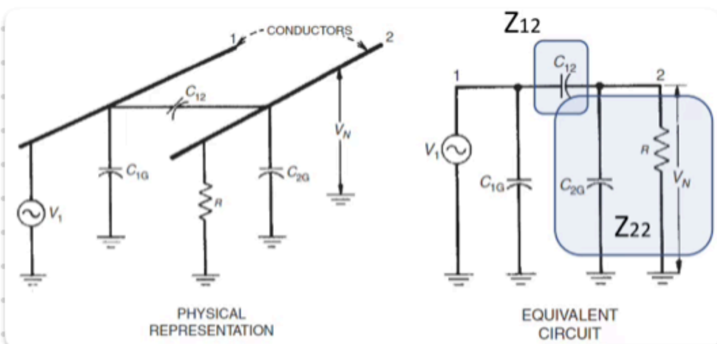
Q1

Q1 [5 pts]: Show the detailed steps to get (derive) the equation shown in T6 slide 3 for V_N from the equivalent circuit model shown on the same slide.



Derive, $V_N = j\omega R C_{12} V_1$

1. Rewrite Z_{22} in terms of C_{2G} and R impedance,

$$Z_{12} = \frac{1}{j\omega C_{12}} \qquad Z_{2G} = \frac{1}{j\omega C_{2G}}$$

$$Z_{22} = \frac{1}{j\omega C_{2G} + \frac{1}{R}} = \frac{R}{j\omega C_{2G} R + 1}$$

2. Treat Z_{12} and Z_{22} as a voltage divider to solve for V_N,

$$V_N = \left(\frac{Z_{22}}{Z_{12} + Z_{22}}\right) V_1 = \frac{\frac{R}{j\omega C_{2G} R + 1}}{\frac{R}{j\omega C_{2G} R + 1} + \frac{1}{j\omega C_{12}}} V_1 = \frac{j\omega C_{12} R}{j\omega C_{12} R + j\omega C_{2G} R + 1} V_1$$

$$V_N = \frac{j\omega C_{12} R}{j\omega R(C_{12} + C_{2G}) + 1} V_1$$

3. Assume $R << j\omega(C_{12} + C_{2G})$,

$$V_N = \frac{j\omega C_{12} R}{\cancelto{0}{j\omega R(C_{12} + C_{2G})} + 1} V_1 = \frac{j\omega C_{12} R}{1} V_1 = j\omega R C_{12} V_1$$

Q2

$$V(0,t) = \frac{Z_c}{R_s+Z_c} V_s(t) = \frac{55}{11+55}(5) = \frac{275}{66} = 4.167 \text{ V}$$

$$I(0,t) = \frac{V_s(t)}{R_s+Z_c} = \frac{5}{80} = \frac{1}{16} A = 62.5 mA$$

a) $$\Gamma_s = \frac{R_s - Z_c}{R_s+Z_c} = \frac{11-55}{11+55} = \frac{-44}{66} = -\frac{2}{3}$$

$$\Gamma_L = \frac{R_L - Z_c}{R_L + Z_c} = \frac{275-55}{275+55} = \frac{220}{330} = \frac{2}{3}$$

b) @ 1µs, $V_{1L} = V_{0S} + V_{0S}\Gamma_L = 4.167 + (4.167)(2/3) = 4.167 + 2.78 \text{ V} = 6.947 \text{ V}$

$V_{1R} = 2.78 V$

@ 2µs $V_{1S} = 6.947 + (-\frac{2}{3})(2.78) = 5.0834 V$

@ 3µs $V_{2L} = V_{1S} + (V_{1R})(\Gamma_s)(\Gamma_L) = 5.0834 - 1.236 = 3.848 V$

$V_{2R} = -1.236 V$

@ 4µs $V_{2S} = V_{2L} + (V_{2R})(\Gamma_s) = 3.848 + (-1.236)(-2/3) = 4.672 V$

@ 5µs $V_{3L} = V_{2S} + (V_{2R})(\Gamma_s)(\Gamma_L) = 4.672 + 0.549 = 5.221 V$

$V_{3R} = 0.549 V$

@ 6µs $V_{3S} = V_{3L} + (V_{3R})(\Gamma_s) = 5.221 - 0.366 = 4.855 V$

@ 7µs $V_{4L} = V_{3S} + (V_{3R})(\Gamma_s)(\Gamma_L) = 4.855 - 0.244 = 4.611 V$

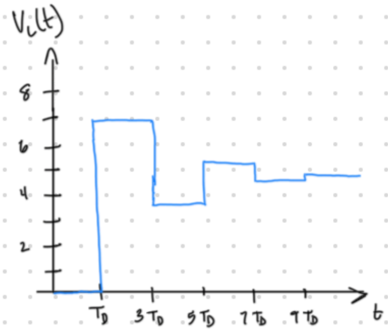
$V_{4R} = -0.244 V$

@ 8µs $V_{4S} = V_{4L} + (V_{4R})(\Gamma_s) = 4.611 + 0.162 = 4.774 V$

@ 9µs $V_{5L} = V_{4S} + (V_{4R})(\Gamma_s)(\Gamma_L) = 4.774 + 0.1084 = 4.882 V$

$V_{5R} = 0.1084 V$

@ 10µs $V_{5S} = V_{5L} + (V_{5R})(\Gamma_s) = 4.882 - 0.072 = 4.810 V$



c) $V_L(\ell,t) = \frac{Z_c}{R_s+Z_c}(1+\Gamma_L)\Big[V_s(t-T_D) + \Gamma_S\Gamma_L V_s(t-3T_D)$

$+ (\Gamma_S\Gamma_L)^2 V_s(t-5T_D)$

$+ (\Gamma_S\Gamma_L)^3 V_s(t-7T_D)$

$+ (\Gamma_S\Gamma_L)^4 V_s(t-9T_D)\Big]$

$$\frac{Z_c}{R_s+Z_c}(1+\Gamma_L) = \frac{55}{11+55}\left(1+\frac{2}{3}\right) = \left(\frac{5}{6}\right)\left(\frac{5}{3}\right) = \frac{25}{18}$$

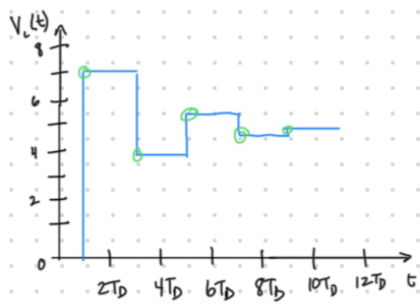
@ $1\mu s$, $V_L(1\mu s, T_D) = \frac{25}{18}(5) = (1.389)(5) = 6.9\overline{4}$ V

@ $3\mu s$, $V_L(1\mu s, 3T_D) = \frac{25}{18}\left(5+\left(-\frac{4}{9}\right)(5)\right) = \frac{25}{18}\left(5+(-0.\overline{4})(5)\right) = \frac{25}{18}\left(5-2.\overline{2}\right) = 3.86$ V

@ $5\mu s$, $V_L(1\mu s, 5T_D) = \frac{25}{18}\left(5+\left(-\frac{4}{9}\right)(5)+\left(\frac{-4}{9}\right)^2(5)\right) = \left(\frac{25}{18}\right)\left(2.\overline{7} + (0.1975)(5)\right) = \frac{25}{18}\left(2.\overline{7}+0.99\right) = \frac{25}{18}(3.77) = 5.23$ V

@ $7\mu s$, $V_L(1\mu s, 7T_D) = \frac{25}{18}\left(5+\left(\frac{-4}{9}\right)(5)+\left(\frac{-4}{9}\right)^2(5)+\left(-\frac{4}{9}\right)^3(5)\right) = \frac{25}{18}\left(3.77+(5)\left(-\frac{4}{9}\right)^3\right) = \frac{25}{18}\left(3.77-0.439\right) = \frac{25}{18}(3.33) = 4.63$ V

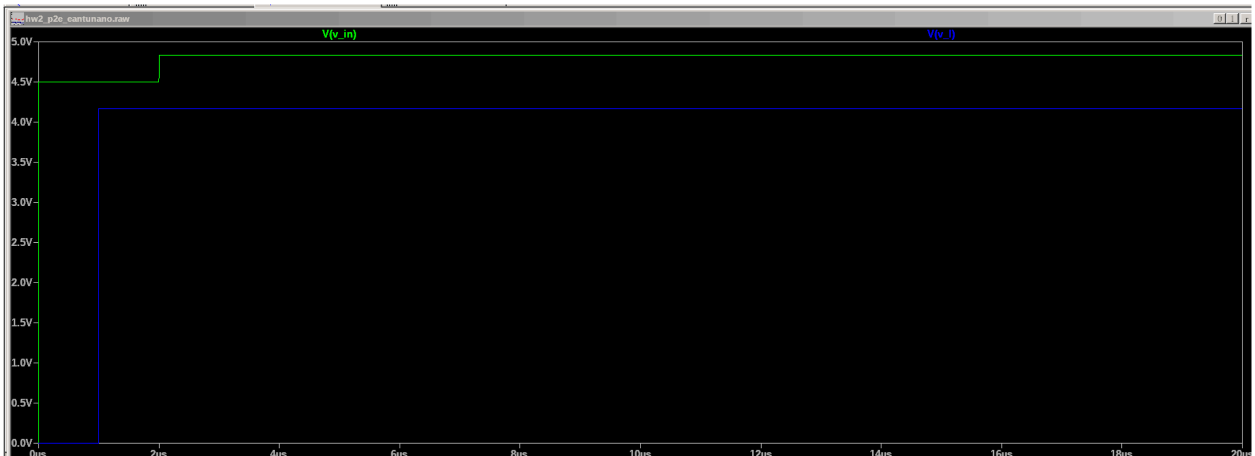
@ $9\mu s$, $V_L(1\mu s, 9T_D) = \frac{25}{18}\left(5+\left(\frac{-4}{9}\right)(5)+\left(\frac{-4}{9}\right)^2(5)+\left(-\frac{4}{9}\right)^3(5)+\left(\frac{-4}{9}\right)^4(5)\right) = \frac{25}{18}\left(3.33+(5)(-4/9)^4\right) = \frac{25}{18}\left(3.33+0.195\right) = \frac{25}{18}(3.525) = 4.89$ V

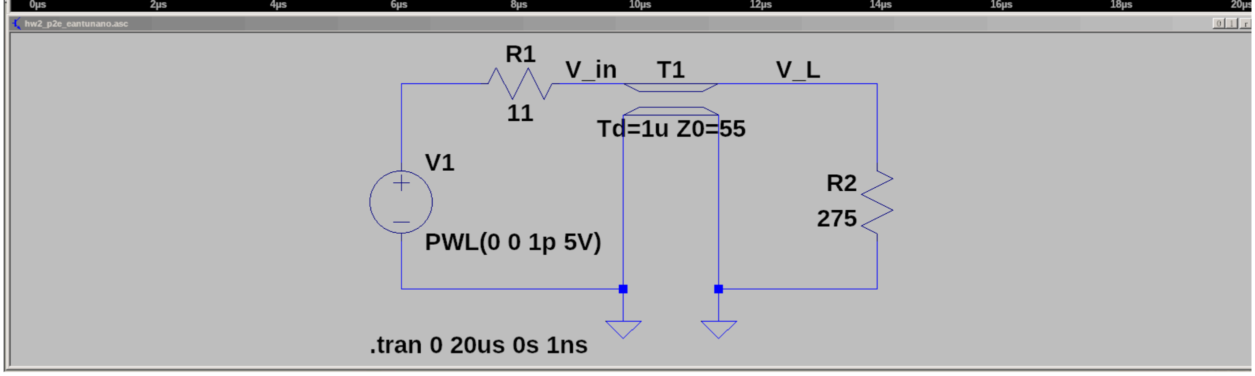


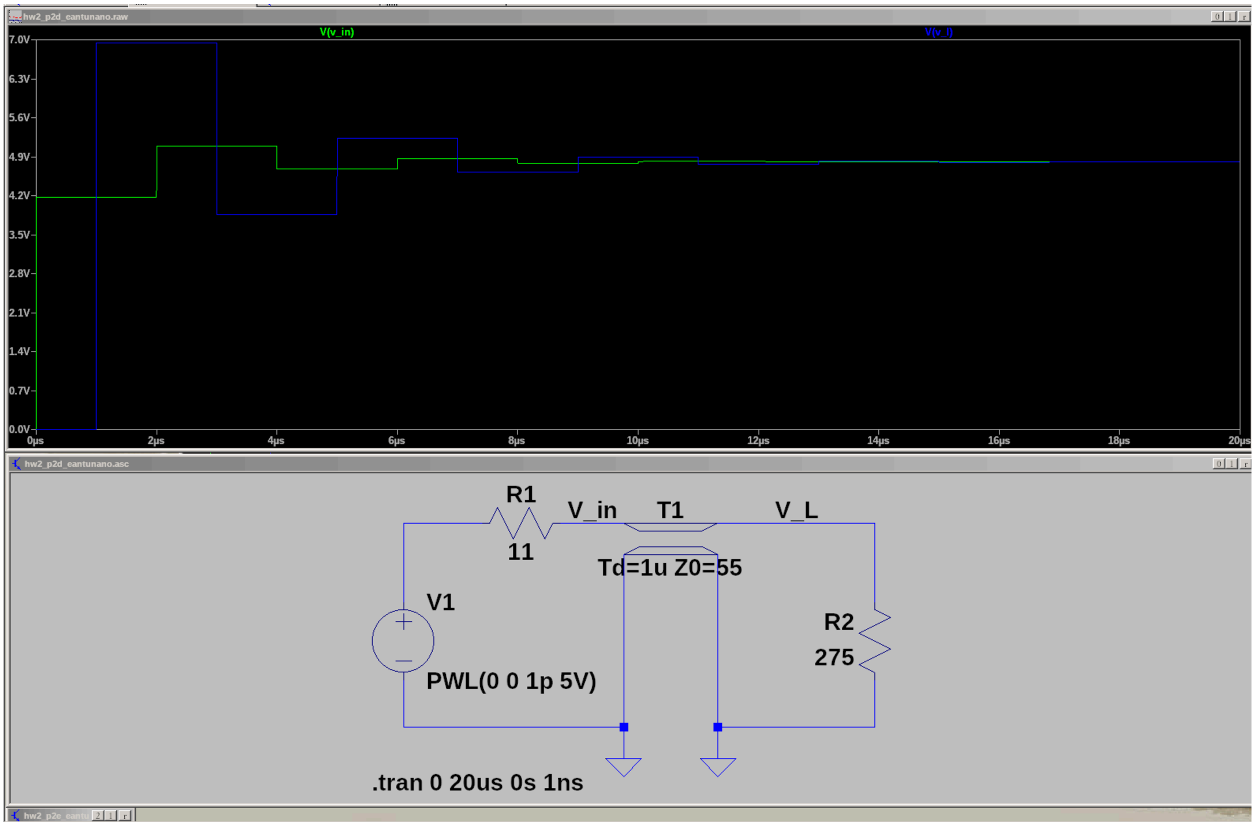
e) After the line's T_D, the $V_L(t)$ saturated because the source impedance matched the transmission line's characteristic impedance. There was no ringing on the load. There was a small ring on $V_{in}(t)$, as the reflected load voltage caused $V_{in}(t)$ to match $V_s(t)$ on $2T_D$.

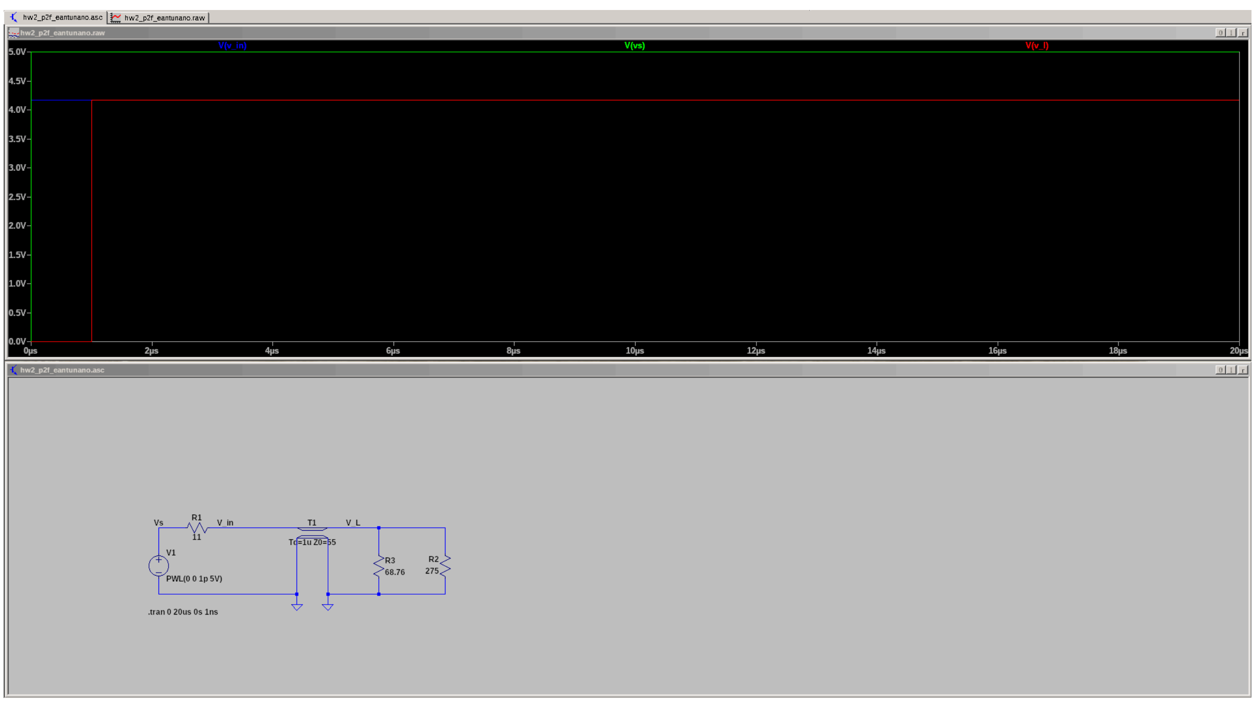
f) Similar to e), the load voltage immediately saturated to the maximum V_L with no ringing, at T_D. Unlike e), $V_{in}(t)$ settled to its maximum value at time 0.

g) In general, the voltage rating for the receiver should be set to the value of V_L at time T_D, the time step with the worst-case ringing. Use the following formula for the exact voltage rating, $V_L(l, T_D) = \frac{Z_0}{R_s + Z_0}(1+\Gamma_L)(V_s(t-T_D))$






hw2_p2d_eantunano.raw
V(v_in)
V(v_l)
7.0V
6.3V
5.6V
4.9V
4.2V
3.5V
2.8V
2.1V
1.4V
0.7V
0.0V
0µs
2µs
4µs
6µs
8µs
10µs
12µs
14µs
16µs
18µs
20µs
hw2_p2d_eantunano.asc
R1
11
V_in
T1
V_L
Td=1u Z0=55
V1
PWL(0 0 1p 5V)
R2
275
.tran 0 20us 0s 1ns


hw2_p2f_eantunano.asc
hw2_p2f_eantunano.raw
V(v_in)
V(vs)
V(v_l)
5.0V
4.5V
4.0V
3.5V
3.0V
2.5V
2.0V
1.5V
1.0V
0.5V
0.0V
0µs
2µs
4µs
6µs
8µs
10µs
12µs
14µs
16µs
18µs
20µs
Vs
R1
11
V_in
T1
V_L
Td=1u Z0=55
V1
PWL(0 0 1p 5V)
R3
68.76
R2
275
.tran 0 20us 0s 1ns

Q3

a) $\lambda = \frac{v_0}{f} = \frac{3E8}{5E6} = 0.6E2 = 60$

$$f\left(\frac{L}{v}\right) = 5E6\left(\frac{78}{60}\right) = \boxed{1.3}$$

b) $\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$; $Z_0 = Z_C$ both signs used interchangably depending on textbook

$$\Gamma_S = \frac{20 - j30 - 50}{20 - j30 + 50} = \frac{-30 - j30}{70 - j30} \rightarrow \frac{-30 - j30}{70 - j30}\left(\frac{70 + j30}{70 + j30}\right) \overset{\text{simplify by multiplying by denominator conjugate}}{=} \frac{-2100 - j2100 - j900 + 900}{4900 + 900} = \frac{-1200 - j3000}{5800} =$$

$$-0.207 - j0.517 \xrightarrow{\text{std 2 polar form}} \begin{array}{l} r = \sqrt{(-0.207)^2 + (-0.517)^2} = 0.557 \\ \theta = \operatorname{atan}\left(\frac{-0.517}{-0.207}\right) = \operatorname{atan}\left(\frac{0.517}{0.207}\right) + \pi = 1.190 \end{array} \xrightarrow{\text{Euler Format}} \boxed{0.557e^{j1.19}}$$

accounts for numerator & denominator both being negative (3rd Quadrant)

$$\Gamma_0 = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200 - j500 - 50}{200 - j500 + 50} = \frac{150 - j500}{250 - j500} = \frac{150 - j500}{250 - j500}\left(\frac{250 + j500}{250 + j500}\right) = \frac{37500 + j75000 - j125000 + 250000}{62500 + 250000} = \frac{287500 - j50000}{312500}$$

denominator conjugate

$$= 0.92 - j0.16 \xrightarrow{\text{std 2 polar form}} \begin{array}{l} r = \sqrt{(0.92)^2 + (0.16)^2} = 0.934 \\ \theta = \operatorname{atan}\left(\frac{-0.16}{0.92}\right) = \operatorname{atan}\left(\frac{-0.16}{0.92}\right) = -0.172 \end{array} \xrightarrow{\text{Euler Format}} \boxed{0.934e^{-j0.172}}$$

c) $\beta = \omega\sqrt{LC} = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$

$$\beta = \frac{2\pi}{60} = \frac{\pi}{30}$$

$$Z_{in}(-\ell) = Z_0\left(\frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}\right)$$

$$Z_{in}(.78) = 50\left(\frac{200 - j500 + j(50)(\tan(\frac{\pi}{30}\cdot 78))}{50 + j(200 - j500)\tan(\frac{\pi}{30}\cdot 78)}\right)$$

$$= 50\left(\frac{200 - j500 + j\tan(\frac{13\pi}{5})(50)}{50 + j(200 - j500)\tan(\frac{13\pi}{5})}\right)$$

$$= 50\left(\frac{200 - j500 + j(-3.078)50}{50 + j(200 - j500)(-3.078)}\right)$$

$$= 50\left(\frac{200 - j500 - j153.88}{50 - j615.5 - 1538.84}\right)$$

$$= 50\left(\frac{200 - j653.88}{-1488.84 - j615.5}\right)$$

$$= 50\left(\frac{200 - j653.88}{-1488.84 - j615.5}\right)\left(\frac{-1488 + j615.5}{-1488 + j615.5}\right)$$

$$= 50\left(\frac{-297600 + j972973.44 + j123100 + 402463.14}{2214144 + 378840.25}\right)$$

$$= 50\left(\frac{104863.14 + j1096073.44}{2592984.55}\right)$$

$$\boxed{Z_{in} = 2.02 + j21.14}$$

d) $V_{in} = V_s\left(\frac{Z_{in}}{Z_{in}+Z_s}\right)$

$$V_{in} = V_s\left(\frac{Z_{in}}{Z_{in}+Z_s}\right)$$

$$= 50\angle 0^\circ\left(\frac{2.02 + j21.14}{2.02 + j21.14 + 20 - j30}\right)$$

$$= 50\angle 0^\circ\left(\frac{2.02 + j21.14}{22.02 - j8.86}\right)$$

$$= 50\angle 0^\circ\left(\frac{21.24\angle 84.54^\circ}{23.74\angle -21.92^\circ}\right)$$

$r_n = \sqrt{2.02^2 + 21.14^2} = 21.24$

$\theta_n = \tan^{-1}\left(\frac{21.14}{2.02}\right) = 84.54^\circ$

$r_d = \sqrt{22.02^2 + 8.86^2} = 23.74$

$\theta_d = \tan^{-1}\left(\frac{-8.86}{22.02}\right) = -21.92^\circ$

$$= 50\angle 0^\circ(0.895\angle 106.46^\circ)$$

$$V_{in} = 44.75\angle 106.46^\circ$$

$$V_{in} = 44.75e^{j1.86}$$

$\frac{106.46}{180}\pi = 1.86 \text{ rad}$

$V_{in} = 44.75(\cos(1.86) + j\sin(1.86)) \xrightarrow{\text{time-domain}} r(\cos(\omega t+\phi) + \sin(\omega t+\phi))$

$$\boxed{V_{in}(t) = 44.75(\cos(10\pi E6\, t + 1.86) + j\sin(10\pi E6\, t + 1.86))}$$

Lec 4, Slide 34

$V(z) = V^+(e^{-j\beta z} + \Gamma_0 e^{j\beta z})$

$$V_L = V_{in}e^{-j\beta l}(1 + \Gamma_L)$$

$$= 44.75\angle 106.45\, e^{-j(72)(\frac{\pi}{30})}(1 + 0.92 - j0.16)$$

$$= 44.75\angle 106.45\, e^{-j8.17}(1.92 + j0.16)$$

$360^\circ = 6.2832$ rad

$$= 44.75\angle 106.45\, e^{-j466.79} \quad 1.93\angle 4.76^\circ$$

$$= 44.75\angle 106.45\, e^{-j466.79-360} \quad 1.93\angle 4.76^\circ$$

$$= 86.39\angle 111.55^\circ\, e^{-j106.79^\circ}$$

$$= 86.39\angle 111.55^\circ \quad 1\angle -106.79$$

$$= 86.39\angle 4.76^\circ$$

$$V_L = 86.39e^{j0.083}$$

$\xrightarrow{\text{time-domain}} r(\cos(\omega t+\phi) + \sin(\omega t+\phi))$

$$\boxed{V_L(t) = 86.39(\cos(10\pi E6 t + 0.083) + j\sin(10\pi E6\, t + 0.083))}$$

e) $P_{av} = \frac{1}{2} \text{Re}\{ V I^* \}$

$$= \frac{1}{2} \text{Re}\left\{ \frac{\left(86.39 e^{j0.0839}\right)^2}{200 - j500} \right\}$$

$$= \frac{19923.66 e^{j3.546}}{538.52 \angle -68.2} \qquad \frac{3.54 \cdot 180}{\pi} = 202.827$$

$$= \frac{19837.83 \angle 202.827}{538.52 \angle -68.2}$$

$$= \frac{1}{2} \text{Re}\{ 13.849 \angle 271.03° \}$$

$$= \frac{1}{2} r \cos(\theta)$$

$$= \frac{1}{2} (13.849) \cos(271.03)$$

$$= \frac{1}{2} (2.99)$$

$$\boxed{P_{av} = 1.435 \text{ W}}$$

f) $$\text{VSWR} = \left| \frac{V_{max}}{V_{min}} \right| = \left| \frac{I_{max}}{I_{min}} \right| = \frac{1 + |\Gamma_0|}{1 - |\Gamma_0|}$$

$$= \frac{1 + \left| 0.934 e^{-j0.172} \right|}{1 - \left| 0.934 e^{-j0.172} \right|}$$

$$= \frac{1 + 0.934}{1 - 0.934}$$

$$\boxed{\text{VSWR} = 29.3}$$

$$T_D = \left(\frac{1}{f}\right)(\lambda) = \left(\frac{1}{5\,\text{MHz}}\right)(1.3) = 260\,\text{ns}$$

Negative imaginary number indicates capacitor $Z = \left(\frac{1}{j\omega C}\right)$
Positive imaginary indicates inductor $Z = j\omega L$

$$Z_S = \frac{1}{j\omega C_S}$$

$$C_S = \frac{1}{j\omega Z_S}$$

$$= \frac{1}{j(2\pi)(5E6)(-j30)}$$

$$= \frac{1}{2\pi(5E6)(30)}$$

$$C_S = 1\,\text{nF}$$

$$C_L = \frac{1}{2\pi(5E6)(500)}$$

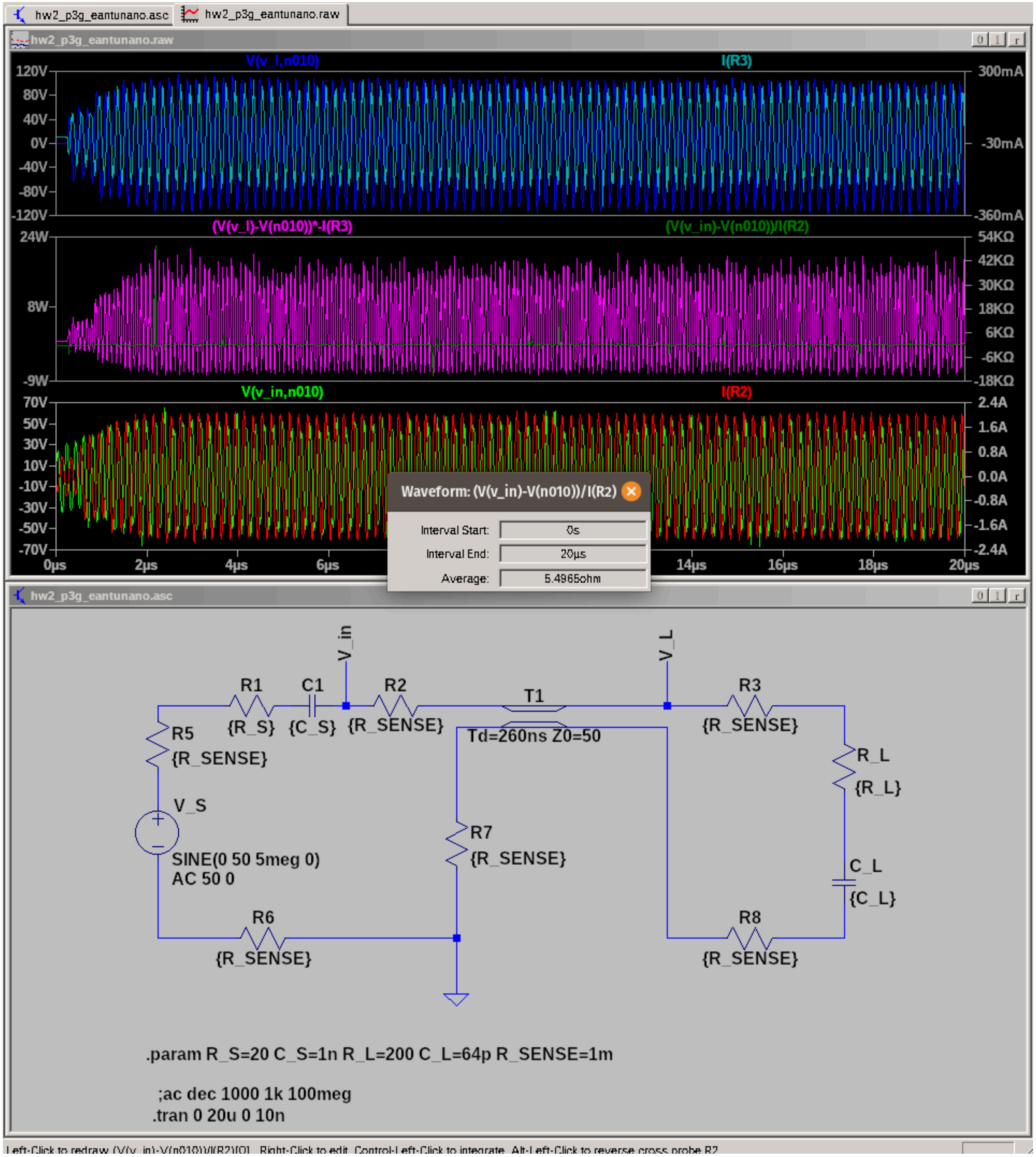
$$= \frac{1}{2\pi(5)(5)(10^6)(10^2)}$$

$$= \frac{1}{(10^6)(10^2)(10^2)(1.571)}$$

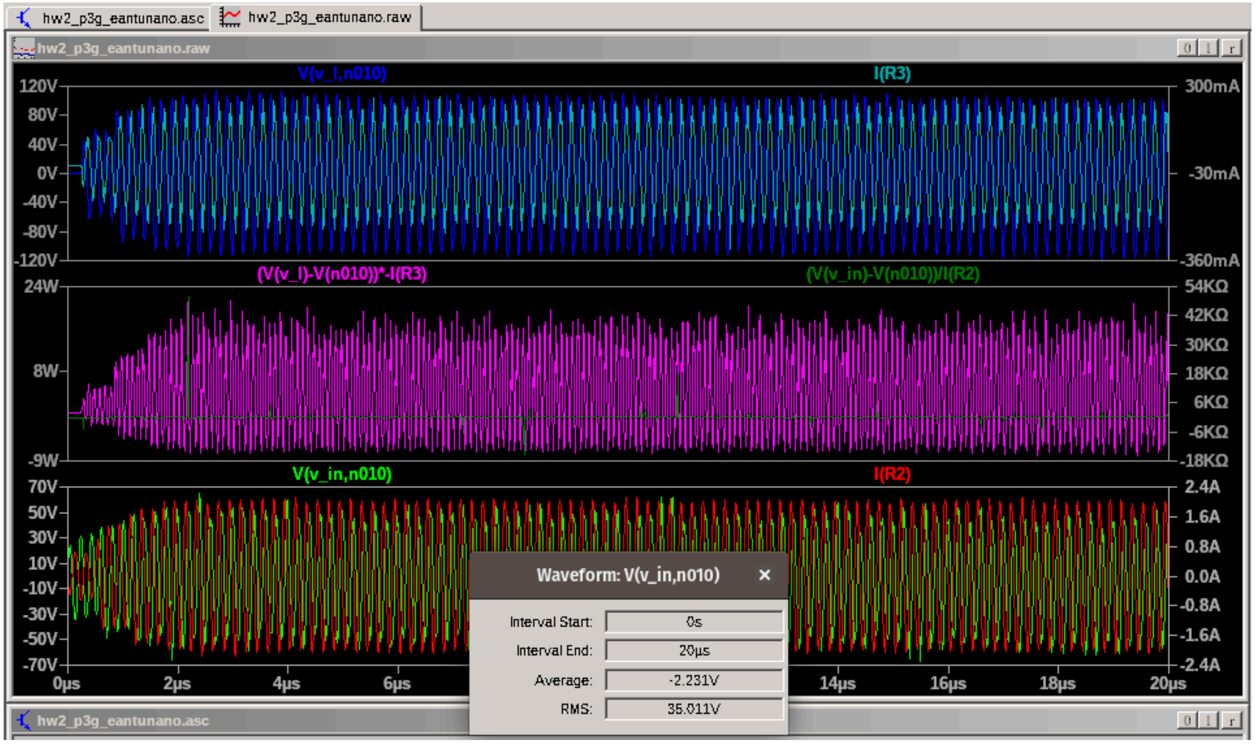
$$= 0.64 \times 10^{-10}$$

$$C_L = 64\,\text{pF}$$

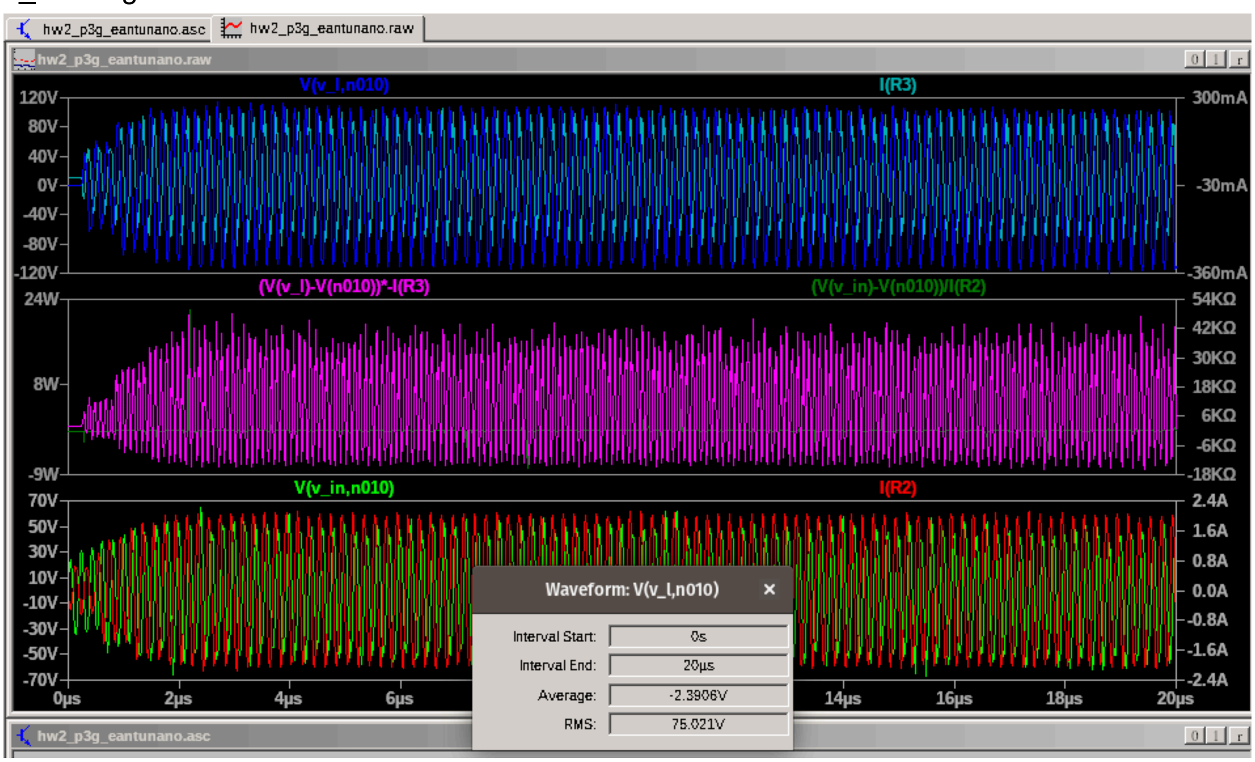
Impedance



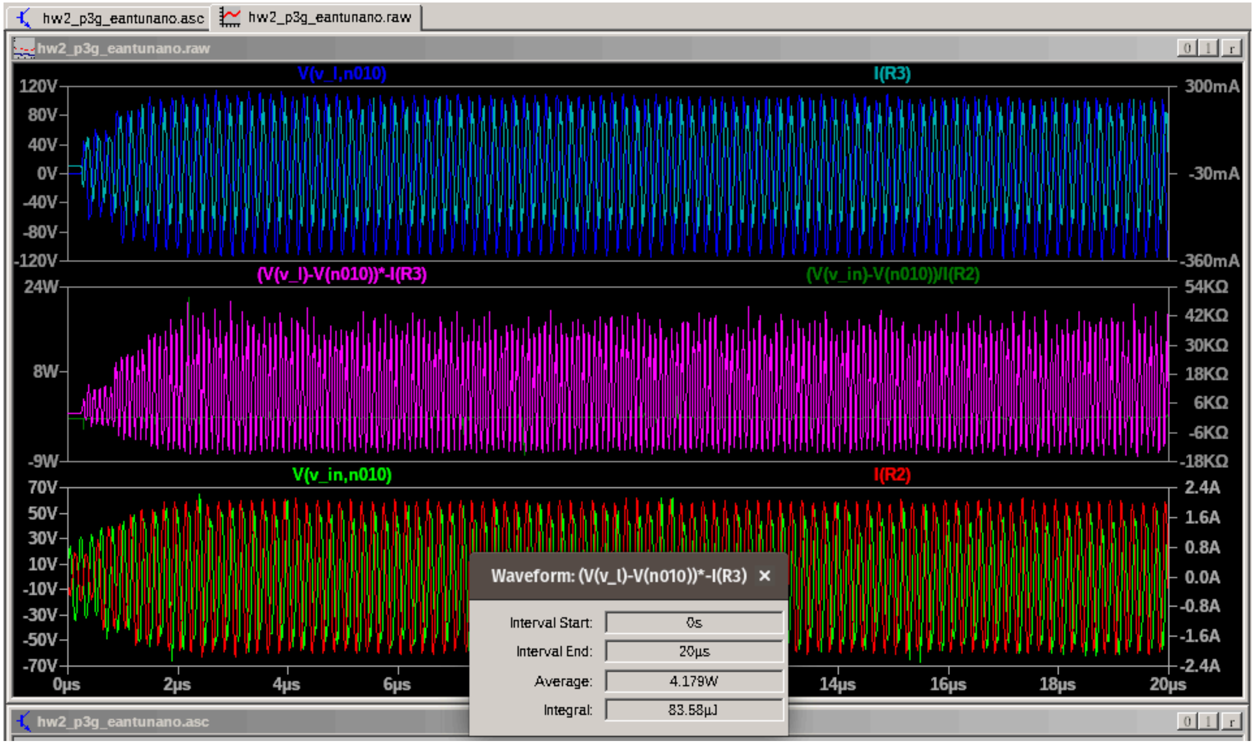
Vin Value


hw2_p3g_eantunano.asc
hw2_p3g_eantunano.raw
V(v_l,n010)
I(R3)
(V(v_l)-V(n010))*-I(R3)
(V(v_in)-V(n010))/I(R2)
V(v_in,n010)
I(R2)
Waveform: V(v_in,n010)
Interval Start: 0s
Interval End: 20µs
Average: -2.231V
RMS: 35.011V

V_L voltage

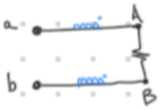

hw2_p3g_eantunano.asc
hw2_p3g_eantunano.raw
V(v_l,n010)
I(R3)
(V(v_l)-V(n010))*-I(R3)
(V(v_in)-V(n010))/I(R2)
V(v_in,n010)
I(R2)
Waveform: V(v_l,n010)
Interval Start: 0s
Interval End: 20µs
Average: -2.3906V
RMS: 75.021V

Average Power

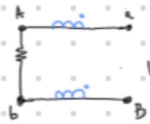


Q4

a)



$300k \angle 90^\circ$ @ 50 MHz



$1M \angle 90^\circ$ @ 50 MHz

$$\hat{Z}_1 = \frac{\hat{V}_1}{\hat{I}_1} = \frac{j\omega(L\hat{I}_1 + m\hat{I}_2)}{\hat{I}_1}$$

Considering $\hat{I}_1 = \hat{I}_c$ and $\hat{I}_2 = \hat{I}_c$ then,

$$\hat{Z}_{cm} = j\omega(L+m) + j\omega(L+m)$$

$$300k \angle 90^\circ = j(2\pi f)(L+m) + j(2\pi f)(L+m)$$
$$= j2(\pi E8)(L+m)$$
$$300k(\cos(90^\circ) + j\sin(90^\circ)) = j2\pi E8(L+m)$$
$$300k(j(1)) = \downarrow$$
$$\frac{j3E5}{j2\pi E8} = L+M$$

$$L+M = 0.478\,mH$$

$$\hat{Z}_{DM} = j\omega(L-m) + j\omega(L-m)$$

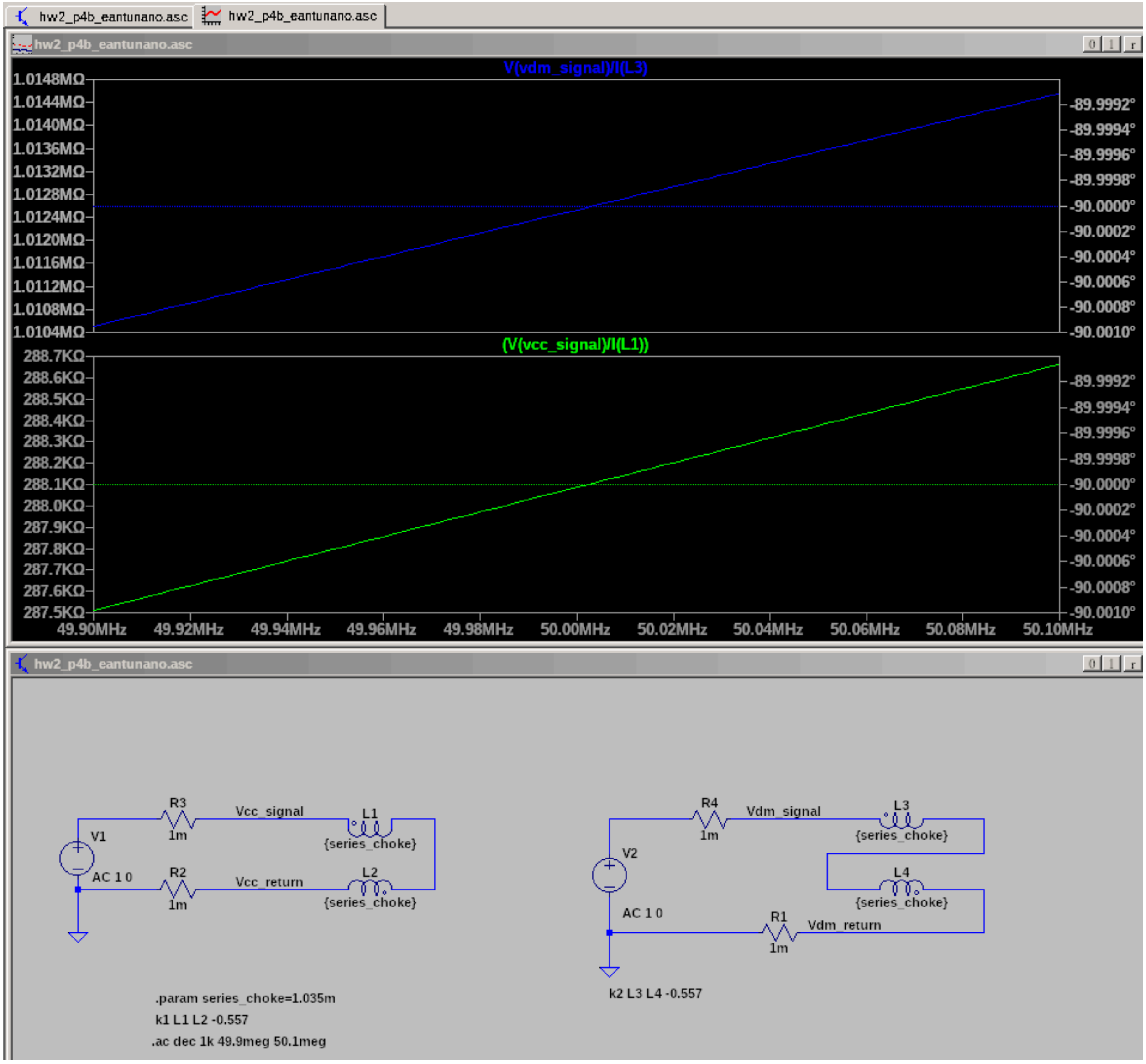
$$1M \angle 90^\circ = j(2\pi f)(L-m) + j2\pi f(L-M)$$
$$= j2\pi(5E7)[\quad]$$
$$= j\pi E8(L-m) + j\pi E7(L-m)$$
$$j1E6 = j2\pi E8(L-m)$$
$$\frac{j1E6}{j2\pi E8} = L-m$$

$$L-M = 1.592\,mH$$

$$m = 0.478\,mH - L$$
$$L - 0.478\,mH + L = 1.592\,mH$$
$$2L = 2.07\,mH$$
$$\boxed{L = 1.035\,mH}$$
$$m = 0.478\,mH - 1.035\,mH$$
$$\boxed{M = -0.557\,mH}$$

Top Graph: Differential Impedance

Bottom Graph: Common Mode Impedance

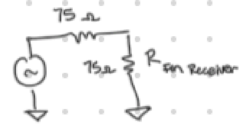


Q5

a)

Assuming room temperature operation,

K = Boltzmann's constant $(1.38 \times 10^{-23}\ J/K)$
T = temp in kelvins
B = bandwidth in Hz
R = resistance in ohms



$$V_{t_o} = \sqrt{4KTRB}$$
$$= \sqrt{4(1.38 \times 10^{-23})(290)(75)(50 \times 10^{3})}$$
$$= 0.245\ \mu V$$

$$V_{t-\text{fm receiver}} = \tfrac{1}{2}(0.245)$$
$$= 0.1225\ \mu V \xrightarrow{\mu V \text{ to } mV} = 0.0001225\ mV$$
$$= 20\log_{10}(0.1225) \qquad = 20\log_{10}(0.0001225)$$
$$= -18.24\ dB\mu V \qquad = -78.24\ dBmV$$

The FM Receiver will add 8 dB of noise then,

$$V_{in-FM} = -78.24 + 8$$
$$V_{in-FM} = -70.24\ dBmV$$

18dB SNR, then signal level @ FM Receiver input should be,

$$V_{sig}|_{dB} = SNR_{|dB} + V_{in-FM}$$
$$= 18 - 70.24$$
$$= -52.24\ dBmV$$

$$V_{sig-ant}|_{dB} = 6 - 52.24$$

$$= -46.24\ dBmV$$

$$\boxed{V_{sig-ant}|_{V} = 10^{\left(\frac{-46.24}{20}\right)} = 0.00487\ mV}$$

b) FM receiver does not require as clean or noiseless signal compared to a TV. So, the input voltage can have a lower SNR, thus leading to a lower input signal than a TV, which has a much larger SNR.