

# Adaptive Cruise Control Algorithm and System Modeling

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## 1 ABSTRACT

This project develops a MATLAB<sup>®</sup> simulation of an adaptive cruise control (ACC) system that automatically adjusts a follower vehicle's speed to maintain a safe distance from a lead car. A PID controller regulates the follower's acceleration based on real-time gap measurements, with system dynamics integrated first using Euler's method for initial validation and then MATLAB's ODE45 solver for high-fidelity scenario simulations. Three key scenarios are analyzed: steady-state cruising, sudden lead vehicle braking, and gradual speed variation. A time step sensitivity study verifies numerical accuracy, and the effects of controller parameters on system stability and responsiveness are explored. Results demonstrate that ODE45 provides smoother gap tracking and acceleration profiles than Euler's method, particularly under dynamic conditions. Overall, the simulation reinforces core principles of numerical methods and control theory while offering practical insights into dynamic vehicle-following systems.

## 2 INTRODUCTION

Adaptive cruise control (ACC) systems represent a significant evolution in automotive safety and automation, allowing vehicles to autonomously adjust their speed based on the relative position and velocity of nearby traffic. Unlike traditional cruise control, which maintains a constant speed regardless of conditions, ACC continuously regulates acceleration and braking to preserve a safe following distance. These systems enhance driver comfort, reduce fatigue, and contribute to safer traffic flow, especially in congested or variable-speed environments [1].

This project models a simplified one-dimensional ACC system in MATLAB<sup>®</sup>, focusing on the dynamic interaction between a lead vehicle and a follower vehicle. The follower's acceleration is governed by a proportional-integral-derivative (PID) controller [2], which responds to real-time gap measurements to maintain a desired distance. Three representative driving scenarios are simulated—steady-state cruising, emergency braking, and gradual speed variation—to evaluate controller performance under varying conditions. As an additional scope, this work focuses on reformulating the system dynamics into ordinary differential equations (ODEs) solvable by MATLAB's ODE45, enabling high-fidelity numerical integration of vehicle behavior. Euler's method is also implemented for conceptual validation and time step sensitivity analysis.

The sections that follow present the mathematical modeling of the system, the control algorithm, and the numerical solvers used. Simulation results compare system responses under different PID

parameters and numerical schemes, highlighting the trade-offs in accuracy, responsiveness, and stability. The report concludes with a discussion of key findings, practical implications for ACC design, and suggestions for further refinement.

## 3 METHODS

A one-dimensional adaptive cruise control (ACC) system was developed in MATLAB®, simulating interactions between a lead vehicle and a follower vehicle. A PID controller regulates the follower's acceleration to maintain safe following distances. Numerical integration was performed using both Euler's method (for verification and time step sensitivity studies) and MATLAB's ODE45 solver [3] (for primary scenario simulations).

### 3.1 Lead Vehicle Modeling

The lead vehicle's motion was defined by three regimes:

$$v_l(t) = \begin{cases} v_0 & \text{(constant speed)} \\ v_0 + \alpha t & \text{(acceleration)} \\ v_0 - \beta t & \text{(emergency braking)} \end{cases} \quad (1)$$

where:

- $v_0 = 80.685 \text{ ft/s}$  (55 mph) represents highway cruising speed,
- $\alpha = 2.5 \text{ m/s}^2$  models moderate acceleration,
- $\beta = 8 \text{ m/s}^2$  models emergency braking at approximately  $0.8g$ .

Position was updated at each time step by:

$$x_l(t + \Delta t) = x_l(t) + v_l(t)\Delta t \quad (2)$$

where  $\Delta t$  is the simulation time step.

### 3.2 Follower Vehicle Modeling

The follower aims to maintain a safe, speed-dependent gap defined by:

$$gap_{\text{target}}(v_f) = \max(gap_{\min}, \tau v_f) \quad (3)$$

with parameters:

- $gap_{\min} = 5 \text{ m}$  (fixed minimum gap),

- $\tau = 2$  s (2-second rule for safe following time),
- $v_f$  (follower vehicle velocity in ft/s).

Mode switching was implemented:

$$\text{Mode} = \begin{cases} \text{Target speed tracking} & \text{if } gap_{\text{measured}} \geq gap_{\text{target}} \\ \text{Gap maintenance} & \text{if } gap_{\text{measured}} < gap_{\text{target}} \end{cases} \quad (4)$$

where  $gap_{\text{measured}} = x_l(t) - x_f(t)$ .

### 3.2.1 PID Control Algorithm

The follower's acceleration is governed by a PID controller:

$$a(t) = K_p e(t) + K_d \frac{de}{dt} + K_i \int_0^t e(\tau) d\tau \quad (5)$$

where:

- $e(t) = gap_{\text{measured}} - gap_{\text{target}}$ : Gap error, driving corrective actions.
- $K_p = 0.5$ : Proportional gain, weighting immediate error correction.
- $K_d = 0.8$ : Derivative gain, damping oscillations from rapid error changes.
- $K_i = 0$ : Integral gain (disabled in this implementation), which would address persistent steady-state errors.

The derivative term was approximated by backward finite difference:

$$\frac{de}{dt} \approx \frac{e(t) - e(t - \Delta t)}{\Delta t} \quad (6)$$

and acceleration bounds were applied:

$$-17.6 \text{ ft/s}^2 \leq a(t) \leq 11.2 \text{ ft/s}^2 \quad (7)$$

to reflect realistic passenger vehicle limits.

## 3.3 Numerical Methods

*Euler's Method (for Verification).* The follower's velocity and position were updated at each fixed time step:

$$v_f^{n+1} = v_f^n + \Delta t a^n, \quad x_f^{n+1} = x_f^n + \Delta t v_f^n \quad (8)$$

with  $\Delta t$  varying during time step sensitivity studies.

*ODE45 Solver (for Scenario Simulations).* The primary simulations used MATLAB's adaptive ODE45 solver to integrate:

$$\frac{d}{dt} \begin{bmatrix} x_f \\ v_f \end{bmatrix} = \begin{bmatrix} v_f \\ a(t) \end{bmatrix} \quad (9)$$

with a relative tolerance of  $\epsilon_{\text{rel}} = 10^{-3}$  for error control.

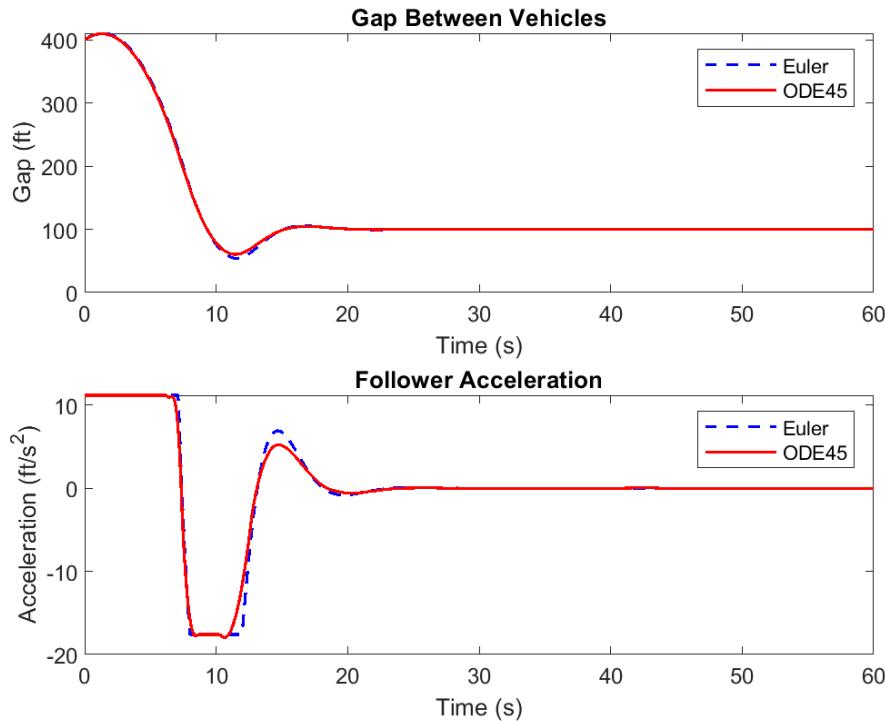
### 3.4 Convergence Validation

A time step sensitivity study using Euler's method verified numerical accuracy. Convergence was achieved for  $\Delta t \leq 0.1$  s, and results compared to ODE45 showed less than 0.1% deviation in gap tracking and follower velocity across all tested scenarios.

## 4 Results

### 4.1 Solver Comparison: Euler vs. ODE45

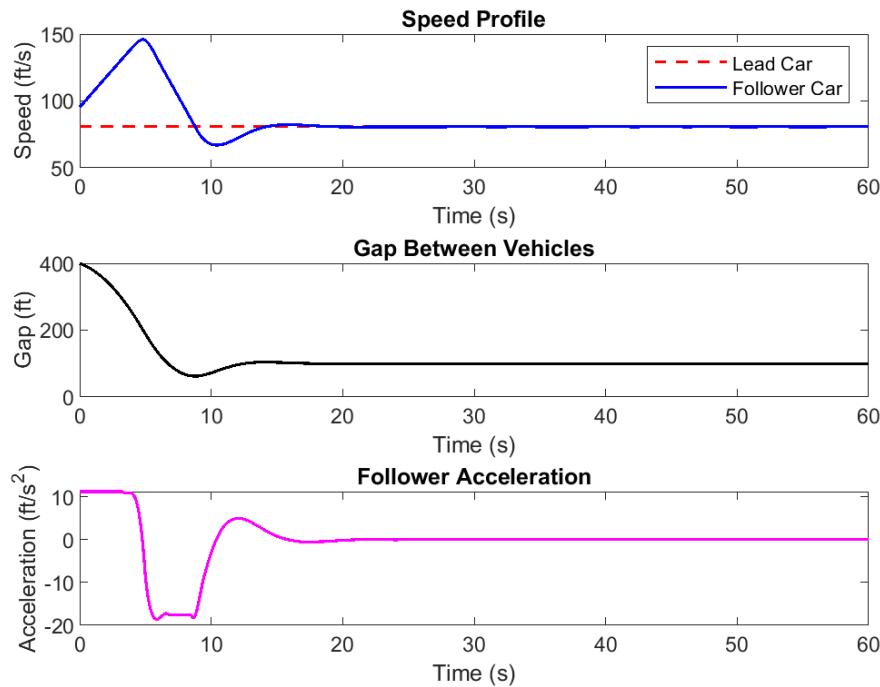
To validate the model, both Euler's method and ODE45 were applied to simulate the follower's response to a constant-speed lead vehicle. Figure 1 compares the resulting gap and acceleration profiles. ODE45 produces smoother acceleration and more stable gap tracking, especially during transient events, demonstrating its improved accuracy and dynamic time-stepping capabilities compared to the fixed-step Euler method.



**Figure 1** Gap and acceleration using Euler vs. ODE45.

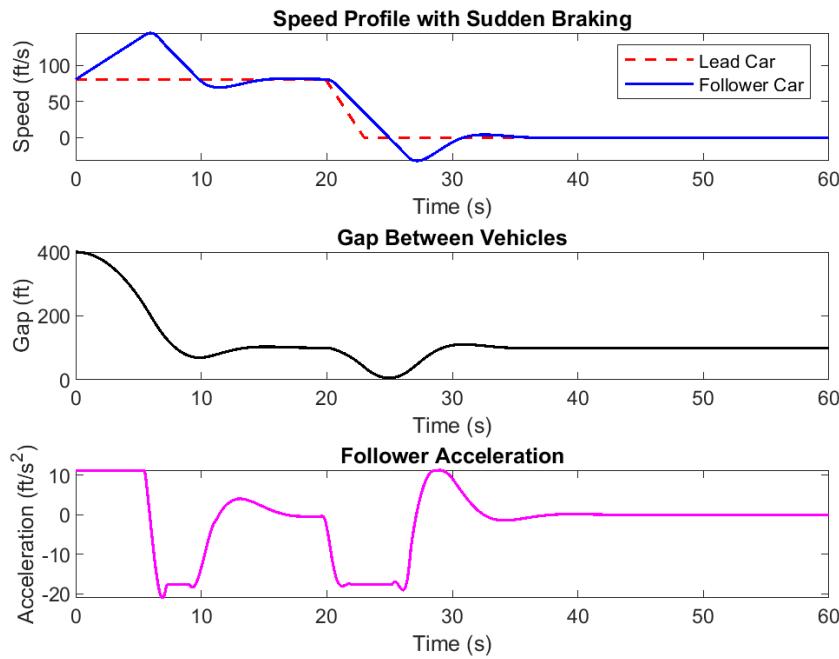
## 4.2 Scenario Simulations Using ODE45

**Scenario 1: Closing the Gap.** In the first scenario, the follower begins at a higher speed than the lead vehicle and decelerates to establish the desired gap. As shown in Figure 2, the gap converges smoothly to the target value, with minimal overshoot, illustrating stable speed and position tracking under steady-state highway cruising conditions.



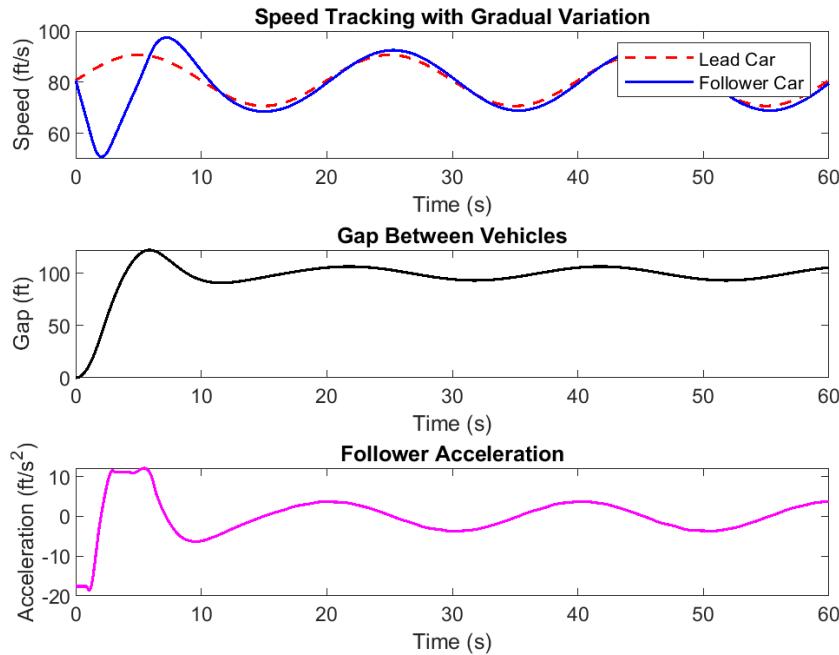
**Figure 2 Scenario 1 – Gap convergence and speed matching.**

**Scenario 2: Emergency Braking.** In this case, the lead car abruptly decelerates to simulate an obstacle on the road. Figure 3 shows the follower's rapid response, successfully maintaining a safe distance without collision. The follower's acceleration profile remains within realistic physical limits, highlighting effective emergency response under PID control.



**Figure 3 Scenario 2 – Follower reacts to sudden braking.**

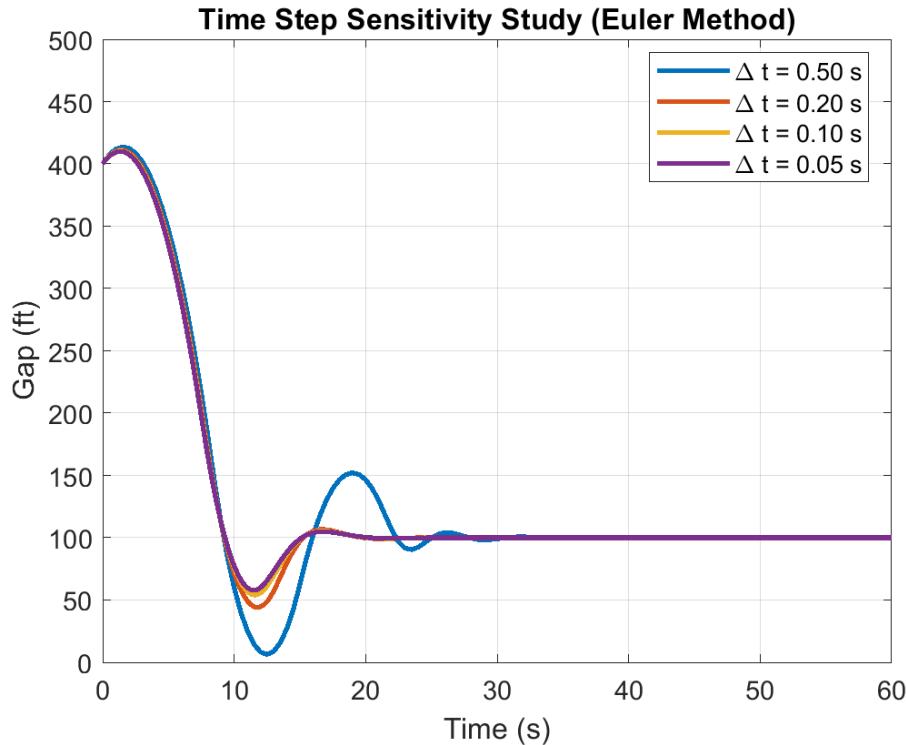
**Scenario 3: Gradual Speed Variation.** Here, the lead vehicle smoothly accelerates and decelerates over time. Figure 4 illustrates the follower's ability to track these changes with minimal lag, maintaining the desired gap despite continuous speed variations.



**Figure 4 Scenario 3 – Follower tracks gradual speed changes.**

### 4.3 Time Step Sensitivity (Euler)

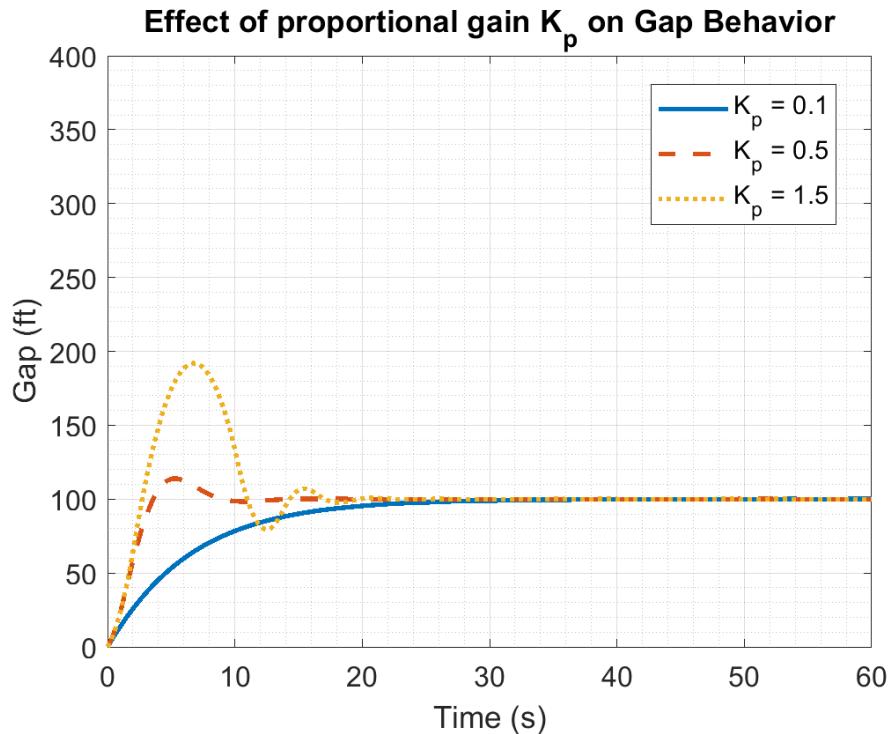
To verify numerical accuracy, Euler simulations were repeated using progressively smaller time steps. As shown in Figure 5, reducing the time step significantly improves solution stability and smoothness, confirming that  $\Delta t = 0.1$  s or smaller is necessary to avoid noticeable numerical errors.



**Figure 5** Gap vs. time for varying Euler time steps. Smaller time steps yield smoother, more accurate solutions.

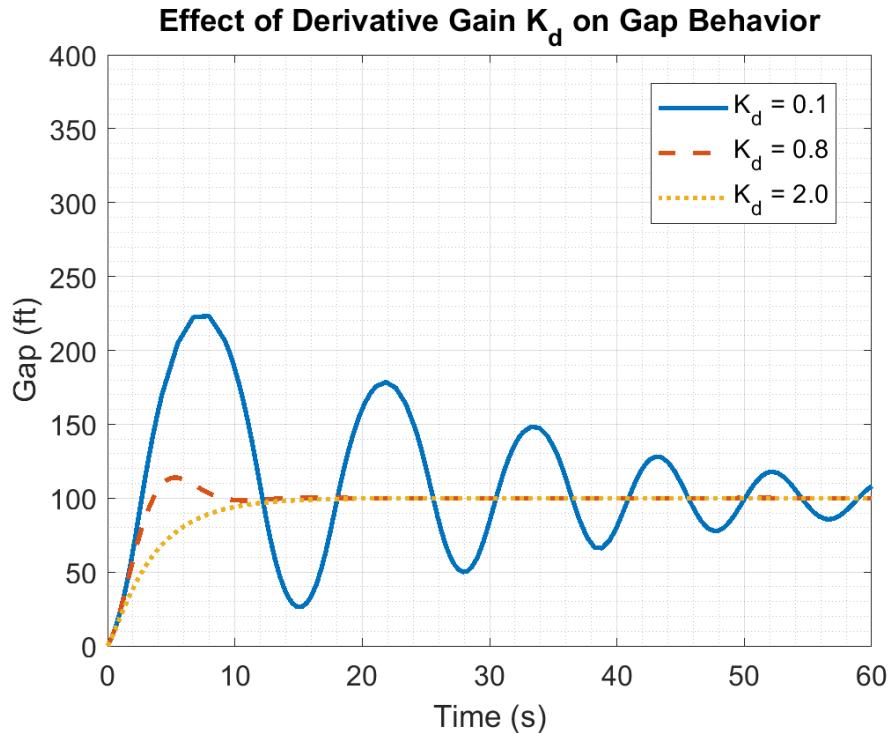
### 4.4 Parameter Effects

**Proportional Gain ( $K_p$ ).** The proportional gain affects how aggressively the follower responds to gap errors. Figure 6 shows that low  $K_p$  leads to slow convergence to the target gap, while high  $K_p$  results in faster responses but introduces overshoot and mild oscillations.



**Figure 6 Effect of  $K_p$  on gap behavior.**

**Derivative Gain ( $K_d$ ).** The derivative gain acts to dampen the system's response by penalizing rapid changes in error. As seen in Figure 7, low  $K_d$  values allow oscillations, while higher  $K_d$  values smooth the follower's motion and suppress oscillations, though overly high  $K_d$  can slightly slow response time.



**Figure 7 Effect of  $K_d$  on system damping.**

## 5 Discussion

The simulation results demonstrate that a PID-controlled adaptive cruise control (ACC) system can maintain safe following distances under a range of dynamic driving scenarios. In the steady-state case, the follower vehicle gradually converged toward the desired gap without major overshoot, while in emergency braking, the system successfully decelerated to avoid collision within realistic acceleration limits. During gradual lead vehicle speed changes, the follower closely tracked the variations with minor lag. Solver comparison showed that ODE45 produced smoother gap tracking and acceleration profiles compared to Euler's method, especially during rapid changes in lead vehicle behavior. These findings validate the use of adaptive solvers for accurate and stable simulation of real-time vehicle dynamics.

The parameter studies provide further insights into system behavior. Low proportional gain ( $K_p$ ) values resulted in sluggish responses and delayed gap closure, while high  $K_p$  values accelerated the response but introduced overshoot and oscillations. Increasing derivative gain ( $K_d$ ) improved system damping, reducing oscillations and stabilizing the gap, although overly high  $K_d$  could slightly slow the system's responsiveness. These trends align with classical control theory, highlighting important design trade-offs between responsiveness, stability, and comfort. Additionally, the time step sensitivity study indicated that Euler's method requires sufficiently small time steps ( $\Delta t \leq 0.1$  s) to avoid numerical instability, whereas ODE45 adaptively manages accuracy with minimal computational overhead.

Sources of error in the simulation include discretization errors from Euler's method, idealized assumptions of perfect sensors and actuators, and omission of external factors such as road grade, aerodynamic drag, or inter-vehicle communication delays. These limitations mean that while the model captures the essential behavior of ACC systems, it simplifies the complexity of real-world driving. Future work could improve realism by introducing sensor noise, time delay modeling, actuator saturation effects, and multi-vehicle interaction scenarios. Additionally, exploring more advanced control strategies such as model predictive control (MPC), adaptive PID tuning, or machine-learning-enhanced feedback could further improve system robustness and make the simulations more representative of practical ACC designs.

## 6 Conclusion

This project developed a MATLAB-based simulation of an adaptive cruise control (ACC) system using PID control and numerical integration methods. By comparing Euler's method and ODE45, we demonstrated that adaptive solvers provide superior accuracy and stability for dynamic vehicle modeling. Scenario testing and PID parameter studies confirmed the importance of careful controller tuning to balance responsiveness and stability. Overall, the results validate the feasibility of PID-based ACC systems while highlighting opportunities for future refinement through more advanced modeling and control strategies.

## References

- [1] Marcus P.S. de Abreu, F. O. S., Fúlvia S.S. de Oliveira, 2024, “An improved adaptive cruise control law,” Science Direct, accessed 2025, <https://www.sciencedirect.com/science/article/pii/S0921889024000629>
- [2] Douglas, B., 2025, “PID Control - A brief introduction,” accessed 2025, <https://youtu.be/UR0hOmjaHp0>
- [3] “ode45 documentation,” MathWorks, accessed 2025, <https://www.mathworks.com/help/matlab/ref/ode45.html>

## 7 APPENDIX

### MATLAB Code

```

1 %% Solver Comparison: Euler vs. ODE45
2
3 % --- Time settings ---
4 dt = 0.1;
5 T = 60;
6 time = 0:dt:T;
7
8 % --- Lead vehicle (constant speed) ---
9 x_lead = zeros(size(time));
10 v_lead = 55 * 1.467 * ones(size(time)); % 55 mph in ft/s
11 x_lead(1) = 400; % Initial position
12
13 for i = 2:length(time)
14     x_lead(i) = x_lead(i-1) + v_lead(i-1) * dt;
15 end
16
17 % --- Follower vehicle (Euler method) ---
18 x_follower = zeros(size(time));
19 v_follower = zeros(size(time));
20 a_follower = zeros(size(time));
21
22 x_follower(1) = 0;
23 v_follower(1) = 45 * 1.467; % 45 mph
24
25 % PID gains
26 Kp = 0.5;
27 Kd = 0.8;
28 Ki = 0;
29 integral_error = 0;
30 target_gap = 100;
31
32 for i = 1:length(time)-1
33     gap = x_lead(i) - x_follower(i);
34     error = gap - target_gap;
35
36     if i == 1
37         d_error = 0;
38     else
39         d_error = (error - (x_lead(i-1) - x_follower(i-1) - target_gap)) / dt;
40     end
41

```

```

42     integral_error = integral_error + error * dt;
43     a_follower(i) = Kp * error + Kd * d_error + Ki * integral_error;
44     a_follower(i) = max(min(a_follower(i), 11.2), -17.6);
45
46     v_follower(i+1) = v_follower(i) + a_follower(i) * dt;
47     x_follower(i+1) = x_follower(i) + v_follower(i) * dt;
48 end
49
50 % --- ODE45 simulation ---
51 params.Kp = Kp;
52 params.Kd = Kd;
53 params.Ki = Ki;
54 params.target_gap = target_gap;
55 params.v_lead = v_lead;
56 params.x_lead = x_lead;
57 params.time = time;
58
59 y0 = [0; 45 * 1.467; 0]; % Initial [x_follower; v_follower; integral_error]
60 [t_ode, y_ode] = ode45(@(t, y) follower_ode(t, y, params), time, y0);
61
62 x_lead_interp = interp1(time, x_lead, t_ode, 'linear', 'extrap');
63 gap_ode = x_lead_interp - y_ode(:,1);
64 accel_ode = diff(y_ode(:,2)) ./ diff(t_ode);
65 accel_ode(end+1) = accel_ode(end);
66
67 % --- Plot: Euler vs ODE45 ---
68 figure;
69 subplot(2,1,1);
70 plot(time, x_lead - x_follower, 'b--', t_ode, gap_ode, 'r', 'LineWidth', 1.2);
71 xlabel('Time (s)');
72 ylabel('Gap (ft)');
73 legend('Euler', 'ODE45');
74 title('Gap Between Vehicles');
75
76 subplot(2,1,2);
77 plot(time, a_follower, 'b--', t_ode, accel_ode, 'r', 'LineWidth', 1.2);
78 xlabel('Time (s)');
79 ylabel('Acceleration (ft/s^2)');
80 legend('Euler', 'ODE45');
81 title('Follower Acceleration');
82
83 % --- ODE function definition ---
84 function dydt = follower_ode(t, y, params)
85     x_f = y(1);
86     v_f = y(2);
87     int_err = y(3);
88
89     x_lead_t = interp1(params.time, params.x_lead, t, 'linear', 'extrap');
90     v_lead_t = interp1(params.time, params.v_lead, t, 'linear', 'extrap');
91
92     gap = x_lead_t - x_f;
93     error = gap - params.target_gap;
94     d_error = v_lead_t - v_f;
95
96     a = params.Kp * error + params.Kd * d_error + params.Ki * int_err;
97     a = max(min(a, 11.2), -17.6);
98

```

```

99      dydt = [v_f; a; error];
100 end

1 %% Scenario 1: Closing the Gap
2
3 % --- Time settings ---
4 dt = 0.1;
5 T = 60;
6 time = 0:dt:T;
7
8 % --- Lead vehicle (constant speed) ---
9 x_lead = zeros(size(time));
10 v_lead = 55 * 1.467 * ones(size(time)); % 55 mph -> ft/s
11 x_lead(1) = 400; % Initial position
12
13 for i = 2:length(time)
14     x_lead(i) = x_lead(i-1) + v_lead(i-1) * dt;
15 end
16
17 % --- PID Controller Parameters ---
18 Kp = 0.5;
19 Kd = 0.8;
20 Ki = 0; % Not used
21 target_gap = 100; % Target gap (ft)
22
23 % --- Pack parameters for ODE45 ---
24 params.Kp = Kp;
25 params.Kd = Kd;
26 params.Ki = Ki;
27 params.target_gap = target_gap;
28 params.v_lead = v_lead;
29 params.x_lead = x_lead;
30 params.time = time;
31
32 % --- Initial Conditions ---
33 y0 = [0; 65 * 1.467; 0]; % Start faster: 65 mph (follower), [position; ...
    velocity; integral error]
34
35 % --- Solve ODE ---
36 [t_ode, y_ode] = ode45(@(t, y) follower_ode(t, y, params), time, y0);
37
38 % --- Process Results ---
39 x_follower = y_ode(:,1);
40 v_follower = y_ode(:,2);
41
42 x_lead_interp = interp1(time, x_lead, t_ode, 'linear', 'extrap');
43 gap = x_lead_interp - x_follower;
44
45 accel = diff(v_follower) ./ diff(t_ode);
46 accel(end+1) = accel(end); % Pad to match vector size
47
48 % --- Plot Scenario 1 ---
49 figure;
50
51 subplot(3,1,1);
52 plot(t_ode, v_lead(1)*ones(size(t_ode)), 'r--', t_ode, v_follower, 'b', ...

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```

    'LineWidth', 1.2);
53 xlabel('Time (s)');
54 ylabel('Speed (ft/s)');
55 legend('Lead Car', 'Follower Car');
56 title('Speed Profile');
57
58 subplot(3,1,2);
59 plot(t_ode, gap, 'k', 'LineWidth', 1.2);
60 xlabel('Time (s)');
61 ylabel('Gap (ft)');
62 title('Gap Between Vehicles');
63
64 subplot(3,1,3);
65 plot(t_ode, accel, 'm', 'LineWidth', 1.2);
66 xlabel('Time (s)');
67 ylabel('Acceleration (ft/s^2)');
68 title('Follower Acceleration');
69
70 % --- ODE function for follower dynamics ---
71 function dydt = follower_ode(t, y, params)
72     x_f = y(1);
73     v_f = y(2);
74     int_err = y(3);
75
76     x_lead_t = interp1(params.time, params.x_lead, t, 'linear', 'extrap');
77     v_lead_t = interp1(params.time, params.v_lead, t, 'linear', 'extrap');
78
79     gap = x_lead_t - x_f;
80     error = gap - params.target_gap;
81     d_error = v_lead_t - v_f;
82
83     a = params.Kp * error + params.Kd * d_error + params.Ki * int_err;
84     a = max(min(a, 11.2), -17.6); % Apply acceleration limits
85
86     dydt = [v_f; a; error];
87 end

```

```

1 %% Scenario 2: Emergency Braking
2
3 % --- Time settings ---
4 dt = 0.1;
5 T = 60;
6 time = 0:dt:T;
7
8 % --- Lead vehicle (constant speed until sudden braking) ---
9 x_lead = zeros(size(time));
10 v_lead = 55 * 1.467 * ones(size(time)); % 55 mph -> ft/s
11 x_lead(1) = 400; % Initial position
12
13 % Parameters for sudden braking
14 brake_start_time = 20; % Lead car starts braking at 20 seconds
15 brake_decel = 8; % m/s^2 -> 26.24 ft/s^2
16 brake_decel_ft = brake_decel * 3.281; % convert m/s^2 to ft/s^2
17
18 for i = 2:length(time)
19     if time(i) < brake_start_time

```

```

20         v_lead(i) = v_lead(i-1); % constant speed
21     else
22         v_lead(i) = max(0, v_lead(i-1) - brake_decel_ft * dt); % decelerate
23     end
24     x_lead(i) = x_lead(i-1) + v_lead(i-1) * dt;
25 end
26
27 % --- PID Controller Parameters ---
28 Kp = 0.5;
29 Kd = 0.8;
30 Ki = 0; % Not used
31 target_gap = 100; % Target gap (ft)
32
33 % --- Pack parameters for ODE45 ---
34 params.Kp = Kp;
35 params.Kd = Kd;
36 params.Ki = Ki;
37 params.target_gap = target_gap;
38 params.v_lead = v_lead;
39 params.x_lead = x_lead;
40 params.time = time;
41
42 % --- Initial Conditions ---
43 y0 = [0; 55 * 1.467; 0]; % Follower initially matches lead car (55 mph)
44
45 % --- Solve ODE ---
46 [t_ode, y_ode] = ode45(@(t, y) follower_ode(t, y, params), time, y0);
47
48 % --- Process Results ---
49 x_follower = y_ode(:,1);
50 v_follower = y_ode(:,2);
51
52 x_lead_interp = interp1(time, x_lead, t_ode, 'linear', 'extrap');
53 gap = x_lead_interp - x_follower;
54
55 accel = diff(v_follower) ./ diff(t_ode);
56 accel(end+1) = accel(end); % Pad to match vector size
57
58 % --- Plot Scenario 2 ---
59 figure;
60
61 subplot(3,1,1);
62 plot(t_ode, interp1(time, v_lead, t_ode), 'r--', t_ode, v_follower, 'b', ...
    'LineWidth', 1.2);
63 xlabel('Time (s)');
64 ylabel('Speed (ft/s)');
65 legend('Lead Car', 'Follower Car');
66 title('Speed Profile with Sudden Braking');
67
68 subplot(3,1,2);
69 plot(t_ode, gap, 'k', 'LineWidth', 1.2);
70 xlabel('Time (s)');
71 ylabel('Gap (ft)');
72 title('Gap Between Vehicles');
73
74 subplot(3,1,3);
75 plot(t_ode, accel, 'm', 'LineWidth', 1.2);

```

```

76 xlabel('Time (s)');
77 ylabel('Acceleration (ft/s^2)');
78 title('Follower Acceleration');
79
80 % --- ODE function for follower dynamics ---
81 function dydt = follower_ode(t, y, params)
82     x_f = y(1);
83     v_f = y(2);
84     int_err = y(3);
85
86     x_lead_t = interp1(params.time, params.x_lead, t, 'linear', 'extrap');
87     v_lead_t = interp1(params.time, params.v_lead, t, 'linear', 'extrap');
88
89     gap = x_lead_t - x_f;
90     error = gap - params.target_gap;
91     d_error = v_lead_t - v_f;
92
93     a = params.Kp * error + params.Kd * d_error + params.Ki * int_err;
94     a = max(min(a, 11.2), -17.6); % Apply acceleration limits
95
96     dydt = [v_f; a; error];
97 end

1 %% Scenario 3: Gradual Speed Variation
2
3 % --- Time settings ---
4 dt = 0.1;
5 T = 60;
6 time = 0:dt:T;
7
8 % --- Lead vehicle (gradual speed variation: sinusoidal profile) ---
9 v_base = 55 * 1.467; % Base speed: 55 mph in ft/s
10 amp = 10; % Amplitude of variation (ft/s)
11 freq = 0.05; % Frequency (Hz), or cycles/sec
12
13 v_lead = v_base + amp * sin(2*pi*freq*time); % Smooth sinusoidal speed profile
14 x_lead = cumtrapz(time, v_lead); % Integrate to get position
15
16 % --- PID Controller Parameters ---
17 Kp = 0.5;
18 Kd = 0.8;
19 Ki = 0;
20 target_gap = 100;
21
22 % --- Pack parameters for ODE45 ---
23 params.Kp = Kp;
24 params.Kd = Kd;
25 params.Ki = Ki;
26 params.target_gap = target_gap;
27 params.v_lead = v_lead;
28 params.x_lead = x_lead;
29 params.time = time;
30
31 % --- Initial Conditions ---
32 y0 = [0; 55 * 1.467; 0]; % Initial follower state [x, v, int_err]
33

```

```

34 % --- Solve ODE ---
35 [t_ode, y_ode] = ode45(@(t, y) follower_ode(t, y, params), time, y0);
36
37 % --- Post-processing ---
38 x_follower = y_ode(:,1);
39 v_follower = y_ode(:,2);
40 x_lead_interp = interp1(time, x_lead, t_ode, 'linear', 'extrap');
41 v_lead_interp = interp1(time, v_lead, t_ode, 'linear', 'extrap');
42 gap = x_lead_interp - x_follower;
43 accel = diff(v_follower) ./ diff(t_ode);
44 accel(end+1) = accel(end);
45
46 % --- Plot Scenario 3 ---
47 figure;
48
49 subplot(3,1,1);
50 plot(t_ode, v_lead_interp, 'r--', t_ode, v_follower, 'b', 'LineWidth', 1.2);
51 xlabel('Time (s)');
52 ylabel('Speed (ft/s)');
53 legend('Lead Car', 'Follower Car');
54 title('Speed Tracking with Gradual Variation');
55
56 subplot(3,1,2);
57 plot(t_ode, gap, 'k', 'LineWidth', 1.2);
58 xlabel('Time (s)');
59 ylabel('Gap (ft)');
60 title('Gap Between Vehicles');
61
62 subplot(3,1,3);
63 plot(t_ode, accel, 'm', 'LineWidth', 1.2);
64 xlabel('Time (s)');
65 ylabel('Acceleration (ft/s^2)');
66 title('Follower Acceleration');
67
68 % --- ODE function ---
69 function dydt = follower_ode(t, y, params)
70     x_f = y(1);
71     v_f = y(2);
72     int_err = y(3);
73
74     x_lead_t = interp1(params.time, params.x_lead, t, 'linear', 'extrap');
75     v_lead_t = interp1(params.time, params.v_lead, t, 'linear', 'extrap');
76
77     gap = x_lead_t - x_f;
78     error = gap - params.target_gap;
79     d_error = v_lead_t - v_f;
80
81     a = params.Kp * error + params.Kd * d_error + params.Ki * int_err;
82     a = max(min(a, 11.2), -17.6);
83
84     dydt = [v_f; a; error];
85 end

1 %% Time Step Sensitivity (Euler)
2
3 % --- Simulation settings ---

```

```

4 T = 60; % total simulation time (seconds)
5 dt_values = [0.5, 0.2, 0.1, 0.05]; % different time steps to test
6
7 % --- Lead car parameters (constant speed) ---
8 v_lead_const = 55 * 1.467; % 55 mph in ft/s
9 x_lead_init = 400; % Initial position
10
11 % --- Follower car initial conditions ---
12 v_follower_init = 45 * 1.467; % 45 mph in ft/s
13
14 % --- PID parameters ---
15 Kp = 0.5;
16 Kd = 0.8;
17 Ki = 0;
18 target_gap = 100; % ft
19
20 % --- Prepare figure ---
21 figure;
22 hold on;
23 colors = lines(length(dt_values)); % Automatically assign distinct colors
24
25 for j = 1:length(dt_values)
26     dt = dt_values(j);
27     time = 0:dt:T;
28
29     % Preallocate arrays
30     x_lead = zeros(size(time));
31     v_lead = v_lead_const * ones(size(time));
32     x_follower = zeros(size(time));
33     v_follower = zeros(size(time));
34     a_follower = zeros(size(time));
35     integral_error = 0;
36
37     % Initial conditions
38     x_lead(1) = x_lead_init;
39     x_follower(1) = 0;
40     v_follower(1) = v_follower_init;
41
42     for i = 1:length(time)-1
43         % Lead car (constant speed)
44         x_lead(i+1) = x_lead(i) + v_lead(i) * dt;
45
46         % Gap and error
47         gap = x_lead(i) - x_follower(i);
48         error = gap - target_gap;
49
50         if i == 1
51             d_error = 0;
52         else
53             d_error = (error - (x_lead(i-1) - x_follower(i-1) - target_gap)) / dt;
54         end
55         integral_error = integral_error + error * dt;
56
57         % PID control for acceleration
58         a = Kp * error + Kd * d_error + Ki * integral_error;
59         a = max(min(a, 11.2), -17.6); % acceleration bounds
60

```

```

61      % Euler integration for follower
62      v_follower(i+1) = v_follower(i) + a * dt;
63      x_follower(i+1) = x_follower(i) + v_follower(i) * dt;
64  end
65
66  % Plot gap vs. time for this dt
67  plot(time, x_lead - x_follower, 'LineWidth', 2, 'Color', colors(j,:), ...
68      'DisplayName', sprintf('\Delta t = %.2f s', dt));
69 end
70
71 xlabel('Time (s)', 'FontSize', 12);
72 ylabel('Gap (ft)', 'FontSize', 12);
73 title('Time Step Sensitivity Study (Euler Method)', 'FontSize', 14, ...
    'FontWeight', 'bold');
74 legend('Location','best', 'FontSize', 10);
75 grid on;
76 set(gca, 'FontSize', 11);
77 xlim([0 T]);
78 ylim([0 500]);
79 box on;
80 hold off;

```

```

1 %% PID Parameter Effects K_p
2
3 % --- Time settings ---
4 dt = 0.1;
5 T = 60;
6 time = 0:dt:T;
7
8 % --- Lead vehicle (constant speed) ---
9 v_lead_const = 55 * 1.467; % 55 mph in ft/s
10 v_lead = v_lead_const * ones(size(time));
11 x_lead = cumtrapz(time, v_lead); % Integrate to get position
12
13 % --- Follower car settings ---
14 Kd = 0.8; % Derivative gain (fixed)
15 Ki = 0; % Integral gain (fixed)
16 target_gap = 100; % Target gap (ft)
17
18 % --- Range of Kp values to study ---
19 Kp_values = [0.1, 0.5, 1.5]; % Low, medium, high
20
21 % --- Prepare plot ---
22 figure;
23 hold on;
24 colors = lines(length(Kp_values));
25 styles = {'-', '--', ':'};
26
27 for j = 1:length(Kp_values)
28     % Pack parameters for ODE
29     params.Kp = Kp_values(j);
30     params.Kd = Kd;
31     params.Ki = Ki;
32     params.target_gap = target_gap;
33     params.v_lead = v_lead;
34     params.x_lead = x_lead;

```

```

35     params.time = time;
36
37     % Initial conditions
38     y0 = [0; 45 * 1.467; 0]; % Start slower than lead car
39
40     % ODE45 simulation
41     [t_ode, y_ode] = ode45(@(t, y) follower_ode(t, y, params), time, y0);
42
43     % Process gap
44     x_follower = y_ode(:,1);
45     x_lead_interp = interp1(time, x_lead, t_ode, 'linear', 'extrap');
46     gap = x_lead_interp - x_follower;
47
48     % Plot
49     plot(t_ode, gap, 'LineWidth', 2, 'Color', colors(j,:), ...
50          'LineStyle', styles{j}, 'DisplayName', sprintf('K_p = %.1f', ...
51          Kp_values(j)));
52 end
53 xlabel('Time (s)', 'FontSize', 13);
54 ylabel('Gap (ft)', 'FontSize', 13);
55 title('Effect of proportional gain K_p on Gap Behavior', 'FontSize', 15, ...
      'FontWeight', 'bold');
56 legend('Location','best', 'FontSize', 11);
57 grid on;
58 grid minor;
59 set(gca, 'FontSize', 12);
60 xlim([0 T]);
61 ylim([0 400]);
62 box on;
63 hold off;
64
65 % --- ODE function ---
66 function dydt = follower_ode(t, y, params)
67     x_f = y(1);
68     v_f = y(2);
69     int_err = y(3);
70
71     x_lead_t = interp1(params.time, params.x_lead, t, 'linear', 'extrap');
72     v_lead_t = interp1(params.time, params.v_lead, t, 'linear', 'extrap');
73
74     gap = x_lead_t - x_f;
75     error = gap - params.target_gap;
76     d_error = v_lead_t - v_f;
77
78     a = params.Kp * error + params.Kd * d_error + params.Ki * int_err;
79     a = max(min(a, 11.2), -17.6);
80
81     dydt = [v_f; a; error];
82 end
83
84 %% PID Parameter Effects K_d
85
86 %% Time settings
87 dt = 0.1;
88 T = 60;

```

```

6 time = 0:dt:T;
7
8 % --- Lead vehicle (constant speed) ---
9 v_lead_const = 55 * 1.467; % 55 mph in ft/s
10 v_lead = v_lead_const * ones(size(time));
11 x_lead = cumtrapz(time, v_lead); % Integrate to get position
12
13 % --- Follower car settings ---
14 Kp = 0.5; % Proportional gain (fixed now)
15 Ki = 0; % Integral gain (fixed)
16 target_gap = 100; % Target gap (ft)
17
18 % --- Range of Kd values to study ---
19 Kd_values = [0.1, 0.8, 2.0]; % Low, medium, high
20
21 % --- Prepare plot ---
22 figure;
23 hold on;
24 colors = lines(length(Kd_values));
25 styles = {'-', '--', ':'};
26
27 for j = 1:length(Kd_values)
28     % Pack parameters for ODE
29     params.Kp = Kp;
30     params.Kd = Kd_values(j);
31     params.Ki = Ki;
32     params.target_gap = target_gap;
33     params.v_lead = v_lead;
34     params.x_lead = x_lead;
35     params.time = time;
36
37     % Initial conditions
38     y0 = [0; 45 * 1.467; 0]; % Start slower than lead car
39
40     % ODE45 simulation
41     [t_ode, y_ode] = ode45(@(t, y) follower_ode(t, y, params), [0 T], y0);
42
43     % Process gap
44     x_follower = y_ode(:,1);
45     x_lead_interp = interp1(time, x_lead, t_ode, 'linear', 'extrap');
46     gap = x_lead_interp - x_follower;
47
48     % Plot
49     plot(t_ode, gap, 'LineWidth', 2, 'Color', colors(j,:), ...
50          'LineStyle', styles{j}, 'DisplayName', sprintf('K_d = %.1f', ...
51          Kd_values(j)));
52 end
53 xlabel('Time (s)', 'FontSize', 13);
54 ylabel('Gap (ft)', 'FontSize', 13);
55 title('Effect of Derivative Gain K_d on Gap Behavior', 'FontSize', 15, ...
      'FontWeight', 'bold');
56 legend('Location','best', 'FontSize', 11);
57 grid on;
58 grid minor;
59 set(gca, 'FontSize', 12);
60 xlim([0 T]);

```

```
61 ylim([0 400]);
62 box on;
63 hold off;
64
65 % --- ODE function ---
66 function dydt = follower_ode(t, y, params)
67     x_f = y(1);
68     v_f = y(2);
69     int_err = y(3);
70
71     x_lead_t = interp1(params.time, params.x_lead, t, 'linear', 'extrap');
72     v_lead_t = interp1(params.time, params.v_lead, t, 'linear', 'extrap');
73
74     gap = x_lead_t - x_f;
75     error = gap - params.target_gap;
76     d_error = v_lead_t - v_f;
77
78     a = params.Kp * error + params.Kd * d_error + params.Ki * int_err;
79     a = max(min(a, 11.2), -17.6);
80
81     dydt = [v_f; a; error];
82 end
```