Electromagnetic Compatibility (EMC)

Topic 11 Radiated Emissions and Susceptibility

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Introduction

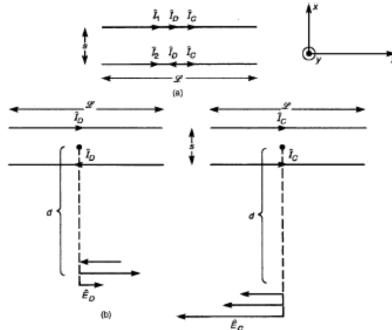
- Recall radiated emissions limits
- Domestic radiated emissions span the frequencies from 30MHz to more than 1 GHz
- FCC measurement distance was 3m for class B products and 10m for class A ones
- CISPR 22 provide a measurement distance of 10m for class B and Class A products
- @ 30MHz, $10m = 1 \lambda$
- @ 1 GHz, 30cm = 1λ
- Thus at lower frequencies, product might be in the *near-field* region, while for higher frequencies it is in the *far-field*
- We will work with simple first-order models to predict the levels of radiated emissions, they assume that the antenna is in the far field of the product
- While products might radiate and cause interference to other nearby products, they can be susceptible to radiation from other products as well.

Differential-mode and Common-mode currents (again!)

- Wires and PCB traces in products can become unintentional Antennas radiating emissions all around
- Time varying currents can generate radiation of electromagnetic fields, thus any conductor carrying a time-varying current will radiate
- Recall that (Figure (a)):

$$\begin{vmatrix} I_{1} = I_{CM} + I_{DM} \\ I_{2} = I_{CM} - I_{DM} \end{vmatrix} \Rightarrow I_{DM} = \frac{I_{1} - I_{2}}{2} , \quad I_{CM} = \frac{I_{1} + I_{2}}{2}$$

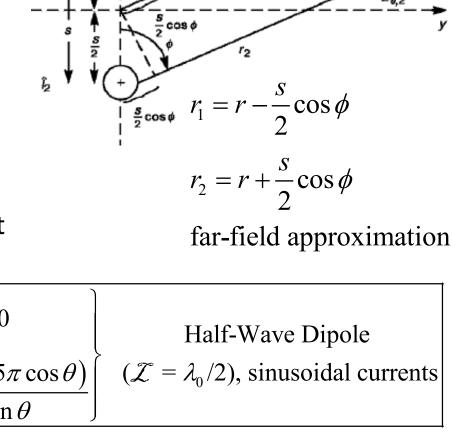
- Fields from DM currents are almost non-existent or with very small values because they are of similar magnitude and opposite direction (Figure(b))
- Fields from CM current add up, thus reinforcing their magnitudes and are the ones contributing to radiation (also called antenna-mode currents) (Figure (c))



- We will come up with simple emission models for a pair of parallel wires or PCB traces (lands) due to the currents on such conductors
- Consider the two conductors placed on the x-axis the currents flowing in the same direction (+ mean into the page, z-axis for the figure shown)
- We want to find the total radiated electric field (vector quantity) from this two conductor radiator (antenna)
- The maximum electric field (vector field) will be on the x-y plane and will be the sum of the two fields

$$\hat{E}_{\theta,Total} = \hat{E}_{\theta,1} + \hat{E}_{\theta,2} \quad , \quad \hat{E}_{\theta,i} = \hat{M}\hat{I}_i \frac{e^{-j\beta r_i}}{r_i} F(\theta)$$

Where the term M is a function of the antenna type, and F() is the geometrical factor that depends on the antenna element placement in space (i.e. array factor of an antenna array)



$$\hat{M} = j \frac{\eta_0 \beta_0}{4\pi} \mathcal{I} = j2\pi \times 10^{-7} f \mathcal{I}$$
Hetzian Dipole
$$F(\theta) = \sin \theta$$
(electrically short $< \lambda/10$)

$$\hat{M} = j \frac{\eta_0}{2\pi} = j60$$
Half-Wave Dipole
$$F(\theta) = \frac{\cos(0.5\pi \cos \theta)}{\sin \theta}$$

$$(\mathcal{Z} = \lambda_0/2), \text{ sinusoidal currents}$$

• The total field from two identical elements (hertzian or half-wave Dipoles), and considering $|r|^{\sim}|r_1|=|r_2|$ we get

$$\begin{split} \hat{E}_{\theta,Total} &= \hat{M} \left(\hat{I}_1 \frac{e^{-j\beta r_1}}{r_1} + \hat{I}_2 \frac{e^{-j\beta r_2}}{r_2} \right) \\ \hat{E}_{\theta,Total} &= \hat{M} \left(\hat{I}_1 \frac{e^{-j\beta_0 \left(r - \frac{s}{2}\cos\phi \right)}}{r_1} + \hat{I}_2 \frac{e^{-j\beta_0 \left(r + \frac{s}{2}\cos\phi \right)}}{r_2} \right) \\ \hat{E}_{\theta,Total} &= \hat{M} \frac{e^{-j\beta_0 r}}{r} \left(\hat{I}_1 e^{j\beta_0 \left(\frac{s}{2}\cos\phi \right)} + \hat{I}_2 e^{-j\beta_0 \left(\frac{s}{2}\cos\phi \right)} \right) \end{split}$$

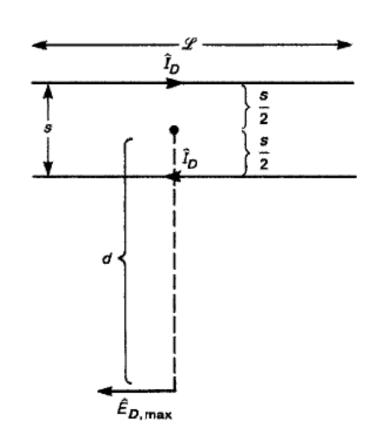
- Note that all the above is based on the far-field assumption, and if you violate this
 assumption, i.e. you operate in the near field, then the above formulas should not be
 used.
- We will now substitute the DM and CM currents in the conductors, and check the obtained E-field expression for each case. Then we can estimate the radiated emissions and fields accordingly and examine against the limits allowed.

DM current emission model

- We will model each conductor as a Hertzian Dipole for simplicity of expressions obtained (we can re-do for half-wave dipoles if needed)
- Thus, we assume:
 - The lengths of the lines carrying the current are sufficiently short
 - The measurement point is in the far field (sufficiently far away)
 - The current distribution in the wires is constant
- For the Hertzian Dipole (very short length), a cable of 1 m, will be 1λ @ 300 MHz, and @ 100MHz, the cable length is $\lambda/3$, thus we can consider a constant current on it. A PCB land of 30cm, is $\lambda/10$ @100MHz, thus the current will be reasonably constant at 200MHz.
- Let $I_1 = -I_2 = I_{DM}$, then, $\hat{E}_{DM, max} = j2\pi \times 10^{-7} \frac{f \mathcal{I} \hat{I}_{DM}}{d} e^{-j\beta_0 d} \left\{ e^{j\beta_0 \frac{s}{2}} e^{-j\beta_0 \frac{s}{2}} \right\}$

$$\hat{E}_{DM,\text{max}} = -4\pi \times 10^{-7} \frac{f \mathcal{Z} \hat{I}_{DM}}{d} e^{-j\beta_0 d} \sin\left(\beta_0 \frac{s}{2}\right)$$

• Thus, if wire spacing is small,
$$\hat{E}_{DM, \max} = 1.316 \times 10^{-14} \frac{f^2 \mathcal{Z} |\hat{I}_{DM}| s}{d}$$



, using
$$\beta_0 \frac{s}{2} = \frac{\pi s}{\lambda_0} = \frac{\pi s f}{c}$$
 and $\sin \left(\beta_0 \frac{s}{2}\right) \approx \beta_0 \frac{s}{2}$

Example 11.1

Consider a Ribbon cable constructed with 28-AWG wires separated by 50mils. Suppose the length of the wires is 1m and that they carry a 30MHz DM current. The level of DM current will give a radiated emission in the plane of the wires and broadside to the cable(worst case) that equals the FCC Class B limits (40 dB μ V/m or 100 μ V/m @ 30MHz) can be found as,

Sol:

d = 3m (from FCC class B limit to give 100 μ V/m)

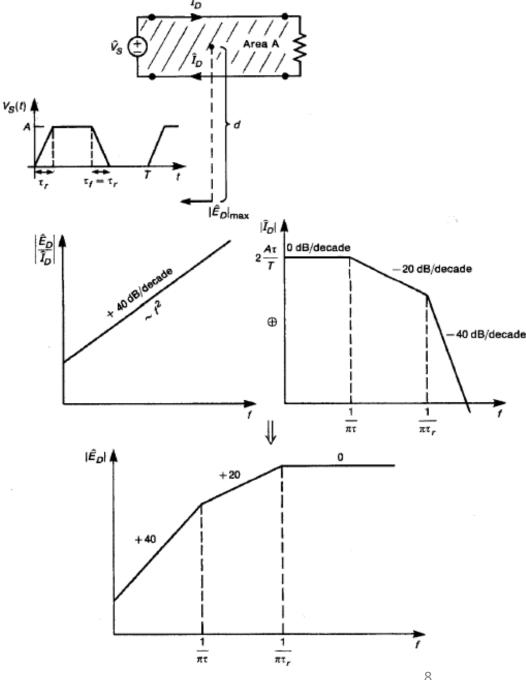
$$100 \times 10^{-6} \left(\frac{V}{m}\right) = 1.316 \times 10^{-14} \frac{\left|\hat{I}_{DM}\right| \left(30 \times 10^{6}\right)^{2} \left(1\right) \left(1.27 \times 10^{-3}\right)}{3}$$

$$\rightarrow \hat{I}_{DM} = 19.95 mA$$

- Consider a trapezoidal waveform (i.e. clock signal) driving a 2-wire line as shown.
- The transfer function relating the maximum electric field to the current, varies with the loop area (Ls) and the square of the frequency,

Where $K=1.316x10^{-14}/d$ for the FCC class B at 3m.

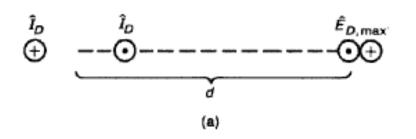
- The transfer function will increase with 40dB/decade because of the f² term.
- Multiplying the above transfer function with that of a Trapezoidal signal (adding the Bode plot magnitudes in dB) yields the Spectrum to the right.
- Thus, radiated emissions from DM currents are confined towards higher frequencies, usually above 200MHz.

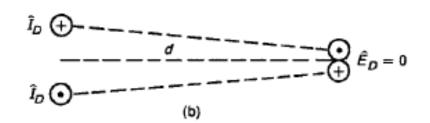


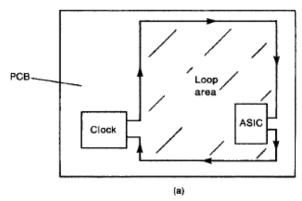
In summary,

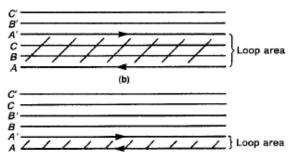
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- radiated emissions from DM currents are confined towards higher frequencies, usually above 200MHz.
- Also, maximum radiation occurs in the plane broadside to the wires.
- The maximum radiated E-fields vary with
 - $(1) f^2$
 - (2) Loop Area (A = $s\mathcal{I}$)
 - (3) level of I_{DM}
- To reduce radiated emissions, we can
 - (1) reduce current levels (I_{DM})
 - (2) reduce loop area (A)
- A common mistake that can lead to large DM emissions →









CM current emission Model

- Again, we will approximate using Hertzian Dipole (very short Dipole)
- The maximum emissions point will be the one on the plane of the conductors
- Considering that I1=I2=I_{CM}, then the radiated field becomes,

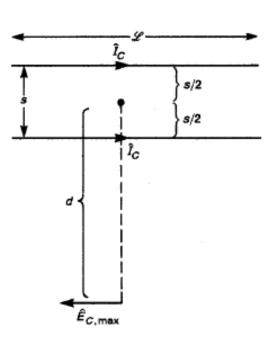
$$\hat{E}_{C,\text{max}} = j2\pi \times 10^{-7} \frac{f \hat{I}_C \mathcal{Z}}{d} e^{-j\beta_0 d} \left\{ e^{-j\beta_0 d} + e^{j\beta_0 d} \right\}$$

$$\hat{E}_{C,\text{max}} = j4\pi \times 10^{-7} \frac{f\hat{I}_C \mathcal{I}}{d} e^{-j\beta_0 d} \cos\left(\frac{\beta_0 s}{2}\right)$$

• With small wire spacing (small s), we can write

$$\left| |\hat{E}_{C,\text{max}}| = 1.257 \times 10^{-6} \frac{f \mathcal{Z} |\hat{I}_{CM}|}{d} \right|, \text{ using } \beta_0 \frac{s}{2} = \frac{\pi s}{\lambda_0} \text{ and } \cos\left(\beta_0 \frac{s}{2}\right) \approx 1$$

- For CM currents and electrically small wire separations, the radiated pattern if virtually omnidirectional around the wire. This is because we can replace the two wires with one carrying twice the current (2I_{CM}).
- A Current Probe is widely used for current wire measurements, it will measure 2I_{CM}.



Example 11.2

Consider a Ribbon cable constructed with 28-AWG wires separated by 50mils. Suppose the length of the wires is 1m and that they carry a 30MHz CM current. The level of the CM current that will give a radiated emission in the plane of the wires and broadside to the cable (worst case) that equals the FCC Class B limits (40 dB μ V/m or 100 μ V/m @ 30MHz) can be found as,

Sol:

d = 3m (from FCC class B limit to give 100 μ V/m)

$$100 \times 10^{-6} \left(\frac{V}{m} \right) = 1.257 \times 10^{-6} \frac{\left| \hat{I}_{CM} \right| \left(30 \times 10^{6} \right)^{2} (1)}{3}$$

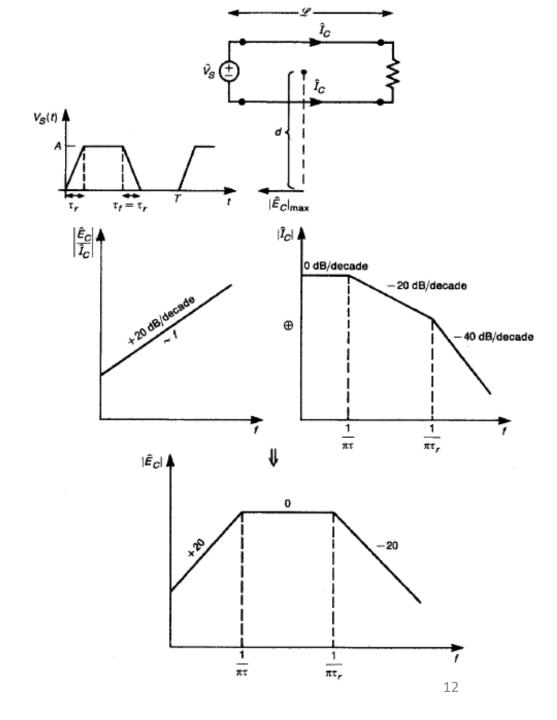
$$\rightarrow \hat{I}_{CM} = 7.97 \,\mu A$$

→ This is a much, much, much smaller current that will cause the product to FAIL radiated emissions compared to its DM counterpart! (See example 11.1)

- Consider a trapezoidal waveform (i.e. clock signal) driving a 2-wire line as shown.
- The transfer function relating the maximum electric field to the current, varies with the line length (\mathcal{I}) and the frequency, $\left|\frac{\hat{E}_{CM,\max}}{\hat{I}_{CM}}\right| = Kf\mathcal{I}$

Where $K=1.257x10^{-6}/d$ for the FCC class B at 3m.

- The transfer function will increase with 20dB/decade because of the f term.
- Multiplying the above transfer function with that of a Trapezoidal signal (adding the Bode plot magnitudes in dB) yields the Spectrum to the right.
- Thus, radiated emissions from CM currents are confined within lower frequency values usually below 300MHz.
- Considering a 100MHz pulse train with 50% duty cycle and 1ns t_r , then $1/\pi\tau = 63.7$ MHz, and $1/\pi t_r = 318.3$ MHz.



In summary,

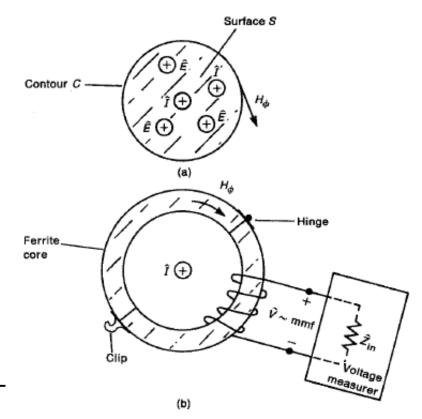
- radiated emissions from DM currents are confined towards higher frequencies, usually above 200MHz.
- Also, maximum radiation occurs in the plane broadside to the wires and is essentially constant around the wires (not sensitive to rotation)
- The maximum radiated E-fields vary with
 - (1) f
 - (2) Line length (\mathcal{I})
 - (3) level of I_{CM}
- To reduce radiated emissions, we can
 - (1) reduce current levels (I_{CM})
 - (2) reduce the line length (\mathcal{I})

Current Probes

- While DM currents can be reliably calculated CM ones are difficult to predict and calculate.
- We use current probes to measure CM currents.
- Current probes operate based on Ampere's Law:

$$\oint_C \vec{H} \bullet d\vec{l} = \int_S \vec{J} \bullet d\vec{s} + \frac{d}{dt} \varepsilon \int_S \vec{E} \bullet d\vec{s}$$

- A magnetic field can be induced around a contour by either conduction currents or displacement currents (due to time-varying Efields) that penetrate an open surface.
- A ferrite core is the center of a current probe that is separated into two halves, that are joined by a hinge. The ferrite core will concentrate the magnetic flux inside it. Several turns of wire are wound around the core, and so a time-varying H-field that is concentrated in the code will induce a an EMF proportional to the this H-field. This induced voltage can be measured and will be proportional to the current inside the wires, i.e. CM current.



Voltage measurer = Spectrum Analyzer

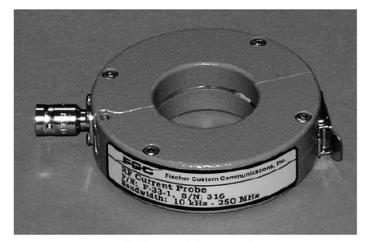
- Again, we will carry approximate calculations to get a feel of the levels obtained.
- To calibrate the probe, run known currents, measure the output voltages and construct the transfer impedance curve as,

$$\hat{Z}_{\scriptscriptstyle T} = \hat{V} / \hat{I}$$

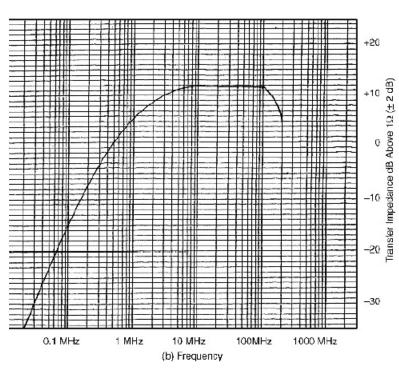
- The probe manufacturer provides such curves for their probes.
- Usually, the transfer impedance is given in dB relative to 1Ω

$$\left|\hat{Z}_{T}\right|_{dB-\Omega} = \left|\hat{V}\right|_{dB\mu V} - \left|\hat{I}\right|_{dB\mu A}$$

- A typical curve of the transfer impedance of a current probe is shown
- The value is constant around 12 dB Ω from 10 to 100MHz.
- Typical values of 15 dB Ω from 10 to 200MHz are used in the following analysis
- An important parameter of the transfer impedance curve is the termination impedance of the probe. The curve is valid ONLY when the probe is terminated in the same impedance as was used in the course of its calibration (i.e. $50~\Omega$)



(a) Photograph of current probe.



- Consider determining the probe voltage level that corresponds to a CM current on the cable that will give a radiated emissions just meeting the regulatory limit.
- The probe will measure the net CM current on the cable/wire, while the magnetic fluxes of the DM current cancel out.
- Thus the current probe will not measure DM currents unless placed around each individual wire, not both at the same time.
- Suppose the current probe has two wires pass through it, thus

$$|\hat{E}_{C,\text{max}}| = \frac{1.257}{2} \times 10^{-6} \frac{f \mathcal{Z} |\hat{I}_{CM}|}{d} = 6.28 \times 10^{-7} \frac{f \mathcal{Z} |\hat{I}_{C,net}|}{d}$$

• Note that we divided $I_{CM}/2$ to get the contribution of an individual current as the current probe sees double the CM currents one from each wire.

Example 11.3

Suppose that a current probe is clamped around a 1m cable and the voltage is measured at 30MHz to produce an emission equal to the FCC Class B limit. Find the net CM current value. Then find the level of the measured voltage from this current value if the probe transfer impedance is 15Ω .

Sol:

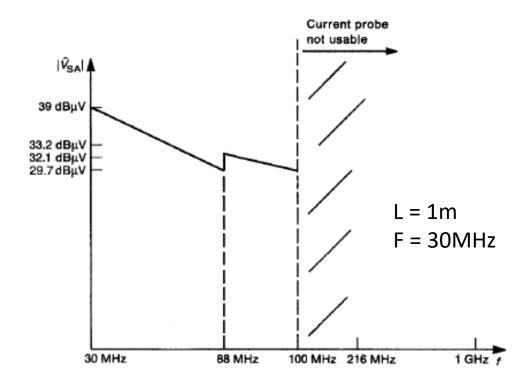
On the BOARD!

- The current probe is an excellent EMC diagnostic tool though out the design process.
- It is cost effective and simple to measure the emissions using a SA and a current probe though all the design phases rather than using a complete chamber setup.
- Anticipated fixes can be made early on and the addition of ferrite beads or common-mode chokes of other methods to reduce CM currents.
- This way the chances of success for complete pre-compliance testing can be increased for every step of the design process.
- A calibration chart for determining the level of the probe voltage that will correspond to a current that will equal regulatory limits can be found through,

$$\left| \hat{E}_{C,\text{max}} \right| = 6.28 \times 10^{-7} \frac{\left| \hat{V}_{SA} \right| f \mathcal{Z}}{\left| \hat{Z}_{T} \right| d}$$

In dB, and solving for probe voltage

$$\begin{aligned} |\hat{V}_{SA}|_{dB\mu V} &= |\hat{E}|_{\text{limit}, dB\mu V/m} + |\hat{Z}_{T}|_{dB\Omega} + 20\log_{10} d \\ &- 20\log_{10} f_{MHz} - 20\log_{10} \mathcal{Z} + 4.041 \end{aligned}$$



Example 11.4

Consider a 1m cable and a current probe with transfer impedance of $15dB\Omega$. In order to comply with the FCC Class B limit (d=3m) at 30 MHz of $40dB\mu V/m$, what would be the probe voltage reading?

Sol:

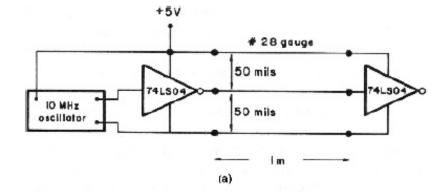
$$\begin{aligned} \left| \hat{V}_{SA} \right|_{dB\mu V} &= \left| \hat{E} \right|_{\text{limit}, dB\mu V/m} + \left| \hat{Z}_{T} \right|_{dB\Omega} + 20 \log_{10} d - 20 \log_{10} f_{MHz} - 20 \log_{10} \mathcal{I} + 4.041 \\ \left| \hat{V}_{SA} \right|_{dB\mu V} &= 40 dB\mu V / m + 15 dB\Omega + 20 \log_{10}(3) - 20 Log_{10}(30) + 4.041 \\ \left| \hat{V}_{SA} \right|_{dB\mu V} &= 39 dB\mu V \quad \text{(as found before!)} \end{aligned}$$

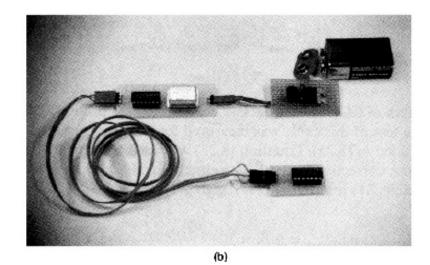
Experimental Results

- Simple experiments are conducted to show the validity of the models developed
- Objectives are:
 - (1) validate accuracy of the proposed models given the current levels and currents provided
 - (2) Show that a current probe can provide accurate values of CM current levels on cables and PCBs
 - (3) a ferrite toroid can be an effective solution for reducing CM current emissions
- Two experiments will be considered
 - (1) with cable connections
 - (2) with PCB traces
- Both will use a dedicated battery and will be an isolated circuit

Experiment 1

- A 10MHz oscillator in a 14-pin DIP package driving an inverter gate and the output is connected to another inverter gate via a 1m 3-wire ribbon cable (28-AWG) with center to center separation of 50mils.
- The clock pulses are carried in the middle conductor, the +5 supply on the outer conductor, and the other outer conductor serves as a return path. The +5V supply is provided from a 9V battery with a 7805 voltage regulator. Thus the power is isolated to confine the observations and analysis and avoid external noise contributors.
- Radiated emissions were measured in a semi-anechoic chamber for EMC compliance.
- A biconical antenna covering the range 20-200MHz was used.
- The circuit was parallel to the chamber floor and 1m above it as shown in the setup on the next page.
- More details about experiment in [1]





[1] C. R. Paul and D. R. Bush, "Radiated emissions from common-mode currents," Proceedings of the International Symposium on Electromagnetic Compatibility 1987, Atlanta, GA.

- A current probe placed at the mid-pint of the cable with $15 dB\Omega$ transfer impedance was used to measure CM current on the cable to predict the radiated emissions.
- A spectrum analyser (SA) was used to plot the spectrum of the voltage in the frequency range of interest. The cable loss of the connecting cable to the SA is included in the formula

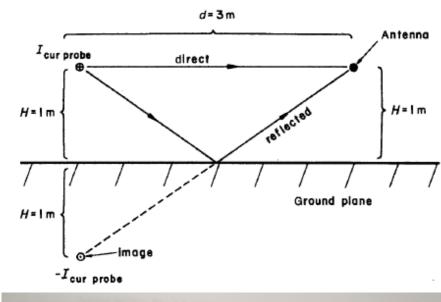
$$\left| \hat{I}_{probe} \right|_{dB\mu A} = \left| \hat{V}_{SA} \right|_{dB\mu V} + cable \left| loss_{dB} - \left| \hat{Z}_{T} \right|_{dB\Omega}$$

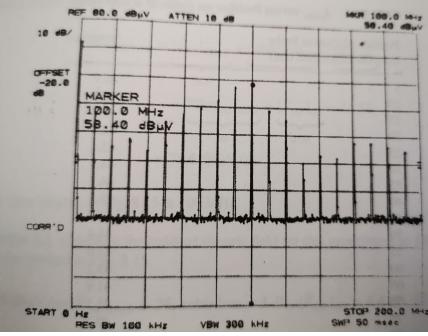
• To correct for the reflections from the ground plane, a correction factor is incorporated (F_{GP}). The values are obtained from tables based on the frequency of operation and the measurement setup. This gives, $\left|\hat{E}_{C,\max}\right| = 6.28 \times 10^{-7} \frac{\left|\hat{I}_{C,net}\right| f \mathcal{I}}{J} \hat{F}_{GP}$

d

• Combining the above with 1m line lengths, and d=3m,

$$\left| \hat{E}_{C} \right|_{dB\mu V/m} = \left| \hat{V}_{SA} \right|_{dB\mu V} + cable _loss_{dB} - \left| \hat{Z}_{T} \right|_{dB\Omega} + 20 \log_{10} \left(f_{MHz} \right) + \left| \hat{F}_{GP} \right|_{dB} - 13.58$$





Current probe measurements

- Measured spectrum in the chamber using the antenna versus the predicted values using the current probe (x marks) at each frequency.
- We can see values from the prediction equations using the current probe measurements are within 3dB of the antenna measurements, except at 50, 80 and 130 MHz
- →Good estimates from the current probe with a simple setup and cost efficient... but at the end we have to conduct the compliance tests under the specific environment conditions as highlighted in the standards (i.e. chamber!)
- CM current values were measured at 5cm intervals on the cable (not only middle point) to show of any variations. Table to the side shows that the current values were stable with not much variation at almost all locations \rightarrow The approximation of short antenna ($\lambda/3$) @ 100MHz is valid and provided almost constant current as initially assumed!

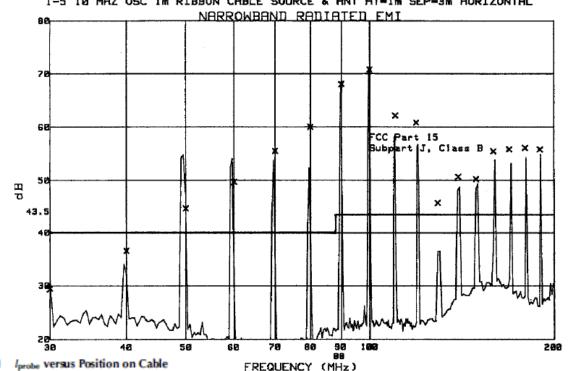


TABLE 8.1 l_{probe} versus Position on Cable (f = 100 MHz).

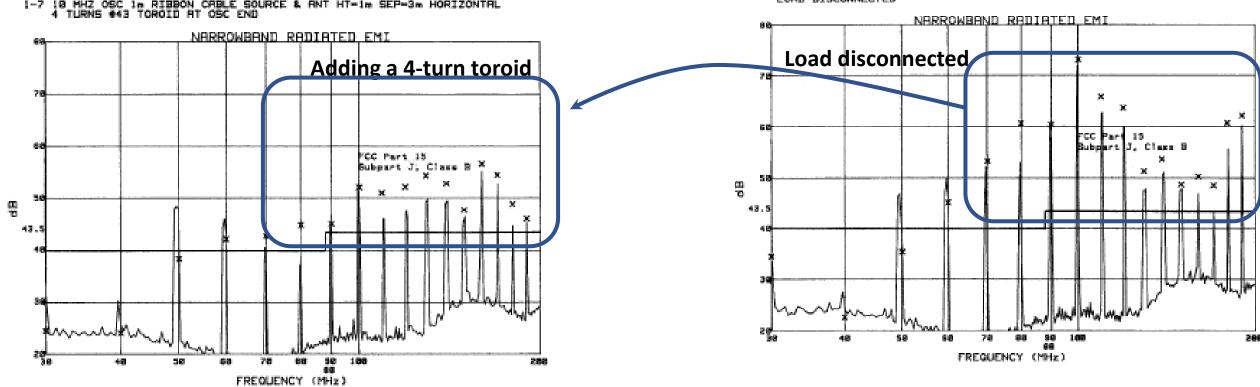
Position of Current Probe from
Oscillator End (cm)

5

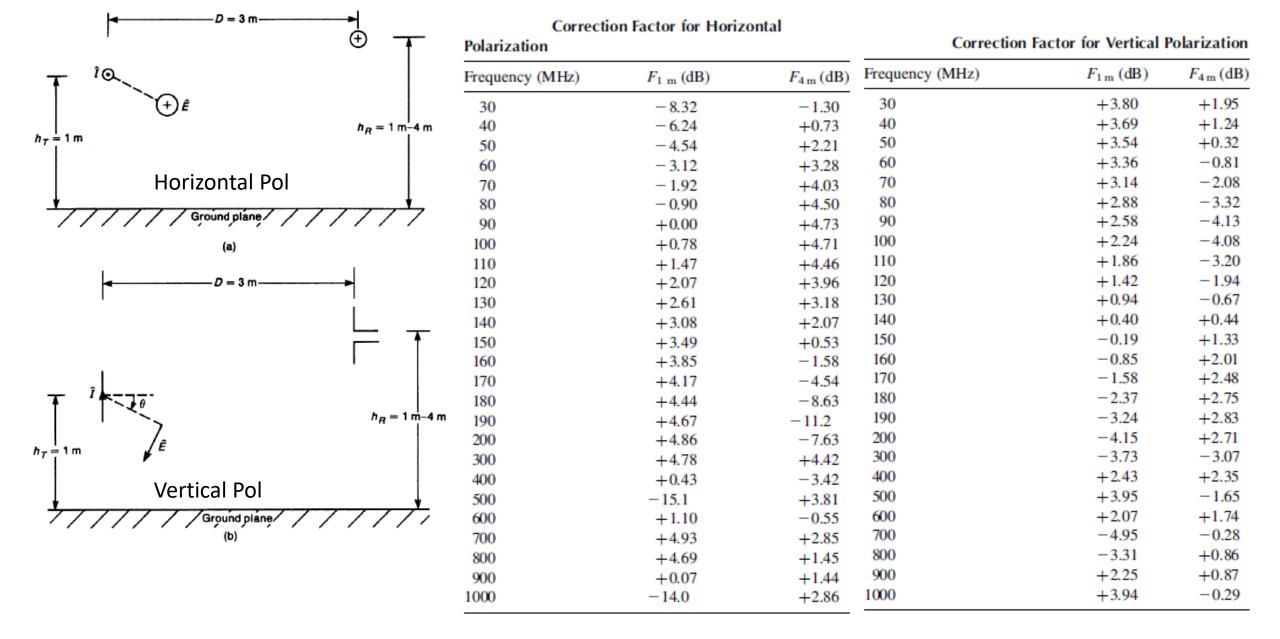
38.7
10
40.7
15
41.9
20
42.6
25
43.4

15 41.9
20 42.6
25 43.4
30 44.3
35 44.7
40 45.1
45 44.7
50 44.4
55 43.9
60 43.2
65 41.9
70 41.1
75 40.2
80 39.5
85 38.4
90 35.5

23



- In order to make sure that the radiated emissions were from CM currents and no contribution from DM ones is obtained, we removed the far end inverter gate (load) was removed and the radiated emissions were measured.
- Note that the levels WITH and WITHOUT the load gate show almost similar levels, emphasizing that CM currents are the ones responsible for the radiated emissions.
- Insertion of a 4-turn toroid can reduce emission values by almost 20dB at some frequencies. This is a practical fix that one should consider when needed.



• Correction factor tables for ground plane reflections for horizontal and vertical polarizations

Example 11.5

A current probe is attached to a SA with an RG55U coaxial cable of 40ft. The loss of this cable is 2.5 dB/100 ft at 100 MHz. The correction factor for the ground plane (GP) with Horizontal polarization and 1 m height is 0.78 dB (see tables on previous slides). The CM current at the middle point of the cable is measured to be $44.4 dB\mu A$. Assuming a current probe transfer impedance of $15 dB\Omega$, what will be the SA reading at 100 MHz? Calculate the radiated E-field at 100 MHz.

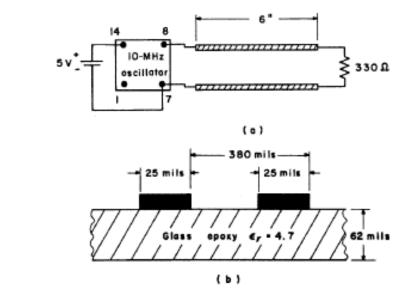
Sol:

On the BOARD!

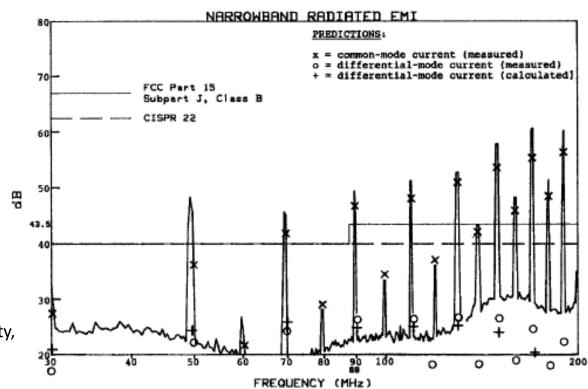
Experiment 2

- This experiment was described in details in [2].
- A pair of 1 oz. copper traces 25mils in width and 6 in. in length were etched on glass epoxy as shown that had a thickness of 62 mils. The center to center spacing between them was 380 mils.
- Z0 of this configuration was computed as 342 Ω . A load of 330 Ω was used.
- The same 10MHz oscillator with 5V supply and DIP package was used. The waveform had 50% duty cycle and 4ns rise time and 2ns fall time. An isolated battery and regulator was used to isolate outside noise sources.
- Measured and predicted values followed the same steps as in Experiment 1 (current probe and chamber)

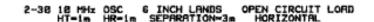
[2] C. R. Paul, "A comparison of the contributions of the common-mode and differential-mode currents in radiated emissions," IEEE Transactions on Electromagnetic Compatibility, 31, 189-193, 1989.

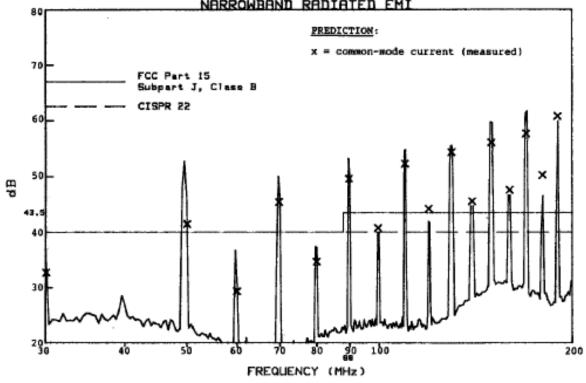






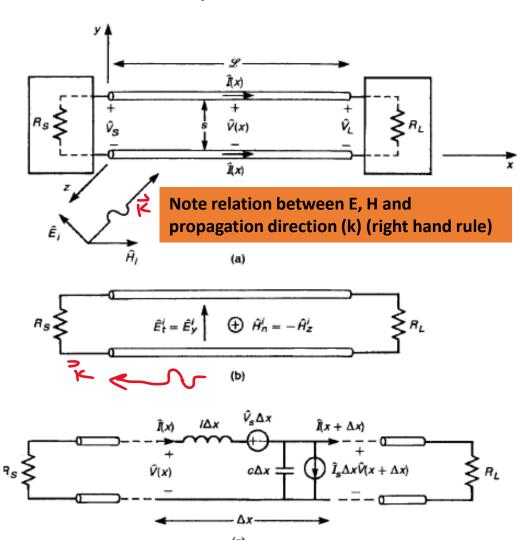
- The emissions predicted (calculated) using the current probe and the chamber measurements show good agreement for the PCB design.
- DM current levels were measured using the current probe and estimated from the transmission-line model. Good agreement is shown.
- When comparing DM contributions to CM ones to the radiated emissions, we can see at least 20dB lower DM contribution than CM ones, thus CM ones are dominant.
- To further confirm that CM currents are dominant, the 330Ω load was removed and the measurement repeated. As shown in the figure, CM currents still dominate.





Simple Susceptibility Models for wires and PCB traces/lands

- Complying with emission limits does not guarantee that your product will be compliant from EMC standpoint, as it might fail susceptibility requirements!
- Again, we will develop a simplified model to be able to predict susceptibility limits with acceptable accuracy before conducting detailed measurements. Exact models can be found is several references in literature.
- Consider the model shown in Figure (a) which shows a two wire system (separated by s) with source (Rs) located at x=0 and load (RL) at x= \mathcal{I} terminations (this can be also a PCB model).
- Our interest is to predict the values of the terminal voltages VS and VL due to the incidence of a sinusoidal, steady-state plane wave with certain polarization and direction of propagation.



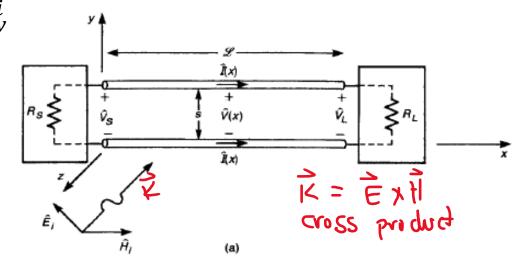
- **Two** components will contribute to the induced voltages Figure (b),
 - (1) E-field component that is Transverse to line $E_t^i = E_y^i$ (in the plane of wires and perpendicular to them and directed upwards)
 - (2) H-field component that is Normal to the plane of the wires, $H_n^i = -H_z^i$

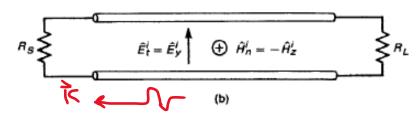
(perpendicular to the plane of wires and into the page)

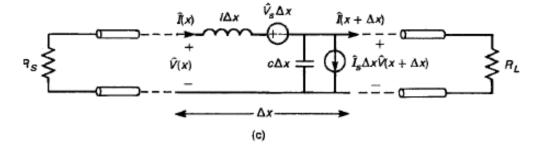
• As we discussed before, the lines will have per unit length (Δx) C and L (Figure (c)), according to:

$$c = \frac{\pi \varepsilon_0 \varepsilon_r}{\ln\left(\frac{s}{r_w}\right)} \quad \text{F/m} \qquad , \quad L = \frac{\mu_0}{\pi} \ln\left(\frac{s}{r_w}\right) \quad \text{H/m}$$

• The per-unit-length induced V_s and I_s generated by the incident wave will follow the following relationships



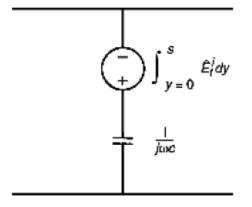




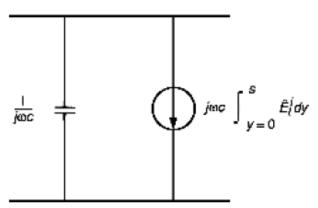
- Faraday's law gives the per-unit-length induced EMF in a loop bounded by the wires as, $\hat{V_s}(x) = j\omega\mu_0 \int\limits_{v=0}^s \hat{H}_n^i dy$
- According to Lentz's Law, the polarity of the induced voltages will be such that it produces a current that will oppose the change in the magnetic field, thus with the field going into the page, the polarity of the source will be as shown in Figure (c) in the previous slide (to the left)
- The per-unit-length induced current source Is is directed in the —y direction and is due to the E-field component that is transverse to the line and directed in the +y direction, and it given by,

$$\hat{I}_s(x) = j\omega c \int_{y=0}^{s} \hat{E}_t^i dy$$

- This current is the dual of the voltage V_s . The induced voltage and its Norton equivalent model are shown to the side.
- To generate the incident wave, we need a distance antenna and to come up with a field expression, we need to use the Friis transmission Equation.







 Consider the two antenna system shown, one is transmitting and the other is receiving separated by d m.

- The transmit and receiving antenna each have a gain $G(\theta, \phi)$ and effective aperture $A_e(\theta, \phi)$. The transmitting antenna sends P_T and the receiving one receives P_R .
- The average power density S_{av} is defined as,

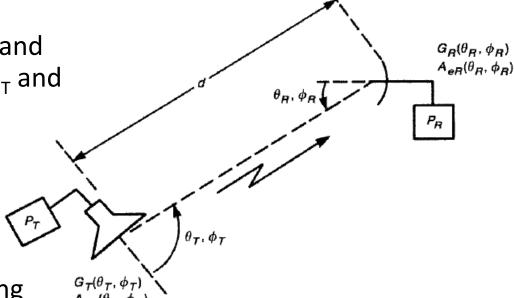
$$S_{av} = \frac{P_T}{4\pi d^2} G_T(\theta_T, \varphi_T) = \frac{1}{2} \frac{\left|\hat{E}\right|^2}{\eta_0}, \quad \eta_0 = 120\pi \Omega$$

• The received power, is the product of the S_{av} and the receiving antenna effective aperture $A_{eR}(\theta,\phi)$

$$P_{R} = S_{av} A_{e} (\theta_{R}, \varphi_{R})$$

• Thus, the two above equations give the Friis transmission equation,

$$\frac{P_{R}}{P_{T}} = \frac{G_{T}(\theta_{T}, \varphi_{T}) A_{eR}(\theta_{R}, \varphi_{R})}{4\pi d^{2}} \Rightarrow \boxed{\frac{P_{R}}{P_{T}} = G_{T}(\theta_{T}, \varphi_{T}) G_{R}(\theta_{R}, \varphi_{R}) \left(\frac{\lambda_{0}}{4\pi d}\right)^{2}}{P_{T}}}$$



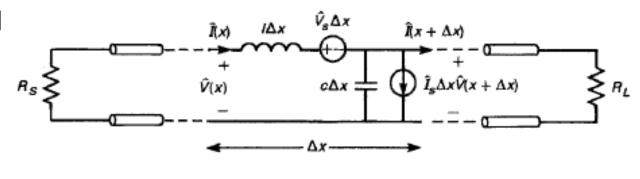
• And we can get expressions for the incident E-field and H-fields as, \hat{r}_{i}

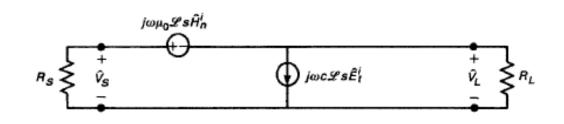
$$\left| \hat{E}^i \right| = \frac{\sqrt{60P_TG}}{d}$$
 and $\left| \hat{H}^i \right| = \frac{\left| \hat{E}^i \right|}{\eta_0}$

 From the T-line model and the per-unit-length values, we can write and exact model using

$$\frac{d\hat{V}(x)}{dx} + j\omega L\hat{I}(x) = -\hat{V}_s(x) = -j\omega\mu_0 \int_{y=0}^s \hat{H}_n^i dy$$
$$\frac{d\hat{I}(x)}{dx} + j\omega c\hat{V}(x) = -\hat{I}_s(x) = -j\omega c \int_{y=0}^s \hat{E}_t^i dy$$

• We will **approximate** the above model for an electrically short wire $\mathcal{I} << \lambda$, and we will lump the values into one section replacing Δx with \mathcal{I} . Thus integrals with respect to y are replaced by s (separation distance), giving after performing superposition,





$$\rightarrow \begin{cases} \hat{V}_{s} \mathcal{I} \cong j\omega \mu_{0} \hat{H}_{n}^{i} A \\ \hat{I}_{s} \mathcal{I} \cong j\omega c \hat{E}_{t}^{i} A \end{cases}$$

$$\hat{V}_{s} = \frac{R_{s}}{R_{s} + R_{L}} j\omega\mu_{0}\hat{H}_{n}^{i}A - \frac{R_{s}R_{L}}{R_{s} + R_{L}} j\omega c\hat{E}_{t}^{i}A$$

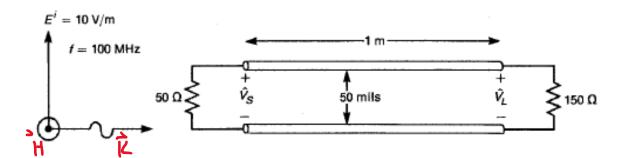
$$\hat{V}_{L} = -\frac{R_{L}}{R_{s} + R_{L}} j\omega\mu_{0}\hat{H}_{n}^{i}A - \frac{R_{s}R_{L}}{R_{s} + R_{L}} j\omega c\hat{E}_{t}^{i}A$$

Example 11.6

Consider a 1m ribbon cable as shown. The wires are 28-AWG (rw=7.5 mils) and are separated by 50mils. The termination impedances are Rs = 50Ω and RL = 150Ω . A 100MHz plane wave is incident with a transverse E-file in the y-direction and the H-field is perpendicular to the plane of the loop (outside of the page). The amplitude of the E-field is 10V/m. Find the equivalent circuit due to the induced voltages and currents, and then find the values of Vs and VL.

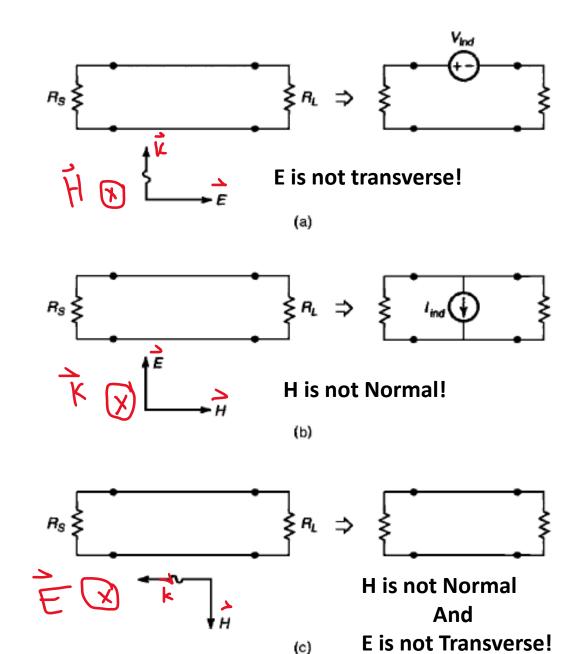
Sol:

On the BOARD!



In summary,

- The two induced sources are due to
 - (1) the component of the incident magnetic field that is NORMAL to the loop formed by the transmission-line
 - (2) the component of the electric field TRANSVERSE to the transmission-line
- If either one is ZERO, then the associated induced source is not present.
- Thus orientation with respect to the incoming wave makes a difference!



NEXT

• EMC Applications