



# Design and optimisation of lattice structures for aerospace applications

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## Colophon

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# Introduction

**Towards lighter structures**

**Objective**

**Outline of the thesis**





Introduction

Extensively present in nature (e.g., bone-microstructure or birds beak), their research interest came thanks to the observation that the stiffness of optimal structures spans multiple scales [1, 2]. In addition to that, Fleck observed that [3] *“one reason for such structural hierarchy in engineering structures is to increase buckling strength: recall that the buckling strength scales with any representative strut length  $l$  according to  $l^{-2}$ , and so the finer the length scale, the higher the buckling strength.”*

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1.2. Ultra-lightweight structures optimization

1.2.1. Feature-Mapping topology optimization

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1. Kohn et al. (1986), ‘Optimal design and relaxation of variational problems’

2. Allaire et al. (1999), ‘On optimal microstructures for a plane shape optimization problem’

3. Fleck et al. (2010), ‘Micro-architected materials’



# Evaluating discretization approaches for ultralight structure optimization

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Introduction

### 2.1. The formulation of a common problem: volume minimization with stress constraints

#### 2.1.1. Continuous discretization NAND minimum volume formulation

Spatial filtering and projection

Von Mises stress evaluation

Constraints aggregation and relaxation

Optimization formulation

Sensitivity calculation

#### 2.1.2. Truss discretization SAND minimum volume formulation

Plastic material formulation

### 2.2. Comparison between continuous and truss discretization

#### 2.2.1. Definition of a common test case

#### 2.2.2. Numerical application

Compliance-volume graph

Compliance-stress graph

Stress-volume graph

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### 4.1.1. Variable linking

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### 4.2.1. On the equivalence of multi load cases and cellular structures

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# Optimizing the cell layout in space

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## 5.2. DMO

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6.1.3. Numerical application

Maximum displacements constraints

Active mechanical constraints

## 6.2. NACA profile extruded

6.2.1. Numerical implementation

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# Conclusion and perspectives

**Conclusion**

**Perspectives**



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# APPENDIX



# Geometry generation for ultra-light structures

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