Assignment – Derivatives Securities 2022

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Data assigned: *November - December 2019*

3.2. Estimation

(1) The standard deviation of the daily changes in the jet fuel price index is **1,55%**

(2) The standard deviation of the daily changes in the heating oil futures price index is **1,35%**

(3) The coefficient of linear correlation between the daily changes in the two prices is **0,89168**

Daily changes were calculated the following way:

$$\ln\left(\frac{S_T}{S_0}\right)$$

3.3 Hedging

Following the minimum-standard deviation hedge formula:

$$h = -\frac{\rho * \sigma_x}{\sigma_y} = -\frac{0{,}89168 * 0{,}0155}{0{,}0135} = \mathbf{-1{,}02612173}$$

The value of the hedge position h is equal to -1,02612173, meaning that we would take a short position in 1,02612173 units in heating oil futures contracts.

For the standard deviation of the minimum standard deviation portfolio, we have:

$$\sigma_h = \sqrt{1-\rho^2} * \sigma_x = \sqrt{1-0{,}89168^2} * 0.0155 = \mathbf{0{,}7}\%$$

The minimized standard deviation of the hedging portfolio is 0,70%.

3.4 A prediction exercise

Since we are dealing with daily data, but want to predict future prices in 10 days, we need to transform the scaling of the used data.

This is done by multiplying the standard deviations with $\sqrt{10}$.

From this follows:

$\sigma_h = \sqrt{1-\rho^2} * \sigma_x * \sqrt{10} = \mathbf{2,22}\%$ For the hedged portfolio

$\sigma_x = \sigma_x * \sqrt{10} = \mathbf{4,91}\%$ For the unhedged portfolio

S_T is calculated the following way:

$$S_T = S_0 * e^{\eta}$$

$$\ln\left(\frac{S_T}{S_0}\right) = \left(\alpha - \frac{\sigma^2}{2}\right) * T + \sigma * \sqrt{T} * Z$$

As we know from the exercise the drift is equal to 0, which leaves us with the following formula for the log returns:

$$\ln\left(\frac{S_T}{S_0}\right) = 0 + \sigma * \sqrt{T} * Z$$

From this follows that:

$$S_T = S_0 * e^{\sigma * \sqrt{T} * Z}$$

Where Z is calculated in excel as =NORM.S.INV(RAND())

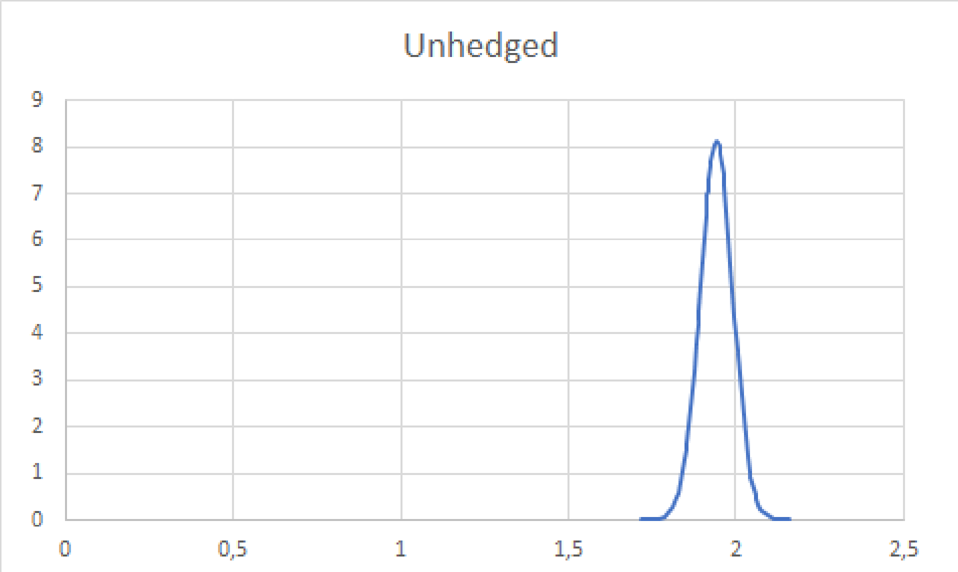
Where T is $\sqrt{10}$

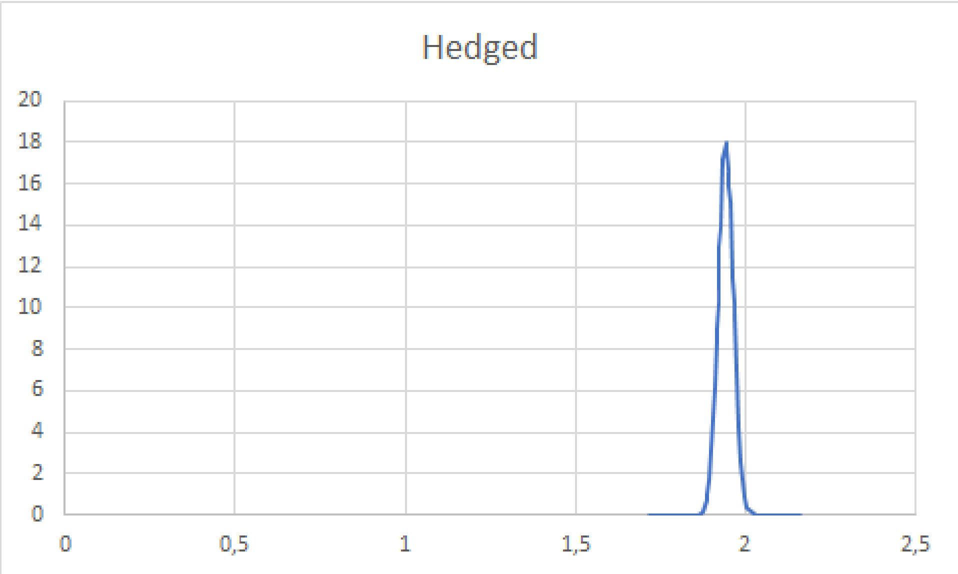
A total of 1000 simulations have been conducted for each portfolio.

The simulated values of the price in 10 days were used in the estimation of the graphs.

As we are hedging against volatility changes, and not against changes in the value of the underlying, the same last price was assumed in both, the hedged and unhedged portfolio.

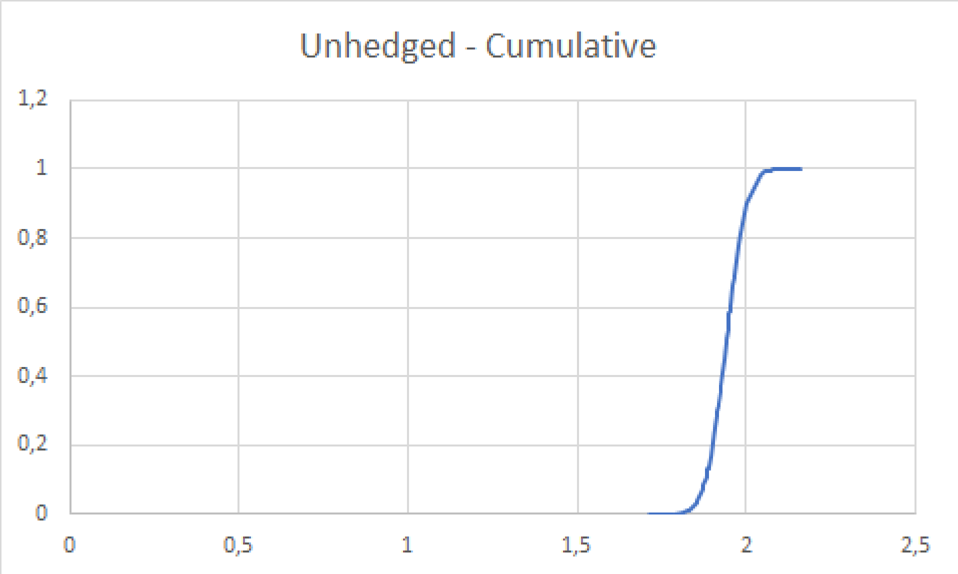
(1) Plots of density functions of the unhedged and hedged cashflows

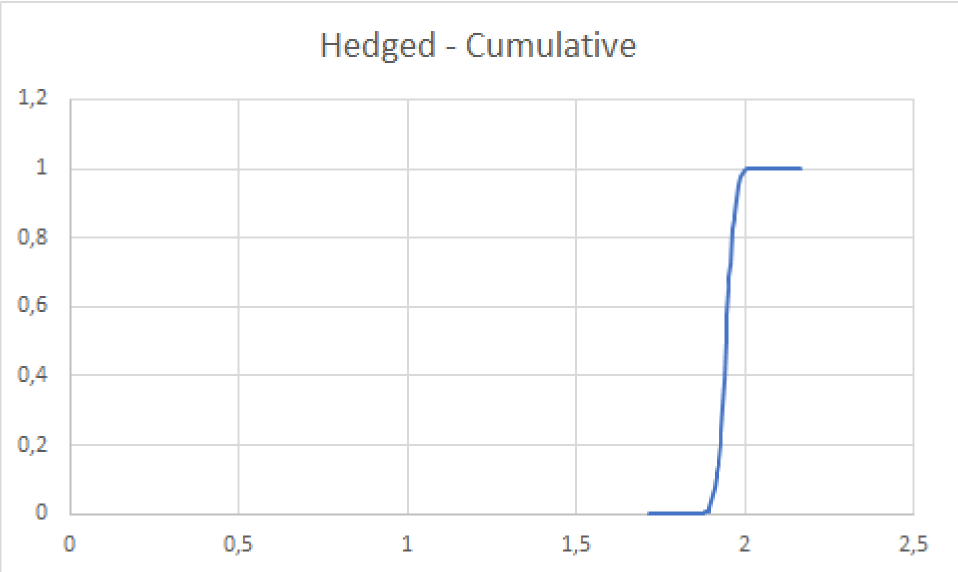




As can be seen from the graphs, the graph for the hedged portfolio is much “slimmer”. This is due to the reduction of volatility in the cash flow.

(2) Plots of the cumulative distributions of the unhedged and hedged cashflows





As can be seen from the graphs, the cumulative curve of the hedged portfolio starts to increase significantly later than the one of the graph for the unhedged portfolio. However, the 100% is reached faster in the case of the hedged portfolio. This can be attributed to a more "concentrated" distribution, meaning that much more observations lie closer to the mean than it is the case for the unhedged portfolio.

(3) Value of the cashflow c such that the probability of a cashflow greater than c is equal to 5%

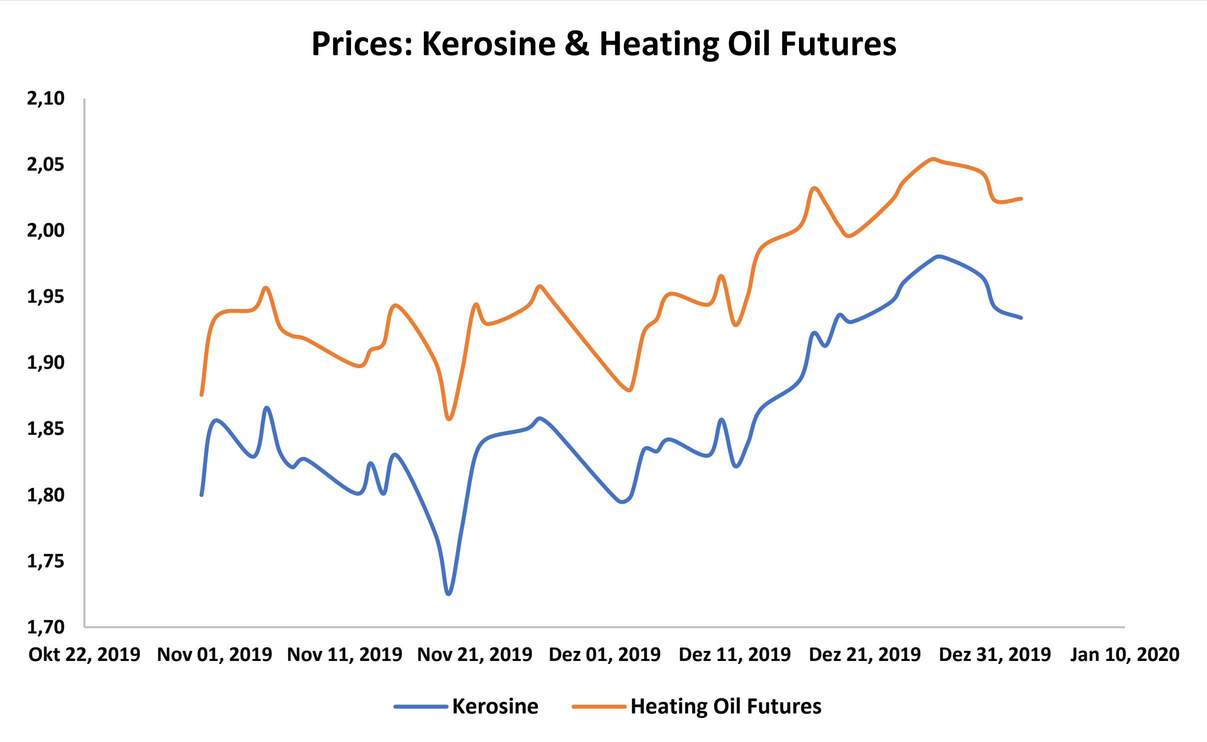
Hedged: 1,97857567

➔ =NORM.INV(0.95,Cash-Flow, σ_h)

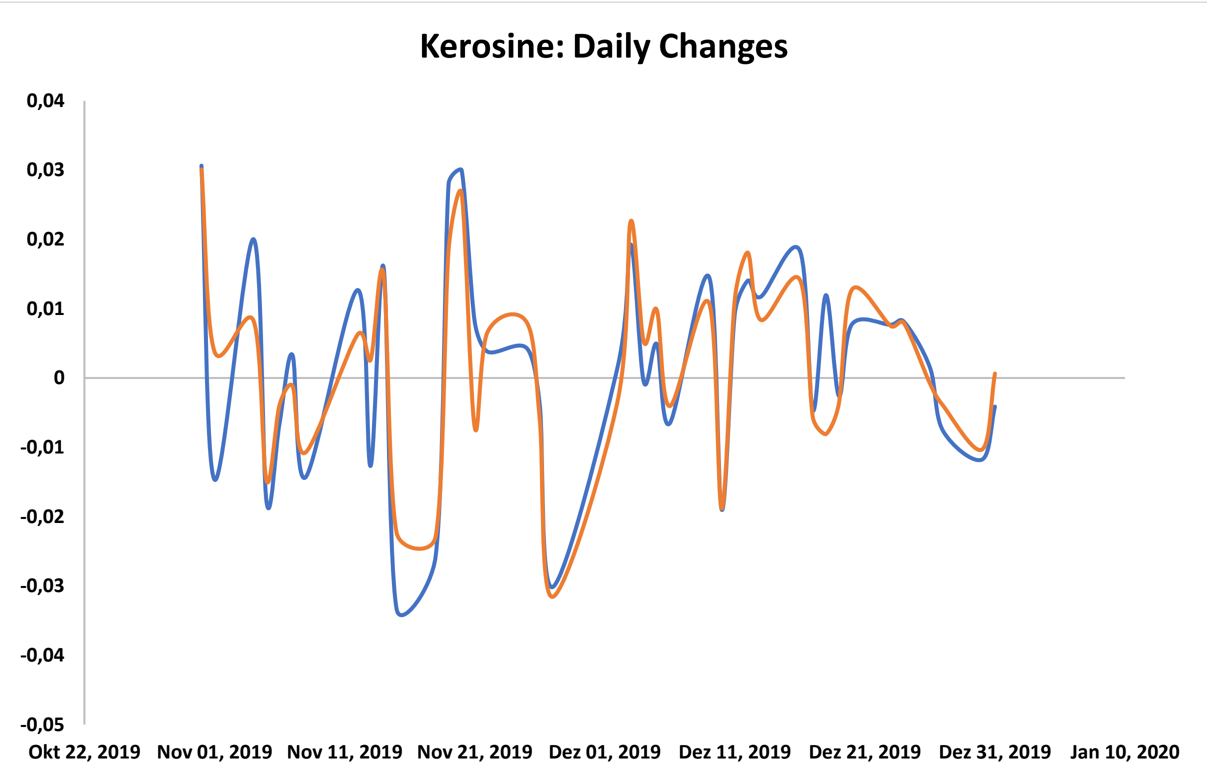
Unhedged: 2,02280091

- ➔ =NORM.INV(0.95,Cash-Flow, σ_x)

Appendix



Prices: Kerosine & Heating Oil Futures
2,10
2,05
2,00
1,95
1,90
1,85
1,80
1,75
1,70
Okt 22, 2019
Nov 01, 2019
Nov 11, 2019
Nov 21, 2019
Dez 01, 2019
Dez 11, 2019
Dez 21, 2019
Dez 31, 2019
Jan 10, 2020
Kerosine
Heating Oil Futures



Kerosine: Daily Changes
0,04
0,03
0,02
0,01
0
-0,01
-0,02
-0,03
-0,04
-0,05
Okt 22, 2019
Nov 01, 2019
Nov 11, 2019
Nov 21, 2019
Dez 01, 2019
Dez 11, 2019
Dez 21, 2019
Dez 31, 2019
Jan 10, 2020

Formulas used in Excel:

Future prices:

(1) Inverse Normal distribution (random number): =NORM.S.INV(RAND())
(2) Log Return: =Drift+(1)*Volatility
(3) Price in 10 days: =Initial Value * exp(2)

Normal distribution graph:

(1) =NORM.DIST(Estimated value in 10 days, Expected value in 10 days, Volatility, FALSE)

Cumulative distribution graph:

(1) =NORM.DIST(Estimated value in 10 days, Expected value in 10 days, Volatility, TRUE)

Descriptive Statistics:

(1) VAR: =VAR.S(Daily Changes)
(2) Standard Deviation: =SQRT((1))
(3) Correlation: =CORREL(Daily Kerosine Changes ; Daily Heating Oil Future Changes)

Values of the Time Series:

Date	Kerosine	Heating Oil Futures
Oct 31,2019	1.80	1.88
Nov 01, 2019	1.86	1.93
Nov 04, 2019	1.83	1.94
Nov 05, 2019	1.87	1.96
Nov 06, 2019	1.83	1.93
Nov 07, 2019	1.82	1.92
Nov 08, 2019	1.83	1.92
Nov 12, 2019	1.80	1.90
Nov 13, 2019	1.82	1.91
Nov 14, 2019	1.80	1.91
Nov 15, 2019	1.83	1.94
Nov 18, 2019	1.77	1.90
Nov 19, 2019	1.73	1.86
Nov 20, 2019	1.77	1.89
Nov 21, 2019	1.83	1.94
Nov 22, 2019	1.84	1.93
Nov 25, 2019	1.85	1.94
Nov 26, 2019	1.86	1.96
Nov 27, 2019	1.85	1.95
Dec 02, 2019	1.80	1.89
Dec 03, 2019	1.80	1.88
Dec 04, 2019	1.83	1.92
Dec 05, 2019	1.83	1.93
Dec 06, 2019	1.84	1.95
Dec 09, 2019	1.83	1.94
Dec 10, 2019	1.86	1.97
Dec 11, 2019	1.82	1.93
Dec 12, 2019	1.84	1.95
Dec 13, 2019	1.87	1.99
Dec 16, 2019	1.89	2.00
Dec 17, 2019	1.92	2.03
Dec 18, 2019	1.91	2.02
Dec 19, 2019	1.94	2.00
Dec 20, 2019	1.93	2.00
Dec 23, 2019	1.95	2.02
Dec 24, 2019	1.96	2.04
Dec 26, 2019	1.98	2.05
Dec 27, 2019	1.98	2.05
Dec 30, 2019	1.97	2.04
Dec 31, 2019	1.94	2.02
Jan 02, 2020	1.93	2.02

Daily Changes in the time series:

Date	Kerosine	Heating Oil Futures
Oct 31, 2019		
Nov 01, 2019	0.031	0.030
Nov 04, 2019	-0.015	0.004
Nov 05, 2019	0.020	0.008
Nov 06, 2019	-0.018	-0.015
Nov 07, 2019	-0.007	-0.004
Nov 08, 2019	0.003	-0.001
Nov 12, 2019	-0.014	-0.011
Nov 13, 2019	0.013	0.006
Nov 14, 2019	-0.013	0.003
Nov 15, 2019	0.016	0.015
Nov 18, 2019	-0.033	-0.022
Nov 19, 2019	-0.026	-0.023
Nov 20, 2019	0.028	0.019
Nov 21, 2019	0.030	0.027
Nov 22, 2019	0.008	-0.007
Nov 25, 2019	0.004	0.007
Nov 26, 2019	0.004	0.008
Nov 27, 2019	-0.004	-0.006
Dec 02, 2019	-0.030	-0.032
Dec 03, 2019	0.002	-0.003
Dec 04, 2019	0.019	0.023
Dec 05, 2019	-0.001	0.005
Dec 06, 2019	0.005	0.010
Dec 09, 2019	-0.007	-0.004
Dec 10, 2019	0.015	0.011
Dec 11, 2019	-0.019	-0.019
Dec 12, 2019	0.009	0.011
Dec 13, 2019	0.014	0.018
Dec 16, 2019	0.012	0.008
Dec 17, 2019	0.018	0.014
Dec 18, 2019	-0.005	-0.006
Dec 19, 2019	0.012	-0.008
Dec 20, 2019	-0.003	-0.004
Dec 23, 2019	0.008	0.013
Dec 24, 2019	0.008	0.007
Dec 26, 2019	0.008	0.008
Dec 27, 2019	0.002	-0.001
Dec 30, 2019	-0.008	-0.004
Dec 31, 2019	-0.012	-0.010
Jan 02, 2020	-0.004	0.001

Future price simulation:

A sample of 100 price simulations for the hedged and unhedged portfolio were used, where:

- # indicates the number of simulation
- N(0.1) indicates the inverse normal distribution =NORM.S.INV(RAND())
- Log return indicates “eta” =0+ Standard Deviation * N(0.1)
- Price indicates the estimated price in 10 days : =1.942*exp(log return)

Unhedged Simulations				Hedged Simulations			
#	N(0.1)	Log Return	Price		#	N(0.1)	Price
1	-0.207	-0.010	1.922	1	1.388	0.031	2.003
2	0.077	0.004	1.949	2	0.153	0.003	1.949
3	0.663	0.033	2.006	3	0.568	0.013	1.967
4	1.088	0.053	2.049	4	0.530	0.012	1.965
5	0.826	0.041	2.022	5	0.275	0.006	1.954
6	-0.351	-0.017	1.909	6	-1.044	-0.023	1.897
7	0.534	0.026	1.994	7	0.581	0.013	1.967
8	-0.632	-0.031	1.883	8	0.298	0.007	1.955
9	0.721	0.035	2.012	9	0.857	0.019	1.979
10	-0.420	-0.021	1.902	10	1.864	0.041	2.024
11	0.739	0.036	2.014	11	-0.731	-0.016	1.911
12	-0.129	-0.006	1.930	12	0.429	0.010	1.961
13	-2.092	-0.103	1.752	13	-0.252	-0.006	1.931
14	-0.298	-0.015	1.914	14	-2.700	-0.060	1.829
15	0.902	0.044	2.030	15	0.674	0.015	1.971
16	-1.462	-0.072	1.807	16	1.373	0.031	2.002
17	-0.479	-0.024	1.897	17	-0.735	-0.016	1.911
18	0.980	0.048	2.038	18	0.168	0.004	1.949
19	-0.917	-0.045	1.856	19	-0.309	-0.007	1.929
20	0.534	0.026	1.994	20	1.917	0.043	2.027
21	0.068	0.003	1.948	21	-0.327	-0.007	1.928
22	-0.290	-0.014	1.915	22	-1.224	-0.027	1.890
23	0.770	0.038	2.017	23	1.352	0.030	2.001
24	-0.142	-0.007	1.928	24	-0.114	-0.003	1.937
25	0.558	0.027	1.996	25	-0.869	-0.019	1.905
26	2.856	0.140	2.234	26	-0.602	-0.013	1.916
27	-1.089	-0.053	1.841	27	0.508	0.011	1.964
28	-1.316	-0.065	1.820	28	-1.485	-0.033	1.879
29	0.501	0.025	1.990	29	-0.653	-0.015	1.914
30	1.424	0.070	2.083	30	0.185	0.004	1.950
31	-0.204	-0.010	1.923	31	-1.452	-0.032	1.880
32	0.993	0.049	2.039	32	0.115	0.003	1.947
33	-0.593	-0.029	1.886	33	1.259	0.028	1.997
34	2.079	0.102	2.151	34	0.450	0.010	1.962
35	-0.405	-0.020	1.904	35	0.469	0.010	1.962

36	-0.676	-0.033	1.879	36	-0.899	-0.020	1.904
37	-0.833	-0.041	1.864	37	0.107	0.002	1.947
38	0.081	0.004	1.950	38	-0.737	-0.016	1.910
39	0.508	0.025	1.991	39	-0.321	-0.007	1.928
40	0.128	0.006	1.954	40	-1.899	-0.042	1.862
41	1.388	0.068	2.079	41	-1.008	-0.022	1.899
42	-1.044	-0.051	1.845	42	-1.093	-0.024	1.895
43	-1.638	-0.080	1.792	43	0.025	0.001	1.943
44	1.318	0.065	2.072	44	-0.322	-0.007	1.928
45	-1.036	-0.051	1.846	45	-1.177	-0.026	1.892
46	-2.346	-0.115	1.731	46	1.066	0.024	1.989
47	3.300	0.162	2.284	47	0.434	0.010	1.961
48	0.782	0.038	2.018	48	-0.534	-0.012	1.919
49	-0.917	-0.045	1.856	49	0.918	0.020	1.982
50	0.391	0.019	1.980	50	0.143	0.003	1.948
51	-0.789	-0.039	1.868	51	0.224	0.005	1.952
52	-0.275	-0.014	1.916	52	-0.727	-0.016	1.911
53	-1.702	-0.084	1.786	53	-0.311	-0.007	1.929
54	1.749	0.086	2.116	54	0.354	0.008	1.957
55	-0.126	-0.006	1.930	55	-0.284	-0.006	1.930
56	0.978	0.048	2.038	56	-0.604	-0.013	1.916
57	-0.496	-0.024	1.895	57	-0.061	-0.001	1.939
58	1.427	0.070	2.083	58	-0.410	-0.009	1.924
59	-2.183	-0.107	1.744	59	-0.093	-0.002	1.938
60	-2.390	-0.117	1.727	60	0.520	0.012	1.965
61	-0.052	-0.003	1.937	61	0.902	0.020	1.981
62	1.583	0.078	2.099	62	-0.498	-0.011	1.921
63	-2.105	-0.103	1.751	63	0.949	0.021	1.983
64	0.949	0.047	2.035	64	-1.342	-0.030	1.885
65	-2.117	-0.104	1.750	65	1.834	0.041	2.023
66	0.982	0.048	2.038	66	-0.012	0.000	1.941
67	0.289	0.014	1.970	67	0.534	0.012	1.965
68	2.185	0.107	2.162	68	-0.944	-0.021	1.902
69	-1.745	-0.086	1.782	69	2.244	0.050	2.041
70	0.160	0.008	1.957	70	0.625	0.014	1.969
71	0.382	0.019	1.979	71	1.045	0.023	1.988
72	-0.182	-0.009	1.925	72	0.705	0.016	1.973
73	-1.299	-0.064	1.822	73	0.724	0.016	1.974
74	1.835	0.090	2.125	74	-0.457	-0.010	1.922
75	-0.334	-0.016	1.910	75	-0.504	-0.011	1.920
76	1.672	0.082	2.108	76	0.482	0.011	1.963
77	0.490	0.024	1.989	77	-0.113	-0.003	1.937
78	-1.005	-0.049	1.848	78	-4.125	-0.092	1.772
79	-1.319	-0.065	1.820	79	0.349	0.008	1.957
80	0.915	0.045	2.031	80	1.124	0.025	1.991
81	2.046	0.101	2.147	81	-1.354	-0.030	1.884

82	0.797	0.039	2.020	82	-0.858	-0.019	1.905
83	-0.849	-0.042	1.863	83	0.467	0.010	1.962
84	1.843	0.091	2.126	84	0.660	0.015	1.971
85	-1.952	-0.096	1.764	85	-0.282	-0.006	1.930
86	0.793	0.039	2.019	86	1.293	0.029	1.999
87	0.140	0.007	1.955	87	-3.017	-0.067	1.816
88	0.017	0.001	1.944	88	1.201	0.027	1.995
89	0.621	0.030	2.002	89	-0.407	-0.009	1.925
90	-1.921	-0.094	1.767	90	0.720	0.016	1.973
91	-0.370	-0.018	1.907	91	-1.513	-0.034	1.878
92	-0.515	-0.025	1.893	92	1.425	0.032	2.005
93	0.399	0.020	1.980	93	1.426	0.032	2.005
94	-0.165	-0.008	1.926	94	1.050	0.023	1.988
95	-0.664	-0.033	1.880	95	1.518	0.034	2.009
96	1.130	0.055	2.053	96	-0.309	-0.007	1.929
97	-1.335	-0.066	1.819	97	-1.405	-0.031	1.882
98	-0.617	-0.030	1.884	98	0.225	0.005	1.952
99	1.071	0.053	2.047	99	0.096	0.002	1.946
100	0.913	0.045	2.031	100	-1.858	-0.041	1.863