homework_7 ## Niccolò Parodi 4668271

7.1

$$X \sim N\ (3;16)$$

$$a = \frac{1}{\sigma} = \frac{1}{4}$$

$$b = -\frac{\mu}{\sigma} = -\frac{3}{4}$$

7.2

$$Var\ (X) = \frac{1}{\lambda^2}$$

conoscendo:

$$E[X] = \frac{1}{\lambda}$$

$$Var\ [X] = E[X^2] - (E[x])^2$$

$$E[X^2] = \int x^2 \lambda e^{-\lambda x} dx = -x^2 e^{-\lambda x} \Big|_0^{+\infty} - 2\int x\,(-e^{-\lambda x})\,dx =$$

$$= \frac{2}{\lambda} \int \lambda x \left(-e^{-\lambda x}\right) dx = \frac{2}{\lambda} \cdot \frac{1}{\lambda} = \frac{2}{\lambda^2}$$

$$Var\ [X] = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

7.3

$$f(x) = \begin{cases} Cx(1-x) & x \in [0,1] \\ 0 & \text{altrimenti} \end{cases}$$

a) $\int f_x(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} Cx(1-x)\,dx + \int_{1}^{+\infty} 0\,dx \Rightarrow$

$$\Rightarrow \quad C\int_0^1 x - x^2 \Rightarrow \frac{1}{C} = \frac{1}{6} \rightarrow C = 6$$

b) $E[X] = \frac{(a+b)}{2} = \frac{0+1}{2} = \frac{1}{2}$

c) $F_Y(x) = P\{Y < y\} = P\{\sqrt{X} < y\} = P\{X < y^2\} \rightarrow F_x(x^2)$

$$f_y(x) = \frac{dF_Y(x)}{dx} = \frac{dF_X(x^2)}{dx} = \begin{cases} \frac{F_X(x^2)}{2x} & x \geq 0 \\ -\pi & x < 0 \end{cases}$$

7.4

$f_x(x) \rightarrow X$

$f_Y(y) \rightarrow Y = g(x) = |X|$

$F_Y(y) = P\{Y \leq y\} = P\{g(x) \leq y\} = P\{|X| \leq y\} = P\{X \leq |y|\} =$

$= F_x(|X|)$

$$f_Y(|y|) = \frac{dF_Y(y)}{dy} = \frac{dF_X(|x|)}{dx} = \begin{cases} f_x(x) & x > 0 \\ -f(x) & x < 0 \end{cases}$$