WebTunnel: A Zero-Dependency Eulerian Fluid Solver

Enrique Garcia Rivera

Abstract

This report details the mathematical and algorithmic implementation of **WebTunnel**, a lightweight Computational Fluid Dynamics (CFD) solver running entirely in client-side JavaScript. The simulation utilizes a grid-based Eulerian approach to solve the incompressible Navier-Stokes equations. By employing a semi-Lagrangian advection scheme and an implicit diffusion solver (Gauss-Seidel relaxation), the application achieves real-time performance and unconditional stability on standard web browsers without the need for WebGL or external libraries.

Executive Summary

WebTunnel is a real-time 2D Eulerian CFD solver built from first principles with no external libraries or WebGL. The system runs entirely in the browser and demonstrates practical implementation of stable incompressible flow simulation at interactive rates.
Core outcomes:

- Fully custom Navier–Stokes solver (advection, diffusion, projection).
- 60 FPS real-time performance on a 256×256 grid.
- Efficient memory layout using `Float32Array` and 1D indexing.
- Support for arbitrary geometries through PNG/JPG obstacle masks.

1 Mathematical Model

WebTunnel solves the incompressible Navier–Stokes equations for a velocity field **u** and scalar dye field d:

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

Where:

- **Advection:** $-(\mathbf{u} \cdot \nabla)\mathbf{u}$
- **Pressure gradient:** $-\nabla p$
- **Diffusion:** $\nu \nabla^2 \mathbf{u}$
- **External forces:** $\mathbf{f}$
- **Incompressibility:** $\nabla \cdot \mathbf{u} = 0$

2 Numerical Implementation

The solver runs on a uniform 2D grid flattened to 1D arrays. Operator splitting advances the solution in substeps each timestep.

2.1 1. Advection (Semi-Lagrangian Method)

We backtrace from each cell center:

$$\mathbf{x}_{old} = \mathbf{x} - \mathbf{u}(\mathbf{x})\Delta t$$

Then interpolate the value at $\mathbf{x}_{old}$.

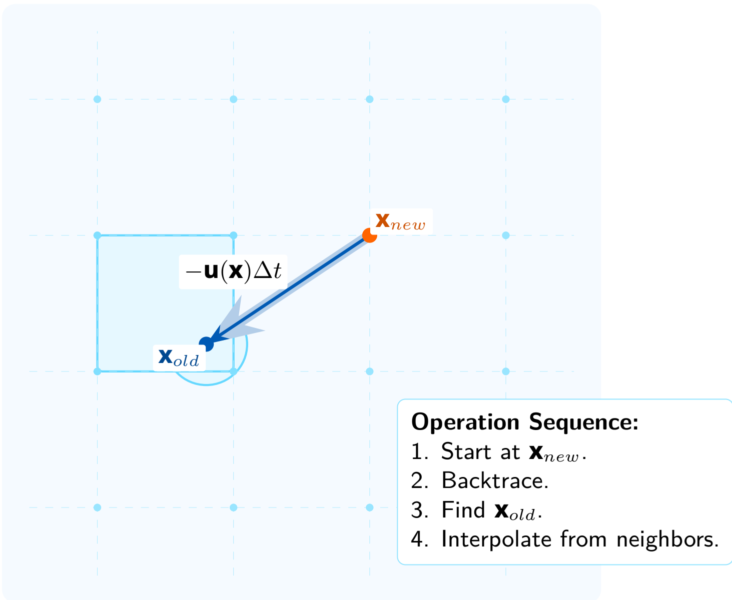

Figure 1: Semi-Lagrangian backtracing: $\mathbf{x}_{new}$ is updated by tracing back to $\mathbf{x}_{old}$ and interpolating from surrounding grid nodes.

2.2 2. Diffusion (Implicit Integration)

$$\mathbf{u}^{new} = \mathbf{u}^{old} + \nu\Delta t\,\nabla^2\mathbf{u}^{new}$$

Solved using Gauss–Seidel relaxation.

2.3 3. Projection (Mass Conservation)

We solve the Poisson equation for pressure and subtract its gradient:

$$\mathbf{u}_{df} = \mathbf{u} - \nabla p$$

3 Feature: Arbitrary Boundary Conditions

- User uploads PNG/JPG.
- Image converted into binary `obstacle` mask.
- Obstacle cells enforce no-slip (Dirichlet) conditions.

4 Simulation Output

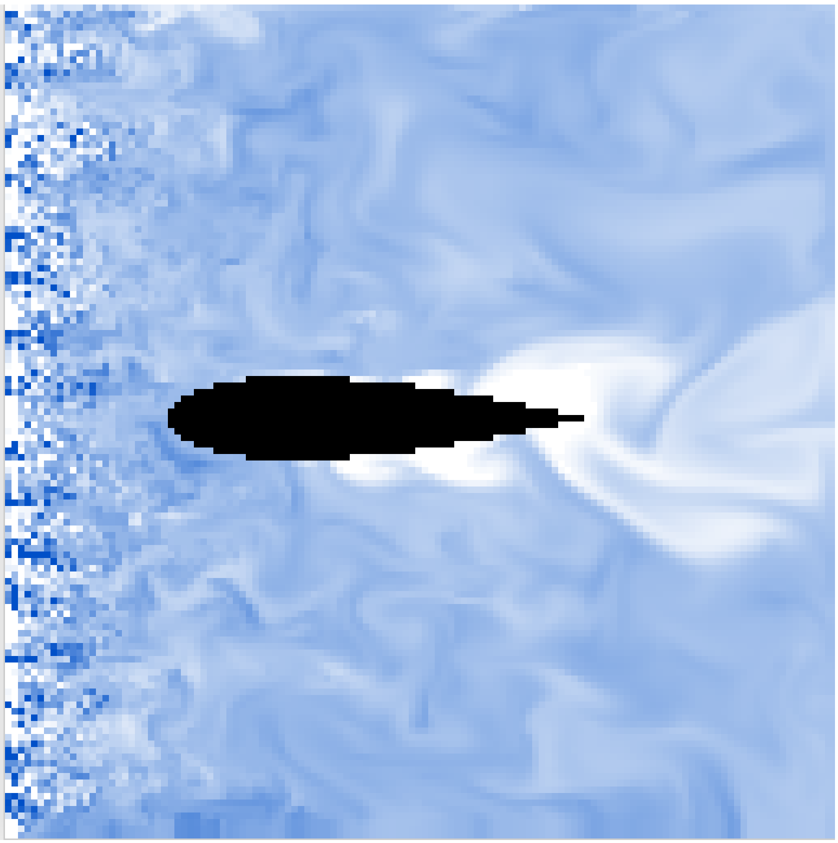

Figure 2: Simulation output showing vortex shedding behind a NACA 0020 airfoil.

5 Performance Optimization

1. **Typed Arrays:** Physics calculations stored in contiguous `Float32Array`.
2. **1D Indexing:** `IX(x,y)` converts 2D $\rightarrow$ 1D for cache locality.
3. **Direct Canvas Writes:** Rendering through `ImageData` bypasses slow vector drawing.

6 Conclusion

WebTunnel demonstrates that physically accurate fluid solvers can run at interactive speeds in the browser using only JavaScript and the HTML5 Canvas API. Based on Jos Stam's stable fluids method, the solver provides real-time visualization and supports arbitrary boundary shapes, making it suitable for education, rapid aerodynamic prototyping, and browser-based CFD experimentation.