

$$1. \quad Y_1 = (X_1 + X_3) \quad Y_2 = (X_3 + X_4) \quad Y_3 = (X_2 + X_4)$$

media = 0 e varianza = 1 per ciascuna X

correlazione?

a) Y_1 e Y_2

$$\begin{aligned} E[Y_1 Y_2] &= E[(X_1 + X_3)(X_3 + X_4)] = \\ &= E[X_1 X_3 + X_3^2 + X_1 X_4 + X_3 X_4] = E[X_3^2] = 1 \end{aligned}$$

b) Y_1 e Y_3

$$\begin{aligned} E[Y_1 Y_3] &= E[(X_1 + X_3)(X_2 + X_4)] = \\ &= E[X_1 X_3 + X_3 X_2 + X_1 X_4 + X_3 X_4] = 0 \end{aligned}$$

2.

$$P(X=1, Y=3) = \frac{1}{5}$$

$$P(X=3, Y=3) = \frac{1}{20}$$

$$P(X=3, Y=4) = \frac{1}{2}$$

$$P(X=1, Y=4) = \frac{1}{4}$$

$$X = \{1, 3\} \quad Y = \{3, 4\}$$

$$P_X(X=1) = \frac{1}{5} + \frac{1}{4} = \frac{9}{20}$$

$$P_X(X=3) = \frac{1}{20} + \frac{1}{2} = \frac{11}{20}$$

$$P_Y(Y=3) = \frac{1}{5} + \frac{1}{20} = \frac{1}{4}$$

$$P_Y(Y=4) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$E[XY] = 1 \cdot 3 \cdot \frac{1}{5} + 3 \cdot 3 \cdot \frac{1}{20} + 3 \cdot 4 \cdot \frac{1}{2} + 1 \cdot 4 \cdot \frac{1}{4} = \frac{161}{20}$$

3. μ ? con precisione 0,5 cm e confidenza al 90%

$$\sigma = 1 \text{ cm}$$

$$P\left\{\left|\sum_i \frac{X_i}{n} - \mu\right| \geq 0,5\right\} \leq \frac{\sigma^2}{n \cdot 0,25} = \frac{4\sigma^2}{n}$$

il numero di misure necessarie è $n = 40\sigma^2$