MATH40082
Mini Task 2

Version 9910821

Background

A trader has asked you to calculate the value of an interest rate derivative contract using a non-standard model. In this particular model, it is assumed that $r(t)$ evolves according to an Itô diffusion process and as such the value of the derivative contract $V(r,t)$ can be found by solving

$$V(r,t) = P(r,t,T)E[g(R_{r,t,T})] \quad (1)$$

where $R_{r,t,T}$ denotes a normal random variable with mean $f(r,t,T)$ and variance $v^2(t,T)$. Here P is the value of pure discount bond paying \$1 at time $t = T$ under the risk neutral measure.

The trader has already solved the SDE under the risk-neutral measure and given you explicit functions for f, $v^2(t,T)$ and P. They are:-

$$\begin{aligned}
P(r,t,T) =& \exp\left[\frac{2}{3}k^2(t,T) - \frac{1}{4}n(r,t,T)\right], \\
f(r,t,T) =& m(r,t,T) - \frac{1}{2}q(t,T), \\
v^2(t,T) =& \frac{\sigma^2}{3\kappa}(1 - e^{-3\kappa(T-t)})
\end{aligned}$$

where

$$\begin{aligned}
m(r,t,T) =& e^{-\kappa(T-t)}r + (1 - e^{-\kappa(T-t)})\theta, \\
n(r,t,T) =& r(T-t) - \frac{\theta - r}{2\kappa}(1 - e^{-4\kappa(T-t)}), \\
k^2(t,T) =& \frac{\sigma^2}{2\kappa^3}(2e^{-\kappa(T-t)} - 3e^{-2\kappa(T-t)} + 4\kappa(T-t) + 1), \\
q(t,T) =& \frac{\sigma^2}{4\kappa^2}(1 - e^{-\kappa(T-t)})^3.
\end{aligned}$$

The derivative that needs to be priced will be a cash-or-nothing call or put with an interest rate strike price of X_r. In the case of a call at maturity the holder will receive \$1 if $r > X_r$ and nothing otherwise. In the case of a put at maturity the holder will receive \$1 if $r < X_r$ and nothing otherwise.

Then the value of a call option can be written as

$$V(r,t) = P(r,t,T)(1 - N(h))$$

and a put option is given by

$$V(r,t) = P(r,t,T)N(h)$$

where the value of h must be determined. In order to find h you may use the fact that $R_{r,t,T}$ is normally distributed and $N(x)$ is equal to the probability that $y < x$ when y is drawn from a normal random distribution with mean 0 and variance 1. *Hint: you need $N(h)$ to be equivalent to $P(R_{r,t,T} < X_r)$.*

Tasks

1. Work out the formula for the value of h.

2. State the value $V(r_0, t = 0, T)$ of the following financial contract. The contract you should value is a put option. The current value of the interest rate is $r_0 = 0.052$, the maturity of the option is $T = 2$ and the strike price $X_r = 0.06$. The model parameters are $\kappa = 0.0944$, $\theta = 0.0616$ and $\sigma = 0.0317$.

3. Using the parameters from Task 2, write a program to calculate bond price P and the option price V for different interest rates and output the results to file. You must generate three columns of data:

   – the value of r,

   – the value of $P(r, t = 0, T)$,

   – and the value of $V(r, t = 0, T)$.

   Output each of the above three columns for around 100 values of r in the range $r \in [0, 0.2]$. Use a plotting package (such as excel/matlab/gnuplot) to plot P against r and V against r on the same figure.

Instructions

This coursework should not take more than 2 hours or so to complete. You may use any codes that I have provided in Lab Class 2 as part of your solution (any codes/examples will be uploaded to the forum). Please write your own code and final solution, you may work together but **do not** transfer codes or solution files (via email/USB etc.).

Please complete the tasks and hand in your solution on "TurnItIn" (on the Blackboard system) by 5pm Sunday 16th February. If you have any problems with the system at all you can also email your solution directly to `paul.johnson-2@manchester.ac.uk`. If you are DASS registered and are eligible for an extension, please let me know. Solutions handed in AFTER 5pm SUNDAY 16th February will be docked 1 marks plus an additional 1 marks each day thereafter until a mark of zero is reached.

For the solution you should create a document (accepted formats are doc/docx/pdf) containing

- A title with your student id number (9******) – **do not mention your name**;
- Write down the formula for h (not the value) as well as stating the value of the option you calculate for B with parameters as given in Task 2. (1 mark)
- a figure of your results - it must be given a label with a number and caption as if it were in a technical report, as well labelling correctly so that each of the lines may be distinguished (2 marks);
- your full program to Task 3. - it doesn't need annotations but you should add a small number of comments within the code (2 marks);
- **No text or accompanying comment is required**.

This exercise is worth 5% of the total mark for the course.