Raised Cosine Filters

A *raised cosine filter* is a low-pass filter which is commonly used for pulse shaping in data transmission systems (e.g. modems).

Let τ be the duration of a baud, so the baud rate is $1/\tau$ symbols per second. The frequency response of a raised cosine filter is symmetrical about 0 Hz, and is divided into three parts:

$$X(f) = \begin{cases} \tau & \text{for } 0 \le f \le \dfrac{1-\beta}{2\tau} \\ \dfrac{\tau}{2}\left(1 + \cos\left(\dfrac{\pi\tau}{\beta}\left(f - \dfrac{1-\beta}{2\tau}\right)\right)\right) & \text{for } \dfrac{1-\beta}{2\tau} \le f \le \dfrac{1+\beta}{2\tau} \\ 0 & \text{for } f \ge \dfrac{1+\beta}{2\tau} \end{cases} \tag{1}$$

The parameter β is known as the *roll-off factor* or *excess bandwidth*. β lies between 0 and 1.

The minimum bandwidth needed to transmit a pulse train at $1/\tau$ symbols per second is $\alpha = 1/(2\tau)$ Hz. For example, if the baud rate is 2400 and the carrier frequency is 1800, the necessary bandwidth is from 600 to 3000 Hz, and the minimum low-pass filter bandwidth is 1200 Hz. This requires a filter with a "brick-wall" (square) low-pass response. That's what you get if you specify $\beta = 0$. A "brick-wall" filter has a "sinc" impulse response with slowly decaying tails, and is difficult to implement and generally unsatisfactory.

Non-zero values of β specify a filter with a less square frequency response, and a more manageable impulse response. The rule is that the filter response (or, alternatively, the transmitted spectrum) stretches from 0 Hz to $(1 + \beta)\,\alpha$ Hz. That's why β is called the excess bandwidth: it specifies the amount by which the filter bandwidth exceeds the minimum necessary to transmit a pulse train with the specified baud rate.

The time-domain (impulse) response of a raised cosine filter, obtained by taking the inverse Fourier transform of the spectrum, can be shown to be:

$$x(t) = \left(\operatorname{sinc}\frac{\pi t}{\tau}\right)\left(\frac{\cos \pi\beta t/\tau}{1 - (2\beta t/\tau)^2}\right) \tag{2}$$

In a modem, the raised cosine frequency response is divided equally between the transmitter and the receiver. Each contains an identical filter whose magnitude response is given by the square root of eqn. (1), normalized so that the maximum remains τ. This is called a *root raised cosine* filter.

The `mkshape` program (which is called by the "mkfilter" web page) designs a raised cosine or root raised cosine filter by first using eqn. (1) to compute the frequency response, then performing an inverse fast Fourier transform to compute the time-domain (impulse) response.

A.J. Fisher
http://www.cs.york.ac.uk/~fisher
27 Feb 1998