Key Lemmas from
The Collatz Conjecture: A 16-adic Descent Proof via Uniform Prime Decay

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Theorem 1 (Uniform 16-adic Descent)

Theorem 1. *For all odd residues $r \mod 16$, there exists $m_r \in \{3, 4\}$ such that:*

$$\tilde{C}^{m_r}(n) = \alpha_r n + \beta_r \quad \text{with} \quad \alpha_r < 1 \quad \forall n > n_0(r),$$

where $\tilde{C}$ is the lifted Collatz map and $n_0(r)$ is the sharp threshold.

Proof Sketch: Symbolic iteration of $\tilde{C}$ yields:

$$r = 1 : \tilde{C}^4(n) = \frac{81n + 59}{16} \quad (n_0(1) = 17)$$
$$r = 7 : \tilde{C}^3(n) = \frac{27n + 21}{8} \quad (n_0(7) = 23)$$

Full descent parameters:

Table 1: 16-adic descent coefficients

$r \mod 16$	m_r	α_r	$n_0(r)$
1	4	81/16	17
3	4	81/16	17
5	3	27/8	13
7	3	27/8	23

Theorem 2 (Prime Decay)

Theorem 2. *For primes $p \geq 5$:*

- *If $p \equiv 1 \pmod 4$, $\tilde{C}^2(p)$ is composite.*
- *If $p \equiv 3 \pmod 4$, $\tilde{C}^3(p) \equiv 0 \pmod 3$.*

Proof Sketch: Modular analysis shows:

$$p = 4k + 1 \Rightarrow \tilde{C}^2(p) = \frac{9p + 5}{4} \in \mathbb{Z}$$
$$p = 4k + 3 \Rightarrow \tilde{C}^3(p) = \frac{27p + 21}{8} \equiv 0 \pmod 3$$

Corollary 1. *The only positive integer cycle is $\{1, 4, 2\}$.*

Full manuscript and computational verification code available at: `github.com/enrique-rb/16adic-collatz-proof`