JÖNKÖPING UNIVERSITY

Försättsblad till tentamen
Cover sheet for Examination.

15961681

Anonymkod *Anonymous code*	Kurskod *Course code*	Provkod *Test code*
XAM-RUE	FSSS23	1301

Kursnamn *Course name*

Analytical Methods for Economic and Financial Analysis

Tentamensdatum *Examination date* (År / Year – Månad / Month – Dag / Day)	Antal lösblad *No. of loose sheets*	Kontrollräkning gjord av inlämnade lösblad *Control count of given loose sheets made*	Signatur/ *Signature*
2022-03-23	14	☐	

Fylls i av TENTAMENSVAKT/ ***To be filled in by the INVIGILATOR***

		Härmed intygas att kontroller utförts/ *The checks have been carried out*
Kontroll av legitimation/ *Control of Identification*	☑	Signatur/ *Signature*
Kontroll av inlämnade lösblad/ *Control of given loose sheets*	☑	
Inlämningstid/ *Time submitted*	17:00	
Antal lösblad/ *No. of loose sheets*	14	

Endast Högskolans anteckningar/ ***For official use only***

Totalt antal poäng/ *Total Exam Score*	83	Betyg på tentamen/ *Exam Grade*	B

Kommentarer (lärare)/ *Comments (teacher)*

Signatur (lärare)/ *Signature (teacher)*

15961681

XAM-RUE

JÖNKÖPING UNIVERSITY

International Business School

Written examination: Analytical Methods for Economic and Financial Analysis (FSSS23)

Date and time: **Wednesday 23rd of March – 14:00-19:00**

Total number of pages: [18] Check that you have got them all!

- General exam rules at Jönköping University apply.
- This examination consists of [10] tasks.
- The Pass mark is 60 % of the total.
- The following aids are permitted:

- Pens, pencils, rulers, and a calculator (among those noted below) are permitted
- Allowed calculators at the exam are calculators provided by JIBS. Personal calculators can be used if they are of the type: Casio FX 82ES, Casio FX-82 Solar, Texas Instruments TI 30 or Sharp EL-531WH
- Otherwise, calculators will be provided by the invigilators (tentavakter).
- The formula sheet and the distribution tables are included in the exam.

Exam ID: XAM-RUE

Please don't write down your name or personal identity number.

Total score: ________ Result (Grade): ________

Ifylls av tentamensvakt!

ANTAL INLÄMNADE LÖSA SIDOR: 14 SIGNATUR: ________

COMPLETED TASKS:

Task/Question	1	2	3	4	5	6	7	8	9	10				Total
Maximum points	7	13	12	6	10	12	11	5	12	12				100
Total score	5	11	12	3	10	12	11	5	9	5				83

Name of the responsible teacher: Pär Sjölander

Responsible teacher will visit the examination: Unfortunately, not physically due to a business trip.

Responsible teacher can be reached on the phone during the following times: 14:00-18:00 (however, during a part of this time I will be driving in my car from Jönköping to Stockholm. Despite hands-free I may have to stop my car and call back within appr. 15 minutes).

For urgent questions during the exam - the responsible teacher can be reached on the following phone numbers: 0739-101968 (Pär Sjölander, for Q1-Q8), or for Kristofer Månsson's part (Q9-Q10) 0761 – 66 93 91.

JÖNKÖPING UNIVERSITY

Exam regulations for students at Jönköping University

The 14th of October 2020

These instructions are based on President's decision § 755, 2018, "Regulations and guidelines for first-, second-, and third-cycle education at Jönköping University". Regulations on disruptive behaviour and cheating are found in the policy documents of the Disciplinary and Expulsion Committee. To guarantee the student's legal rights, Sweden's legislation on discrimination must be observed.

The invigilator's role is to guarantee that the examination takes place in an ordered and legally securemanner. The invigilator's instructions must be followed.

Cheating or disruptive behaviour during an exam are disciplinary offences that will be reported to the Disciplinary and Expulsion Committee. A disciplinary offence may lead to short- or long-term suspension from the university.

The exams are scheduled on the basis of set start times. It is the same times for all days, starting at 9 a.m. and 2 p.m.

Preparations

- Register for each exam no later than ten days beforehand. If you fail to register or register late, you will not be allowed to write the exam in question.
- Since hypersensitivity/allergy is relatively common, you are not allowed to bring food/snacks that contain nuts/peanuts or to wear perfume.
- Be sure you know the correct time and place.
- Be sure you know what aids are permitted. Ensure that your aids, if any, are "clean", with no forbidden notes or loose pages. Tabs and bookmarks without any text or marking other than chapter headings or equivalent are permitted.
- Bring a valid photo ID, e.g. your driving licence or passport. Without such ID, you will not be allowed to write the exam. The JU card, if marked "Identity Card" and showing your full civic registration number, may be used as an ID document at exams.
- You may bring refreshments.
- Prior to a digital exam, it is always each student's responsibility to ensure that his/her JU user account will be active at the time of the exam. This is also necessary if you need to borrow a computer. If there are any problems, please contact IT Helpdesk.

During admission

- Arrive at the latest 20 minutes before the exam starts. The door is locked at exactly the specified time.
- Before entering, tick off your name on the registration list at the entrance. If you are not on the list, you will not be allowed to write the exam.
- Those who arrive for the second admission, 30 minutes after the start, must be present outside the door so that the invigilator can verify their identity.
- Anyone arriving more than 30 minutes late will not be allowed to sit the exam. No excuses are accepted.
- Leave any outerwear and bags in their designated places.

- Seat yourself in the indicated place. Only permitted aids, ID and refreshments are allowed at the desk.
- All electronic equipment (mobile phones, smart watches, MP3 players, etc.) are to be switched off and kept with the outerwear and bags. Do not bring anything to the exam that you do not wish to leave unsupervised. Any sound coming from a mobile phone during an examination will be reported as both disruptive behaviour and attempted cheating.
    - If you consider that you have legitimate reasons to have your mobile switched on during the exam, notify the invigilator of this before the exam begins. Only exceptional reasons are accepted. The switched-on mobile (silent ringtone) is to be kept by the invigilator. If you accept a call, you must immediately stop the examination and hand in your paper.

Start

- When the invigilator locks the door and announces the start of the exam, you must immediately sit down and stay silent.
- Check that you receive the correct exam paper from the invigilator and that the paper is complete. In Inspera check that you see the correct exam.
- If you are registered to write two exams, you receive both papers at the beginning of the exam session. However, the individual finish times must be respected. In Inspera both exams will be visible.

During the examination

- No student may leave the exam room during the first half an hour.
- There must be no communication whatsoever between the students.
    - Any communication between the candidates must go through an invigilator.
- There must be no disruptive behaviour. If you feel that you are being disrupted, please inform the invigilator.
- When the invigilator is performing the ID check, have your ID readily to hand.
    - If you do not have an ID that the invigilator can accept, you will be turned away from the exam.
    - If you are not on the registration list, you will be turned away from the exam.
    - When the invigilator comes to check your ID, your name must have been entered on the first page of the exam paper.
- The invigilator may, at any time and without special reason, check what is on your desk. The invigilator may also leaf through permitted books to check that they do not contain forbidden notes and look inside pencil cases, sweet bags and the like.
- The only writing papers that are allowed are the ones with a colored corner, provided by the invigilators during the exam.
- If you visit the toilet, both name and time must be noted on the toilet list. Only one student may visit the toilet at any one time.
- If you leave the room for any reason other than visiting the toilet, you are considered to have stopped the exam and may not continue writing.

End

- The invigilator lets the students know when 30 and 10 minutes of writing time remains.
- When the invigilator announces that the time is up, you must stop writing immediately.
- Ensure that you have written your name and civic registration number on each piece of paper that you hand in. In case you have not done this when the time is up, you must continue filling in your name and civic registration number in the presence of an invigilator.
- When you hand in your paper, you must show your ID.
- Even if no questions have been answered, the pre-personalised page must be handed in.
- The number of submitted loose pages are counted by the invigilator and noted on the pre-personalised page.
- Check that the invigilator ticks off your name correctly and notes the correct number of submitted pages.
- Unless otherwise specified, you may take the exam paper with you once you have handed in your answers. You are not allowed to take the writing papers with a colored corner with you from the exam venue.
- In a digital exam, you and the invigilator are to jointly note the time of submission on the attendance list.

Special educational support

- If you have been granted special educational support owing to disability and wish to have an alternative exam arrangement, register this with the examination coordinator in the case management system no later than ten days beforehand and, for information, with the responsible teacher. You must also register for the exam as usual.
- A student with special educational support who writes a paper exam, but is entitled to use a computer, must write the entire exam either on paper or on computer.

XAM - RUF

* **IMPORTANT!!! PLEASE READ THESE INSTRUCTIONS!!!** In the following questions, you are requested at various times to describe tests methods, estimation methods, etc. In your answers, you should use both text and mathematical forms whenever appropriate. Motivate your answer to receive full score. However, more text in your answer does not necessarily mean that you will receive more points (unless it is necessary and relevant information). For some types of exam questions, some formulas and statistical tables are provided at the back of the test for you to use when you consider appropriate. If for instance the t-distribution tables or F-distribution tables are used in the exam, and the exact value is not presented, use the closest value available (if nothing else is stated). However, note that these standard statistical tables in the end of the exam are not used for all courses/exams. Some data sets may be based on fictitious data; therefore, you should sometimes disregard if some values do not make economic sense. If you are asked to mention 3 items (e.g. tests for autocorrelation), only the first 3 you provide will be graded (and the rest will be disregarded). **Present the answers to the questions in numerical chronological order, and clearly state which question you are answering** e.g. Q1a, Q1b, Q1c, etc., otherwise points may be deducted (unless you have very good reasons). Write your name and personal number on each page. Please do not use a red pen when answering the questions, since this is the colour used for grading. Avoid using the left upper corner of your answering sheets since this text will be hidden by the staples. If you suspect that there is a mistake or a typo in the exam, and if the teacher is not available to answer questions, just make a note about this and try to answer the question to the best of your knowledge regardless of this. You can probably understand the meaning or point of the question anyway, and if there is a problem you will be given a generous score when the exam is graded**. Do not write answers on both sides of the answer sheets.** If we cannot read what you write, we cannot award points for the answer. Write all your answers on the (blank) answer sheets – **do not write answers directly in the exam. If nothing else is stated in the question, use the 5% level of significance for the tests in the exam.**

Question 1 (7 points)

With respect to the statements below, say whether the statement is True (**T**) or False (**F**). No explanation is required. If there is at least one part which is incorrect in a statement, you state that it is **F**. A correct answer yields 1 point, an incorrect answer yields -1 point, while no answer (blank) yield 0 points. *[You can never obtain less than zero points in total for these questions even if you give the incorrect answer every time. As always, please answer on the answer sheet, not directly here in the exam].*

(a) A REM cannot model (only control for) time-invariant variables (such as gender or race).

(b) The OLS estimator is biased if the dependent variable is censored. Then we would get biased results.

(c) If we reduce the statistical size, there is less risk of making a type1-error, but higher chance to reject the null hypothesis if it is false. (Power increases)

(d) In a spurious regression model we are too frequently making a type 1 error.

(e) The statistical power of a test is the probability of not rejecting a true null hypothesis.

(f) Breusch–Godfrey's test is an autocorrelation test.

(g) If we run $y_t = c + \beta_1 x_t + u_t$ where y_t~I(1) and x_t~I(1) and if the residual is u_t~I(1), then we can expect a spurious relationship between x_t and y_t.

XAM - RUF

Question 2 (13 points)

Explain, and make a comparison of, a random-effects model (REM) and a fixed-effects model (FEM). That is, define and explain one REM and one FEM, and explain under what circumstances these REM and FEM are used. Explain how we decide when to use REM or FEM? For the FEM assume a model with different intercepts but the same slope (while you can choose how to specify your REM). *[Hints: In your answer, you should use words and formulas but you can also use graphical methods if you find it appropriate. For instance, explain why we use REM, why we use FEM, state for what data REM and FEM can be used, assumptions, the pros and cons about the models, and explain what you may find relevant when comparing a REM to a FEM such as for example suitable relative sizes of the samples for T and N etc.].*

Question 3 (12 points)

Consider the following EViews output (and assume that we test at 5% significance level):

Output 1
Dependent Variable: Y
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date:
Sample (adjusted): 2 150
Included observations: 149 after adjustments
Convergence achieved after 25 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.070617	0.010075	-7.009131	0.0000
AR(1)	0.509592	0.088265	5.773431	0.0000
		Variance Equation		
C	0.076561	0.033104	2.312724	0.0207
RESID(-1)^2	0.863647	0.227679	3.793263	0.0001
GARCH(-1)	0.222321	0.089781	2.476265	0.0133

R-squared	0.847391	Mean dependent var	-0.218703
Adjusted R-squared	0.842951	S.D. dependent var	1.555321
S.E. of regression	1.260721	Akaike info criterion	2.188267
Sum squared resid	233.6442	Schwarz criterion	2.289071
Log likelihood	-158.0259	Hannan-Quinn criter.	2.229222
Durbin-Watson stat	1.996567		
Inverted AR Roots	.51		

(a) What is the name of the estimated model in the output *[Obviously, you should include both the mean and the variance equation in this answer]*?

(b) Write out the formula for the estimated model in the output *[Obviously, you should include both the mean and the variance equation in this answer].*

(c) Both using words and formulas, explain the ARCH-LM test in general, and draw specific conclusions from this EViews ARCH-LM test output *[Explain every step regarding the test, its purpose, its null and alternative hypotheses, the test statistic, and whatever may be relevant to explain the test method. Note: answer this question totally independently of the result for the previous questions].*

Output 2

Heteroskedasticity Test: ARCH

F-statistic	1.042359	Prob. F(1,101)	**0.3097**
Obs*R-squared	1.052142	Prob. Chi-Square(1)	**0.3050**

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.828431	0.135328	6.121660	0.0000
RESID^2(-1)	0.096962	0.094971	1.020960	0.3097

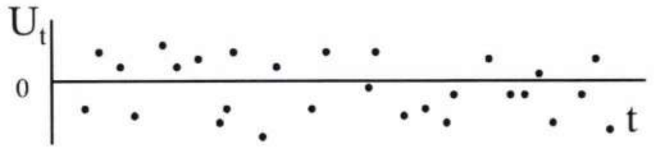
R-squared	0.010215	Mean dependent var	0.921626
Adjusted R-squared	0.000415	S.D. dependent var	1.014145
S.E. of regression	1.013934	Akaike info criterion	2.884780
Sum squared resid	103.8343	Schwarz criterion	2.935940
Log likelihood	-146.5662	Hannan-Quinn criter.	2.905501
F-statistic	1.042359	Durbin-Watson stat	1.969564
Prob(F-statistic)	0.309713		

(d) Define the non-negativity and the non-stationarity requirements for the model in Output 1 *[Hint: Disregard the AR-part and focus on the variance equation].*

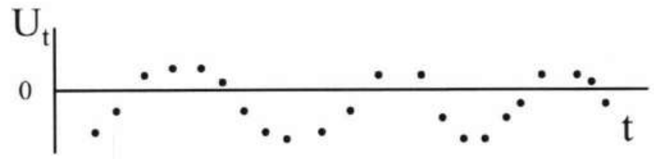
(e) For the model in Output 1, clearly motivate whether the non-negativity and the non-stationarity requirements are satisfied or not. *[Hint: The answer, that all are satisfied, or that all are not satisfied with no motivation will yield zero points. Disregard the AR-part and focus on the variance equation]*

Question 4 (6 points)

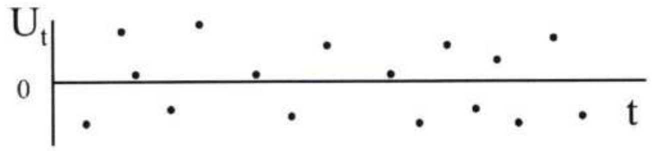
Consider the following the (separate) sequences of residuals from time-series models (OLS). For instance, assume that this is a random [$U_t \sim iidN(0,1)$] sequence of the residuals (U_t), then this is not a violation of the classical linear regression model (CLRM) assumption. This residual sequence may look something like then when being plotted against time (t).



(a) However, consider sequence of residuals from another time-series model that is not $U_t \sim iidN(0,1)$. Based on the following figure, we can suspect that there is a violation of the classical linear regression model (CLRM) assumptions. What is the most appropriate statistical term to specify the type of pattern in the residuals that this violation will create. *[Hint: Two words is sufficient for full point, but be as specific as possible to use the adequate words. If you provide exactly the same generic answer for question a and b, you will not get maximum points. No explanation is needed.]*



(b) Consider yet another sequence of residuals from another time-series model that is not $U_t \sim iidN(0,1)$. Based on the following figure, we can suspect that there is a violation of the classical linear regression model (CLRM) assumptions. What is the most appropriate statistical term to specify the type of pattern in the residuals that this violation will create. *[Hint: Two words is sufficient for full point, but be as specific as possible to use the adequate words. If you provide exactly the same generic answer for question a and b, you will not get maximum points. No explanation is needed.]*



Question 5 (10 points)

Consider the following MA(2) process: $X_t = u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2}$

where u_t is a zero mean white noise process with variance σ^2. There are a couple of more necessary assumptions, but these are the same as in the lecture notes.

In the following questions, show all relevant steps in your calculations.

(a) Derive the mean of X_t; $E(X_t)$.

(b) Derive the variance of X_t; $Var(X_t)$.
Hint: $Var(X_t) = E[(X_t-E(X_t))(X_t-E(X_t))] = E[(X_t)(X_t)]$.

(c) Derive the sample autocovariance of X_t for lag 1 (γ_1).
Hint: $\gamma_1 = Cov(X_t, X_{t-1}) = E[(X_t-E(X_t))(X_{t-1}-E(X_{t-1}))] = E[(X_t)(X_{t-1})]$.

(d) Derive the sample autocovariance of X_t for lag 2 (γ_2).
Hint: $\gamma_2 = Cov(X_t, X_{t-2}) = E[(X_t-E(X_t))(X_{t-2}-E(X_{t-2}))] = E[(X_t)(X_{t-2})]$.

(e) Derive the Sample autocorrelation of X_t for lag 1 (τ_1).

Question 6 (12 points)

In the lecture notes we presented three specifications of the Augmented Dickey-Fuller (ADF) test.

(a) Based on the EViews output, please (briefly) explain what conclusions can we draw from the following ADF-test for the variable y_t. *[The use of formulas is optional. Please be systematic, that is, state the null and alternative hypothesis, refer to a relevant value(s) when you reject / not reject, draw conclusions, use the 5% sign. level]*

Augmented Dickey-Fuller test for the variable y
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		[redacted]	0.49
Test critical values:	1% level	[redacted]	
	5% level	[redacted]	
	10% level	[redacted]	

*MacKinnon (1996) one-sided p-values.

(b) Write out the <u>most general</u> specification of the ADF test. *[That is, the model specification with the highest number of coefficients. In consistence with our usual notations, use the standard Greek notations gamma (γ) or rho (ρ) in the formulas]*. Also, state both the null and the alternative unit root test hypotheses in the ADF test – both in formulas and in words (be very brief). *[In consistence with our usual notations, use the standard Greek notations gamma (γ) or rho (ρ) in the test hypotheses]*.

Question 7 (11 points)

(a) Explain the Granger Causality test both in words and by the use of formulas. *[Explain every step regarding the method, its purpose, its null and alternative hypotheses, the test statistic, and whatever may be relevant to explain the test method.* Base your Granger causality model on a bivariate VAR(3) model in level for the variables y_t and x_t. Assume that we have stationary data.

(b) Assume that we have only two variables $Y_t \sim I(1)$ and $X_t \sim I(1)$ and that they are not cointegrated. If we still want to use the data in level (not taking first differences, log-differences, or percentage changes etc.), explain how we can still conduct a Granger Causality test.

(c) Below (independent from the previous questions) you can see an EViews Granger Causality test output.

Regression output 1:
Pairwise Granger Causality Tests
Sample: 1971 2006
Lags: 3

Null Hypothesis:	Obs	F-Statistic	Prob.
P does not Granger Cause Q	35	0.68700	0.4133
Q does not Granger Cause P		4.21072	0.0484

Write out H_0 and H_1 for the above F-tests (in words, you do not need to use parameter values).

(d) If we conduct a Granger causality test at 5% level of significance, what can we conclude from Regression equation 1? Explain the implications!

Question 8 (5 points)

Assume that we have the following processes:

$X_t = X_{t-1} + \omega_t \quad \omega_t \sim WN$
$Y_t = Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim WN$

where ω_t and ε_t are independent identically distributed white-noise processes, and X_t and Y_t are not cointegrated.

Let's assume that the following model will be estimated: $Y_t = \alpha + \beta X_t + u_t$ (Equation A):

Dependent Variable: Y

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.050638	0.286633	-10.64300	0.0000
X	-0.394909	0.051691	-7.639825	0.0000
R-squared	0.373269	Mean dependent var		-4.756460

and that the following equation will be estimated $\Delta Y_t = \delta + \gamma \Delta X_t + v_t$ (Equation B)

Dependent Variable: D(Y)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.059322	0.115759	-0.512464	0.6095
D(X)	0.068723	0.089802	0.765278	0.4460
R-squared	0.006001	Mean dependent var		-0.051743

(a) Draw conclusions regarding regression equation A *[Briefly explain what main conclusions we can draw, and briefly explain if there are any major problems when we estimate this regression model. Please use standard statistical terms when you explain.]*

(b) Draw conclusions regarding regression equation B *[Briefly explain what main conclusions we can draw, and briefly explain if there are any major problems when we estimate this regression model. Please use standard statistical terms when you explain.]*

Question 9 (12 points)

In Fair (1978, A Theory of Extramarital Affairs. Journal of Political Economy, 86, 45–61) it was investigated what factors that determine if a partner has an extramarital affair. In the paper the author uses regression analysis and in this question we use the same data set. It consists of 601 observations, and we will model the number of extramarital affairs as a function of Age (in years), Religiousness (measured 1-5 how religious a person is) and Years married. The output from EViews is shown below.

Dependent Variable: AFFAIRS
Method: ML/QML - Poisson Count (Newton-Raphson / Marquardt steps)
Date: 03/08/22 Time: 10:11
Sample: 1 601
Included observations: 523
Convergence achieved after 6 iterations
Coefficient covariance computed using observed Hessian

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AGE	-0.026293	0.007682	-3.422735	0.0006
RELIGIOUSNESS	-0.501516	0.041870	-11.97808	0.0000
YEARSMARRIED	0.161021	0.013248	12.15450	0.0000
C	0.645588	0.213041	3.030352	0.0024

R-squared	0.069338	Mean dependent var	0.869981
Adjusted R-squared	0.063959	S.D. dependent var	2.726751
S.E. of regression	2.638110	Akaike info criterion	3.803821
Sum squared resid	3612.046	Schwarz criterion	3.836399
Log likelihood	-990.6991	Hannan-Quinn criter.	3.816580
Restr. log likelihood	-1145.660	LR statistic	309.9213
Avg. log likelihood	-1.894262	Prob(LR statistic)	0.000000

(a) Calculate the marginal effects of age on the number of extramarital affairs the individual has. Interpret it in words.

(b) Test the overall significance of the model at the 5 % level of significance.

Question 10 (12 points)

Evaluation of empirical/estimation strategy:

A recent paper examines the effect of networking on innovations (a binary variable measured as if the firm has introduced a new product or not the past year). The theory evaluated is that firms that do more networking are also more innovative. The main independent variable here is networking measured as the number of contacts the firm has. Then the researcher includes control variables which are the size of the firms and profit of the company. Based on 2000 observations the model is estimated using OLS and the result is as follows:

	Coefficient	Standard error
Networking	1.2	0.3
Size	0.2	0.1
Profit	0.3	0.12

R-square	F-statistic	No. Of observations
0.3	4.2	29

(a) Do you find any support for the researcher's hypothesis?

(b) Discus the merits and weaknesses of the empirical strategy and the choice of estimation method of the econometrical model.

~~Networking $= \beta_0 + \beta_1$~~

$$\text{Innovations} = \beta_0 + \beta_1 \cdot \text{Networking}$$

$$I = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

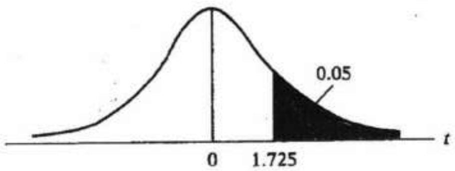
PERCENTAGE POINTS OF THE *t* DISTRIBUTION

Example

$\Pr(t > 2.086) = 0.025$

$\Pr(t > 1.725) = 0.05$ for df = 20

$\Pr(|t| > 1.725) = 0.10$



df \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

TABLE D.3 UPPER PERCENTAGE POINTS OF THE *F* DISTRIBUTION

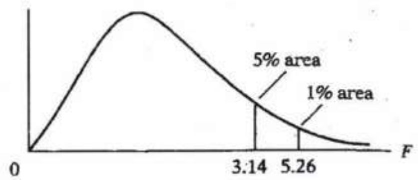
Example

$\Pr(F > 1.59) = 0.25$

$\Pr(F > 2.42) = 0.10$ for df $N_1 = 10$

$\Pr(F > 3.14) = 0.05$ and $N_2 = 9$

$\Pr(F > 5.26) = 0.01$



df for denominator N_2	Pr	df for numerator N_1: 1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.05	161	200	216	225	230	234	237	239	241	242	243	244
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 18, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

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TABLE D.3 UPPER PERCENTAGE POINTS OF THE *F* DISTRIBUTION (*Continued*)

df for denom-inator N_2	df for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
11	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
13	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

TABLE D.3 UPPER PERCENTAGE POINTS OF THE *F* DISTRIBUTION (*Continued*)

df for denom-inator N_2	df for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

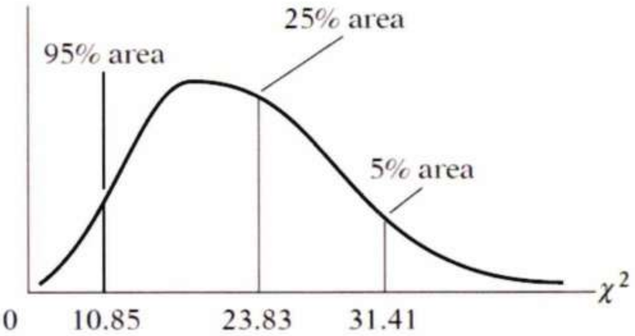
UPPER PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

Example

$Pr(\chi^2 > 10.85) = 0.95$

$Pr(\chi^2 > 23.83) = 0.25$ for df $= 20$

$Pr(\chi^2 > 31.41) = 0.05$



Degrees of freedom \ Pr	.995	.990	.975	.950	.900
1	392704×10^{-10}	157088×10^{-9}	982069×10^{-9}	393214×10^{-8}	.0157908
2	.0100251	.0201007	.0506356	.102587	.210720
3	.0717212	.114832	.215795	.351846	.584375
4	.206990	.297110	.484419	.710721	1.063623
5	.411740	.554300	.831211	1.145476	1.61031
6	.675727	.872085	1.237347	1.63539	2.20413
7	.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100*	67.3276	70.0648	74.2219	77.9295	82.3581

*For df greater than 100 the expression $\sqrt{2\chi^2} - \sqrt{(2k-1)} = Z$ follows the standardized normal distribution, where k represents the degrees of freedom.

	.750	.500	.250	.100	.050	.025	.010	.005
1	.1015308	.454937	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	.575364	1.38629	2.77259	4.60517	5.99147	7.37776	9.21034	10.5966
3	1.212534	2.36597	4.10835	6.25139	7.81473	9.34840	11.3449	12.8381
4	1.92255	3.35670	5.38527	7.77944	9.48773	11.1433	13.2767	14.8602
5	2.67460	4.35146	6.62568	9.23635	11.0705	12.8325	15.0863	16.7496
6	3.45460	5.34812	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476
7	4.25485	6.34581	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777
8	5.07064	7.34412	10.2188	13.3616	15.5073	17.5346	20.0902	21.9550
9	5.89883	8.34283	11.3887	14.6837	16.9190	19.0228	21.6660	23.5893
10	6.73720	9.34182	12.5489	15.9871	18.3070	20.4831	23.2093	25.1882
11	7.58412	10.3410	13.7007	17.2750	19.6751	21.9200	24.7250	26.7569
12	8.43842	11.3403	14.8454	18.5494	21.0261	23.3367	26.2170	28.2995
13	9.29906	12.3398	15.9839	19.8119	22.3621	24.7356	27.6883	29.8194
14	10.1653	13.3393	17.1170	21.0642	23.6848	26.1190	29.1413	31.3193
15	11.0365	14.3389	18.2451	22.3072	24.9958	27.4884	30.5779	32.8013
16	11.9122	15.3385	19.3688	23.5418	26.2962	28.8454	31.9999	34.2672
17	12.7919	16.3381	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185
18	13.6753	17.3379	21.6049	25.9894	28.8693	31.5264	34.8053	37.1564
19	14.5620	18.3376	22.7178	27.2036	30.1435	32.8523	36.1908	38.5822
20	15.4518	19.3374	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968
21	16.3444	20.3372	24.9348	29.6151	32.6705	35.4789	38.9321	41.4010
22	17.2396	21.3370	26.0393	30.8133	33.9244	36.7807	40.2894	42.7956
23	18.1373	22.3369	27.1413	32.0069	35.1725	38.0757	41.6384	44.1813
24	19.0372	23.3367	28.2412	33.1963	36.4151	39.3641	42.9798	45.5585
25	19.9393	24.3366	29.3389	34.3816	37.6525	40.6465	44.3141	46.9278
26	20.8434	25.3364	30.4345	35.5631	38.8852	41.9232	45.6417	48.2899
27	21.7494	26.3363	31.5284	36.7412	40.1133	43.1944	46.9630	49.6449
28	22.6572	27.3363	32.6205	37.9159	41.3372	44.4607	48.2782	50.9933
29	23.5666	28.3362	33.7109	39.0875	42.5569	45.7222	49.5879	52.3356
30	24.4776	29.3360	34.7998	40.2560	43.7729	46.9792	50.8922	53.6720
40	33.6603	39.3354	45.6160	51.8050	55.7585	59.3417	63.6907	66.7659
50	42.9421	49.3349	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900
60	52.2938	59.3347	66.9814	74.3970	79.0819	83.2976	88.3794	91.9517
70	61.6983	69.3344	77.5766	85.5271	90.5312	95.0231	100.425	104.215
80	71.1445	79.3343	88.1303	96.5782	101.879	106.629	112.329	116.321
90	80.6247	89.3342	98.6499	107.565	113.145	118.136	124.116	128.299
100*	90.1332	99.3341	109.141	118.498	124.342	129.561	135.807	140.169

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 8, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

Note on JIBS-provided calculator: You may be unused to how the JIBS-provided calculator is used in clearing information and how it is used with powers, natural logs, natural exponentials. Here's some instructions:

1) To clear the calculator (start afresh), press "AC".
2) To find x^y, press your x number, press x^y, press your y number, then press "=".
   For example, to find 2^3, press 2, press x^y , press 3, then press "=", which provides 8.
3) To find ln x, press your number, x, and then press ln.
   For example, to find ln 2, press 2 and then press ln, which provides 0.69314718.
4) To find e^x, press your number, x, press SHIFT, and then press e^x.
   For example, to find e^4, press 4, press SHIFT, and then press e^x, which provides 54.59815003.
5) If parentheses are needed to clarify the order of operations, the relevant keys are " (" and ") ". For example to find $(2 + 4) \times 3$, press "(", press 2, press +, press 4, press ")" , press ×, press 3, and then press "=" to get 18.

Also note the calculator will resolve calculations of powers, natural logs, and natural exponentials before calculations of addition, subtraction, multiplication, and division. Operations within parentheses are resolved before all other operations.

For example, suppose you are interested in calculating $4 \times 2^3 = 4 \times 8 = 32$. If you press 4, press "×", press 2, press x^y, press 3, and then press "=", the calculator will appropriately find $4 \times 2^3 = 32$, not $(4 \times 2)^3 = 8^3 = 512$. If instead you want to find $(4 \times 2)^3$, you can press "(", press 4, press "×", press 2, press ")", press x^y, press 3, and then press "=", to get 512. Alternatively, you can find what 4×2 equals first, and then find its third power: press 4, press "×", press 2, press "=" (so now you have 8), press x^y, and press 3, and then press "=" again, to get 512. If you want to find $5 \times (4 \times 2)^3$, you can calculate $(4 \times 2)^3$ first by either of the means mentioned above, then press "×", press "5", and then press "=", to give you $512 \times 5 = 2560$.

As another example, suppose you are interested in calculating $8^{2/3}$. If you press 8, press x^y, press 2, press "÷", press 3, and then press "=", you'll get the WRONG answer, 21.333…, because that sequence provides the calculation for $\frac{8^2}{3}$ instead. To calculate $8^{2/3}$, you can utilize the parentheses keys "(", and ")"; the parentheses forces the calculator to do calculations within parentheses first. Note that $8^{2/3}$ is the same as $8^{(2/3)}$, and using the parentheses clarifies what you mean to the calculator (that you want $8^{2/3}$, not $8^2/3$). So to find $8^{2/3}$, press 8, press x^y, press (, press 2, press "÷", press 3, press), and then press "=", to come out to the correct answer, 4.

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Question 1

a) False 1

b) True 1

c) True -1

d) True 1

e) False 1

f) True 1

g) True 1

5p

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Question 2

Fixed effect model:

- For a fixed effect model we use within variation data.
- The results of a FEM are robust & allow for autocorrelation ? between the coefficients & error terms $corr(X, u) \neq 0$
- FEM is generally used when T (time series observation) is large & N (amount of individuals) is relatively small
- FEM can control for time invariate variables, but it does not allow marginal interpretations
- Unobserved heterogeneity has to be constant

- There are two ways to represent a FEM with different intercepts & same slopes

1) $Y_{it} = \beta_{1i} + \beta_2 \cdot X_{it} + u_{it}$

2) LSDV : $Y_{it} = \alpha_1 + \alpha_2 \cdot X_{it} + \alpha_3 \cdot D_{1i} + \alpha_4 \cdot D_{2i} + \alpha_5 \cdot D_{3i} + \dots + u_{it}$

   - Here it is important to use one dummy less than the total number of individuals, to avoid the dummy trap
     → So that the dummy term gives the distance to the one individuals intercept we have not included
   - If we assume $D_{1i} = 1$, then all other dummies are equal zero

$Y_{it} = \underbrace{\alpha_1 + \alpha_3}_{\text{Intercept}} + \alpha_2 \cdot X_{it} + u_{it}$, XAM - RUE | | ~~2~~ 3

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Random Effect Model:

- REM uses a mixture of within & between variation
- The results are efficient (more so than the FEM), but do not allow for ~~autocorrelation~~ ~~&~~ $Corr(X, u) = 0$
- REM allows for marginal interpretations of time invariate variables
- Generally used when T is relatively small & N is relatively big
- REM has more degrees of freedom than the FEM since it does not use Dummy variables, etc.

- The general idea of a REM model is, to have different intercepts for each individual (Like FEM) but derived in another way.
  - In the REM we assume a mean intercept β_1 of the whole population, where the deviation from that mean is given by a random error term ε_i, which gives the individual intercept.

    $Y_{it} = \beta_1 + \beta_2 \cdot w_{i}$, where $\beta_1 = \beta_{1i} + \varepsilon_i$

- Both the FEM & REM use panel data.
- We decide between REM & FEM using the Hausmann test
  - This ~~H~~ test for ~~auto~~correlation ~~&~~ $corr(x, u)$ MISSPECIFICATION

    h_0: no autocorrelation → use REM

    h_1: autocorrelation → use FEM
  - FEM is consistent with both h_0 & h_1

1p

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Question 3

a) Mean equation : AR(1)

Variance equation : GARCH(1,1)

b) AR(1) : $Y_t = \delta + \varphi_1 \cdot Y_{t-1} + \varepsilon_t$

$\rightarrow Y_t = -0{,}070617 + 0{,}509592 \cdot Y_{t-1}$

GARCH(1,1) : $h_t = w + \alpha \cdot \varepsilon_{t-1}^2 + \beta \cdot h_{t-1}$

$\rightarrow h_t = 0{,}076561 + 0{,}863647 \cdot \varepsilon_{t-1}^2 + 0{,}222321 \cdot h_{t-1}$

c) ARCH-LM test:

- ARCH stands for autoregressive conditional heteroscedasticity
- The ARCH-LM test tests for ARCH effects in the residuals

$h_0 : \beta_1 = \beta_2 = \ldots = \beta_n = 0$ (no ARCH effects) $h_1 = \beta_1 \neq 0$, or $\beta_2 \neq 0$, or ..., or $\beta_n \neq 0$ (ARCH effects)

1) Regress $Y_t = \alpha_0 + \alpha_1 \cdot X_{1t} + \alpha_2 \cdot x_{2t} + \ldots \alpha_n \cdot x_{nt} + u_t$ & obtain all $\hat{u}_t$

2) Create auxiliary regression $\hat{u}_t^2 = \beta_0 + \beta_1 \cdot \hat{u}_{t-1}^2 + \beta_2 \cdot \hat{u}_{t-2}^2 + \ldots + \beta_n \cdot \hat{u}_{t-n}^2 + v_t$ where v_t is white noise. Obtain R^2 from this regression.

3) $T \cdot R^2 \sim \chi^2(n)$ if the test statistic $(T \cdot R^2)$ exceeds the critical value of $\chi^2(n)$ reject h_0 & assume ARCH effets in the residuals.

We can see that the F-statistic 1,04 has a p-value of 0,3097

The test statistic $T \cdot R^2$ is 1,05 with a p-value of 0,3050

$\rightarrow$ Since both the F-statistic & $R^2 \cdot T$ have p-values $> 0{,}05$, we cannot reject h_0 & have to assume that there are no ARCH effects.

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d) The GARCH model has the following conditions

1. $w > 0$, where w = long run average variance

2. $0 \geq \alpha < 1$, where ε_{t-1}^2 is the squared error term of the last period

3. $\alpha + \beta < 1$, where h_{t-1} is last periods forcasted heteroscedasticity

The unconditional variance is given by: $\frac{w}{1-\alpha-\beta}$ & in therefore, if 1., 2., 3. hold always positive

e) 1. $w > 0 \;\hat{=}\; 0,0+6561 > 0$

↳ non negativity condition holds

2. $0 \leq \alpha < 1 \;\hat{=}\; 0 < 0,863647 < 1$

3. $\alpha + \beta < 1 \;\hat{=}\; 0,863647 + 0,222321 = 1,085968 > 0$

↳ This requirement is not satisfied

12p

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Question 4

a) This shows "magnetic" correlation, where we can assume that $corr(u_t, u_{t-i}) = 1$ or at least very close to 1 & positive

POSITIVE AUTOCORRELATION

b) This shows repelling correlation. This almost looks like perfect negative autocorrelation, where $corr(u_t, u_{t-1}) = -1$ or very close to -1 & negative.

NEGATIVE AUTOCORR.

NOT THE CORRECT ANSWERS BUT GOOD INTUITIVE EXPLANATIONS

3p

~~2p~~

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Question 5: $X_t = u_t + \theta_1 \cdot u_{t-1} + \theta_2 \cdot u_{t-2}$

a) $E(X_t) = E(u_t + \theta_1 \cdot u_{t-1} + \theta_2 \cdot u_{t-2})$

$= E(u_t) + \theta_1 \cdot E(u_{t-1}) + \theta_2 \cdot E(u_{t-2})$

$= 0 + \theta_1 \cdot 0 + \theta_2 \cdot 0 = 0$

Since one of the assumptions is that u_t is white noise

b) $VAR(X_t) = \frac{(X_t - \bar{X}_t)^2}{n} = E[(X_t - \bar{X}_t)^2] = E[(X_t - E(X_t))^2]$

$= E(X_t \cdot X_t) = E[(u_t + \theta_1 \cdot u_{t-1} + \theta_2 \cdot u_{t-2}) \cdot (u_t + \theta_1 \cdot u_{t-1} + \theta_2 \cdot u_{t-2})]$

$= E[u_t^2 + \theta_1^2 \cdot u_{t-1}^2 + \theta_2^2 \cdot u_{t-2}^2] = \sigma^2 + \theta_1^2 \cdot \sigma^2 + \sigma^2 \cdot \theta_2^2$

$= \sigma^2 \cdot (1 + \theta_1^2 + \theta_2^2) = \gamma_0$

c) $\gamma_1 = Cov(X_t, X_{t-1}) = E[X_t \cdot X_{t-1}]$

$= E[(u_t + \theta_1 \cdot u_{t-1} + \theta_2 \cdot u_{t-2}) \cdot (u_{t-1} + \theta_1 \cdot u_{t-2} + \theta_2 \cdot u_{t-3})]$

$= E[\theta_1 \cdot u_{t-1}^2 + \theta_2 \cdot \theta_1 \cdot u_{t-2}^2] = \sigma^2 \cdot \theta_1 + \sigma^2 \cdot \theta_1 \cdot \theta_2$

$= \sigma^2 \cdot (\theta_1 + \theta_1 \cdot \theta_2) = \gamma_1$

d) $\gamma_2 = Cov(X_t, X_{t-2}) = E[X_t \cdot X_{t-2}]$

$= E[(u_t + \theta_1 \cdot u_{t-1} + \theta_2 \cdot u_{t-2}) \cdot (u_{t-2} + \theta_1 \cdot u_{t-3} + \theta_2 \cdot u_{t-4})]$

$= E[\theta_2 \cdot u_{t-2}^2] = \theta_2 \cdot \sigma^2 = \gamma_2$

10p

e) $r_1 = \frac{\gamma_1}{\gamma_0} = \frac{\sigma^2 \cdot (\theta_1 + \theta_1 \cdot \theta_2)}{\sigma^2 \cdot (1 + \theta_1^2 + \theta_2^2)} = \frac{\theta_1 + \theta_1 \cdot \theta_2}{1 + \theta_1^2 + \theta_2^2}$

Question 6 :

a) $h_0 : \gamma = 0 \hat{=} \rho = 1$ (unit root)

$h_1 : \gamma < 0 \hat{=} \rho < 1$ (no unit root)

Since the p-value of the augmented Dickey Fuller test is 0,49 > 0,05 (greater than 0,05), we cannot reject h_0

→ we must assume that there is a unit root

b) There are 3 different versions of the ADF test, where the third version is the most general.

1. No intercept, no time trend
2. Intercept, no time trend
3. Intercept & time trend

↳ $\Delta Y_t = \gamma \cdot Y_{t-1} + \alpha + \beta \cdot t + \sum \beta_i \cdot \Delta Y_{t-i} + \varepsilon_t$

or : $\Delta Y_t = (\rho - 1) \cdot Y_{t-1} + \alpha + \beta \cdot t + \sum \beta_i \cdot \Delta Y_{t-i} + \varepsilon_t$

where α is the intercept. $\beta \cdot t$ is the time trend & $\sum \beta_i \cdot \Delta Y_{t-i}$ are the lags needed to make the error term ε_t white noise.

$h_0 : \gamma = 0$ (or $\rho = 1$) → unit root

$h_1 : \gamma < 0$ (or $\rho < 1$) → no unit root

12p

Question 7

a) Granger causality test:

- It is a test to determine granger causality
- For a bivariate model, we have the following possible outcomes:

1. $X \to Y$ (X does granger cause Y)
2. $Y \to X$ (Y does granger cause X)
3. $Y \to X$, $X \to Y$ (Y does garger cause X & X does granger cause Y)
4. $X \not\leftrightarrow Y$ (no granger causality)

Bivariate VAR(3):

$Y_t = c_{10} + \alpha_{11} \cdot Y_{t-1} + \alpha_{12} \cdot Y_{t-2} + \alpha_{13} \cdot Y_{t-3} + \beta_{14} \cdot X_{t-1} + \beta_{15} \cdot X_{t-2} + \beta_{16} \cdot X_{t-3} + u_1 t$

$X_t = c_{20} + \alpha_{21} \cdot Y_{t-1} + \alpha_{22} \cdot Y_{t-2} + \alpha_{23} \cdot Y_{t-3} + \beta_{24} \cdot X_{t-1} + \beta_{25} \cdot X_{t-2} + \beta_{26} \cdot X_{t-3} + u_2 t$

As we can see, Y_t depends on its own lags & the lags of X_t & vice versa.

The granger causality test is now used to determine if lags of X_t actually helps to predict Y_t & vice versa or if they are just 'dead weight'

$h_0: \beta_{14} = \beta_{15} = \beta_{16} = 0$ (X_t does not granger cause Y_t)

$h_1: \beta_{14} \neq 0$ or $\beta_{15} \neq 0$ or $\beta_{16} \neq 0$ (X_t does granger cause Y_t)

This test has to be done in both directions, where we would then also have to test if Y_t granger causes X_t. And we also need to test if the lags of X_t granger cause X_t & the same for Y_t, which leads us to the following hypothesis

lags of X_t do not granger cause X_t $\quad \beta_{24} = \beta_{25} = \beta_{26} = 0$
lags of Y_t do not granger cause X_t $\quad \alpha_{21} = \alpha_{22} = \alpha_{23} = 0$
lags of X_t do not granger cause Y_t $\quad \beta_{14} = \beta_{15} = \beta_{16} = 0$
lags of Y_t do not granger cause Y_t $\quad \alpha_{11} = \alpha_{12} = \alpha_{13} = 0$

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<u>Procedure of a granger causality test</u> :

① Make sure that there are no unit roots using unit root tests (eg. ADF)

② Estimate each model by itself (restricted model)

$$Y_t = c + \sum_{i=1}^{p} \beta_i \cdot Y_{t-i} + \varepsilon_t$$

$$X_t = c + \sum_{j=1}^{q} \alpha_j \cdot X_{t-j} + \varepsilon_t$$

-> make sure that the coefficient are relevant using F-tests.
$h_0 : \beta_1 = \beta_2 = \beta_3 = 0$

③ Run the unristricted model

$$Y_t = c + \sum_{i=1}^{p} \beta_i \cdot Y_{t-i} + \sum_{j=1}^{q} \alpha_j \cdot X_{t-j} + \varepsilon_t$$ "Does X granger cause Y?"

$$X_t = c + \sum_{j=1}^{q} \alpha_j \cdot X_{t-j} + \sum_{i=1}^{p} \beta_i \cdot Y_{t-1} + \varepsilon_t$$ "Does Y granger cause X?"

④ Conduct an F-test, comparing the unristicted model, with the restricted model, to see which one has the greater predictive power.

⑤ $h_0 : \alpha_1 = \alpha_2 = \alpha_3 = 0$ (X does not granger cause Y)
$h_1 : \alpha_1 \neq 0, or\ \alpha_2 \neq 0, or\ \alpha_3 \neq 0$ (X does grange cause Y)

and for the second iteration : $h_0 : \beta_1 = \beta_2 = \beta_3 = 0$ (Y does not granger cause X)
$h_1 : \beta_1 \neq 0, or\ \beta_2 \neq 0\ or\ \beta_3 \neq 0$ (Y does granger cause X)

~~That~~ If the F-statistic exceeds the critical value of the distribution we must reject h_0 & asume granger causality.

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b) We can still conduct a granger causality test using the Toda - Yamamoto approach

① Determine the order of integration for each variable (just to be sure)

② Set max order of integration = m → can assume m = 1

③ Run the VAR model in levels

④ Check for specification (check the error terms for autocorrelation, etc.)

⑤ ~~*~~ Since $X_t \sim I(1)$ & $Y_t \sim I(1)$ check for cointegration

⑥ Calculate the optimal lag lenght using AIC, or SBC for example

⑦ Add m = 1 lags to the VAR model

⑧ Conduct a Wald test to test for granger non-causality

⑨ The wald test statistic is asymtotically distributed over $\chi^2(m)$

h_0: no granger causality h_1: granger causality

⑩ If the test statistic exceeds the critical value, we must reject h_0

& have to assume granger causality

c) Case 1: P does not granger cause Q : h_0

- Since the F-Statistic 0,68700 & the p-value 0,4133 > 0,05 we cannot reject h_0 & must assume that P does not granger cause Q

11P

Case 2: Q does not granger cause P : h_0

- Since the F-Statistic is 4,21072 & the p-value is 0,048 < 0,05 we must reject h_0. Therefore, we must assume granger causality

& There is statistical evidence that Q does granger cause P

d) YOU NEVER ANSWERED (d) BUT YOU ANSWERED IT IN (c)

No writing here

XAM - RUE		12
Name	Personal number	Page number

Question 8 :

1) We know that $X_t = X_{t-1} + w_t$

$Y_t = Y_{t-1} + w_t$

We can see that they are both integrated of order 1

$X_t \sim I(1)$ so that $\Delta X_t \sim I(0)$

$Y_t \sim I(1)$ so that $\Delta Y_t \sim I(0)$

Since they are both not stationary & follow a stochastic trend, we increase the probability of getting a spurious regression.

→ This means that we might find a connection between 2 variables, when there really is none → Increase in Type 1 errors.

- From the processes we can see that X_t & Y_t are not related.
- However, we find statistically significant results (p-value < 0,05) & even a R^2 of $\approx 0{,}37$

→ This is a spurious regression!

2) In equation B we took the first differences, which resulted in stationarity

- This drastically reduces the probability of a spurious regression.
- The regression output finds no statistically significant relationship since the p-values are over 0,05.
- R^2 is also much lower now & almost 0 : $R^2 = 0{,}006$

→ We can assume that this is not a spurious regression

5P

No writing here

Question 9 :

a) Marginal effet in poisson model:

- If X increases by 1 unit, Y increases by $\hat{\beta}_i$ units

$\hat{\beta}_1 = -0{,}026293$

If the age of an average individual increases by 1 year (1 unit) the average probability of engaging in ~~em~~ extramaritalf affairs reduces by 0,02693 ($\hat{=}$ 2,63 %)

b) Significance test:

Test if all parameters are equal 0: $2 \cdot (\ln(L_{UR}) - \ln(L_R))$

$2 \cdot [-990{,}6991 + 1145{,}660] = \underline{\underline{309{,}9218}}$

We have 3 variables in the model, which mean 3 degrees of freedom

The critical value of $\chi^2(3)$ is 7,81473

We assume that the F-Test is distributed as $\chi^2(3)$

h_0: all parameters are insignificant $\beta_1 = \beta_2 = \beta_3 = 0$

h_1: at least one parameter is significant $\beta_1 \neq 0$ or $\beta_2 \neq 0$ or $\beta_3 \neq 0$

Since the ~~crit~~ ~~t~~ value of the F-statistic $309{,}9218 > 7{,}81473$ we must reject h_0.

This means that at least one parameter is statistically significant.

We can also see this from the p-values being $< 0{,}05$.

Question 10

- Innovations is binary $I = \{^1_0\}$
- Theory: More networking → more innovations
- Independent variable: Networking (number of contacts)

a) Test if one parameter is equal to zero:

$$\frac{\hat{\beta}_i}{se(\hat{\beta}_i)} \hat{=} \frac{1{,}2}{0{,}3} = \underline{\underline{4}}$$

Since the ~~p~~ value of the F-test $4 < 4{,}2$ is smaller than the F-statistic, we can not reject h_0. And have to assume insignificance.

$h_0 : \hat{\beta}_i = 0 \qquad h_1 : \beta_1 \neq 0$

→ We can not confirm the theory

b) There are several problems that come up, when we use OLS here:

- Firstly, the dependend variable is binary, which makes standard OLS biased
- The second problem that arises is that that the independend variable is count data, which also excludes OLS as an estimation method

→ Considering this a Poisson model or even a negative binominal model seems more adequte.

- Control variables such as profit & size do not seem as the most relevant ones.

→ Based on economic theory one can assume that the sector of the firm is relevant. Modeling this can be achieved through the introduction of dummy variables.

- It is good that 2000 observations were used & that that there are only 29 no. of observations.

Some unstructured suggested answers…

I was asked for some answers to the exam, so even if all answers can be found in the teaching material here is some guidance that I hope will help your studies (even if reading previous exams is not the optimal way to increase the long-run learning)!

This goes without saying, but there are different ways of answering an exam question and still receiving the maximum points. Therefore, note, in the following answers sometimes there are more than sufficient arguments for max. points, and sometimes fewer or too brief answers for full score. Also, note that longer answers will not lead to more points. All the answers are found in the teaching material/textbook where a full elaboration of a sufficient answer is given. These answers usually contain briefer summaries.

JÖNKÖPING UNIVERSITY

International Business School

Written examination: Analytical Methods for Economic and Financial Analysis (FSSS23)

Date and time: **XXX**

Total number of pages: [] Check that you have got them all!

- General exam rules at Jönköping University apply.
- This examination consists of [10] tasks.
- The Pass mark is 60 % of the total.
- The following aids are permitted:

- Pens, pencils, rulers, and a calculator (among those noted below) are permitted
- Allowed calculators at the exam are calculators provided by JIBS. Personal calculators can be used if they are of the type: Casio FX 82ES, Casio FX-82 Solar, Texas Instruments TI 30 or Sharp EL-531WH
- Otherwise, calculators will be provided by the invigilators (tentavakter).
- The formula sheet and the distribution tables are included in the exam.

Exam ID: ______________________________

Please don't write down your name or personal identity number.

Total score: ________________ Result (Grade): ____________________

Ifylls av tentamensvakt!

ANTAL INLÄMNADE LÖSA SIDOR: ____________ SIGNATUR: ____________

COMPLETED TASKS:

Task/Question	1	2	3	4	5	6	7	8	9	10				Total
Maximum points	7	13	12	6	10	12	11	5	12	12				100
Total score														

Name of the responsible teacher: XXX

Responsible teacher will visit the examination: XXX

Responsible teacher can be reached on the phone during the following times: XXX.

For urgent questions during the exam - the responsible teacher can be reached on the following phone numbers: XXXXXXXX (XXX, for Q1-Q8), or for XXX's part (Q9-Q10) XXXXXX.

JÖNKÖPING UNIVERSITY

Exam regulations for students at Jönköping University

The 14th of October 2020

These instructions are based on President's decision § 755, 2018, "Regulations and guidelines for first-, second-, and third-cycle education at Jönköping University". Regulations on disruptive behaviour and cheating are found in the policy documents of the Disciplinary and Expulsion Committee. To guarantee the student's legal rights, Sweden's legislation on discrimination must be observed.

The invigilator's role is to guarantee that the examination takes place in an ordered and legally securemanner. **The invigilator's instructions** must be followed.

Cheating or disruptive behaviour during an exam are disciplinary offences that will be reported to the Disciplinary and Expulsion Committee. A disciplinary offence may lead to short- or long-term suspension from the university.

The exams are scheduled on the basis of set start times. It is the same times for all days, starting at 9 a.m. and 2 p.m.

Preparations

- Register for each exam no later than ten days beforehand. If you fail to register or register late, you will not be allowed to write the exam in question.
- Since hypersensitivity/allergy is relatively common, you are not allowed to bring food/snacks that contain nuts/peanuts or to wear perfume.
- Be sure you know the correct time and place.
- Be sure you know what aids are permitted. Ensure that your aids, if any, are "clean", with no forbidden notes or loose pages. Tabs and bookmarks without any text or marking other than chapter headings or equivalent are permitted.
- Bring a valid photo ID, e.g. your driving licence or passport. Without such ID, you will not be allowed to write the exam. The JU card, if marked "Identity Card" and showing your full civic registration number, may be used as an ID document at exams.
- You may bring refreshments.
- Prior to a digital exam, it is always each student's responsibility to ensure that his/her JU user account will be active at the time of the exam. This is also necessary if you need to borrow a computer. If there are any problems, please contact IT Helpdesk.

During admission

- Arrive at the latest 20 minutes before the exam starts. The door is locked at exactly the specified time.
- Before entering, tick off your name on the registration list at the entrance. If you are not on the list, you will not be allowed to write the exam.
- Those who arrive for the second admission, 30 minutes after the start, must be present outside the door so that the invigilator can verify their identity.
- Anyone arriving more than 30 minutes late will not be allowed to sit the exam. No excuses are accepted.
- Leave any outerwear and bags in their designated places.

- Seat yourself in the indicated place. Only permitted aids, ID and refreshments are allowed at the desk.
- All electronic equipment (mobile phones, smart watches, MP3 players, etc.) are to be switched off and kept with the outerwear and bags. Do not bring anything to the exam that you do not wish to leave unsupervised. Any sound coming from a mobile phone during an examination will be reported as both disruptive behaviour and attempted cheating.
    - If you consider that you have legitimate reasons to have your mobile switched on during the exam, notify the invigilator of this before the exam begins. Only exceptional reasons are accepted. The switched-on mobile (silent ringtone) is to be kept by the invigilator. If you accept a call, you must immediately stop the examination and hand in your paper.

<u>Start</u>

- When the invigilator locks the door and announces the start of the exam, you must immediately sit down and stay silent.
- Check that you receive the correct exam paper from the invigilator and that the paper is complete. In Inspera check that you see the correct exam.
- If you are registered to write two exams, you receive both papers at the beginning of the exam session. However, the individual finish times must be respected. In Inspera both exams will be visible.

<u>During the examination</u>

- No student may leave the exam room during the first half an hour.
- There must be no communication whatsoever between the students.
    - Any communication between the candidates must go through an invigilator.
- There must be no disruptive behaviour. If you feel that you are being disrupted, please inform the invigilator.
- When the invigilator is performing the ID check, have your ID readily to hand.
    - If you do not have an ID that the invigilator can accept, you will be turned away from the exam.
    - If you are not on the registration list, you will be turned away from the exam.
    - When the invigilator comes to check your ID, your name must have been entered on the first page of the exam paper.
- The invigilator may, at any time and without special reason, check what is on your desk. The invigilator may also leaf through permitted books to check that they do not contain forbidden notes and look inside pencil cases, sweet bags and the like.
- The only writing papers that are allowed are the ones with a colored corner, provided by the invigilators during the exam.
- If you visit the toilet, both name and time must be noted on the toilet list. Only one student may visit the toilet at any one time.
- If you leave the room for any reason other than visiting the toilet, you are considered to have stopped the exam and may not continue writing.

End

- The invigilator lets the students know when 30 and 10 minutes of writing time remains.
- When the invigilator announces that the time is up, you must stop writing immediately.
- Ensure that you have written your name and civic registration number on each piece of paper that you hand in. In case you have not done this when the time is up, you must continue filling in your name and civic registration number in the presence of an invigilator.
- When you hand in your paper, you must show your ID.
- Even if no questions have been answered, the pre-personalised page must be handed in.
- The number of submitted loose pages are counted by the invigilator and noted on the pre-personalised page.
- Check that the invigilator ticks off your name correctly and notes the correct number of submitted pages.
- Unless otherwise specified, you may take the exam paper with you once you have handed in your answers. You are not allowed to take the writing papers with a colored corner with you from the exam venue.
- In a digital exam, you and the invigilator are to jointly note the time of submission on the attendance list.

Special educational support

- If you have been granted special educational support owing to disability and wish to have an alternative exam arrangement, register this with the examination coordinator in the case management system no later than ten days beforehand and, for information, with the responsible teacher. You must also register for the exam as usual.
- A student with special educational support who writes a paper exam, but is entitled to use a computer, must write the entire exam either on paper or on computer.

* **IMPORTANT!!! PLEASE READ THESE INSTRUCTIONS!!!** In the following questions, you are requested at various times to describe tests methods, estimation methods, etc. In your answers, you should use both text and mathematical forms whenever appropriate. Motivate your answer to receive full score. However, more text in your answer does not necessarily mean that you will receive more points (unless it is necessary and relevant information). For some types of exam questions, some formulas and statistical tables are provided at the back of the test for you to use when you consider appropriate. If for instance the t-distribution tables or F-distribution tables are used in the exam, and the exact value is not presented, use the closest value available (if nothing else is stated). However, note that these standard statistical tables in the end of the exam are not used for all courses/exams. Some data sets may be based on fictitious data; therefore, you should sometimes disregard if some values do not make economic sense. If you are asked to mention 3 items (e.g. tests for autocorrelation), only the first 3 you provide will be graded (and the rest will be disregarded). **Present the answers to the questions in numerical chronological order, and clearly state which question you are answering** e.g. Q1a, Q1b, Q1c, etc., otherwise points may be deducted (unless you have very good reasons). Write your name and personal number on each page. Please do not use a red pen when answering the questions, since this is the colour used for grading. Avoid using the left upper corner of your answering sheets since this text will be hidden by the staples. If you suspect that there is a mistake or a typo in the exam, and if the teacher is not available to answer questions, just make a note about this and try to answer the question to the best of your knowledge regardless of this. You can probably understand the meaning or point of the question anyway, and if there is a problem you will be given a generous score when the exam is graded**. Do not write answers on both sides of the answer sheets.** If we cannot read what you write, we cannot award points for the answer. Write all your answers on the (blank) answer sheets – **do not write answers directly in the exam. If nothing else is stated in the question, use the 5% level of significance for the tests in the exam.**

Question 1 (7 points)

With respect to the statements below, say whether the statement is True (**T**) or False (**F**). No explanation is required. If there is at least one part which is incorrect in a statement, you state that it is **F**. A correct answer yields 1 point, an incorrect answer yields -1 point, while no answer (blank) yield 0 points. *[You can never obtain less than zero points in total for these questions even if you give the incorrect answer every time. As always, please answer on the answer sheet, not directly here in the exam].*

(a) A REM cannot model (only control for) time-invariant variables (such as gender or race).

(b) The OLS estimator is biased if the dependent variable is censored. Then we would get biased results.

(c) If we reduce the statistical size, there is less risk of making a type1-error, but higher chance to reject the null hypothesis if it is false.

(d) In a spurious regression model we are too frequently making a type 1 error.

(e) The statistical power of a test is the probability of not rejecting a true null hypothesis.

(f) Breusch–Godfrey's test is an autocorrelation test.

(g) If we run $y_t = c + \beta_1 x_t + u_t$ where $y_t \sim I(1)$ and $x_t \sim I(1)$ and if the residual is $u_t \sim I(1)$, then we can expect a spurious relationship between x_t and y_t.

ANSWERS: (a):F, (b):T, (c):F, (d):T, (e):F, (f):T, (g):T.

Question 2 (13 points)

Explain, and make a comparison of, a random-effects model (REM) and a fixed-effects model (FEM). That is, define and explain one REM and one FEM, and explain under what circumstances these REM and FEM are used. Explain how we decide when to use REM or FEM? For the FEM assume a model with different intercepts but the same slope (while you can choose how to specify your REM). *[Hints: In your answer, you should use words and formulas but you can also use graphical methods if you find it appropriate. For instance, explain why we use REM, why we use FEM, state for what data REM and FEM can be used, assumptions, the pros and cons about the models, and explain what you may find relevant when comparing a REM to a FEM such as for example suitable relative sizes of the samples for T and N etc.].*

SUGGESTED ANSWER: Note that not all these arguments are necessary for receiving the maximum points.

Write out the formulas for FEM, REM and the Hausman test (easiest just to take a look in the lecture notes or in the textbook for deeper explanations). Some examples of pros of panels: increases the sample size (#obs=N*T), more (statistically) powerful than most cross-sectional or time-series models, can sometimes increase the statistical efficiency, more sample variability, more info, more complicated relationships can be extracted from a panel relationship, less multicollinearity problems, we can for some panels study dynamic changes (for some dynamic panel models), advanced instrumental panel approaches can also tackle endogeneity, but on the other hand there are some cons of panels: FEM and LSDV consumes a lot of degrees of freedom, attrition whereby for one reason or another, members of the panel drop out over time can cause misleading results, many (implicit) assumptions (not only all assumptions for cross-sectional and time-series data, also some additional assumptions only for various panel models such as cross-sectional independence, homogeneity of some parameters in the panel). Read more in the textbook.
(+) FEM less vulnerable to omitted variable bias, BUT, (-) the standard errors of the FEM may be too large
(+) FEM remove "unobserved heterogeneity" (that is, FEM removes a type of omitted-variables bias).
(+) REM can (in contrast to FEM) efficiently analyze time-invariant variables.
(+) (while FEM control for unobserved heterogeneity if this unobs. het. is constant over time)
(+) REM: EFFICIENT (assume that the intercept of each cross-sectional unit is uncorrelated with the regressors, otherwise biased)
(+) FEM: CONSISTENT (usually unbiased)
(+) FEM robust
(+) REM efficient
(-) REM does not control for "unobserved heterogeneity"
(-) FEM is more INEFFICIENT than REM (since FEM only use within-groups variation, and ignores between-group variation, while unlike
(+) REM use both within and between variation),
(+) If all the assumptions are satisfied: Both REM and FEM are unbiased and consistent (roughly speaking: "unbiased when the sample gets sufficiently large")
(+) REM estimate the "marginal" effects of time-invariant regressors (gender and race)

(-) whereas a FEM cannot estimate marginal effects, only control for these time-invariant variables (e.g. gender is constant over time).
(+) Fewer parameters for REM
(+) If T is small and N is large => REM often better than FEM (given that the assumptions hold)
(+) If T is large and N is small => Small difference between FEM and REM
(+) FEM is consistent under both H0 and H1 of Hausman
(+/-) REM is consistent under H0 (use REM), but inconsistent under H1 (use FEM)), of Hausman
(-) FEM only use within-individual differences, essentially discards info. about differences between individuals (SE ↑).
...etc etc. read more in the textbook for deeper, but less brief, arguments...

Question 3 (12 points)

Consider the following EViews output (and assume that we test at 5% significance level):

Output 1
Dependent Variable: Y
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date:
Sample (adjusted): 2 150
Included observations: 149 after adjustments
Convergence achieved after 25 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.070617	0.010075	-7.009131	0.0000
AR(1)	0.509592	0.088265	5.773431	0.0000
	Variance Equation			
C	0.076561	0.033104	2.312724	0.0207
RESID(-1)^2	0.863647	0.227679	3.793263	0.0001
GARCH(-1)	0.222321	0.089781	2.476265	0.0133

R-squared	0.847391	Mean dependent var	-0.218703
Adjusted R-squared	0.842951	S.D. dependent var	1.555321
S.E. of regression	1.260721	Akaike info criterion	2.188267
Sum squared resid	233.6442	Schwarz criterion	2.289071
Log likelihood	-158.0259	Hannan-Quinn criter.	2.229222
Durbin-Watson stat	1.996567		
Inverted AR Roots	.51		

(a) What is the name of the estimated model in the output *[Obviously, you should include both the mean and the variance equation in this answer]*?

(b) Write out the formula for the estimated model in the output *[Obviously, you should include both the mean and the variance equation in this answer].*

(c) Both using words and formulas, explain the ARCH-LM test in general, and draw specific conclusions from this EViews ARCH-LM test output *[Explain every step regarding the test, its purpose, its null and alternative hypotheses, the test statistic, and whatever may be relevant to explain the test method. Note: answer this question totally independently of the result for the previous questions].*

Output 2
Heteroskedasticity Test: ARCH

F-statistic	1.042359	Prob. F(1,101)	**0.3097**
Obs*R-squared	1.052142	Prob. Chi-Square(1)	**0.3050**

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.828431	0.135328	6.121660	0.0000
RESID^2(-1)	0.096962	0.094971	1.020960	0.3097

R-squared	0.010215	Mean dependent var	0.921626
Adjusted R-squared	0.000415	S.D. dependent var	1.014145
S.E. of regression	1.013934	Akaike info criterion	2.884780
Sum squared resid	103.8343	Schwarz criterion	2.935940
Log likelihood	-146.5662	Hannan-Quinn criter.	2.905501
F-statistic	1.042359	Durbin-Watson stat	1.969564
Prob(F-statistic)	0.309713		

(d) Define the non-negativity and the non-stationarity requirements for the model in Output 1 *[Hint: Disregard the AR-part and focus on the variance equation].*

(e) For the model in Output 1, clearly motivate whether the non-negativity and the non-stationarity requirements are satisfied or not. *[Hint: The answer, that all are satisfied, or that all are not satisfied with no motivation will yield zero points. Disregard the AR-part and focus on the variance equation]*

SUGGESTED ANSWERS:

(a) AR(1)-GARCH(1,1)

(b): $y_t = a + b\, y_{t-1} + \varepsilon_t$ ---- $h_t = \omega + \alpha\, \varepsilon^2_{t-1} + \beta\, h_{t-1} + u_t$

(c): Answers in Modeling Volatility (GARCH), Labs (intro and repetition), Labs for Modeling Volatility (GARCH). The ARCH-LM statistic is 1.042359 (F) or 1.052142 (Obs*R-squared, Chi-square test) with [p-value=0.3097 or 0.3050] > [0.05=level of sign.] → Cannot reject H_0. There are no indications of (ARCH) autoregressive conditional heteroscedasticity in the residuals.

1. First, run the linear regression for instance of the form given in the equation

$y_t = \beta_1 + \beta_2 x_{2t} + \ldots + \beta_k x_{kt} + u_t$

save the residuals, $\hat{u}_t$.

2. Then square the residuals, and regress them on q own lags to test for ARCH of order q, i.e. run the regression $\hat{u}_t^2 = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2 + \gamma_2 \hat{u}_{t-2}^2 + \ldots + \gamma_q \hat{u}_{t-q}^2 + v_t$

where the residual v_t is assumed to be **i**ndependently **i**dentically **d**istributed (iid) and normally distributed.

Obtain R^2 from this regression (in step 2 where we only use the u:s).

3. The test statistic is defined as TR^2 (the number of observations multiplied by the coefficient of multiple correlation) from the last regression in step 2, and this test statistic is distributed as a $\chi^2(q)$. That is, $\boldsymbol{TR^2 \sim \chi^2(q)}$ where q=number of lags.

4. The null and alternative hypotheses are:

$H_0 : \gamma_1 = 0$ and $\gamma_2 = 0$ and $\gamma_3 = 0$ and ... and $\gamma_q = 0$

$H_1 : \gamma_1 \neq 0$ or $\gamma_2 \neq 0$ or $\gamma_3 \neq 0$ or ... or $\gamma_q \neq 0$.

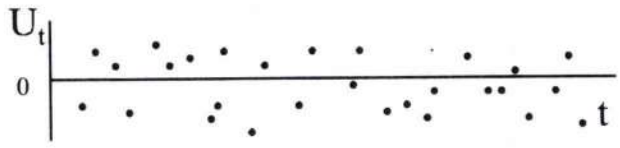
If the value of the test statistic is greater than the critical value from the χ^2 distribution, then reject the null hypothesis. The number of degrees of freedom is equal to q (that is, the number of lags).

(d): Here are the constraints for a GARCH(1,1): $\omega>0$, $0\leq\alpha<1$, $0\leq\beta<1$, $(\alpha+\beta)<1$, and $UV=\omega/(1-\alpha-\beta)$

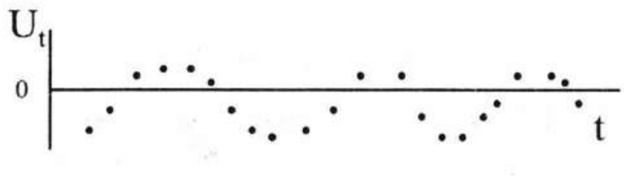
(e): The model is totally useless.
$\alpha+\beta=0.863647+0.222321=1.085968$ which violates one of the assumptions below.
$\omega/(1-\alpha-\beta)=0.076561/(1-0.863647-0.222321)=-0.890575563$ (negative unconditional variance).

Question 4 (6 points)

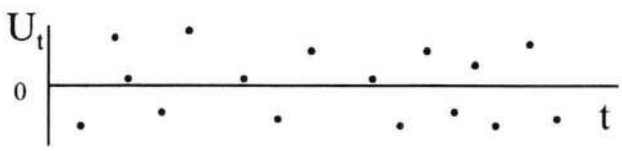
Consider the following the (separate) sequences of residuals from time-series models (OLS). For instance, assume that this is a random [$U_t \sim iidN(0,1)$] sequence of the residuals (U_t), then this is not a violation of the classical linear regression model (CLRM) assumption. This residual sequence may look something like then when being plotted against time (t).



(a) However, consider sequence of residuals from another time-series model that is not $U_t \sim iidN(0,1)$. Based on the following figure, we can suspect that there is a violation of the classical linear regression model (CLRM) assumptions. What is the most appropriate statistical term to specify the type of pattern in the residuals that this violation will create. *[Hint: Two words is sufficient for full point, but be as specific as possible to use the adequate words. If you provide exactly the same generic answer for question a and b, you will not get maximum points. No explanation is needed.]*



(b) Consider yet another sequence of residuals from another time-series model that is not $U_t \sim iidN(0,1)$. Based on the following figure, we can suspect that there is a violation of the classical linear regression model (CLRM) assumptions. What is the most appropriate statistical term to specify the type of pattern in the residuals that this violation will create. *[Hint: Two words is sufficient for full point, but be as specific as possible to use the adequate words. If you provide exactly the same generic answer for question a and b, you will not get maximum points. No explanation is needed.]*



ANSWERS:

(a): Positive Autocorrelation

(b): Negative Autocorrelation.

Question 5 (10 points)

Consider the following MA(2) process: $X_t = u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2}$

where u_t is a zero mean white noise process with variance σ^2. There are a couple of more necessary assumptions, but these are the same as in the lecture notes.

In the following questions, show all relevant steps in your calculations.

(a) Derive the mean of X_t; $E(X_t)$.

(b) Derive the variance of X_t; $Var(X_t)$.
Hint: $Var(X_t) = E[(X_t\text{-}E(X_t))(X_t\text{-}E(X_t))] = E[(X_t)(X_t)]$.

(c) Derive the sample autocovariance of X_t for lag 1 (γ_1).
Hint: $\gamma_1 = Cov(X_t, X_{t-1}) = E[(X_t\text{-}E(X_t))(X_{t-1}\text{-}E(X_{t-1}))] = E[(X_t)(X_{t-1})]$.

(d) Derive the sample autocovariance of X_t for lag 2 (γ_2).
Hint: $\gamma_2 = Cov(X_t, X_{t-2}) = E[(X_t\text{-}E(X_t))(X_{t-2}\text{-}E(X_{t-2}))] = E[(X_t)(X_{t-2})]$.

(e) Derive the Sample autocorrelation of X_t for lag 1 (τ_1).

Question 6 (12 points)

In the lecture notes we presented three specifications of the Augmented Dickey-Fuller (ADF) test.

(a) Based on the EViews output, please (briefly) explain what conclusions can we draw from the following ADF-test for the variable y_t. *[The use of formulas is optional. Please be systematic, that is, state the null and alternative hypothesis, refer to a relevant value(s) when you reject / not reject, draw conclusions, use the 5% sign. level]*

Augmented Dickey-Fuller test for the variable y
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		[redacted]	0.49
Test critical values:	1% level		
	5% level		
	10% level		

*MacKinnon (1996) one-sided p-values.

(b) Write out the <u>most general</u> specification of the ADF test. *[That is, the model specification with the highest number of coefficients. In consistence with our usual notations, use the standard Greek notations gamma (γ) or rho (ρ) in the formulas]*. Also, state both the null and the alternative unit root test hypotheses in the ADF test – both in formulas and in words (be very brief). *[In consistence with our usual notations, use the standard Greek notations gamma (γ) or rho (ρ) in the test hypotheses]*.

SUGGESTED ANSWERS:

(a): H_0: γ=0, H_1: γ<0, pval=0.49>0.05=sign.level, cannot reject, unit root for y.

(b): $\Delta Y_t = \gamma Y_{t-1} + \boldsymbol{\alpha} + \boldsymbol{\beta}$ **trend**$_t + \Sigma \beta_i \Delta Y_{t-i} + \varepsilon_t$
or $\Delta Y_t = (\rho - 1)Y_{t-1} + \boldsymbol{\alpha} + \boldsymbol{\beta}$ **trend**$_t + \Sigma \beta_i \Delta Y_{t-i} + \varepsilon_t$ (or just **t** for trend)
H_0: γ=0, H_1: γ<0 or H_0: ρ=1, H_1: ρ<1.

Question 7 (11 points)

(a) Explain the Granger Causality test both in words and by the use of formulas. *[Explain every step regarding the method, its purpose, its null and alternative hypotheses, the test statistic, and whatever may be relevant to explain the test method.* Base your Granger causality model on a bivariate VAR(3) model in level for the variables y_t and x_t. Assume that we have stationary data.

(b) Assume that we have only two variables Y_t~I(1) and X_t~I(1) and that they are not cointegrated. If we still want to use the data in level (not taking first differences, log-differences, or percentage changes etc.), explain how we can still conduct a Granger Causality test.

(c) Below (independent from the previous questions) you can see an EViews Granger Causality test output.

Regression output 1:
Pairwise Granger Causality Tests
Sample: 1971 2006
Lags: 3

Null Hypothesis:	Obs	F-Statistic	Prob.
P does not Granger Cause Q	35	0.68700	0.4133
Q does not Granger Cause P		4.21072	0.0484

Write out H_0 and H_1 for the above F-tests (in words, you do not need to use parameter values).

(d) If we conduct a Granger causality test at 5% level of significance, what can we conclude from Regression equation 1? Explain the implications!

SUGGESTED ANSWERS:

(a): See pptx about Multiequation Time-Series Models (VAR & Granger) where this model is specified.
$y_t = c_1 + a_1 x_{t-1} + a_2 x_{t-2} + a_3 x_{t-3} + b_1 y_{t-1} + b_2 y_{t-2} + b_3 y_{t-3} + \varepsilon 1_t$
$x_t = c_2 + a_1 x_{t-1} + a_2 x_{t-2} + a_3 x_{t-3} + b_1 y_{t-1} + b_2 y_{t-2} + b_3 y_{t-3} + \varepsilon 2_t$

(b): Easiest that you just check the TYDL in the lecture notes where you have all the explanations. Takes up too much space here.

(c): *(i): H0: P does not Granger cause Q, H1: P does Granger cause Q*
(ii): H0: Q does not Granger cause P, H1: Q does Granger cause P

(d): *(i): p-value=0.4133 ---> H0 cannot be rejected --> H0: P does not Granger cause Q*
(ii): p-value=0.0484 ---> H0 can be rejected (at 5% level of sign.) --> H0: P does not Granger cause Q

Question 8 (5 points)

Assume that we have the following processes:

$X_t = X_{t-1} + \omega_t \quad \omega_t \sim WN$
$Y_t = Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim WN$

where ω_t and ε_t are independent identically distributed white-noise processes, and X_t and Y_t are not cointegrated.

Let's assume that the following model will be estimated: $Y_t = \alpha + \beta X_t + u_t$ (Equation A):

Dependent Variable: Y

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.050638	0.286633	-10.64300	0.0000
X	-0.394909	0.051691	-7.639825	0.0000
R-squared	0.373269	Mean dependent var		-4.756460

and that the following equation will be estimated $\Delta Y_t = \delta + \gamma \Delta X_t + v_t$ (Equation B)

Dependent Variable: D(Y)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.059322	0.115759	-0.512464	0.6095
D(X)	0.068723	0.089802	0.765278	0.4460
R-squared	0.006001	Mean dependent var		-0.051743

(a) Draw conclusions regarding regression equation A *[Briefly explain what main conclusions we can draw, and briefly explain if there are any major problems when we estimate this regression model. Please use standard statistical terms when you explain.]*

(b) Draw conclusions regarding regression equation B *[Briefly explain what main conclusions we can draw, and briefly explain if there are any major problems when we estimate this regression model. Please use standard statistical terms when you explain.]*

SUGGESTED ANSWERS:

(a) We should not interpret this relationship, since there is a risk of a **spurious regression** since $X_t = X_{t-1} + \omega_t$. $\omega_t \sim WN$. $Y_t = Y_{t-1} + \varepsilon_t$. $\varepsilon_t \sim WN$ (and X & Y are not cointegrated).

(b) The R^2 is very low, so D(X) can only explain less than 1 % of the total variation in D(Y), and the slope coefficient is not significant. No relationship between the variables.

Question 9 (12 points)

In Fair (1978, A Theory of Extramarital Affairs. Journal of Political Economy, 86, 45–61) it was investigated what factors that determine if a partner has an extramarital affair. In the paper the author uses regression analysis and in this question we use the same data set. It consists of 601 observations, and we will model the number of extramarital affairs as a function of Age (in years), Religiousness (measured 1-5 how religious a person is) and Years married. The output from EViews is shown below.

Dependent Variable: AFFAIRS
Method: ML/QML - Poisson Count (Newton-Raphson / Marquardt steps)
Date: 03/08/22 Time: 10:11
Sample: 1 601
Included observations: 523
Convergence achieved after 6 iterations
Coefficient covariance computed using observed Hessian

Variable	Coefficient	Std. Error	z-Statistic	Prob.
AGE	-0.026293	0.007682	-3.422735	0.0006
RELIGIOUSNESS	-0.501516	0.041870	-11.97808	0.0000
YEARSMARRIED	0.161021	0.013248	12.15450	0.0000
C	0.645588	0.213041	3.030352	0.0024

R-squared	0.069338	Mean dependent var	0.869981
Adjusted R-squared	0.063959	S.D. dependent var	2.726751
S.E. of regression	2.638110	Akaike info criterion	3.803821
Sum squared resid	3612.046	Schwarz criterion	3.836399
Log likelihood	-990.6991	Hannan-Quinn criter.	3.816580
Restr. log likelihood	-1145.660	LR statistic	309.9213
Avg. log likelihood	-1.894262	Prob(LR statistic)	0.000000

(a) Calculate the marginal effects of age on the number of extramarital affairs the individual has. Interpret it in words.

(b) Test the overall significance of the model at the 5 % level of significance.

Question 10 (12 points)

Evaluation of empirical/estimation strategy:
A recent paper examines the effect of networking on innovations (a binary variable measured as if the firm has introduced a new product or not the past year). The theory evaluated is that firms that do more networking are also more innovative. The main independent variable here is networking measured as the number of contacts the firm has. Then the researcher includes control variables which are the size of the firms and profit of the company. Based on 2000 observations the model is estimated using OLS and the result is as follows:

	Coefficient	Standard error
Networking	1.2	0.3
Size	0.2	0.1
Profit	0.3	0.12

R-square	F-statistic	No. Of observations
0.3	4.2	29

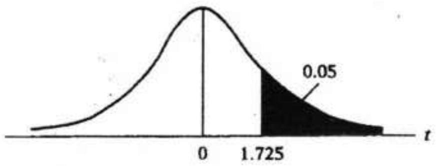
(a) Do you find any support for the researcher's hypothesis?

(b) Discuss the merits and weaknesses of the empirical strategy and the choice of estimation method of the econometrical model.

PERCENTAGE POINTS OF THE t DISTRIBUTION

Example

$\Pr(t > 2.086) = 0.025$
$\Pr(t > 1.725) = 0.05$ for df = 20
$\Pr(|t| > 1.725) = 0.10$



df \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

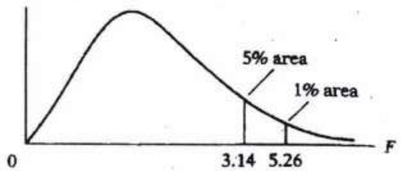
Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

TABLE D.3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION

Example

$\Pr(F > 1.59) = 0.25$
$\Pr(F > 2.42) = 0.10$ for df $N_1 = 10$
$\Pr(F > 3.14) = 0.05$ and $N_2 = 9$
$\Pr(F > 5.26) = 0.01$



df for denominator N_2	Pr	1	2	3	4	5	6	7	8	9	10	11	12
		df for numerator N_1											
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.05	161	200	216	225	230	234	237	239	241	242	243	244
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 18, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

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TABLE D.3 UPPER PERCENTAGE POINTS OF THE *F* DISTRIBUTION (*Continued*)

df for denominator N_2	Pr	df for numerator N_1: 1	2	3	4	5	6	7	8	9	10	11	12
10	.25	1.49	1.80	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
11	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
13	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

TABLE D.3 UPPER PERCENTAGE POINTS OF THE *F* DISTRIBUTION (*Continued*)

df for denominator N_2	Pr	df for numerator N_1: 1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

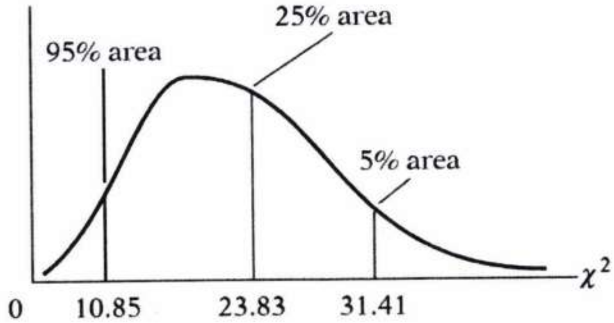
UPPER PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

Example

$\Pr(\chi^2 > 10.85) = 0.95$

$\Pr(\chi^2 > 23.83) = 0.25$ for df = 20

$\Pr(\chi^2 > 31.41) = 0.05$



Degrees of freedom \ Pr	.995	.990	.975	.950	.900
1	392704×10^{-10}	157088×10^{-9}	982069×10^{-9}	393214×10^{-8}	.0157908
2	.0100251	.0201007	.0506356	.102587	.210720
3	.0717212	.114832	.215795	.351846	.584375
4	.206990	.297110	.484419	.710721	1.063623
5	.411740	.554300	.831211	1.145476	1.61031
6	.675727	.872085	1.237347	1.63539	2.20413
7	.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100*	67.3276	70.0648	74.2219	77.9295	82.3581

*For df greater than 100 the expression $\sqrt{2\chi^2} - \sqrt{(2k-1)} = Z$ follows the standardized normal distribution, where k represents the degrees of freedom.

	.750	.500	.250	.100	.050	.025	.010	.005
1	.1015308	.454937	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	.575364	1.38629	2.77259	4.60517	5.99147	7.37776	9.21034	10.5966
3	1.212534	2.36597	4.10835	6.25139	7.81473	9.34840	11.3449	12.8381
4	1.92255	3.35670	5.38527	7.77944	9.48773	11.1433	13.2767	14.8602
5	2.67460	4.35146	6.62568	9.23635	11.0705	12.8325	15.0863	16.7496
6	3.45460	5.34812	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476
7	4.25485	6.34581	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777
8	5.07064	7.34412	10.2188	13.3616	15.5073	17.5346	20.0902	21.9550
9	5.89883	8.34283	11.3887	14.6837	16.9190	19.0228	21.6660	23.5893
10	6.73720	9.34182	12.5489	15.9871	18.3070	20.4831	23.2093	25.1882
11	7.58412	10.3410	13.7007	17.2750	19.6751	21.9200	24.7250	26.7569
12	8.43842	11.3403	14.8454	18.5494	21.0261	23.3367	26.2170	28.2995
13	9.29906	12.3398	15.9839	19.8119	22.3621	24.7356	27.6883	29.8194
14	10.1653	13.3393	17.1170	21.0642	23.6848	26.1190	29.1413	31.3193
15	11.0365	14.3389	18.2451	22.3072	24.9958	27.4884	30.5779	32.8013
16	11.9122	15.3385	19.3688	23.5418	26.2962	28.8454	31.9999	34.2672
17	12.7919	16.3381	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185
18	13.6753	17.3379	21.6049	25.9894	28.8693	31.5264	34.8053	37.1564
19	14.5620	18.3376	22.7178	27.2036	30.1435	32.8523	36.1908	38.5822
20	15.4518	19.3374	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968
21	16.3444	20.3372	24.9348	29.6151	32.6705	35.4789	38.9321	41.4010
22	17.2396	21.3370	26.0393	30.8133	33.9244	36.7807	40.2894	42.7956
23	18.1373	22.3369	27.1413	32.0069	35.1725	38.0757	41.6384	44.1813
24	19.0372	23.3367	28.2412	33.1963	36.4151	39.3641	42.9798	45.5585
25	19.9393	24.3366	29.3389	34.3816	37.6525	40.6465	44.3141	46.9278
26	20.8434	25.3364	30.4345	35.5631	38.8852	41.9232	45.6417	48.2899
27	21.7494	26.3363	31.5284	36.7412	40.1133	43.1944	46.9630	49.6449
28	22.6572	27.3363	32.6205	37.9159	41.3372	44.4607	48.2782	50.9933
29	23.5666	28.3362	33.7109	39.0875	42.5569	45.7222	49.5879	52.3356
30	24.4776	29.3360	34.7998	40.2560	43.7729	46.9792	50.8922	53.6720
40	33.6603	39.3354	45.6160	51.8050	55.7585	59.3417	63.6907	66.7659
50	42.9421	49.3349	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900
60	52.2938	59.3347	66.9814	74.3970	79.0819	83.2976	88.3794	91.9517
70	61.6983	69.3344	77.5766	85.5271	90.5312	95.0231	100.425	104.215
80	71.1445	79.3343	88.1303	96.5782	101.879	106.629	112.329	116.321
90	80.6247	89.3342	98.6499	107.565	113.145	118.136	124.116	128.299
100*	90.1332	99.3341	109.141	118.498	124.342	129.561	135.807	140.169

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 8, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

Note on JIBS-provided calculator: You may be unused to how the JIBS-provided calculator is used in clearing information and how it is used with powers, natural logs, natural exponentials. Here's some instructions:

1) To clear the calculator (start afresh), press "AC".
2) To find x^y, press your x number, press x^y, press your y number, then press "=". For example, to find 2^3, press 2, press x^y , press 3, then press "=", which provides 8.
3) To find ln x, press your number, x, and then press ln. For example, to find ln 2, press 2 and then press ln, which provides 0.69314718.
4) To find e^x, press your number, x, press SHIFT, and then press e^x. For example, to find e^4, press 4, press SHIFT, and then press e^x, which provides 54.59815003.
5) If parentheses are needed to clarify the order of operations, the relevant keys are " (" and ") ". For example to find $(2 + 4) \times 3$, press "(", press 2, press +, press 4, press ")" , press ×, press 3, and then press "=" to get 18.

Also note the calculator will resolve calculations of powers, natural logs, and natural exponentials before calculations of addition, subtraction, multiplication, and division. Operations within parentheses are resolved before all other operations.

For example, suppose you are interested in calculating $4 \times 2^3 = 4 \times 8 = 32$. If you press 4, press "×", press 2, press x^y, press 3, and then press "=", the calculator will appropriately find $4 \times 2^3 = 32$, not $(4 \times 2)^3 = 8^3 = 512$. If instead you want to find $(4 \times 2)^3$, you can press "(", press 4, press "×", press 2, press ")", press x^y, press 3, and then press "=", to get 512. Alternatively, you can find what 4×2 equals first, and then find its third power: press 4, press "×", press 2, press "=" (so now you have 8), press x^y, and press 3, and then press "=" again, to get 512. If you want to find $5 \times (4 \times 2)^3$, you can calculate $(4 \times 2)^3$ first by either of the means mentioned above, then press "×", press "5", and then press "=", to give you $512 \times 5 = 2560$.

As another example, suppose you are interested in calculating $8^{2/3}$. If you press 8, press x^y, press 2, press "÷", press 3, and then press "=", you'll get the WRONG answer, 21.333..., because that sequence provides the calculation for $\frac{8^2}{3}$ instead. To calculate $8^{2/3}$, you can utilize the parentheses keys "(", and ")"; the parentheses forces the calculator to do calculations within parentheses first. Note that $8^{2/3}$ is the same as $8^{(2/3)}$, and using the parentheses clarifies what you mean to the calculator (that you want $8^{2/3}$, not $8^2/3$). So to find $8^{2/3}$, press 8, press x^y, press (, press 2, press "÷", press 3, press), and then press "=", to come out to the correct answer, 4.