piece of kinetic energy expression for calculating a piece of equation of motion to sum up at the end. This method reduce and decompose complexity of expressions in blocks. Separating different contributes has different advantages:

- More clear separation of contributes to dynamics
- Easier assembling and modifications
- Lower computational effort in computation of equations of motions
- Possibility to calculate contributes to mass matrix splitting it in parts

Other methods

Constrained version of dynamic equations

Above see equations of motion that can formulated also in terms of *constrained system* by means of *Lagrange multipliers* method. Given standard form of dynamics equations

$$\tau = H\left(q\right)\ddot{q} + C\left(q,\dot{q}\right)\dot{q} + G\left(q\right)$$

a constrained formulation is obtainable as

$$\begin{gathered}\tau = H\left(q\right)\ddot{q} + C\left(q,\dot{q}\right)\dot{q} + G\left(q\right) \\ g\left(q,t\right)\dot{q} = v\left(t\right)\end{gathered}$$

after differentiation of constraint equations we can be rewritten:

$$\tau = H\left(q\right)\ddot{q} + C\left(q,\dot{q}\right)\dot{q} + G\left(q\right) - \lambda\left(v\right)$$

that in matrix form lead to a system of differential equations:

$$\begin{pmatrix} H & G \\ G & 0 \end{pmatrix}\begin{pmatrix} \ddot{q} \\ -\lambda \end{pmatrix} = \begin{pmatrix} \tau - C - G \\ v \end{pmatrix}$$

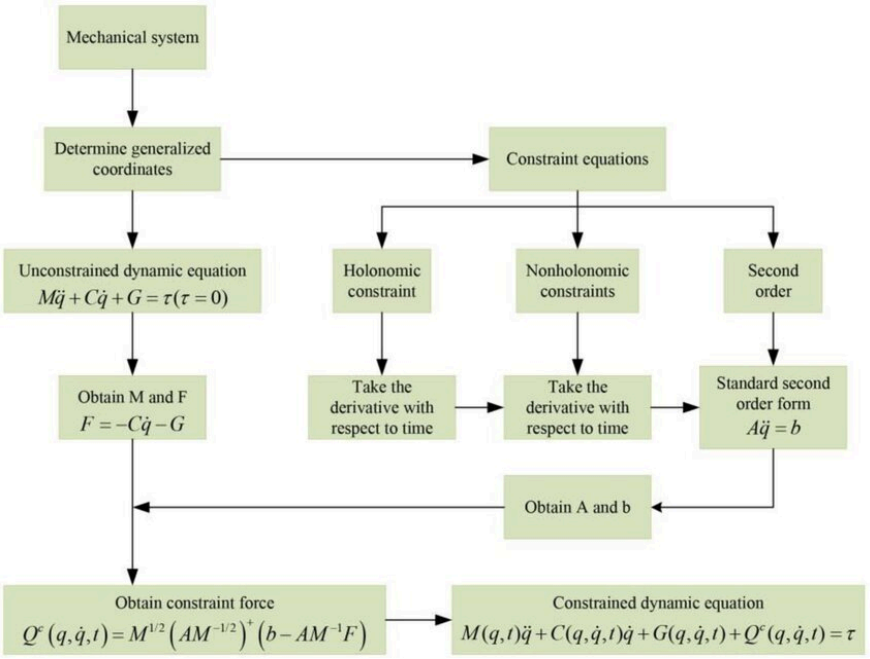
Different solution approaches are possible for this system of constrained equations:

- Augmented approach:
- Direct approach
- Projected approach
- Constraints embedding

for more details see [Jain].

Constrained dynamics: Udwadia–Kalaba

Most of methods seen are for sake of free dynamics motion, although constrained versions are possible. Some methods are specifically thought and designed for constrained dynamics. One of them is Udwadia–Kalaba theory solving process illustrated in the following, knowing some applications in robotics.



Projection method

A method widely applied in dynamic parameters identification field uses jacobians to rewrite forces and torques in terms of projected joints actuation forces/torques. For inertial terms just some simple relations coming from Eulero equations of each body in 6-vector notation (keeping together forces and moments) are used. Summing up all together lead directly to motion equations.
The advantage of this approach is that we can write inertial terms directly as product of standard inertial vector by a matrix of kinematic terms, that results to be by definition as part of regressor, before projections. In mathematical terms

$$\tau = J^T F = \sum_{k=1}^{n} J_k^T F_k$$

$$\begin{bmatrix} \dot{v}_o + \tilde{\omega} v_o - g & \dot{\omega}_o + \tilde{\omega}_o \tilde{\omega}_o & 0_{3,6} \\ 0_{3,1} & -(\dot{v}_o + \tilde{\omega}_o v_o - g)\sim & \dot{\Omega} + \tilde{\omega}_o \Omega \end{bmatrix}$$

where

$$\Omega = \begin{bmatrix} \omega_x & \omega_y & \omega_z & 0 & 0 & 0 \\ 0 & \omega_x & 0 & \omega_y & \omega_z & 0 \\ 0 & 0 & \omega_x & 0 & \omega_y & \omega_z \end{bmatrix}$$

The structure of these equations stress the structure based on cross product of Euler equations, lasting both for linear and angular momenta, has been proposed in the past from [Atkenson Hollerbach] but the direct use for writing regressor is proposed in [Neubauer,

Gattringer, B**reme**r]. A similar conclusion, in slightly different form was reached from [Chae-Atkenson-Hollerbach], whose approach is depicted in further chapters.

The advantage of this formulation are, first the direct formulation of regressor, second the use of a vectorial formulation that gives a compact and simpler form, and contemporarily independent from used frame, even if at at last we should anyway find a connection with trajectory and frames, directly substituting coordinates dependance or indirectly. Postponing to a second time this passage make easier to build a general system of equations customized later to chosen coordinates set and system topology.
In addition a direct use for numerical purpose is possible since no derivatives are implied in building the matrix, though if projector is used, building derivatives are required to connect with joint torques.

On the other side this approach is very convenient dealing with multibody systems because of its modularity. Each body equation can be composed with others using projectors, and a change in the connection just requires adjustment on projection of equations. In addition in this way the contribute of each body to overall system is kept separated and summed up directly both when dealing with regressor and equation of motion.
On the contrary, we can even chose to avoid the use of projection on joint and just use regressor matrix, but implicitly we assume to be able to measure each body torques and forces.

This is not the case for almost any robot identification experimental setup, in literature almost always related to joint toques (easier to obtain from motor currents), and then a not much classic approach. Nevertheless with an extensive and appropriate equipment would not be an impossible approach. At current state of author knowledge none attempt of this kind has been reported in literature, opening a breach for an innovative study development.
Drawbacks are the cost of sensing setup, but on the other side we could have single body full and independent identification. Present price of inertial sensing systems (very cheap and omnipresent even in smartphones and other domestic and personal devices). Other required devices for forces and toques measurements could ideally rely on piezoelectric pressure and torque sensors, not expensive, but requiring ad hoc design and modifications, not suitable for most of commercial applications.

Nonetheless projection method is effectively often proposed in literature, using known projectors, for modeling regressor in commercial and industrial robot identifications. Is remarkable as Kane's equation of motion could be included insame model in a straightforward way.

Summarizing dynamics methods used in literature are

Tabelle 4: Methoden der Dynamik

HUYGENS (konservativ)	$T + V = H$	... 1673
GIBBS & APPELL (hier: holonom)	$\left[\frac{\partial S}{\partial \ddot{\mathbf{z}}}\right]^T = \mathbf{Q}$	1879 & 1899
HAMILTON (hier: konservativ)	$\delta \int_{t_0}^{t_1} (T - V)\, dt = 0$	1834
HAMILTON (hier: konservativ, autonom)	$\dot{\mathbf{p}}^T = -\left[\frac{\partial H}{\partial \mathbf{z}}\right] \quad \dot{\mathbf{z}}^T = \left[\frac{\partial H}{\partial \mathbf{p}}\right]$	1834
LAGRANGE (hier: konservativ)	$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\mathbf{z}}}\right] - \left[\frac{\partial T}{\partial \mathbf{z}}\right] + \left[\frac{\partial V}{\partial \mathbf{z}}\right] = 0$	1788
HAMEL-BOLTZMANN	$\left[\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\mathbf{s}}}\right] - \left[\frac{\partial T}{\partial \mathbf{s}}\right] - \mathbf{Q}^T\right]\delta\mathbf{s} + \frac{\partial T}{\partial \dot{\mathbf{s}}}\left[\frac{d\delta\mathbf{s} - \delta d\mathbf{s}}{dt}\right] = 0$	1904
	ZENTRAL - $\boxed{\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\mathbf{s}}}\,\delta\mathbf{s}\right] - \delta T - \delta W^e = 0}$ GLEICHUNG	
EULER (eliminierte Zwangskräfte)	$\sum_{i=1}^{p}\left\{\left[\frac{\partial \mathbf{v}_i}{\partial \dot{\mathbf{s}}}\right]^T\left[\dot{\mathbf{p}} + \tilde{\boldsymbol{\omega}}\mathbf{p} - \mathbf{f}^e\right] + \left[\frac{\partial \boldsymbol{\omega}}{\partial \dot{\mathbf{s}}}\right]^T\left[\dot{\mathbf{L}} + \tilde{\boldsymbol{\omega}}\mathbf{L} - \mathbf{l}^e\right]\right\}_i = 0$	1750/75