homework_6

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6.1.

$$f(x) = c x^3 \qquad 0 < x < \frac{3}{2}$$

a) $$\int_0^{3/2} x^3 \, dx = \frac{1}{c} \qquad c = \frac{64}{81}$$

b) $$\int_{1/2}^{3} \frac{64}{81} x^3 \, dx = \int_{1/2}^{3/2} \frac{64}{81} x^3 = 0,9876$$

c) $$E[X^2] = \int_0^{3/2} x^2 f(x)\, dx = \int_0^{3/2} x^2 \cdot \frac{64}{81} x^3 \, dx = \frac{64}{81} \cdot \frac{\left(\frac{3}{2}\right)^6}{6} = \frac{3}{2}$$

$$E[X] = \int_0^{3/2} x^4 c \quad dx = \frac{64}{81} \cdot \frac{\left(\frac{3}{2}\right)^5}{5} = \frac{6}{5}$$

$$Var[X] = \frac{3}{2} - \left(\frac{6}{5}\right)^2 = \frac{3}{50}$$

6.2.

$$E[aX+b] = \sum_{i=1}^{\infty} (a x_i + b) P(X = x_i) = a \sum_{i=1}^{\infty} x_i P(X = x_i) +$$

$$b \sum_{i=1}^{\infty} P(X = x_i) = a E[X] + b$$

$$Var(aX+b) = E[(aX + b - aE[X] - b)^2] = E[(aX - aE[X])]^2 =$$

$$= a^2 Var[X]$$

6.3

$$E[2^x] = \int_0^2 \frac{2^x}{2}\,dx \simeq 2{,}1640$$

$$E[2^{2x}] = \int_0^2 \frac{2^{2x}}{2}\,dx \simeq 5{,}4101$$

$$V[2^x] = E[2^{2x}] - (E[2^x])^2 \simeq 0{,}7270$$

6.4

60 min bus passa ogni 15 min
dalle 8 alle 9

$8 < x < 9$

aspettare il bus meno di 5 minuti

$$P(10 < X < 15) = \frac{5}{15} = \frac{1}{3}$$

aspettare il bus più di 10 min

$$P(0 < X < 5) = \frac{5}{15} = \frac{1}{3}$$