Electromagnetic Compatibility (EMC)

Topic 4

Transmission Lines and Signal Integrity

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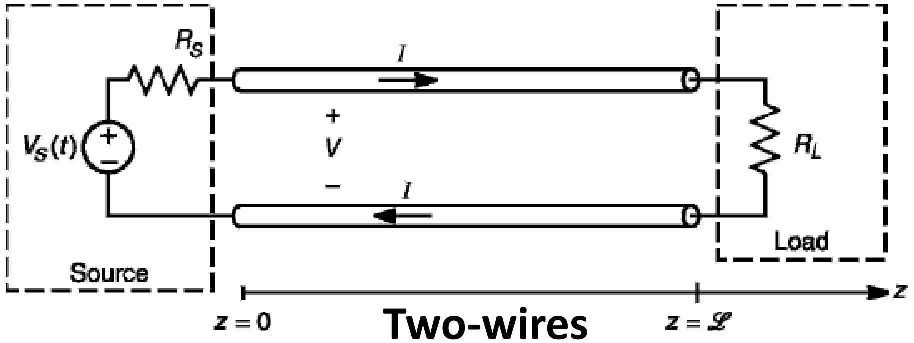
Introduction

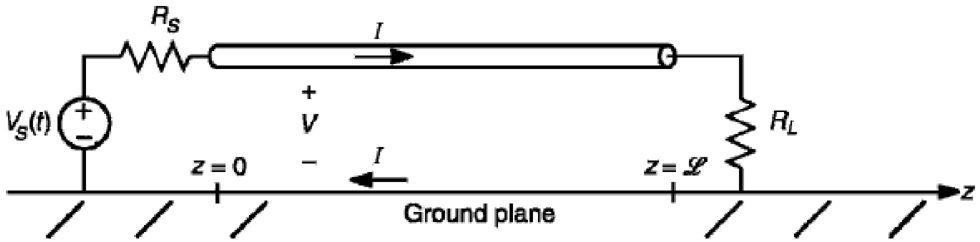
- Any conductor that connects two points in a system with a length that is long enough with respect to the signal rise time (i.e. comparable to a wavelength), will be have like a transmission line (T-line).
- A T-line will add delay to the signal propagation that needs to be accounted for in the system timing requirements and timing budget
- T-lines needs to be matched to avoid reflections (ringing) from the input and output to ensure proper power and signal shape delivery
- T-lines might add dispersion to the signal (i.e. frequency dependent response) which will yield noticeable distortion and increased errors
- Signal Integrity (SI) has to do with ensuring that the waveforms at the input of the line and the output are as close as possible. If the signals are corrupted with respect to one another, we say that we have distortion, or SI issues.
- Thus we should be careful when designing these T-line that will interconnect the various portions of the system and account for their effects and errors to have good SI performance.

Transmission Lines (review)

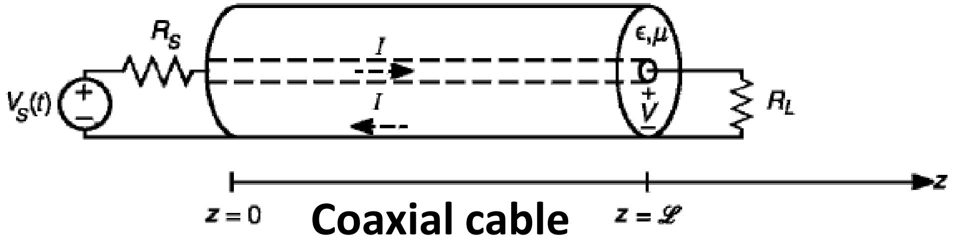
- Used to transfer energy (voltage/current signals/waves)
- primarily in TEM-mode, thus E-field and H-field are perpendicular to one another along the propagation direction
- T-lines are considered for various types of connections and configurations, and needs to be analyzed accordingly
- Not properly considering the line effects can results in poor impedance matching, ringing and crosstalk, thus degrading EMC compliance



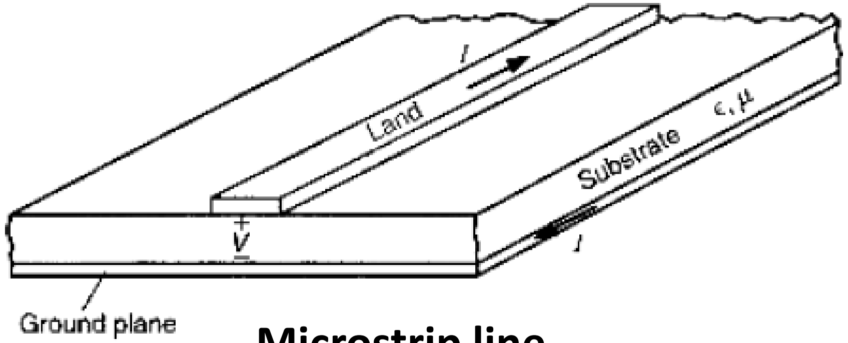
Two-wires
(a)



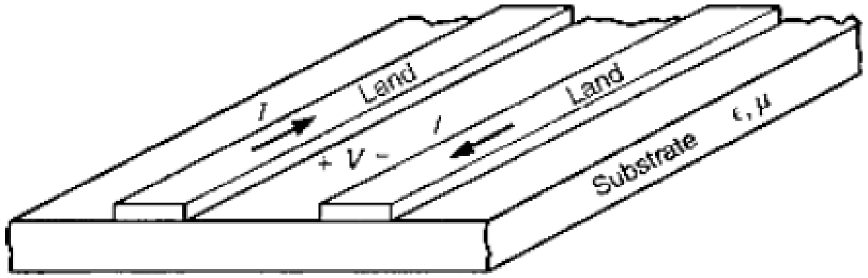
(b)
Line over GND



Coaxial cable
(c)



Microstrip line

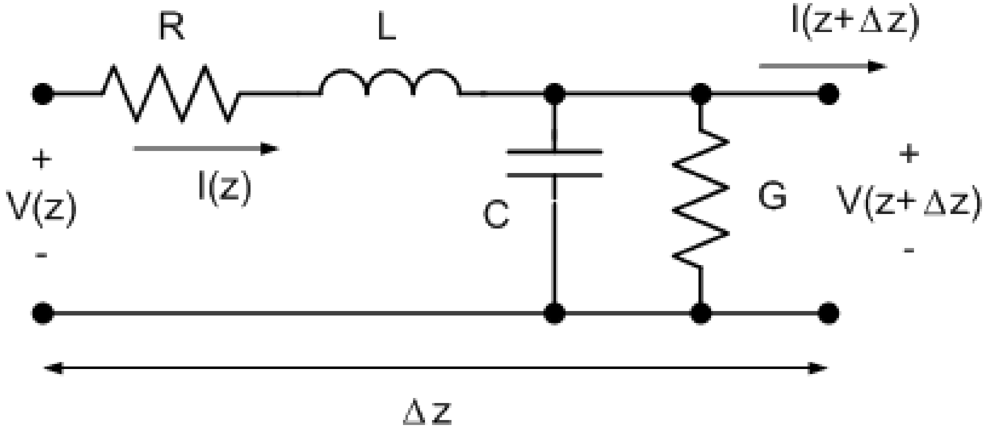


Stripline

$$(R+j\omega L)I(z)\Delta z+V(z+\Delta z)=V(z)$$

then,

$$\lim_{\Delta z\to 0}\left(\frac{-V(z+\Delta z)-V(z)}{\Delta z}\right)=-\frac{dV(z)}{dz}=(R+j\omega L)I(z) \qquad (1)$$



applying KCL,

$$I(z)-V(z+\Delta z)(G+j\omega C)\Delta z=I(z+\Delta z)$$

then,

$$\lim_{\Delta z\to 0}\left(\frac{I(z+\Delta z)-I(z)}{\Delta z}\right)=\frac{dI(z)}{dz}=-(G+j\omega C)V(z) \qquad (2)$$

- Differentiating (1) with respect to I(z), and substituting dI(z)/dz from (2) we get,

$$\frac{d^2V(z)}{dz^2} - k^2V(z) = 0 \qquad (3)$$

- Similarly, for (2)

$$\frac{d^2I(z)}{dz^2} - k^2I(z) = 0 \qquad (4)$$

where k (or γ)is the complex propagation constant,

$$k = k_r + jk_i = \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

- The solutions for (3) and (4) are (phasor form),

$$V(z) = V^+e^{-\gamma z} + V^-e^{\gamma z} \qquad (5)$$

$$I(z) = I^+e^{-\gamma z} + I^-e^{\gamma z} \qquad (6)$$

- (5) and (6) are related, substitute (5) in (1), we get:

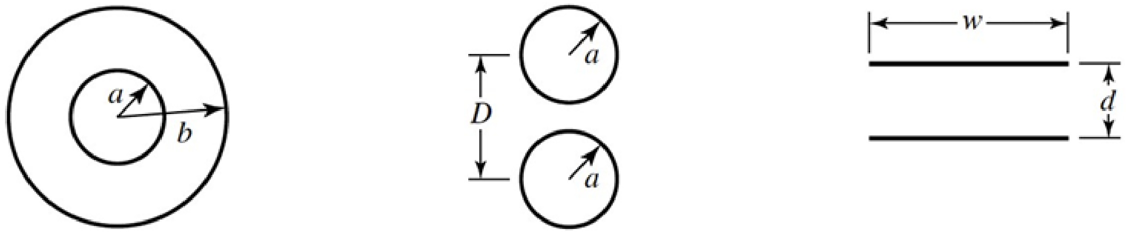
$$I(z) = \frac{k}{(R + j\omega L)}\left(V^+e^{-\gamma z} - V^-e^{\gamma z}\right)$$

- The characteristic impedance of a transmission line is then,

$$Z_0 = \frac{V^+(z)}{I^+(z)} = -\frac{V^-(z)}{I^-(z)} = \frac{(R + j\omega L)}{\gamma} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

- For a homogeneous medium, the propagation velocity is, $v = \dfrac{1}{\sqrt{\mu\varepsilon}}$
- For a lossless T-line, R=G=0, thus $LC = \mu\varepsilon$

Transmission Line Parameters for Some Common Lines



	COAX	TWO-WIRE	PARALLEL PLATE
L	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

A) Microstrip Transmission Lines

- Most widely used in RF/Microwave circuits
- Used in simple designs with single or double layers

narrow lines, $w/h<1$,

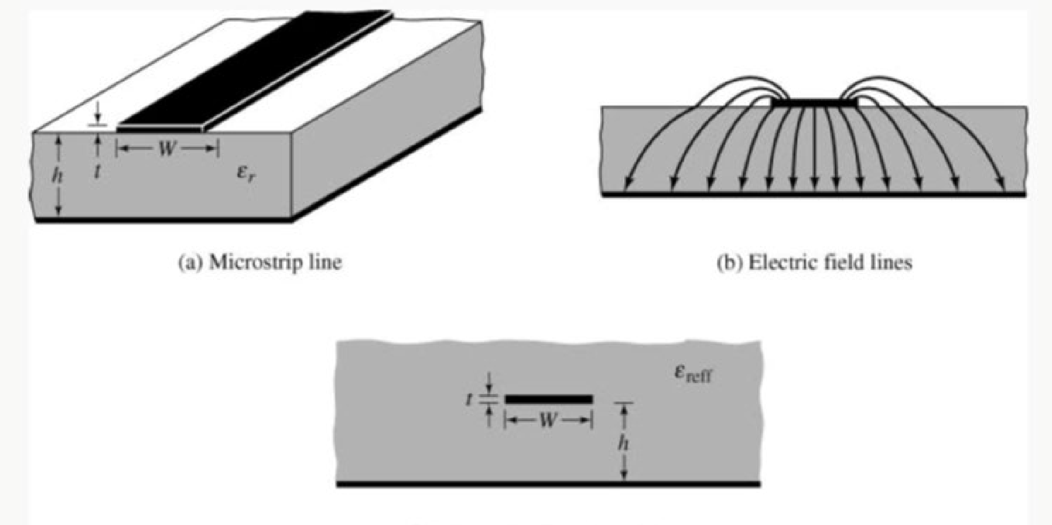
$$\varepsilon_{eff}=\frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\left[\left(1+12\frac{h}{w}\right)^{-1/2}+0.04\left(1-\frac{w}{h}\right)^2\right]$$, phase velocity $v_p=\frac{c}{\sqrt{\varepsilon_{eff}}},\lambda=\frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$

$$Z_0=\frac{Z_f}{2\pi\sqrt{\varepsilon_{eff}}}\ln\left(8\frac{h}{w}+\frac{w}{4h}\right)$$, $Z_f=\sqrt{\frac{\mu_0}{\varepsilon_0}}=376.5\Omega$, propagation constant is $\beta=k_0\sqrt{\varepsilon_{eff}}$

wide lines, $w/h>1$,

$$\varepsilon_{eff}=\frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\left(1+12\frac{h}{w}\right)^{-1/2}$$

$$Z_0=\frac{Z_f}{\sqrt{\varepsilon_{eff}}\left(1.393+\frac{w}{h}+\frac{2}{3}\ln\left(\frac{w}{h}+1.444\right)\right)}$$



(a) Microstrip line
(b) Electric field lines
(c) Effective dielectric constant

- Considering the MS T-Line as a quasi-TEM line, dielectric losses can be found using,

$$\alpha_d = \frac{k_0 \varepsilon_r \left(\varepsilon_{eff} - 1\right) \tan\delta}{2\sqrt{\varepsilon_{eff}}\left(\varepsilon_r - 1\right)} \quad \text{Np/m}$$

1 Np/m = 8.68 dB/m

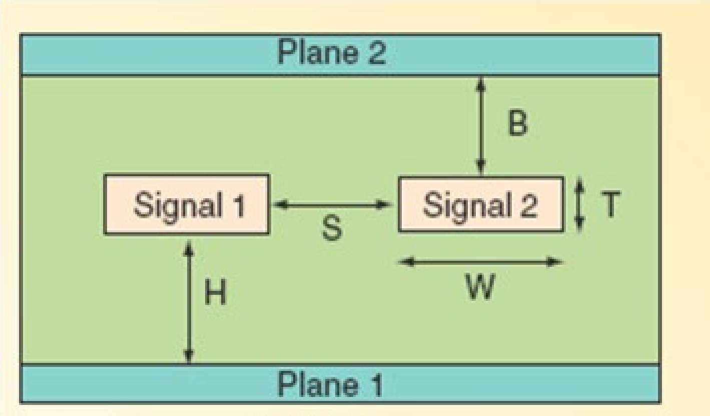
- And conductor losses as, $\alpha_c = \dfrac{R_s}{Z_0 w} \quad \text{Np/m}, \quad \underbrace{R_s = \sqrt{\dfrac{\omega\mu_0}{2\sigma}}}_{\text{surface-resistivity}}$

- With an FR-4 substrate (ε_r=4.7), the velocity is

$$v_p = \frac{c}{\sqrt{\varepsilon_{eff}}} \approx \frac{c}{\sqrt{2.85}} = 1.777 \times 10^8 \quad m/s, \text{ thus}$$

$$T_{PD} = 5.6 \text{ nsec/m, or } 56.3 \text{ ps/cm or } 143 \text{ ps/in}$$

B) Stripline Transmission lines



(a) Stripline

- Usually used in digital PCBs.
- Wave propagation is slower than Microstrips (recall that effective dielectric constant is Less than the dielectric constant itself)

$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{4(H+B)}{0.67\pi(T+0.8W)} \right)$$

$$v_p = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}} = 1.384 \times 10^8 \quad m/s$$

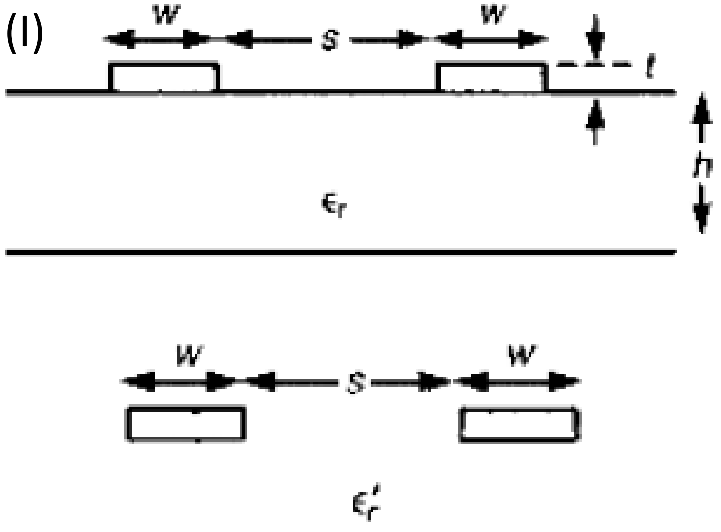
$$T_{PD} = \frac{1}{v_p} = \frac{\sqrt{\varepsilon_r}}{c} = 7.2 \text{ nsec/m, or } 72.3 \text{ psec/cm or } 183.6 \text{ psec/in}$$

C) Other PCB Transmission lines

- **Configuration I** (and assuming t=0):

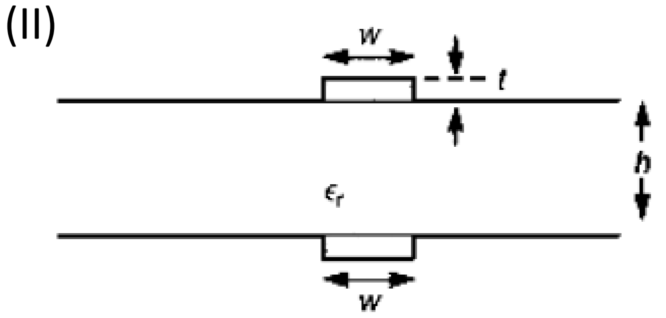
$$Z_c = \begin{cases} \dfrac{120}{\sqrt{\varepsilon_r'}} \ln\left(2\dfrac{1+\sqrt{k}}{1-\sqrt{k}}\right), & \dfrac{1}{\sqrt{2}} \le k \le 1 \qquad , \quad k = \dfrac{s}{s+2w} \\ \dfrac{377\pi}{\sqrt{\varepsilon_r'}\ln\left(2\dfrac{1+\sqrt{k'}}{1-\sqrt{k'}}\right)}, & 0 \le k \le \dfrac{1}{\sqrt{2}} \qquad , \quad k' = \sqrt{1-k^2} \end{cases}$$

$$\varepsilon_r' = \frac{\varepsilon_r+1}{2}\left[\tanh\left(0.775\ln\left(\frac{h}{w}\right)+1.75\right)+\frac{kw}{h}\left(0.04-0.7k+0.01\left(1-0.1\varepsilon_r\right)\left(0.25+k\right)\right)\right]$$



- **Configuration II** (and assuming t=0), not very common:

$$Z_c = \begin{cases} \dfrac{377}{\sqrt{\varepsilon_r}\left\{\dfrac{w}{h}+0.441+\dfrac{\varepsilon_r+1}{2\pi\varepsilon_r}\left[\ln\left(\dfrac{w}{h}+1.451\right)\right]+0.082\dfrac{\varepsilon_r-1}{\varepsilon_r^2}\right\}} & , \text{ for } \dfrac{w}{h}>1 \\ \dfrac{377\sqrt{2}}{\pi\sqrt{\varepsilon_r+1}}\left[\ln\left(\dfrac{4h}{w}\right)+\dfrac{1}{8}\left(\dfrac{w}{h}\right)^2-\dfrac{1}{2}\left(\dfrac{\varepsilon_r-1}{\varepsilon_r+1}\right)\left(0.452+\dfrac{0.242}{\varepsilon_r}\right)\right] & , \text{ for } \dfrac{w}{h}<1 \end{cases}$$



Time Domain Solution

- The time domain solution of the T-line equations refers to the complete solution with no assumptions on the time form of the excitation.
- We saw the sinusoidal steady-state solution (phasor) before.
- The time domain solution is also considered the "transient solution" because the total solution includes the transient plus steady-state components.
- V+ is the forward traveling wave (+z direction), V- is the reverse traveling wave (-z direction)
- Graphical methods are very widely used and beneficial.

$$V(z,t) = V^{+}\left(t-\frac{z}{v}\right) + V^{-}\left(t+\frac{z}{v}\right)$$

$$I(z,t) = \frac{V^{+}}{Z_c}\left(t-\frac{z}{v}\right) - \frac{V^{-}}{Z_c}\left(t+\frac{z}{v}\right)$$

$$Z_c = \sqrt{\frac{L}{C}} \rightarrow \text{real} \quad \text{(lossless)}$$

the load reflection coefficient is

$$\Gamma_{\mathrm{L}} = \frac{V^{-}\left(t+\frac{l}{v}\right)}{V^{+}\left(t-\frac{l}{v}\right)} = \frac{R_L - Z_c}{R_L + Z_c}$$

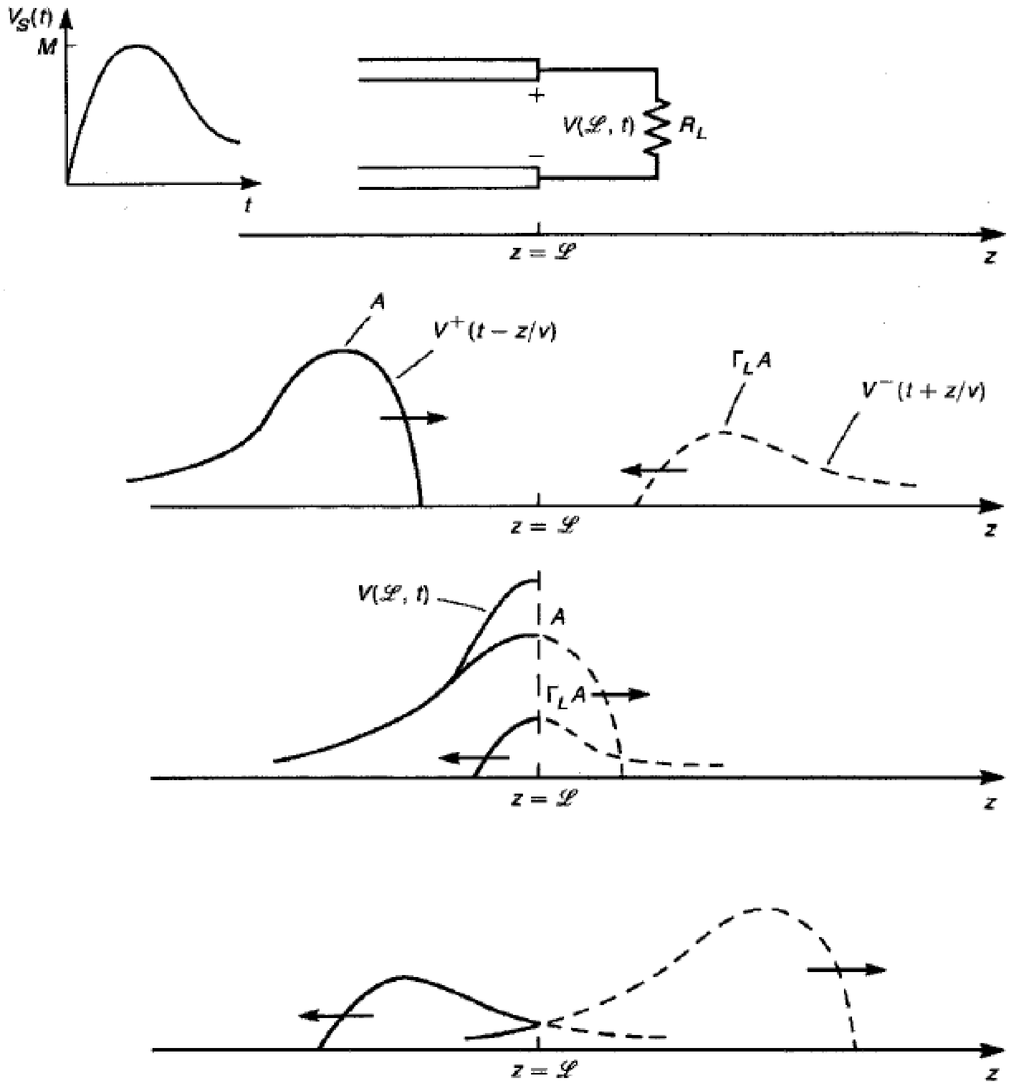
$$\rightarrow V^{-}\left(t+\frac{l}{v}\right) = \Gamma_{\mathrm{L}} V^{+}\left(t-\frac{l}{v}\right)$$

$$\rightarrow I^{-}\left(t+\frac{l}{v}\right) = -\Gamma_{\mathrm{L}} I^{+}\left(t-\frac{l}{v}\right)$$

- Note that the voltage at any point on the line will be the sum of the forward signal and the reflected one.
- For example, at the load, the total voltage is the sum of the two individual waves (incident and reflected) as a particular time instant as shown →
- Roundtrip time is $2T_D = 2\left(\frac{l}{v}\right)$

, thus, for anytime less than $2T_D$, the total voltage and current at z=0 (at input of T-line) will consist only of the forward-traveling waves V+ and I+ (since the reflected signal did not reach input yet.

$$V(0,t) = V^+\left(t - \frac{0}{v}\right) \quad , \; 0 \le t \le \frac{2l}{v}$$



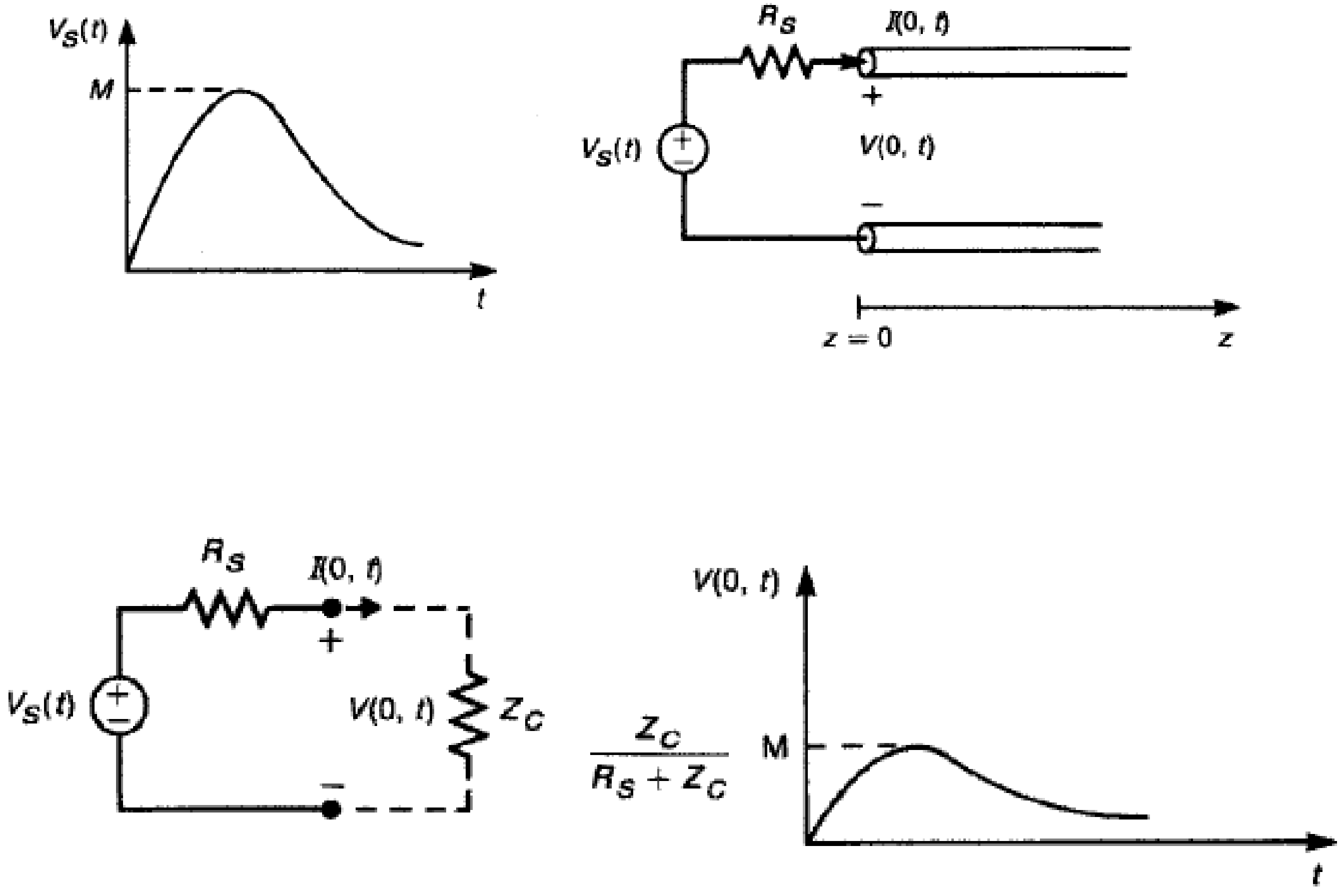
- Relating the input of the T-line to the source, we have

$$V\left(0,t\right)=\frac{Z_c}{R_s+Z_c}V_s(t)$$

- and the current is $$I\left(0,t\right)=\frac{V_s(t)}{R_s+Z_c}$$

- The source reflection coefficient will be

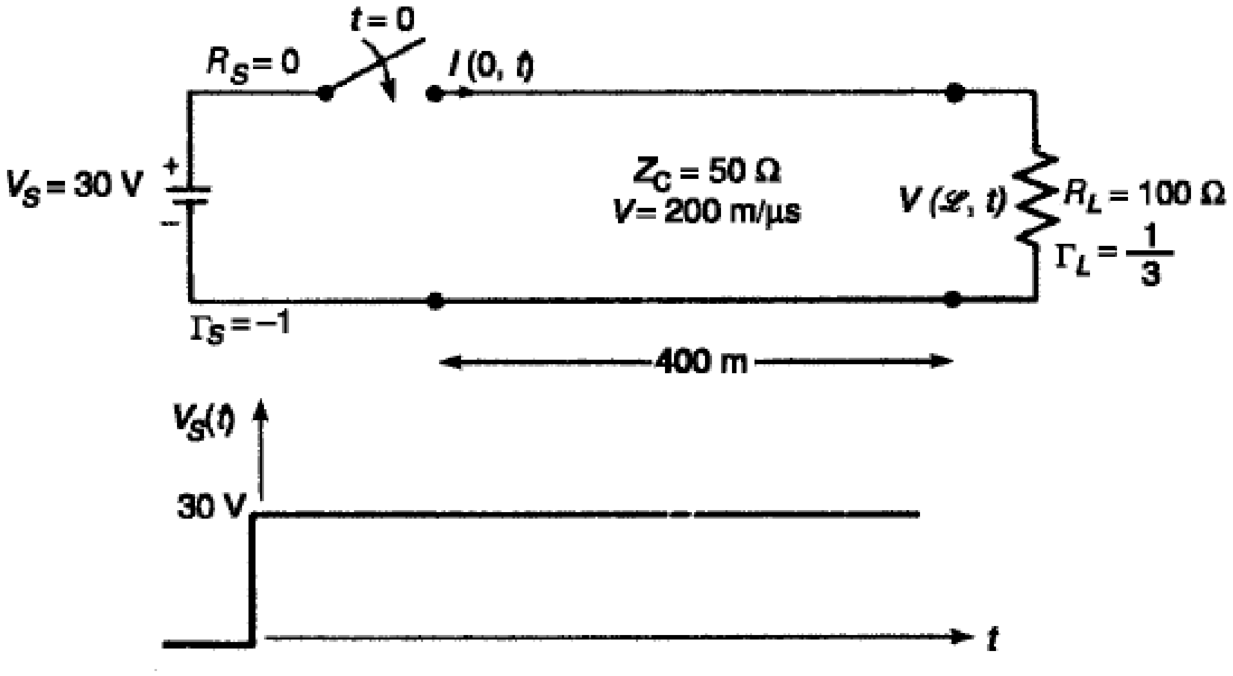
$$\Gamma_s=\frac{R_s-Z_c}{R_s+Z_c}$$



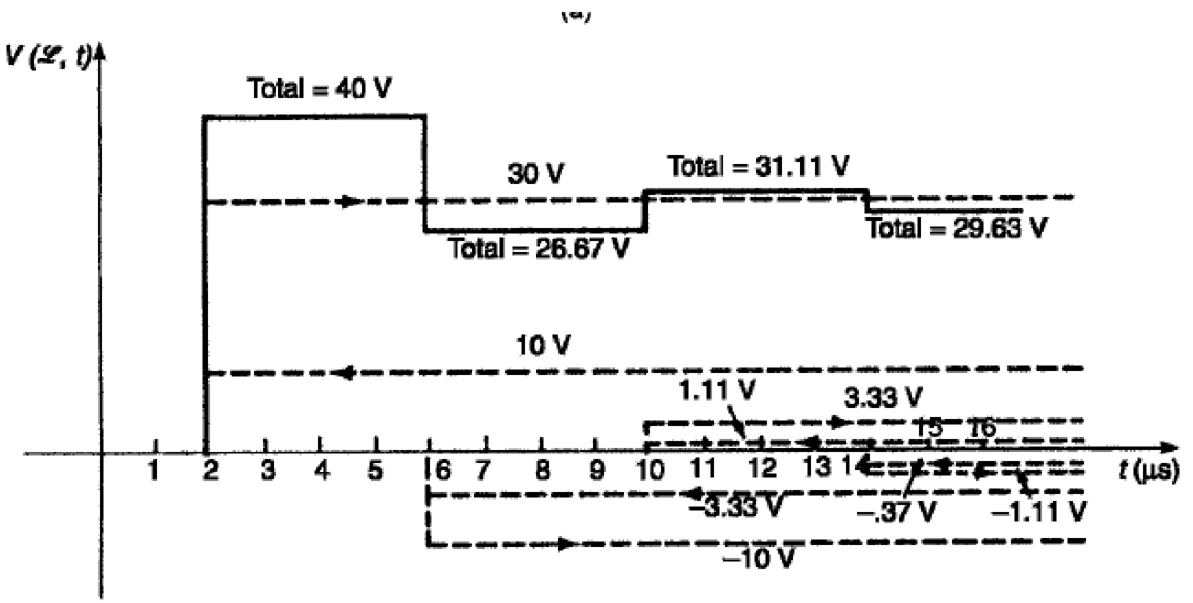
Example 4.1

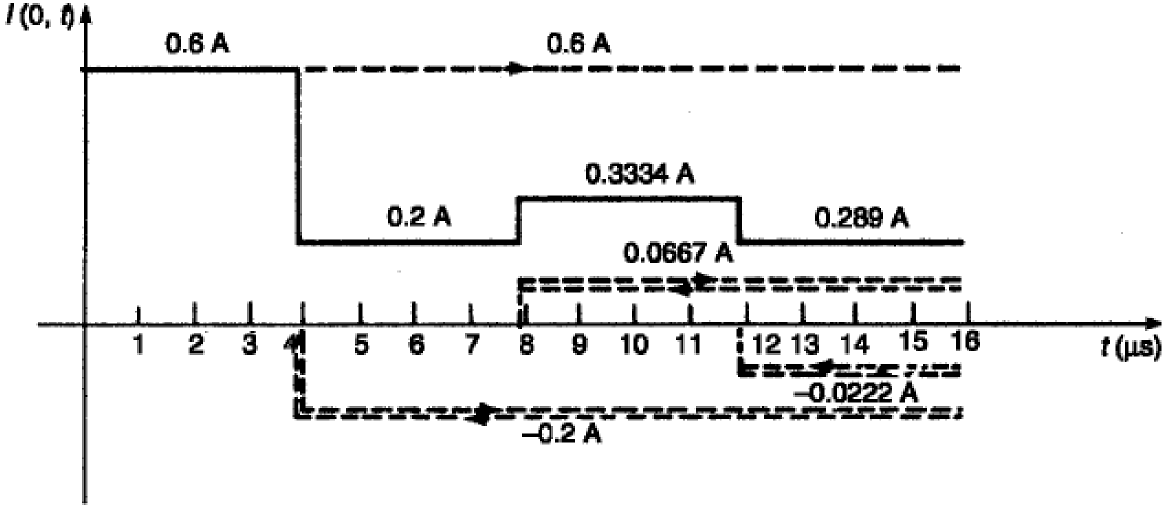
- Consider the circuit with a T-line as shown.
- The leading edge of the voltage reaches the Load after 2µsec (l/v).
- @ 2µsec, $V_L=30+30*\Gamma_L$ = 30+10=40V
- Reflected wave is V_{r1}=10V.
- @4µsec, $Vs=30+V_{r1}+V_{r1}*\Gamma_s$ =30+10-10=30V
- @6µsec, $V_L=40+30*\Gamma_L\,\Gamma_s+30*\Gamma_L\,\Gamma_s\,\Gamma_L$

  V_L=40-10-3.33=26.67V
- Reflected wave to source is V_{r2}=-3.3V
- @8µsec, $Vs=30+V_{r1}+V_{r1}*\Gamma_s+V_{r2}+V_{r2}*\Gamma_s$ =30+10-10-3.33+3.33=30V
- Reflected wave to load is 3.33V….



- The waveforms of VL and Is as a function of time are shown according to the calculations from the previous slide →





- In general,

$$V(0,t)=\frac{Z_c}{R_s+Z_c}\Big[V_s(t)+(1+\Gamma_s)\Gamma_L V_s(t-2T_D)$$
$$+(1+\Gamma_s)(\Gamma_s\Gamma_L)\Gamma_L V_s(t-4T_D)$$
$$+(1+\Gamma_s)(\Gamma_s\Gamma_L)^2\Gamma_L V_s(t-6T_D)+\cdots\Big]$$

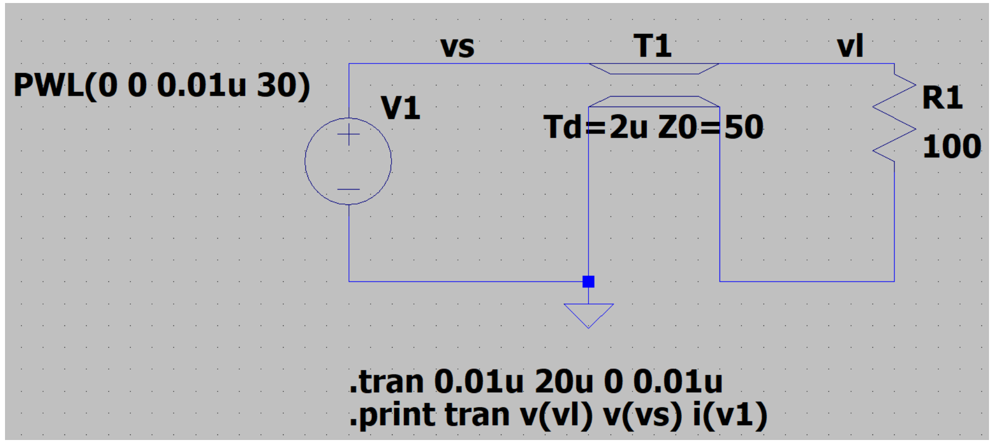
$$V(l,t)=\frac{Z_c}{R_s+Z_c}\Big[(1+\Gamma_L)V_s(t-T_D)+(1+\Gamma_L)\Gamma_s\Gamma_L V_s(t-3T_D)$$
$$+(1+\Gamma_L)(\Gamma_s\Gamma_L)^2 V_s(t-5T_D)$$
$$+(1+\Gamma_L)(\Gamma_s\Gamma_L)^3 V_s(t-7T_D)+\cdots\Big]$$

- If T-line is matched at load, i.e. $\Gamma_L=0$, then

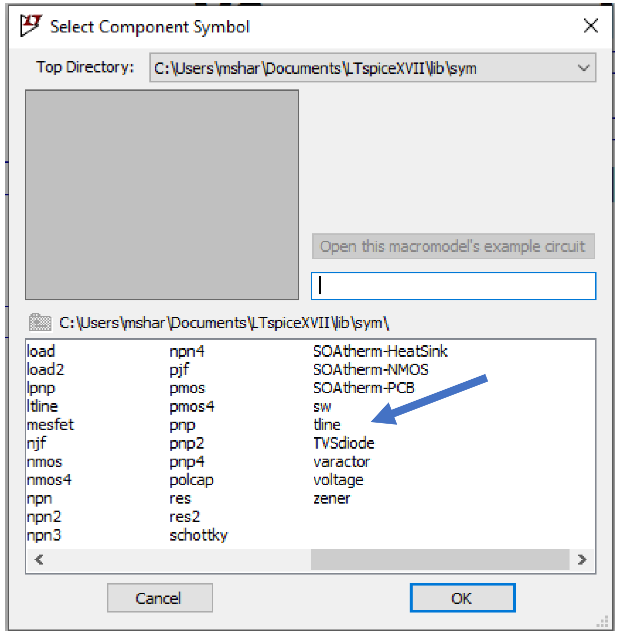
$$V(0,t)=\frac{Z_c}{R_s+Z_c}V_s(t)$$

$$V(l,t)=\frac{Z_c}{R_s+Z_c}V_s(t-T_D)$$

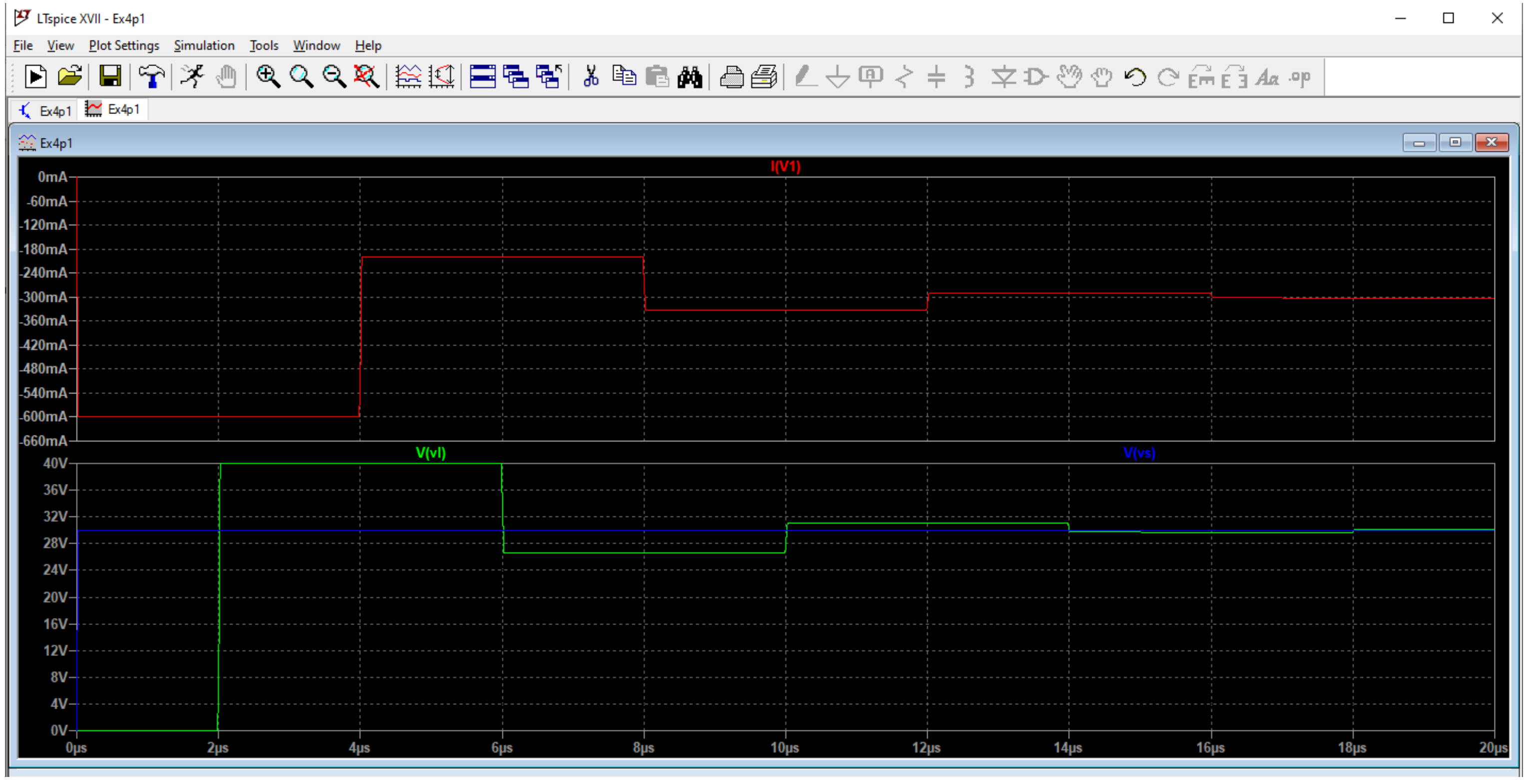
In LTSPICE



PWL(0 0 0.01u 30)
V1
vs
T1
vl
Td=2u Z0=50
R1
100
.tran 0.01u 20u 0 0.01u
.print tran v(vl) v(vs) i(v1)



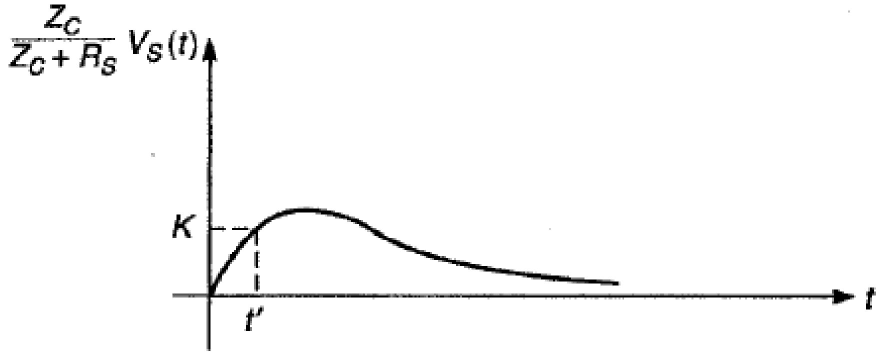
Select Component Symbol
Top Directory: C:\Users\mshar\Documents\LTspiceXVII\lib\sym
Open this macromodel's example circuit
C:\Users\mshar\Documents\LTspiceXVII\lib\sym\
load
load2
lpnp
ltline
mesfet
njf
nmos
nmos4
npn
npn2
npn3
npn4
pjf
pmos
pmos4
pnp
pnp2
pnp4
polcap
res
res2
schottky
SOAtherm-HeatSink
SOAtherm-NMOS
SOAtherm-PCB
sw
tline
TVSdiode
varactor
voltage
zener
Cancel
OK

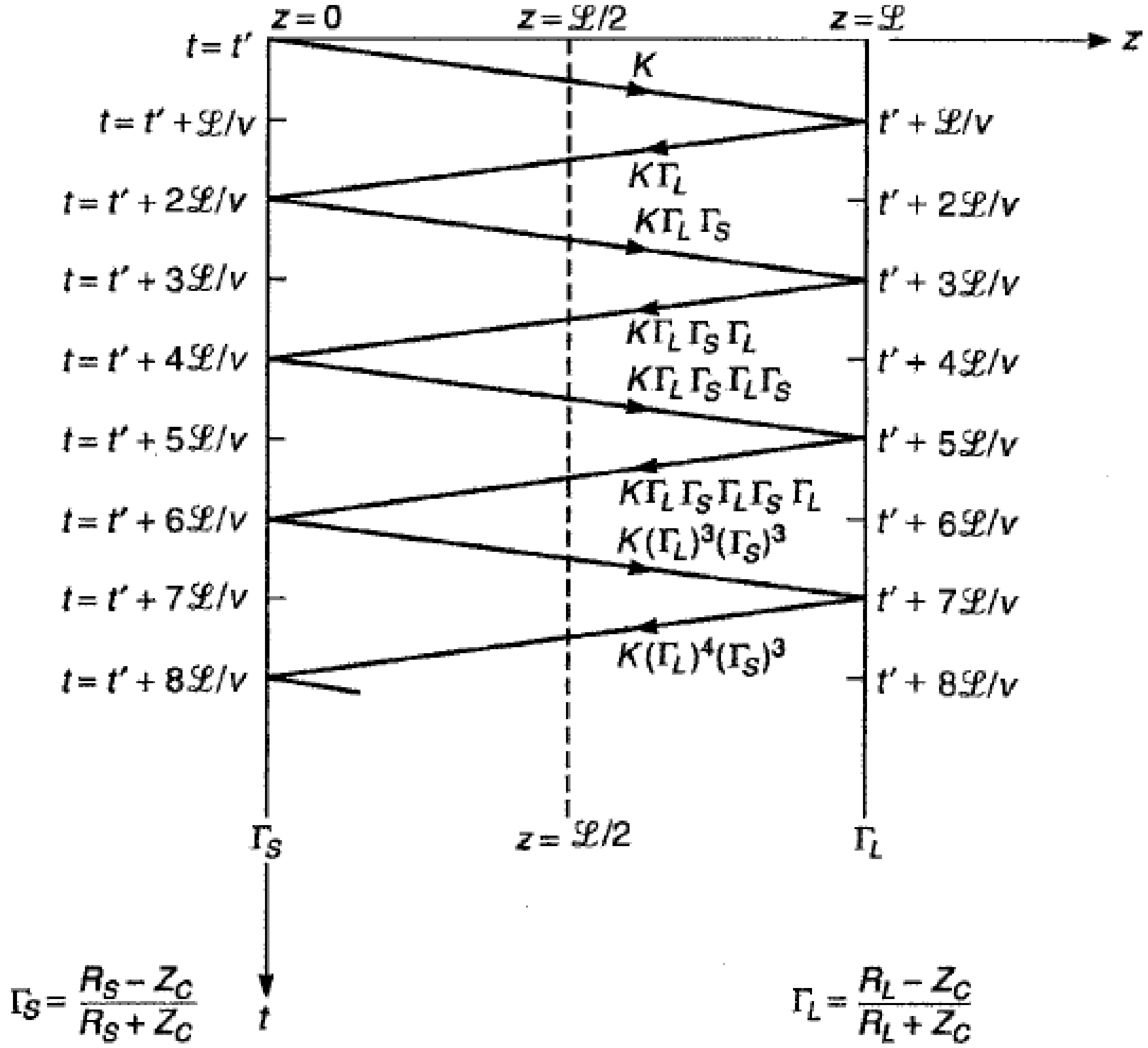


LTspice XVII - Ex4p1
File View Plot Settings Simulation Tools Window Help
Ex4p1
I(V1)
0mA
-60mA
-120mA
-180mA
-240mA
-300mA
-360mA
-420mA
-480mA
-540mA
-600mA
-660mA
V(vl)
V(vs)
40V
36V
32V
28V
24V
20V
16V
12V
8V
4V
0V
0µs
2µs
4µs
6µs
8µs
10µs
12µs
14µs
16µs
18µs
20µs

The Bounce (lattice) Diagram (lossless lines)

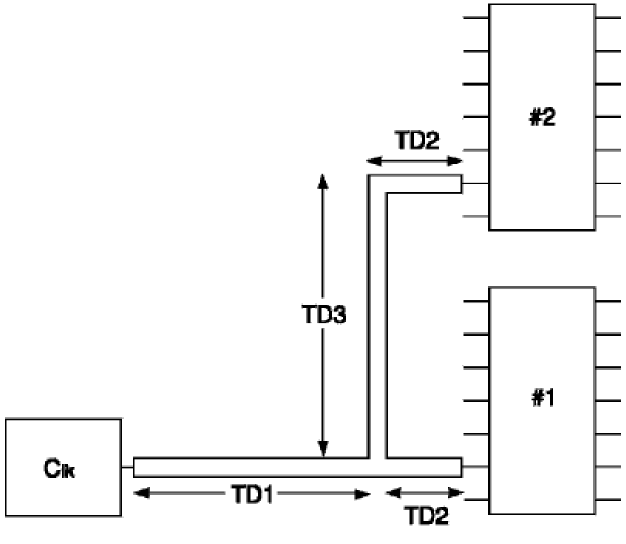
- Can be helpful to predict the levels at the source or load ends as well as at any other point in time
- A graphical method



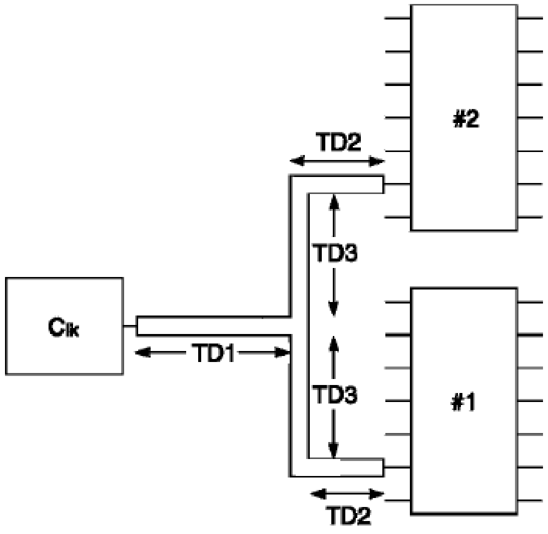


High Speed Digital Interconnects and Signal Integrity

- Signal Integrity (SI) refers to ensuring that a transmitted signal (digital pulse) through a medium (T-line) arrives to its destination with the proper desired level and shape (for proper detection and correct interpretation).
- Time delay due to signal propagation in T-lines can cause series signal detection issues, specially *signal skew (clock skew).*
- In SI, we need to account for all discontinuities within the signal path that might affect the delay, the impedance or the proper return current paths.
- Consider the time delay issue in the figure shown. Note that the clock signal in Figure (a) will reach part 2 much later than part 1, thus we cannot rely on this clock signal as a common reference. A better way to route the traces (on a PCB) is to follow Figure (b).



(a)



(b)

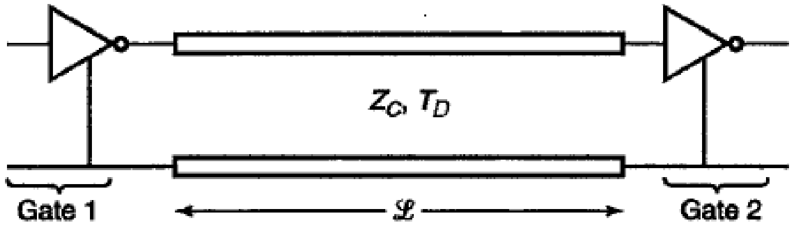
(a) Termination effects

- In CMOS gates, the output resistance (nonlinear) is in the order 10-30 Ω, the input of such gates are capacitive on the order 5-15pF.
- We will represent the source and load as resistances, i.e. a capacitor is open circuit.
- The impedance mismatches will cause ringing effects (recall the bounce diagram). Suppose we have a 5V step function source as shown (Vs(t)). Assume a 50 Ω T-line is used to connect the two gates
- The reflection coefficients are,

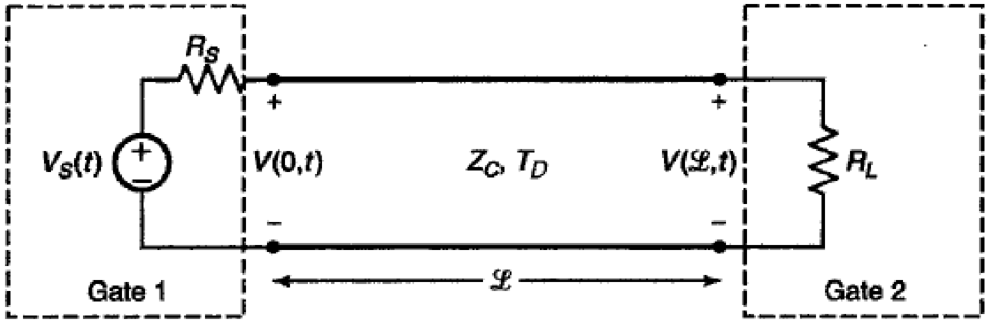
$$\Gamma_s = \frac{10-50}{10+50} = -\frac{2}{3}$$

$$\Gamma_L = \frac{\infty-50}{\infty+50} = +1$$

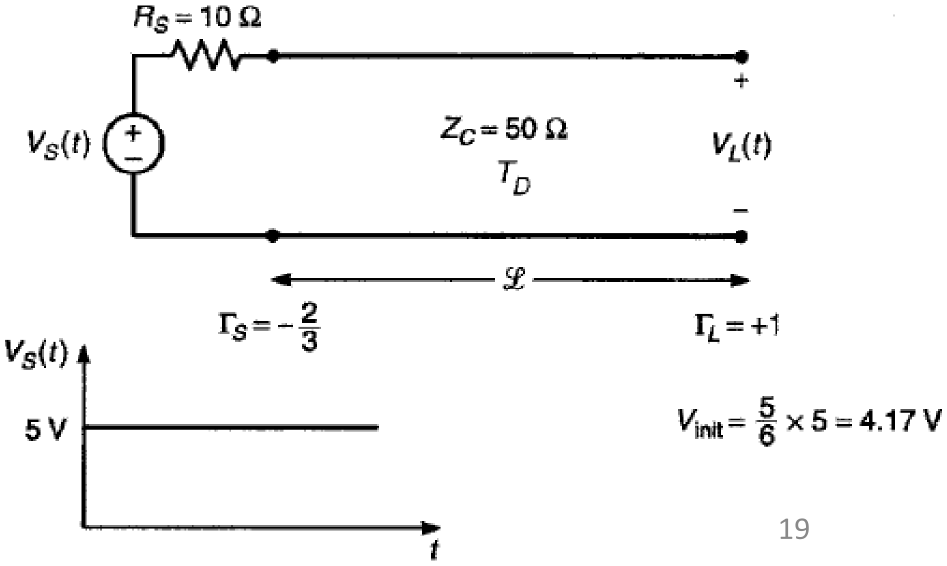
$$V_{init} = \frac{50}{10+50} \times 5 = 4.17\ V$$



(a)

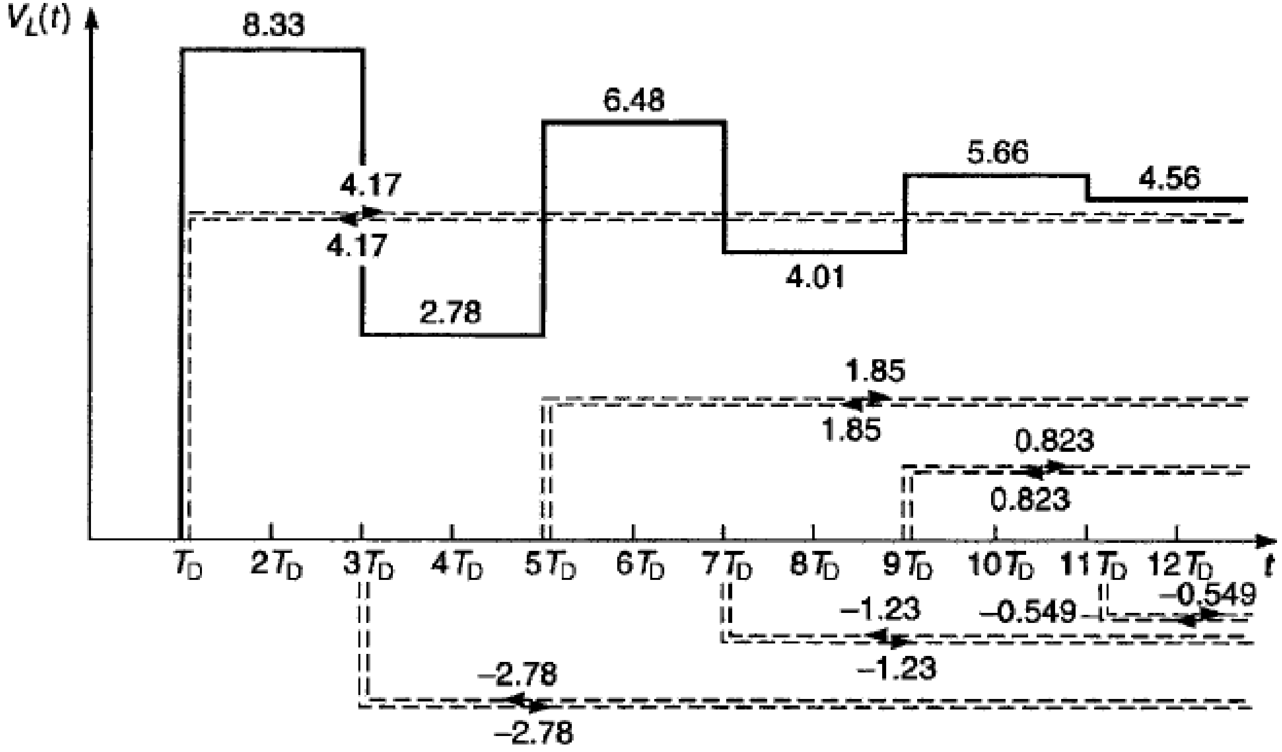


(b)

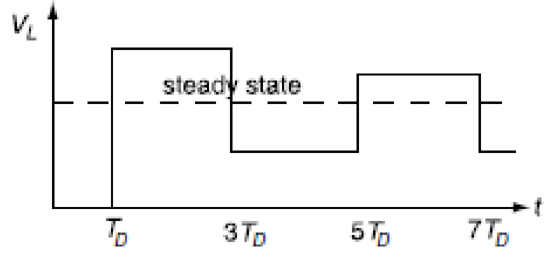
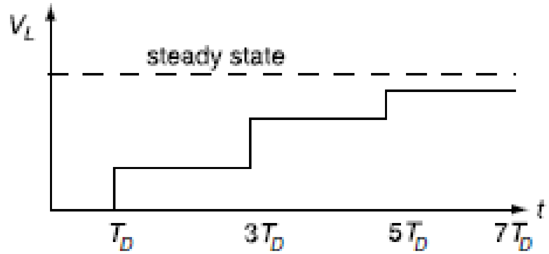


- Graphing the wave at the load side $V_L(t)$, will give the figure on the side.
- We can also use the closed form expression we developed before, as,

$$V_L(l,t) = \frac{Z_c}{R_s + Z_c}(1+\Gamma_L)\Big[V_s(t-T_D) + \Gamma_s\Gamma_L V_s(t-3T_D) + (\Gamma_s\Gamma_L)^2 V_s(t-5T_D) + (\Gamma_s\Gamma_L)^3 V_s(t-7T_D) + \cdots\Big]$$



- Note that in most logic gate circuits, we will have opposite signs for the source and load reflection coefficients, and thus we will have ***ringing***.
- Table shows the signal oscillations as a function of the signs of the Γ's

Γ_S	Γ_L	Load Voltage Waveform
$-$ $R_S < Z_C$ $+$ $R_S > Z_C$	$+$ $R_L > Z_C$ $-$ $R_L < Z_C$	
$+$ $R_S > Z_C$ $-$ $R_S < Z_C$	$+$ $R_L > Z_C$ $-$ $R_L < Z_C$	

- In actual scenarios, the load impedance is **capacitive** (input of a CMOS gate). Assume impedance matching at the source side (we will come to it soon), and thus we should have,

$$\Gamma_s = \frac{50-50}{50+50} = 0$$

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{\frac{1}{sC} - Z_c}{\frac{1}{sC} + Z_c} = \frac{1 - sZ_cC}{1 + sZ_cC} \quad \text{, let } T_c = Z_cC$$

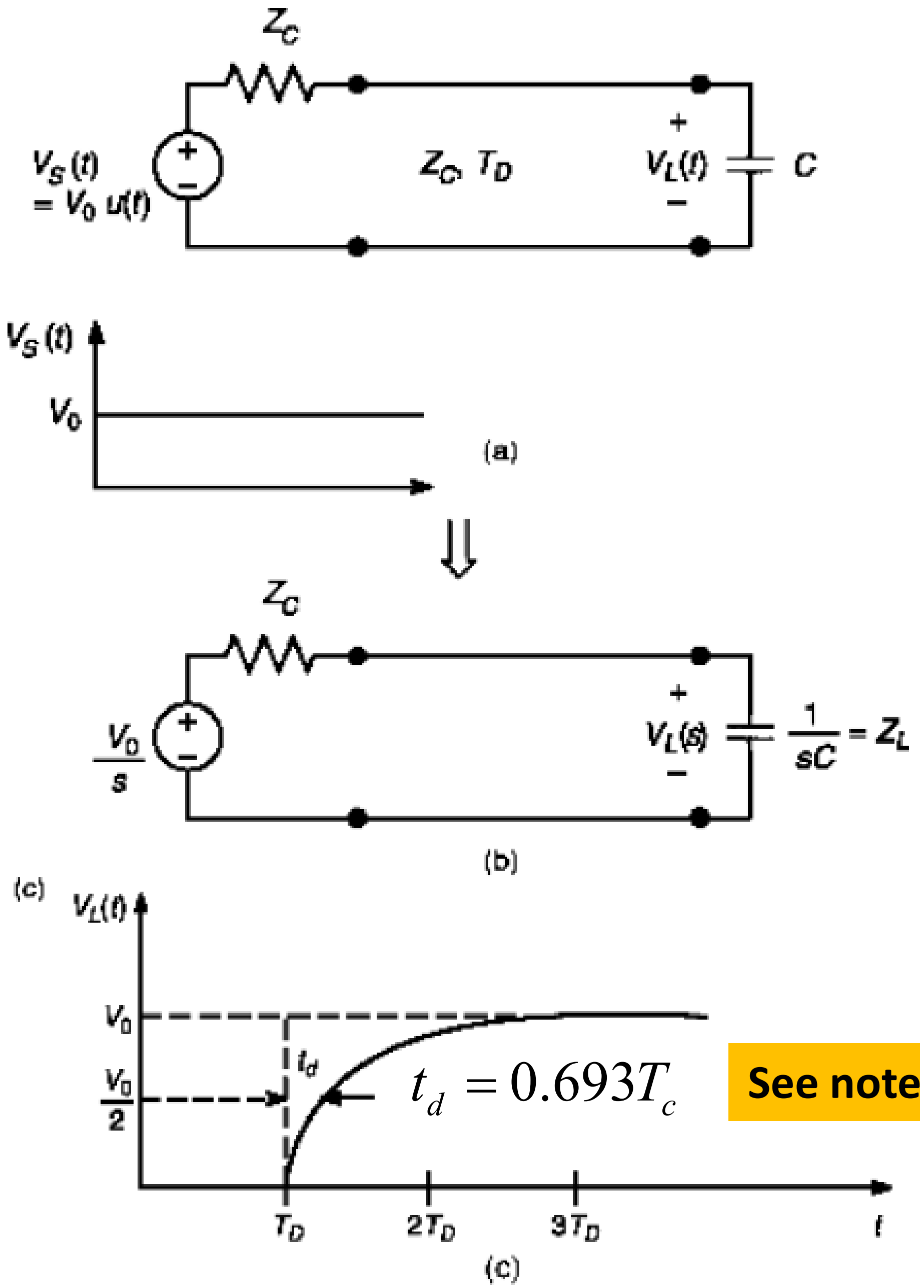
Capacitor initially short circuit, then gradually open circuit

$$V_L(t) = \left(1+\Gamma_L\right)\frac{Z_c}{\left(R_s = Z_c\right)+Z_c}V_0u\left(t-T_D\right)$$

$$\overbrace{\Rightarrow}^{\textit{frequency-domain}\ \text{(Laplace)}} \qquad V_L(s) = \left(1+\Gamma_L(s)\right)\frac{1}{2}V_s(s)e^{-sT_D}$$

$$= \frac{\frac{1}{T_c}}{\left(s+\frac{1}{T_c}\right)s}V_0e^{-sT_D} = \left(\frac{1}{s} - \frac{1}{\left(s+\frac{1}{T_c}\right)}\right)V_0e^{-sT_D}$$

$$\underbrace{\Rightarrow}_{\textit{inverse-Laplace}} \boxed{V_L(t) = V_0u\left(t-T_D\right) - e^{-\frac{(t-T_D)}{T_c}}V_0u\left(t-T_D\right)}$$



See note

- Consider the load impedance is **inductive**. Assume impedance matching at the source side (we will come to it soon), and thus we should have,

$$\Gamma_s = \frac{50-50}{50+50} = 0$$

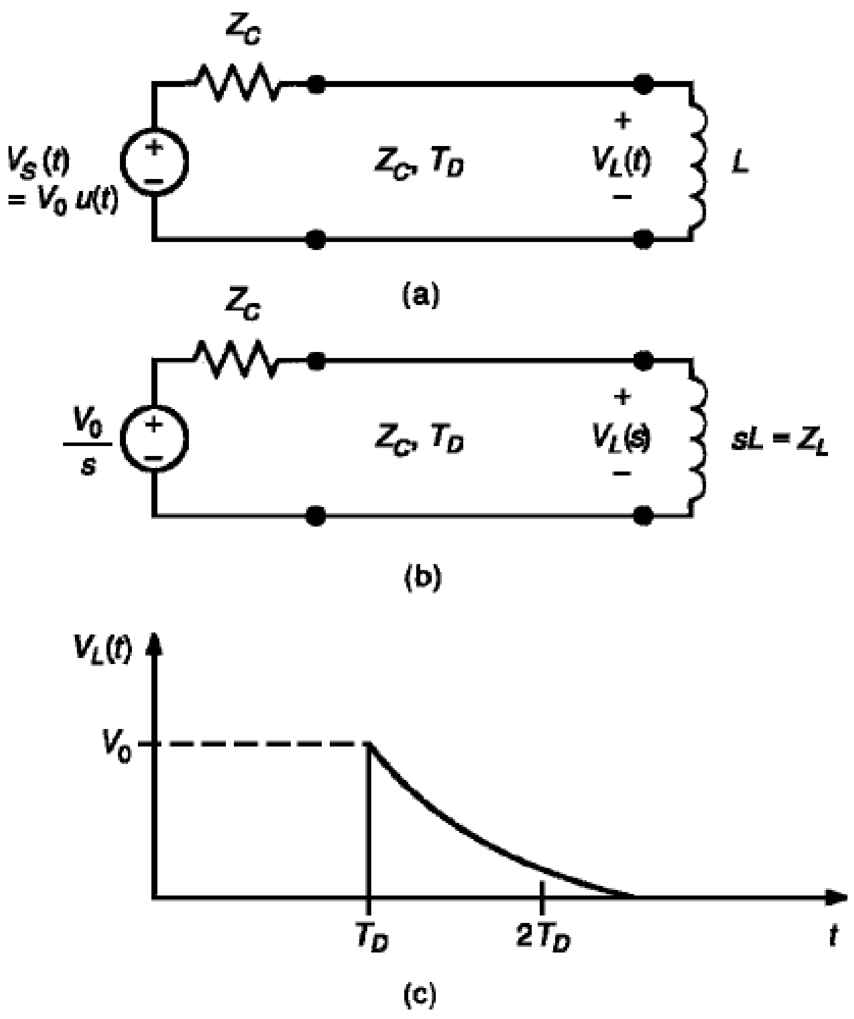
$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{sL - Z_c}{sL + Z_c} = \frac{sT_L - 1}{sT_L + 1} \quad , \text{let } T_L = \frac{L}{Z_c}$$

$$\underbrace{\Rightarrow}_{\textit{frequency-domain} \text{ (Laplace)}} \qquad V_L(s) = \left(1 + \Gamma_L(s)\right)\frac{1}{2}V_s(s)e^{-sT_D}$$

$$= \frac{1}{\left(s + \frac{1}{T_L}\right)} V_0 e^{-sT_D} = \left(\frac{1}{s} - \frac{1}{\left(s + \frac{1}{T_c}\right)}\right) V_0 e^{-sT_D}$$

$$\underbrace{\Rightarrow}_{\textit{inverse-Laplace}} \boxed{V_L(t) = V_0 e^{-\frac{(t-T_D)}{T_L}} u\left(t - T_D\right)}$$

inductor initially open circuit, then gradually short circuit



(b) Matching Schemes for SI

- We have already seen the effect of mismatches (i.e. non-zero Γ).
- Thus to reduce the effect of reflections on the signal levels and settling time (i.e. reaching the final value desired), it is best to match the impedances to that of the T-line at the input and output connections
- A widely common matching scheme is the ***series-matching***. Hence, add ***R*** at the T-line input such that,

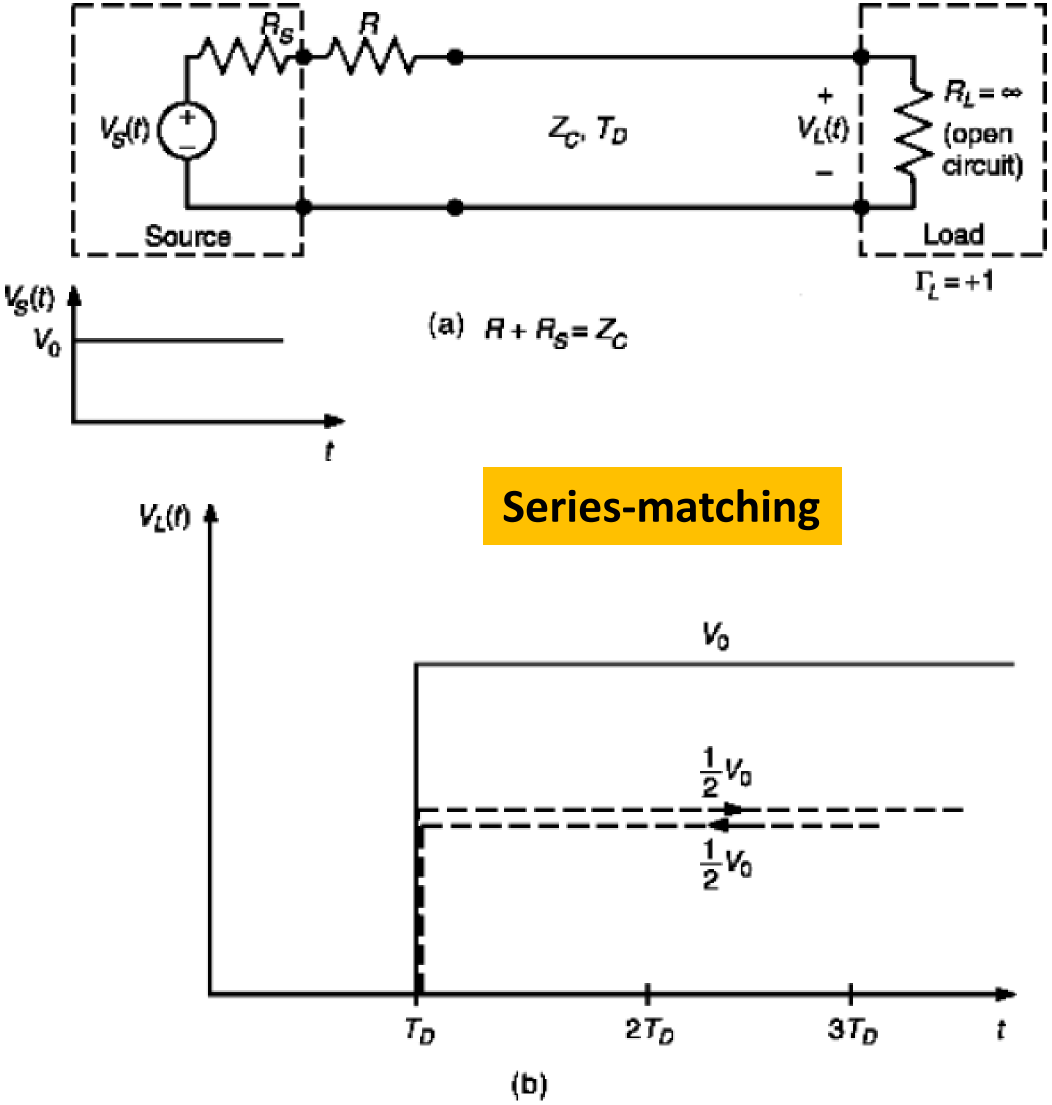
$$R_s + R = Z_c$$

and

$$V_{\text{intial}} = \frac{1}{2} V_0$$

$$\text{since } \Gamma_L = 1$$

$$\rightarrow \boxed{V_L = \underbrace{\frac{1}{2} V_0}_{V^+} + \underbrace{\frac{1}{2} V_0}_{V^-} = V_0}$$



Series-matching

1. **Excellent SI! Load gets its level immediately!**
2. **For open-circuit loads, No current flows**
   → R will not dissipate power

- Another common matching scheme (dependent on technology used), is ***parallel-matching***.
- A resistance ***R*** is placed in parallel with the load. We choose it such that, $R_L \parallel R = Z_c$
- Thus, $$V_{\text{intial}} = \frac{Z_c}{R_s + Z_c} V_0$$

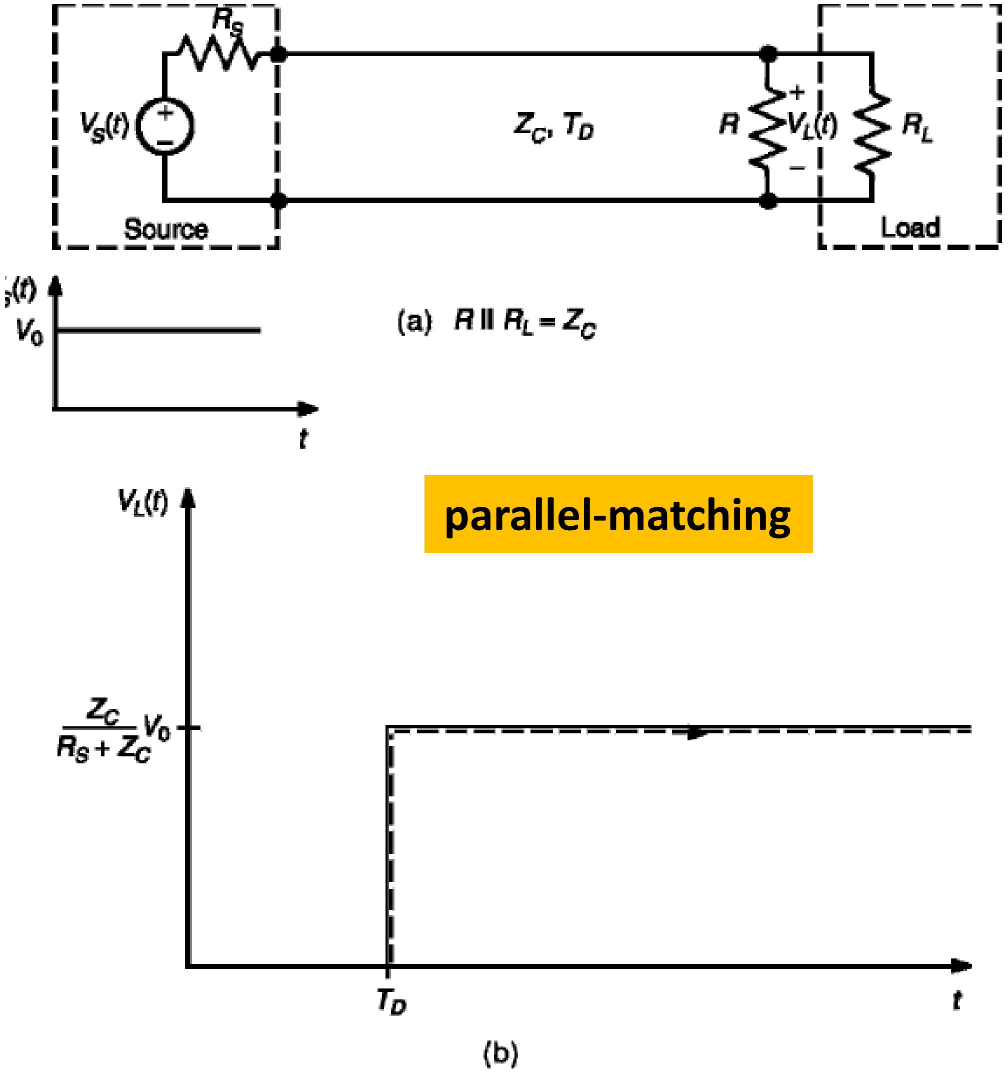
$$\text{since } \Gamma_L = 0$$

$$\rightarrow \boxed{V_L = \frac{Z_c}{R_s + Z_c} V_0}$$

- Two **disadvantages** with this method in this application,

(1) Load voltage is always less than source one

(2) ***R*** will consume power even if R_L = open circuit.



When is matching Not Required?

- Well, when the line is TOO short! Thus we do not have T-line effects!
- Let us develop the bound for this length criteria
- The criteria defining a short line where distributed parameters can be neglected is

$$l < \frac{1}{10}\frac{v}{f_{\max}}$$

- We found that,

$$BW = \frac{1}{\tau_r} \quad , \quad \text{let } f_{\max} = \frac{1}{\tau_r}$$

→ thus, as long as 10 times one way TD is less than the rise-time, then we can ignore matching!

$$\frac{l}{v} = T_D < \frac{1}{10}\tau_r \quad \Rightarrow \quad \boxed{\tau_r > 10T_D}$$

(c) Effect of line discontinuities

- A common discontinuity in the path is where the line crosses section changes, i.e. vias, branching, etc.
- Thus, one part will have an impedance of Z_{c1}, and after the discontinuity Z_{c2}
- At the junction,

left to *right* direction $\rightarrow$

$$\Gamma_{12} = \frac{Z_{c2} - Z_{c1}}{Z_{c2} + Z_{c1}} \quad \text{and} \quad v_{r1} = \Gamma_{12} v_{i1}$$

$$v_{r1} + v_{i1} = v_{t2} \quad \text{and} \quad v_{t2} = T_{12} v_{i1}$$

$$\rightarrow 1 + \Gamma_{12} = T_{12}$$

$$T_{12} = \frac{2Z_{c2}}{Z_{c2} + Z_{c1}}$$

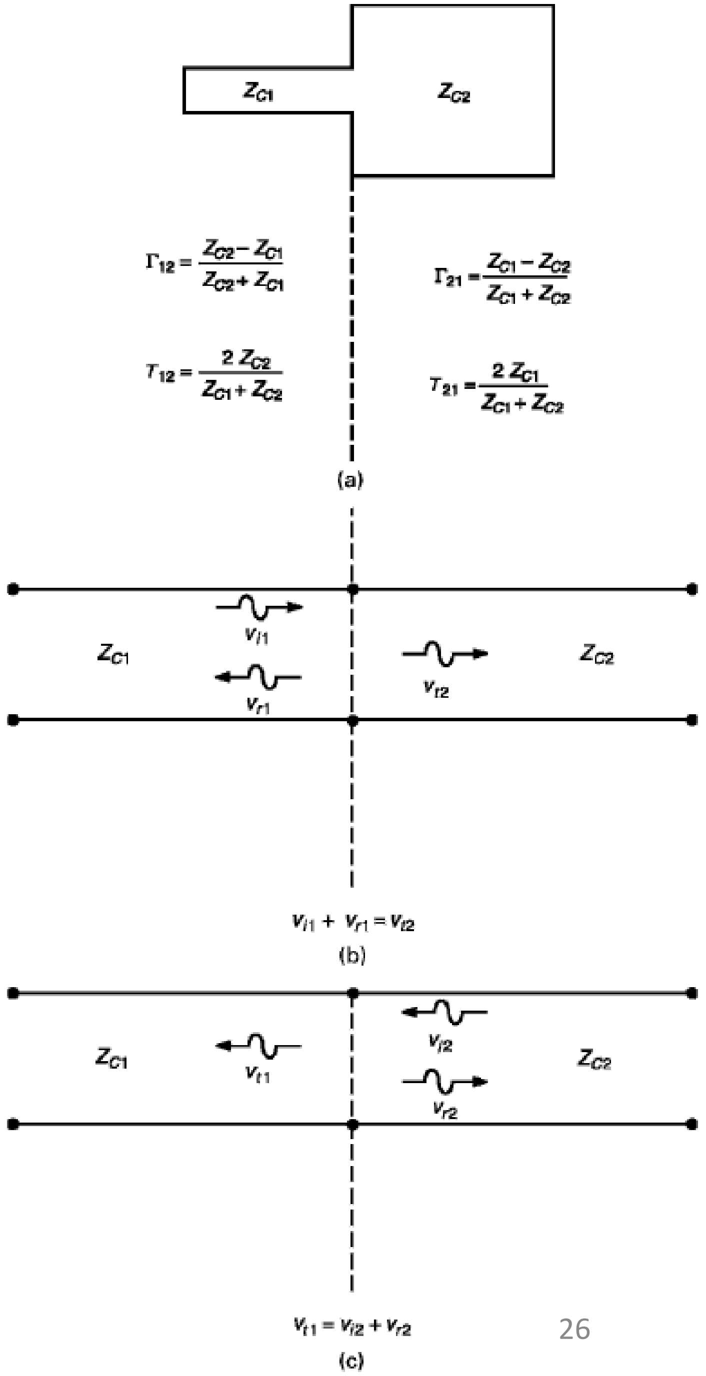
right to *left* direction $\leftarrow$

$$\Gamma_{21} = \frac{Z_{c1} - Z_{c2}}{Z_{c1} + Z_{c2}} \quad \text{and} \quad v_{r2} = \Gamma_{21} v_{i2}$$

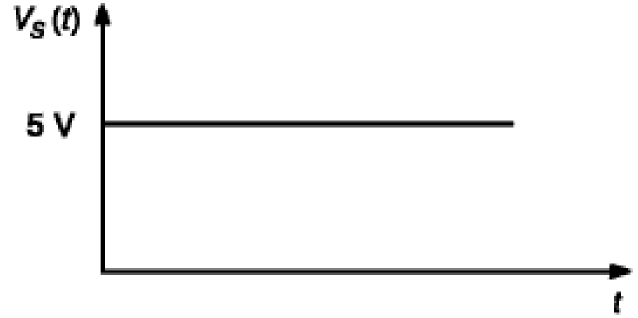
$$v_{r2} + v_{i2} = v_{t1} \quad \text{and} \quad v_{t1} = T_{21} v_{i2}$$

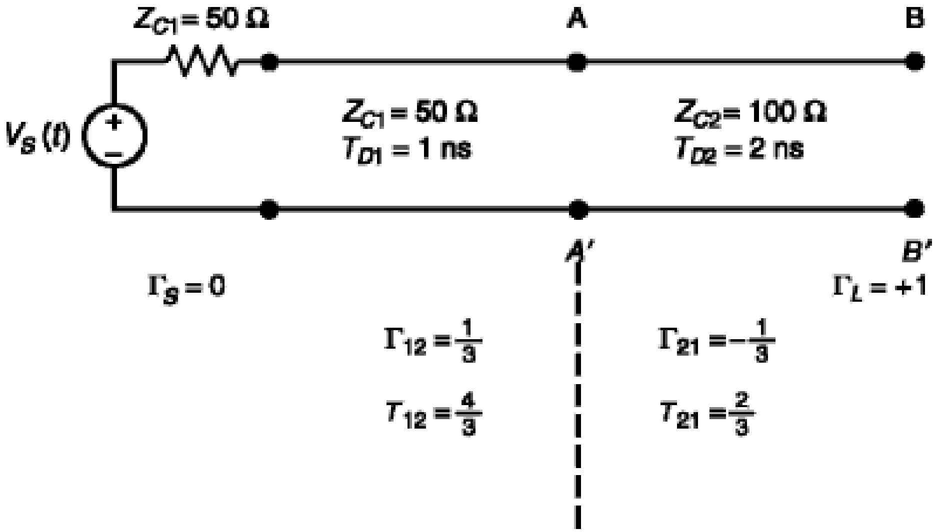
$$\rightarrow 1 + \Gamma_{21} = T_{21}$$

$$T_{21} = \frac{2Z_{c1}}{Z_{c2} + Z_{c1}}$$



Example 4.2



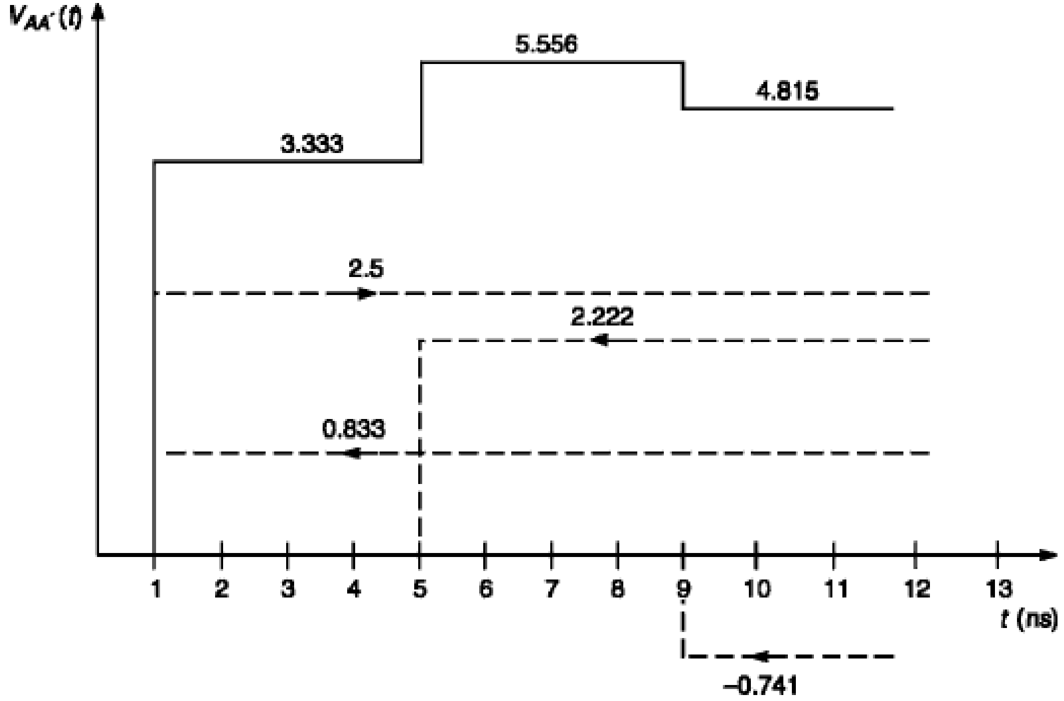


For the connection shown (two different lines), sketch the voltage at the load and at the junction.

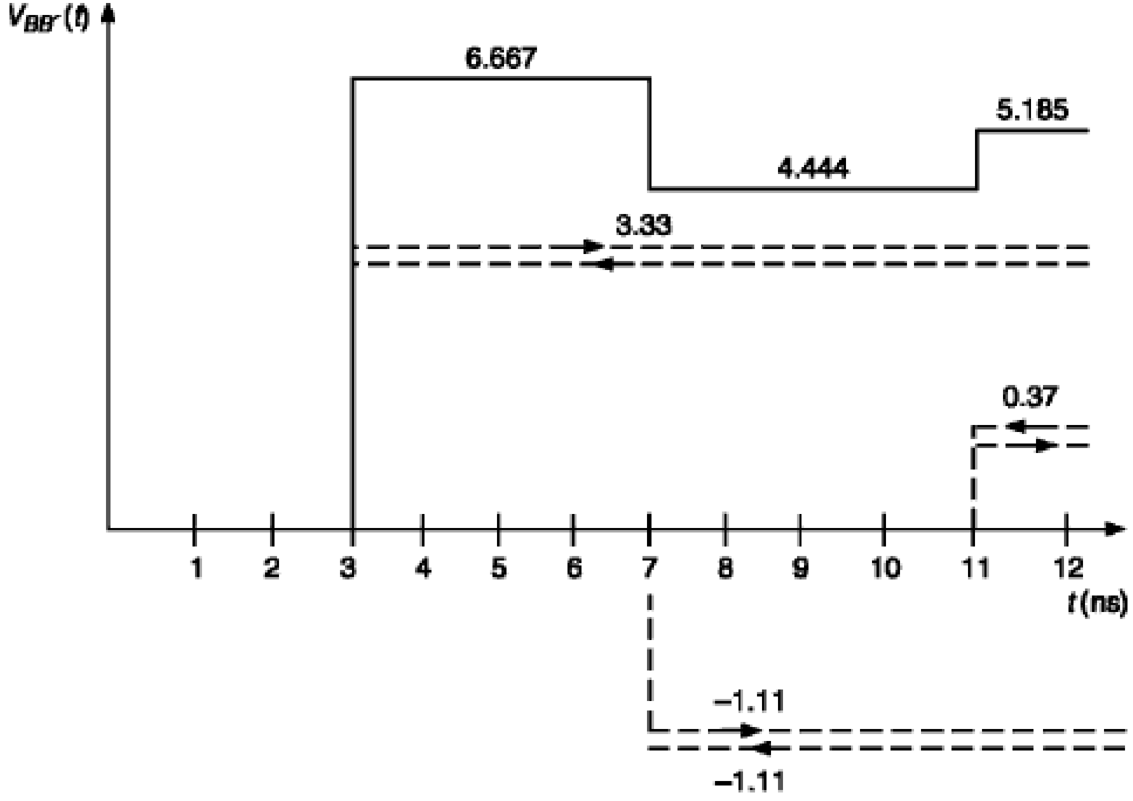
Sol.

On the Board!

Full response on the following slide.



$V_{AA'}(t)$
5.556
4.815
3.333
2.5
2.222
0.833
1 2 3 4 5 6 7 8 9 10 11 12 13
t (ns)
−0.741



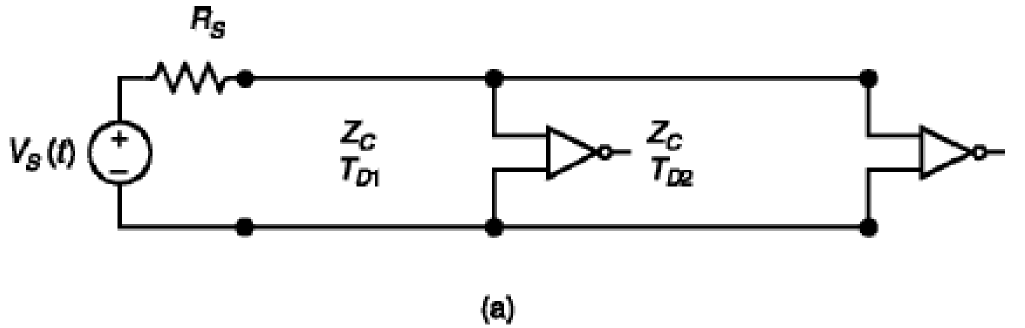
$V_{BB'}(t)$
6.667
5.185
4.444
3.33
0.37
1 2 3 4 5 6 7 8 9 10 11 12
t (ns)
−1.11
−1.11

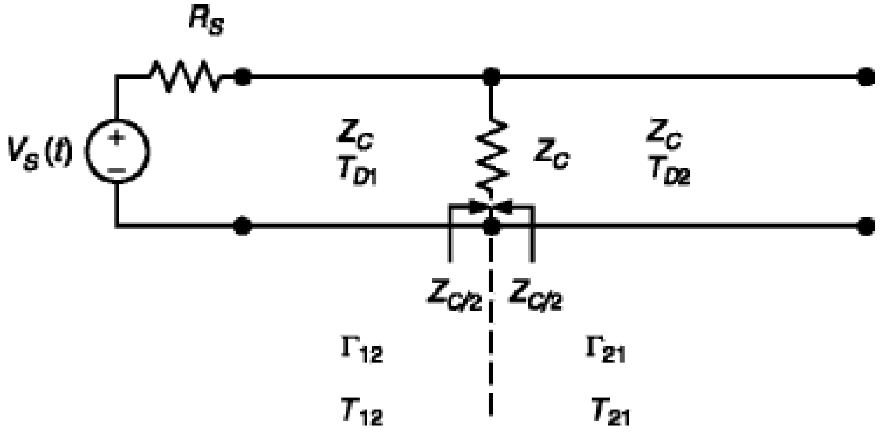
- Discontinuities feeding **multiple lines in series or parallel**.
- **Consider (a)**, this case we treated previously somehow (consider gate inputs as open-circuit):

(a) a series termination at the source will eliminate reflections

(b) a parallel termination at the load will cause power dissipation

(c) a common mistake is to parallel match at the junction (mid-point). Let us analyze placing Z_c at the junction.



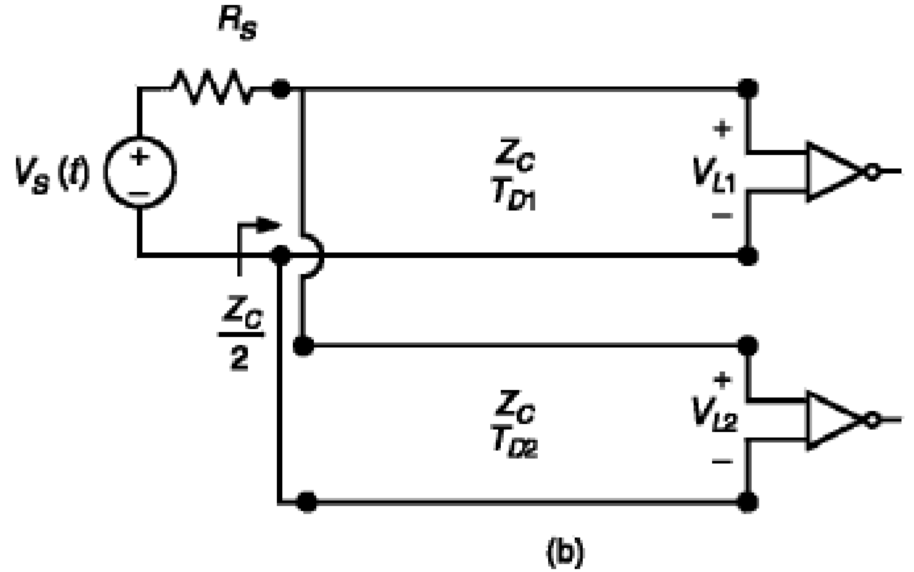


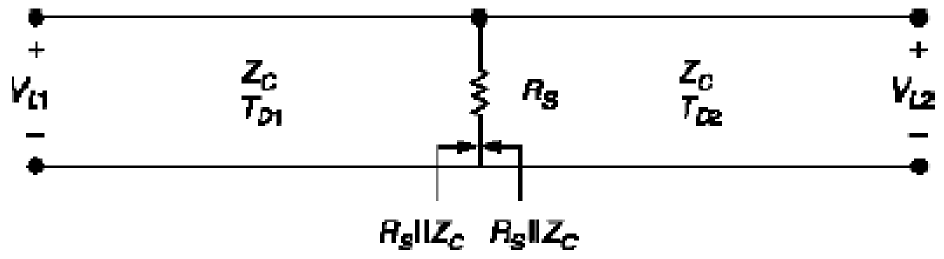
$$\left.\begin{aligned} \Gamma_{12} = \Gamma_{21} &= \frac{\left(\frac{Z_c}{2}\right) - Z_c}{\left(\frac{Z_c}{2}\right) + Z_c} = -\frac{1}{3} \\ T_{12} = T_{21} &= 1 + \Gamma_{21} = \frac{2}{3} \end{aligned}\right\} \text{Not Good! Multiple reflections!!}$$

- **Consider (b)**, this is a parallel distribution where a source drives two loads in parallel.
- We desire to have Vs as the steady state value for both receivers
- The source sees an equivalent impedance of Zc/2, thus

$$V_{init} = \frac{\left(\frac{Z_c}{2}\right)}{R_s + \left(\frac{Z_c}{2}\right)} V_s$$

- At the loads, the open circuit condition reflects the incoming signal completely with delays TD1 and TD2, respectively.
- The two loads will see an equivalent impedance at the junction as
- Thus reflections and transmission between the two loads will exist until final values are reached. This is NOT DESIRABLE!
- Best is to parallel match at loads, at the expense of power dissipation.
- Series distribution is more desirable!





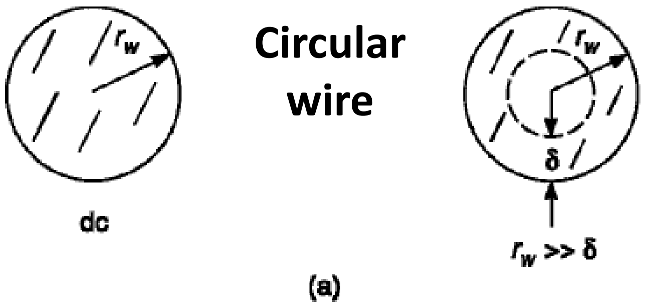
$$\Gamma_{12} = \frac{R_S \| Z_C - Z_C}{R_S \| Z_C + Z_C} = -\frac{Z_C}{2R_S + Z_C} \qquad \Gamma_{21} = \Gamma_{12}$$

$$T_{12} = 1 + \Gamma_{12} = \frac{2R_S}{2R_S + Z_C} \qquad T_{21} = T_{12}$$

(d) Effect of Losses on SI

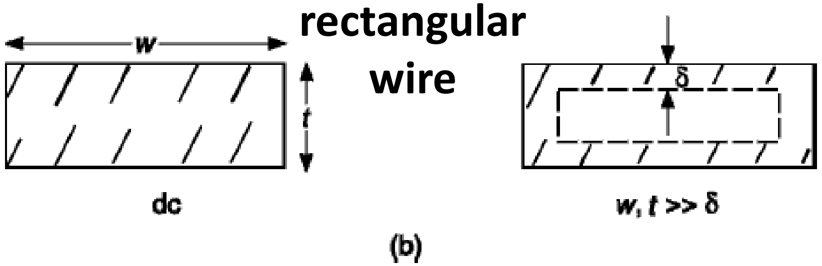
- Losses occur within **conductors** and due to dielectrics. Conductor losses are represented via an increase in the T-line resistance while dielectric losses are represented via an increase in the T-line conductance.
- As frequency increases, the current starts flowing on the surface of the conductor, thus decreasing the cross-sectional area of flow and increasing the resistance.
- **Skin effect**: is the phenomena that shows the concentration of the current on the surface of the conductor at higher frequencies and that is confined within the skin depth (δ).
- At higher frequencies, the conductor resistance increases due to the skin effect as a square-root of the frequency, $\sqrt{f}$

  (i.e. 10dB/decade)



$$r_{DC} = \frac{1}{\sigma \pi r_w^2} \quad (\Omega / m) \qquad r_{hf} \approx \frac{1}{2\sigma \pi r_w \delta} \quad (\Omega / m)$$

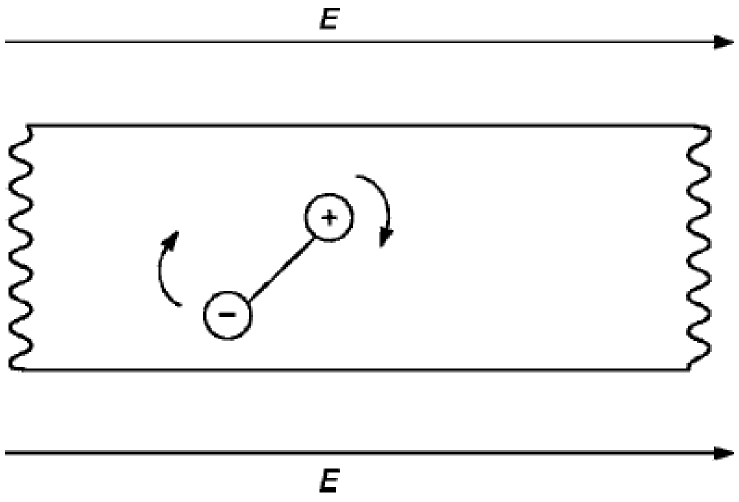
$$\sigma = \underbrace{5.8 \times 10^7}_{copper} (S/m) \qquad \delta = \frac{1}{\sqrt{\pi f \sigma \mu_0}}$$



$$r_{DC} = \frac{1}{\sigma w t} \quad (\Omega / m) \qquad r_{hf} = \frac{1}{2\sigma \delta (w+t)} \quad (\Omega / m)$$

$$\sigma = \underbrace{5.8 \times 10^7}_{copper} (S/m) \qquad \delta = \frac{1}{2} \frac{wt}{(w+t)} \approx \frac{t}{2} \quad \text{when } w >> t$$

- At higher frequencies, the dipoles within the **dielectric material** will not be able to fully rotate and follow the changes in the applied fields, which resembles a damping loss. This is represented via the introduction of the complex part within the permittivity of the material. Also, the free electrons contribute to ohmic losses within it.



- Thus, $$\varepsilon_c = \overbrace{\varepsilon_r^{'}}^{damping-loss} - j \overbrace{\varepsilon_r^{''}}^{ohmic-loss}$$

$$\underbrace{\tan(\theta)}_{loss-\text{tangent}} = \frac{\varepsilon_r^{''}}{\varepsilon_r^{'}}$$

- Now, the capacitance (the part that depends on ε), $j\omega c = j\omega\varepsilon K = j\omega\varepsilon_0\varepsilon_c K$

$$= \underbrace{\omega\varepsilon_0\varepsilon_r^{''}K}_{g} + j\underbrace{\omega\varepsilon_0\varepsilon_r^{'}K}_{x_c}$$

- For a homogeneous medium with fairly constant loss tangent, $g = \omega c \tan(\theta)$
- Thus, g will increase with a rate of 20dB/decade

(direct dependence on f)

- Due to the changes in r and g in the T-line per unit lengths, the impedance and propagation delays gets directly affected according to →

$$\hat{Z}_c = \sqrt{\frac{r + j\omega L}{g + j\omega C}} = \left|\hat{Z}_c\right| \angle \theta_{\hat{Z}_c} \quad (\Omega)$$

$$\hat{\gamma} = \alpha + j\beta = \sqrt{(r + j\omega L)(g + j\omega C)}$$

$$v = \frac{\omega}{\beta} \quad (m/s)$$

- The characteristic impedance (Z), attenuation constant (α), and phase constant (β) are all frequency dependent even if the per unit length resistance and conductance are not.
- This frequency dependence of Z, α, and β, will cause different signals with different frequencies to see different impedances and experience different propagation delays! This will cause signal **dispersion**! Recall that a digital pulse consists of sums of harmonics! This will distort the signals!
- A condition for **distortionless** transmission line is to have,

$$\frac{R}{L} = \frac{G}{C} \quad \rightarrow \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{LC\left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)}$$

$$= \sqrt{LC\left(\frac{R}{L} + j\omega\right)\left(\frac{R}{L} + j\omega\right)} = \sqrt{LC}\left(\frac{R}{L} + j\omega\right) = \underbrace{R\sqrt{\frac{C}{L}}}_{\alpha} + \underbrace{j\omega\sqrt{LC}}_{\beta}$$

Independent of (f)

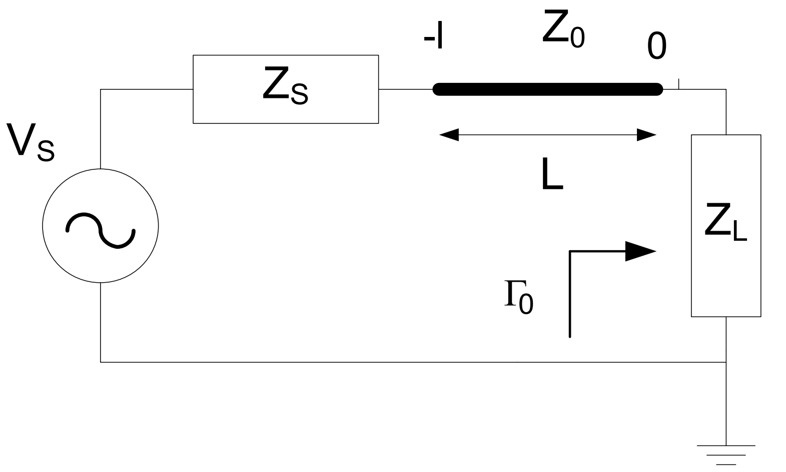
Linearly dependent on (f)

Sinusoidal excitation and phasor solution of T-lines

- Revisiting slide (5),

TEM mode

$$\Gamma_0 = \frac{V^-}{V^+} \quad \text{reflection coefficient}$$



$$V(z) = V^+\left(e^{-\gamma z} + \Gamma_0 e^{\gamma z}\right), I(z) = \frac{V^+}{Z_0}\left(e^{-\gamma z} - \Gamma_0 e^{\gamma z}\right)$$

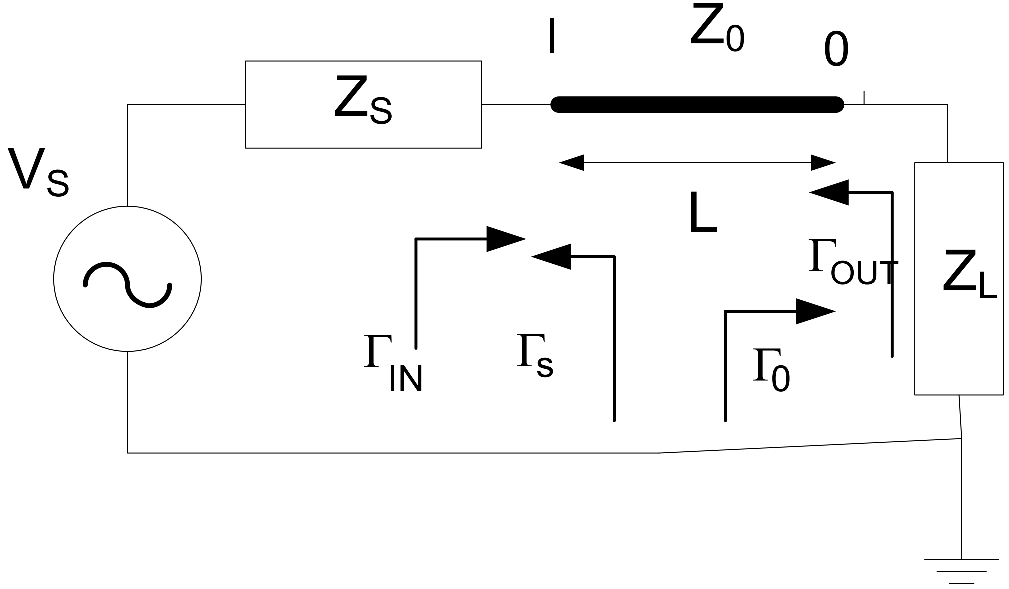
$$lossless,\ R = G = 0 = \alpha, \beta = \omega\sqrt{LC} = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$

$$Z(0) = Z_L = Z_0 \frac{1+\Gamma_0}{1-\Gamma_0} \rightarrow \Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V(z) = V^+\left(e^{-j\beta z} + \Gamma_0 e^{j\beta z}\right), I(z) = \frac{V^+}{Z_0}\left(e^{-j\beta z} - \Gamma_0 e^{j\beta z}\right) \qquad Z_{in}(-l) = Z_0\left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right)$$

$$SWR = \frac{|V_{\max}|}{|V_{\min}|} = \frac{|I_{\max}|}{|I_{\min}|} = \frac{1+|\Gamma_0|}{1-|\Gamma_0|}$$

- Now, at the source side,



$$V_{in} = V_{in}^{+} + V_{in}^{-} = V_{in}^{+}\left(1+\Gamma_{in}\right) = V_s\left(\frac{Z_{in}}{Z_{in}+Z_s}\right)$$

$$\Gamma_{in} = \Gamma(d=l) = \frac{Z_{in}-Z_0}{Z_{in}+Z_0} = \Gamma_0 e^{-2j\beta l}$$

$$T_{in} = 1+\Gamma_{in} = \frac{2Z_{in}}{Z_{in}+Z_0}, T_0 = 1+\Gamma_0 = \frac{2Z_L}{Z_L+Z_0}$$

$$\Gamma_s = \frac{Z_s-Z_0}{Z_s+Z_0}$$

$$\Gamma_{out} = \Gamma_s e^{-2j\beta l}$$

$$P_{av} = \frac{1}{2}\mathrm{Re}\left\{VI^*\right\}$$

$$V_{in} = V_{in}^{+}\left(1+\Gamma_{in}\right)$$

$$I_{in} = \frac{V_{in}^{+}}{Z_0}\left(1-\Gamma_{in}\right)$$

- The input power can be found as,

$$P_{in} = P_{in}^{+} + P_{in}^{-} = \frac{1}{2}\frac{\left|V_{in}^{+}\right|^2}{Z_0}\left(1-\left|\Gamma_{in}\right|^2\right)$$

$$V_{in}^{+} = \frac{V_{in}}{1+\Gamma_{in}} = \frac{V_s}{1+\Gamma_{in}}\left(\frac{Z_{in}}{Z_{in}+Z_s}\right)$$

$$Z_{in} = Z_0\frac{1+\Gamma_{in}}{1-\Gamma_{in}} \quad , \quad Z_s = Z_0\frac{1+\Gamma_s}{1-\Gamma_s}$$

then,

$$V_{in}^{+} = \frac{V_s}{2}\frac{\left(1-\Gamma_s\right)}{\left(1-\Gamma_s\Gamma_{in}\right)}$$

$$P_{in} = \frac{1}{8}\frac{\left|V_s\right|^2}{Z_0}\frac{\left|1-\Gamma_s\right|^2}{\left|1-\Gamma_s\Gamma_{in}\right|^2}\left(1-\left|\Gamma_{in}\right|^2\right)$$

similarly,

$$P_L = \frac{1}{2}\frac{\left|V_L^{+}\right|^2}{Z_0}\left(1-\left|\Gamma_L\right|^2\right)$$

$$\left|V_L^{+}\right| = \left|V_{in}^{+}\right|e^{-\alpha l}$$

then,

$$P_L = \frac{1}{8}\frac{\left|V_s\right|^2}{Z_0}\frac{\left|1-\Gamma_s\right|^2}{\left|1-\Gamma_s\Gamma_{in}\right|^2}e^{-2\alpha l}\left(1-\left|\Gamma_L\right|^2\right)$$

$$recall\ \ that\ \ (a-jb)^2 = (a-jb)(a+jb) = \left|a-jb\right|^2$$

- The ratio between the incident power and reflected power at the input of the T-line is denoted as the return loss (RL),

$$RL = -10\log\left(\frac{P_r}{P_i}\right) = -20\log\left|\Gamma_{in}\right|$$

- The ratio between the transmitted power (to load) to the incident power (input of T-line) is denoted as the insertion loss (IL),

$$IL = -10\log\left(\frac{P_t}{P_i}\right) = -10\log\left(1-\left|\Gamma_{in}\right|^2\right)$$

NEXT TIME

- None ideal components …