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Esercizio 1:

$$f(x) = \sin\left(\frac{x}{2}\right) - \frac{\sin x}{2}$$

(c_1): $x \to s := \sin x$, $s2 := \sin\left(\frac{x}{2}\right) \to y_1 := s2 - s/2$

(c_2): $x \to s := \sin x$, $s2 := \sin\left(\frac{x}{2}\right) \to q := s^2 \to y_2 := 4 \cdot q \cdot s2$

• limite del coefficiente di applicazione

$$f'(x) = \cos\left(\frac{x}{2}\right) \cdot \frac{1}{2} - \frac{1}{2}\cos x = \frac{\cos\left(\frac{x}{2}\right) - \frac{1}{2}\cos x}{2}$$

$$C_f = \frac{x\, f'(x)}{f(x)} = \frac{x \cdot \frac{\cos\left(\frac{x}{2}\right) - \frac{1}{2}\cos x}{2}}{\sin\left(\frac{x}{2}\right) - \frac{\sin x}{2}} = \frac{\frac{x \cdot \cos\left(\frac{x}{2}\right) - \frac{1}{2}\cos x}{\cancel{2}}}{\frac{2 \sin\left(\frac{x}{2}\right) - \sin x}{\cancel{2}}} =$$

$$= \frac{x \cdot \cos\left(\frac{x}{2}\right) - \frac{1}{2}\cos x}{2 \sin\left(\frac{x}{2}\right) - \sin x}$$

$$\lim_{x \to 0^+} C_f = \lim_{x \to 0^+} \frac{x \cdot \cos\left(\frac{x}{2}\right) - \frac{1}{2}\cos x \to 0^+}{2 \sin\left(\frac{x}{2}\right) - \sin x \to 0^+} \longrightarrow \frac{0^+}{\text{qualcosa}} = 0^+$$

qualcosa → qualcosa
qualcosa
→ 0⁺ → qualcosa

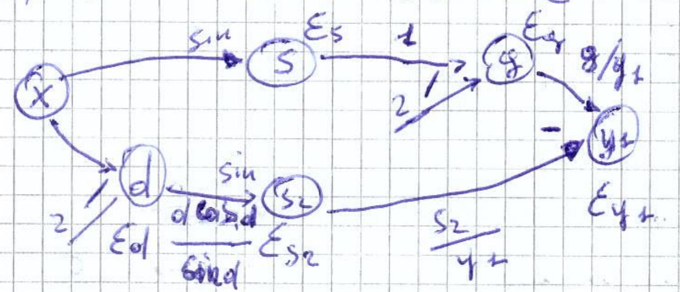
Condizionamento seno e coseno:

$$\cdot\ C_{sen} = \frac{x \cos x}{\sin x}$$

$$\cdot\ C_{cos} = \frac{x(-\sin x)}{\cos x}$$

ci possono venire bene per le etichette degli archi del grafo

1): è stabile o instabile



$$\varepsilon_{y_1} = \left\{1 \cdot \frac{g}{y_1}\right\}\varepsilon_s + \left\{\frac{g}{y_1}\right\}\varepsilon_g +$$

$$+\left\{\frac{d\cos d}{\sin d}\cdot\frac{s_2}{y_1}\right\}\varepsilon_d + \left\{\frac{s_2}{y_1}\right\}\varepsilon_{s_2} + \varepsilon_{y_1}$$

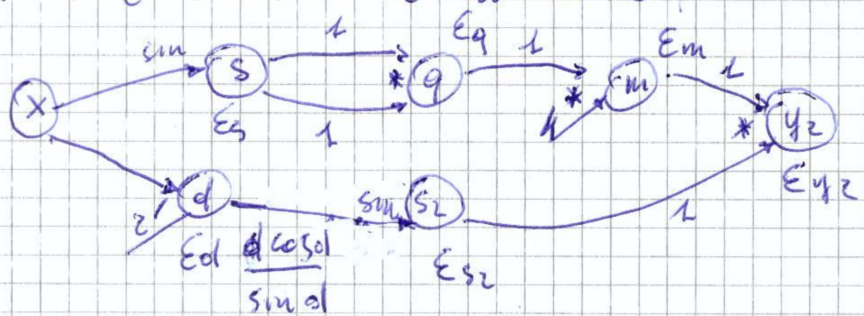
potrebbe essere una cancellazione:

$$\lim_{x\to 0^+} \frac{\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) - \frac{\sin x}{2}} = \lim_{x\to 0^+} \frac{\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) - \frac{1}{2}\sin x} \to 0^+$$

$$\lim_{x\to 0^+} \frac{1}{\sin\left(\frac{x}{2}\right)} \cdot - \frac{\sin\left(\frac{1}{2}x\right)}{\frac{1}{2}\sin(x)} = \frac{1}{2} = \lim_{x\to 0^+} \frac{1}{\sin\left(\frac{x}{2}\right)_{\to 0^+}} \cdot (-2) = \pm\infty$$

l'algoritmo c_1 è <u>INSTABILE</u>

(C2): è stabile o instabile ?



$$\varepsilon_{alg_2} = \left\{1 \cdot 1 \cdot 1 + 1 \cdot 1 \cdot 1\right\} \varepsilon_s + \left\{1 \cdot 1\right\} \varepsilon_q + \left\{\frac{d \cos d}{\sin d} \cdot 1\right\} \varepsilon_d + \left\{1\right\} \varepsilon_{s_2} + \varepsilon_{y_2}$$

tutti i coefficienti vanno bene, verifichiamo vada bene anche $\frac{d \cos d}{\sin d}$ e che non faccia esplodere

$$\lim_{x \to 0^+} \left(\underbrace{\frac{d}{\sin d}}_{\to 1} \cos d \right) = \lim_{x \to 0^+} \cos d = \lim_{x \to 0^+} \cos\left(\frac{\pi}{2}\right) \simeq 1$$

quindi l'algoritmo è <u>STABILE</u>

Esercizio 2:

$$x = \begin{pmatrix} -1 \\ \boxed{-2} \\ 0 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ q \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Se lo voglio fare diventare così parto con le rotazioni che mi azzerano un elemento alla volta

$$\cdot\ G(2,1,\theta) \longrightarrow \quad c = \cos\theta = \frac{x_i}{\sqrt{x_i^2 + x_j^2}}, \quad s = \operatorname{sen}\theta = \frac{-x_j}{\sqrt{x_i^2 + x_j^2}}$$

$$c = \frac{-2}{\sqrt{4+1}} = -\frac{2}{\sqrt{5}}, \quad s = \frac{+1}{\sqrt{4+1}} = +\frac{2}{\sqrt{5}}$$

$$G(2,1,\theta) = \begin{pmatrix} c & s & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Rotazione nel piano $\langle e_2, e_1 \rangle$

$$x = \begin{pmatrix} 0 \\ \boxed{-\sqrt{5}} \\ 0 \\ 0 \\ \boxed{2} \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ q \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \to G(2,5,\theta^1):$$

$$c = \frac{\sqrt{5}}{\sqrt{5+4}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}, \quad s = \frac{-2}{\sqrt{5+4}} = \frac{-2}{\sqrt{9}} = \frac{-2}{3}$$

$$x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3 \end{pmatrix}$$

$$G(2,5,\theta^1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c & 0 & 0 & -s \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & s & 0 & 0 & c \end{pmatrix}$$

Rotazione nel piano $\langle e_2, e_5 \rangle$

Esercizio 3:

x	-2	-2	0	1	2	3
y	-1	-2	0	-1	1	2

$$A = \begin{pmatrix} -2 & 1 \\ -2 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -1 \\ -2 \\ 0 \\ -1 \\ 1 \\ 2 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} -2 & -2 & 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ -2 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 4+4+1+4+9 & -4+6 \\ -2-2+6 & 6 \end{pmatrix} = \begin{pmatrix} 22 & 2 \\ 2 & 6 \end{pmatrix}$$

$$A^T y = \begin{pmatrix} -2 & -2 & 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 0 \\ -1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2+4-1+2+6 \\ -1-2-1+1+2 \end{pmatrix} = \begin{pmatrix} 13 \\ -1 \end{pmatrix}$$

$$\begin{cases} 22\alpha + 2\beta = 13 \\ 2\alpha + 6\beta = -1 \end{cases}$$

$2\alpha + 6\beta = -1$

$\frac{\cancel{2}\alpha}{\cancel{2}} = \frac{-6\beta - 1}{2}$

$\alpha = -3\beta - \frac{1}{2}$

$\frac{22\alpha}{2} + \frac{2}{2}\beta = \frac{13}{2}$

$11\alpha + \beta = 13/2$

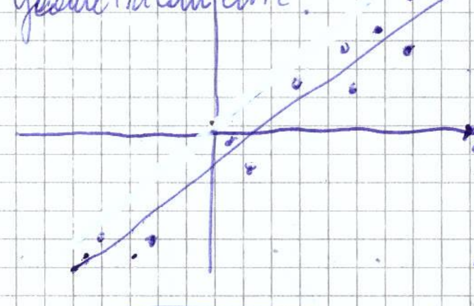
$11\left(-3\beta - \frac{1}{2}\right) = \frac{13}{2}$

$-33\beta - \frac{11}{2} = \frac{13}{2}$

$\frac{-33\beta}{33} = \frac{24}{2}$ (12)

$\beta = \frac{12}{33}$

Geometricamente:



$\alpha = -3\left(\frac{12}{33}\right) - \frac{1}{2}$ (11)

$\alpha = -\frac{12}{11} - \frac{1}{2}$

$\alpha = \frac{-12-11}{22} = -\frac{23}{22}$

$f(x) = -\frac{23}{22}\alpha - \frac{12}{33}$

Esercizio 4:

$$A = \begin{pmatrix} -2 & -1 & -3 \\ -1 & 2 & 1 \\ -3 & 1 & -2 \end{pmatrix}$$ è simmetrica, quindi invertibile, quindi diagonalizzabile

$$A - \lambda I = \begin{pmatrix} -2-\lambda & -1 & -3 \\ -1 & 2-\lambda & 1 \\ -3 & 1 & -2-\lambda \end{pmatrix}$$

$\det(A - \lambda I)$ per trovare il polinomio caratteristico:

$$\begin{vmatrix} -2-\lambda & -1 & -3 \\ -1 & 2-\lambda & 1 \\ -3 & 1 & -2-\lambda \end{vmatrix} \leftarrow$$

$$= +1\begin{vmatrix} -1 & -3 \\ 1 & -2-\lambda \end{vmatrix} + (2-\lambda)\begin{vmatrix} -2-\lambda & -3 \\ -3 & -2-\lambda \end{vmatrix} - 1\begin{vmatrix} -2-\lambda & -1 \\ -3 & 1 \end{vmatrix} =$$

$$= (2+\lambda+3) + (2-\lambda)(+4+\lambda^2+4\lambda) - (-2-\lambda+3) =$$

$$= 2+\lambda+\cancel{3}+8+2\lambda^2+8\lambda-4\lambda-\lambda^3-4\lambda^2+2+\lambda-\cancel{3} =$$

$$-\lambda^3-2\lambda^2+6\lambda+18=0 \rightarrow \lambda^3+2\lambda^2-6\lambda-12=0$$

$$\lambda^2(\lambda+2)-6(\lambda+2)=0$$

$$(\lambda^2-6)(\lambda+2)=0$$

$\lambda^2 = +6$ $\quad \lambda+2=0$

$\sqrt{\lambda^2} = \pm\sqrt{+6}$ $\quad \lambda = -2$

$\lambda^2 = \pm\sqrt{6}$

$$\begin{cases} \lambda = +\sqrt{6} \\ \lambda = -\sqrt{6} \\ \lambda = -2 \end{cases}$$

λ sono gli autovalori: calcoliamo ora gli autovettori

$\lambda = -2$ $\quad A-(-2)I$

$$\begin{pmatrix} -2-2 & -1 & -3 \\ -1 & 2-2 & 1 \\ -3 & 1 & -2-2 \end{pmatrix} = \begin{pmatrix} -4 & -1 & -3 \\ -1 & 0 & 1 \\ -3 & 1 & -4 \end{pmatrix} \longrightarrow \begin{cases} -4x-y-3z=0 \\ -x+z=0 \\ -3x+y-4z=0 \end{cases}$$

→ Quindi dal sistema

$$\begin{cases} -4x - y - 3z = 0 \\ -x + z = 0 \\ -3x + y - 4z = 0 \end{cases}$$ estrapoliamo l'autovettore

$$\begin{cases} -4z - y - 3z = 0 \\ x = z = 0 \\ -3x + y - 4x = 0 \end{cases} \rightarrow \begin{cases} -y = +4z + 3z \\ x = z \\ y = 4z + 3z \end{cases} \rightarrow \begin{cases} y = -7z \\ x = z = 0 \\ y = 7z \end{cases} \rightarrow \begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix} = v_1$$

$$A_{(\lambda=-2)} = \left\langle \begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix} \right\rangle$$

$$v_1 / \|v_1\| = \begin{pmatrix} 0 \\ -7/7 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

da mettere nella matrice $\tilde{X} \Lambda \tilde{X}^T$ dove gli elementi di $\tilde{X}$ sono i vettori con al den. la norma

- $\lambda = +\sqrt{6} \rightarrow A - \lambda I$

$$\begin{pmatrix} -2-\sqrt{6} & 1 & -3 \\ -1 & 2-\sqrt{6} & 1 \\ -3 & 1 & -2-\sqrt{6} \end{pmatrix}$$

$$\begin{cases} (-2-\sqrt{6})x + y - 3z = 0 \\ -x + (2-\sqrt{6})y + z = 0 \\ -3x + y + (-2-\sqrt{6})z = 0 \end{cases}$$

$$\begin{cases} -2x - \sqrt{6}x + y - 3z = 0 \\ -x + 2y - \sqrt{6}y + z = 0 \\ -3x + y - 2z - \sqrt{6}z = 0 \end{cases}$$

NO, errato

$$y = +3x - 2z - \sqrt{6}z$$

$$-x + 2(3x - 2z - \sqrt{6}z) + z = 0$$

$$-x + 6x - 4z - 2\sqrt{6}z + z = 0$$

$$5x = -4z - 2\sqrt{6}z + z$$

$$\frac{5x}{5} = \frac{-3z - 2\sqrt{6}z}{5}$$

$$-2\left(\frac{-3z - 2\sqrt{6}z}{5}\right) - \sqrt{6}\left(\frac{-3z - 2\sqrt{6}z}{5}\right) + 3x - 2z - \sqrt{6}z - 3z = 0$$

$$\frac{+6z + 4\sqrt{6}z}{5} + \frac{3\sqrt{6}z + 2\cdot 6z}{5} + 3\underset{\left(\frac{-3z - 2\sqrt{6}z}{5}\right)}{x} - 2z - \sqrt{6}z = 0$$