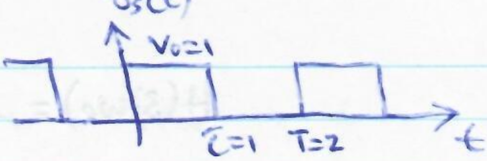
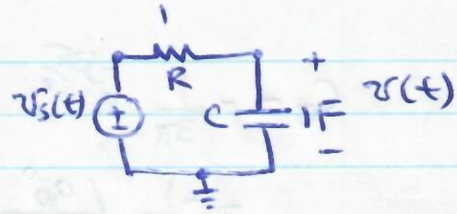
ECE 4320/5230

TOPIC 3

Ex 2:





The transfer function of the filter

$$H(j\omega) = \frac{V(j\omega)}{V_s(j\omega)} = \frac{Z_C}{Z_C + Z_R} = \frac{1/j\omega C}{1/j\omega C + R}$$

$$H(j\omega) = \frac{1}{1+j\omega RC}$$ @ discrete points $\omega = n\omega_0$

$$H(jn\omega_0) = \frac{1}{1+j\omega_0 n RC}$$

Thus,

$$v_n(t) = 2|C_n||H(jn\omega_0)| \cos(n\omega_0 t + \angle C_n + \angle H(jn\omega_0))$$

and

$$v(t) = C_0 H(0) + \sum_{n=1}^{\infty} v_n(t)$$

now,

$$H(nj\omega_0) = \frac{1}{1+jn\omega_0 RC} = \frac{1}{1+jn\underbrace{\frac{2\pi}{T}}_{2}\underbrace{RC}_{1}} = \frac{1}{1+jn\pi}$$

for a pulse train

$$C_n = \frac{V_0\tau}{T} \frac{\sin(n\pi\tau/T)}{(n\pi\tau/T)} e^{-jn\pi\frac{\tau}{T}}$$

$$= \frac{1}{2} \frac{\sin(n\pi/2)}{(n\pi/2)} e^{-jn\pi/2}$$

Evaluating the first few harmonics,

$C_0 = \frac{1}{2}$ $\qquad$ $H(0) = 1$

$C_1 = \frac{1}{\pi} e^{-j\pi/2}$ $\qquad$ $H(j\omega_0) = \frac{1}{1+j\pi} = 0.3033\angle -72.34°$

$C_2 = 0$

$$C_3 = -\frac{1}{3\pi} e^{-j3\pi/2}$$

$$= \frac{-1}{3\pi} \angle 90^\circ$$

$$= \frac{1}{3\pi} \angle -90^\circ$$

$$H(3j\omega_0) = \frac{1}{1+j3\pi}$$

$$= 0.106 \angle -83.94^\circ$$

$$c_4 = 0$$

$$c_5 = \frac{1}{5\pi} e^{-j\pi/2}$$

$$= \frac{1}{5\pi} \angle -90^\circ$$

$$H(j5\omega_0) = \frac{1}{1+j5\pi}$$

$$= 0.0635 \angle -86.36^\circ$$

$$C_6 = 0$$

$$C_7 = \frac{-1}{7\pi} e^{-j7\pi/2}$$

$$= \frac{-1}{7\pi} \angle 90^\circ$$

$$= \frac{1}{7\pi} \angle -90^\circ$$

$$H(j7\omega_0) = \frac{1}{1+j7\pi}$$

$$= 0.04543 \angle -87.4^\circ$$

The source single sided FS is

$$v_s(t) = \frac{1}{2} + \frac{2}{\pi}\cos(\pi t - 90^\circ)$$
$$+ \frac{2}{3\pi}\cos(3\pi t - 90^\circ + \frac{2}{5\pi}\cos(5\pi - 90^\circ).$$
$$+ \frac{2}{7\pi}\cos(7\pi t - 90^\circ) + \cdots$$

$$v(t) = \frac{1}{2} + 0.1931\cos(\pi t - 162.34^\circ)$$
$$+ 0.0224\cos(3\pi t - 173.94^\circ)$$
$$+ 0.0081\cos(5\pi t - 176.36^\circ)$$
$$+ 0.0041\cos(7\pi t - 177.4^\circ) + \cdots$$

Ex4. 1V, 10 MHz, D-C = 50%

(a) 20 ns rise/fall time

$$\text{Duty-cycle } (D) = \frac{\tau}{T}, \quad f_0 = \frac{1}{T}$$

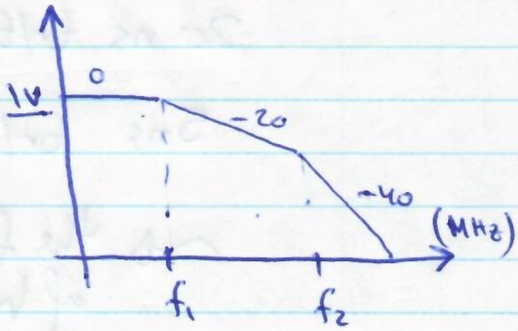
for a trapezoidal waveform, first break frequency (f_1) is

$$f_1 = \frac{1}{\pi \tau} = \frac{1}{\pi (DT)} = \frac{f_0}{\pi D}$$

$$D = \frac{1}{2} \rightsquigarrow f_1 = \frac{10 \times 10^6}{\pi \cdot 1/2} = \boxed{6.37 \text{ MHz}}$$

Second break frequency (f_2),

$$f_2 = \frac{1}{\pi \tau_r} = \frac{10^9}{\pi \cdot 20} = \boxed{15.92 \text{ MHz}}$$



$$\text{Level}_{(110\text{MHz})} = 20 \log_{10}(10^6 \mu V) - 20 \log_{10}\left(\frac{15.9 (\text{MHz})}{6.37 (\text{MHz})}\right) - 40 \log_{10}\left(\frac{110 (\text{MHz})}{15.9 (\text{MHz})}\right)$$

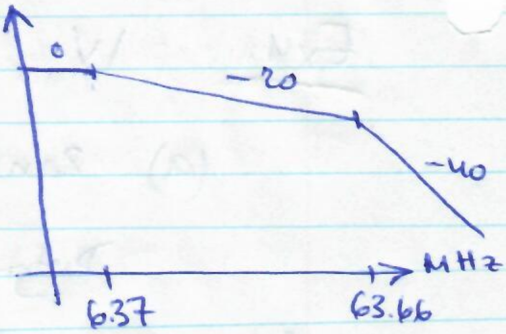
$$= 120 \text{ dB}\mu V - 7.95 \text{ dB}\mu V - 33.6 \text{ dB}\mu V$$

$$= \boxed{78.45 \text{ dB}\mu V}$$

(b) 5 ns rise/fall times.

$$\boxed{f_1 = 6.37 \text{ MHz}} \qquad f_2 = \frac{10^9}{\pi \cdot 5} = \boxed{63.66 \text{ MHz}}$$

interpolate to find Level @ 110 MHz



$$\text{Level}_{(110MHz)} = 20\log_{10}(10^6 \mu V) - 20\log_{10}\left(\frac{63.66}{6.37}\right) - 40\log_{10}\left(\frac{110}{63.66}\right)$$

$$= 120\ dB\mu V - 19.99 - 9.5$$

$$\text{Level}_{(110MHz)} = \boxed{90.51\ dB\mu V}$$

- Comparing the Levels

20 ns t_r/t_f ⤳ 78.45 dBμV

5 ns t_r/t_f ⤳ 90.51 dBμV

⤳ the faster rise time, the higher the Level of harmonics at higher frequencies.

⤳ $\boxed{\text{try to slow the edges!}}$

<u>note:</u> using exact values from the <u>FS. coefficient</u> we get

20 ns ⤳ 73.8 dBμV
5 ns ⤳ 90.4 dBμV

} reasonably close to the Bound.