Electromagnetic Compatibility (EMC)

Topic 3

Time and Frequency domain analysis

Mohammad S. Sharawi, PhD., P.E.

Time and Frequency Domain Analysis

- *Deterministic* signals are those whose time behavior is precisely known
- *Random* signals are those whose time behavior is not known but can be described in a statistical manner.
- We will review Linear systems theory in time and frequency domains
- *Periodic* signals/waveforms are of interest in this course because they can have a direct effect on EMI in electrical systems (i.e. clock signals)
- A *periodic* function has the property,

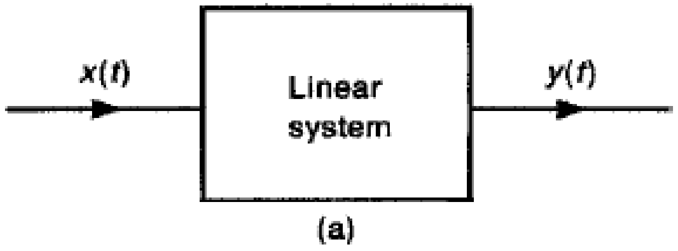
$$x(t \pm kT) = x(t) \quad , \ \text{k=1,2,3,...} \quad , \ f_0 = \frac{1}{T} \quad , \quad \omega_0 = 2\pi f_0 = \frac{2\pi}{T}$$

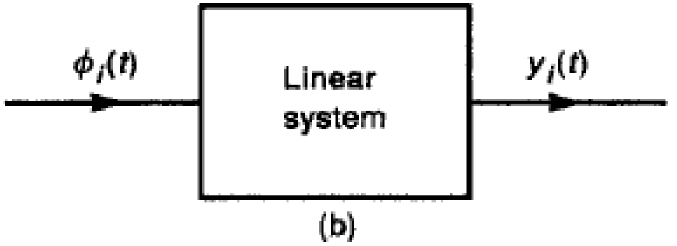
- The **average power** of a periodic signal and its **energy** are,

$$P_{avg} = \frac{1}{T}\int_{t_1}^{t_1+T} x^2(t)dt \qquad , \qquad E = \int_{-\infty}^{\infty} x^2(t)dt$$

- *Periodic* signals are power signals (finite power, infinite energy) while *non-periodic* signals are energy signals (finite energy, zero average power)

Periodic Signals





- Periodic signals can be represented as a linear combination of more basic signals called basis-functions which are periodic as well. Coefficients are called expansion coefficients.

$$x(t) = \sum_{n=0}^{\infty} \underbrace{c_n}_{coefficients} \underbrace{\phi_n(t)}_{basis-functions} = c_0\phi_0(t) + c_1\phi_1(t) + \cdots$$

- A system is considered linear if it satisfies the two properties:
  - (a) If x(t) → y(t), then k x(t) → k y(t)
  - (b) If x1(t) → y1(t) and x2(t)→y2(t) then x1(t) + x2(t) → y1(t) + y2(t) [superposition]

$$\phi_n(t) \to y_n(t)$$

$$y(t) = \sum_{n=0}^{\infty} \underbrace{c_n}_{coefficients} \underbrace{y_n(t)}_{basis-functions} = c_0 y_0(t) + c_1 y_1(t) + \cdots$$

Fourier-Series (FS) of Periodic Signals

- The basis functions are periodic sinusoidal signals
- Each basis has an integer multiple frequency of the fundamental
- Multiple forms for the FS, we will use the exponential form
- Basis functions are:

$$\phi_n(t) = e^{j\omega_0 t} = \cos(n\omega_0 t) + j\sin(n\omega_0 t) \qquad \text{for} \quad -\infty,\ldots,-1,0,1,\ldots,\infty$$

- Thus,

$$x(t) = \sum_{-\infty}^{\infty} \underbrace{c_n}_{complex-coefficients} e^{jn\omega_0 t} = \ldots, c_{-1}e^{-j\omega_0 t} + c_0 + c_1 e^{j\omega_0 t} + \ldots$$

$$c_n = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)e^{-jn\omega_0 t}dt \quad , \quad \text{for n=0 , } c_0 = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)dt = average - value$$

$$c_{-n} = c_n^* = \left|c_n\right| e^{-j\angle c_n}$$

Fourier-Series (FS) of Periodic Signals

- now, we can write,

$$x(t)=\sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t}=c_0+\sum_{n=-1}^{-\infty} c_n e^{jn\omega_0 t}+\sum_{n=1}^{\infty} c_n e^{jn\omega_0 t}$$

$$x(t)=c_0+\sum_{n=1}^{\infty} c_n^* e^{-jn\omega_0 t}+\sum_{n=1}^{\infty} c_n e^{jn\omega_0 t}$$

$$x(t)=c_0+\sum_{n=1}^{\infty} \left|c_n\right| e^{-j(n\omega_0 t+\angle c_n)}+\sum_{n=1}^{\infty} \left|c_n\right| e^{j(n\omega_0 t+\angle c_n)}$$

$$x(t)=c_0+\sum_{n=1}^{\infty} \left|c_n\right| \left(e^{-j(n\omega_0 t+\angle c_n)}+e^{j(n\omega_0 t+\angle c_n)}\right)$$

$$\boxed{\begin{gathered} x(t)=c_0+\sum_{n=1}^{\infty} \underbrace{2\left|c_n\right|}_{\substack{\text{single-sided spectrum}\\ \text{(positive frequencies)}}} \mathbf{cos}(n\omega_0 t+\angle c_n) \\ x(t)=c_0+\sum_{n=1}^{\infty} 2\left|c_n\right| \mathbf{sin}(n\omega_0 t+\angle c_n+90^o) \end{gathered}}$$

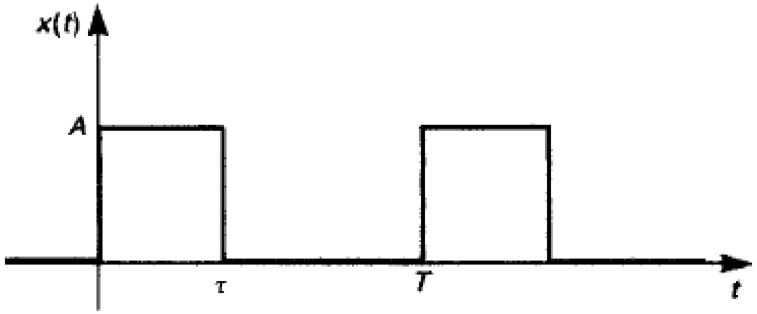
$$\cos(x)=\frac{e^{jx}+e^{-jx}}{2}$$

$$\sin(x)=\frac{e^{jx}-e^{-jx}}{2j}$$

$$\cos(\theta)=\sin(\theta+90^o)$$

Example 1

- Consider the periodic square wave shown, and find its complex FS.



$$c_n = \frac{1}{T}\int_{t_1}^{t_1+T} x(t)e^{-jn\omega_0 t}dt = \frac{1}{T}\int_0^{\tau} Ae^{-jn\omega_0 t}dt + \frac{1}{T}\int_{\tau}^{T} 0e^{-jn\omega_0 t}dt$$

$$\rightarrow c_n = \frac{A}{jn\omega_0 T}\left[1 - e^{-jn\omega_0\tau}\right] = \frac{A}{jn\omega_0 T}e^{-jn\omega_0\frac{\tau}{2}}\left[e^{jn\omega_0\frac{\tau}{2}} - e^{-jn\omega_0\frac{\tau}{2}}\right]$$

$$\rightarrow c_n = \frac{A}{jn\omega_0 T}e^{-jn\omega_0\frac{\tau}{2}}2j\sin\left(n\omega_0\frac{\tau}{2}\right)$$

$$\boxed{\rightarrow c_n = \frac{A\tau}{T}e^{-jn\omega_0\frac{\tau}{2}}\left(\frac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}}\right)}$$

$$\rightarrow |c_n| = \frac{A\tau}{T}\left|\frac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{\left(n\omega_0\frac{\tau}{2}\right)}\right| = \overbrace{\frac{A\tau}{T}\left|\frac{\sin\left(\frac{n\pi\tau}{T}\right)}{\left(\frac{n\pi\tau}{T}\right)}\right|}^{\text{spectrum envelope}}$$

$$and \quad \angle c_n = \pm\left(n\omega_0\frac{\tau}{2}\right) = \pm\left(\frac{n\pi\tau}{T}\right)$$

Note that to get the single-sided spectrum, we need to double the two-sided components except the DC one.

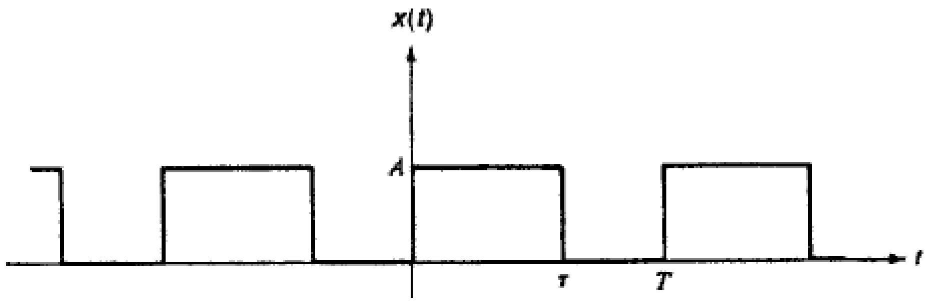
- Assume that $D = \left(\dfrac{\tau}{T}\right)$ (duty cycle of wave)

$$D = 0.5 = 50\%$$

$$let$$

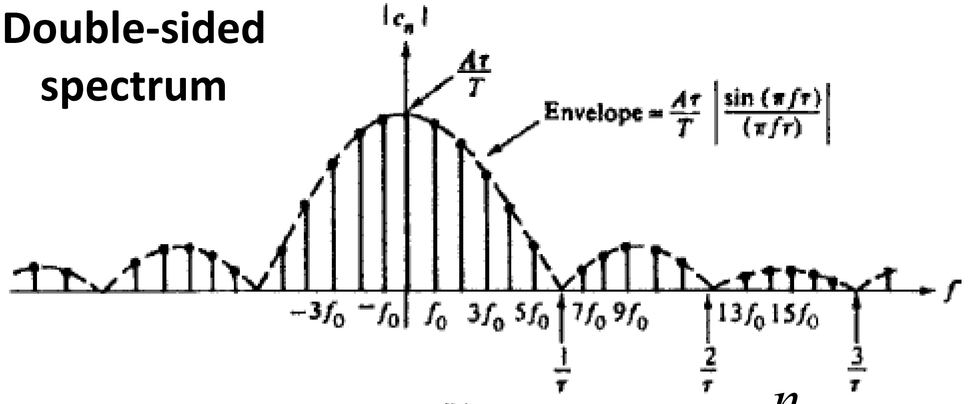
$$\frac{n}{T} = f \quad \text{(to get continuous envelope)}$$

amplitudes are: $\dfrac{A\tau}{T}\left|\dfrac{\sin(\pi f \tau)}{(\pi f \tau)}\right|$



(a)

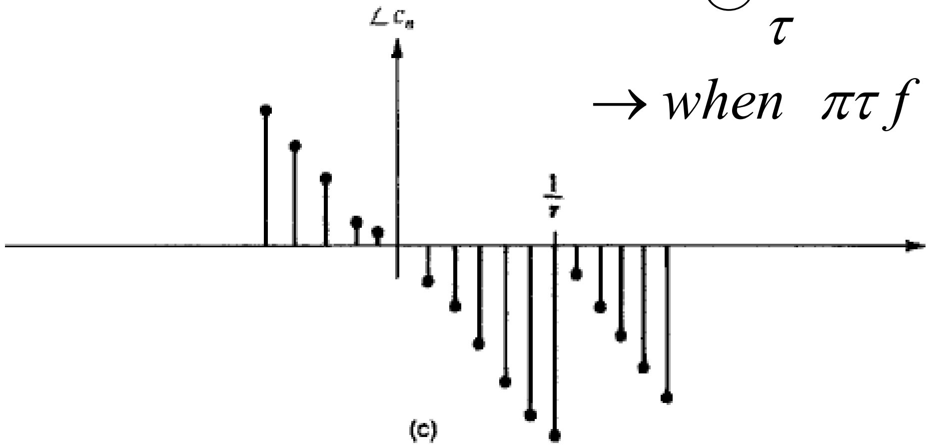
Double-sided spectrum



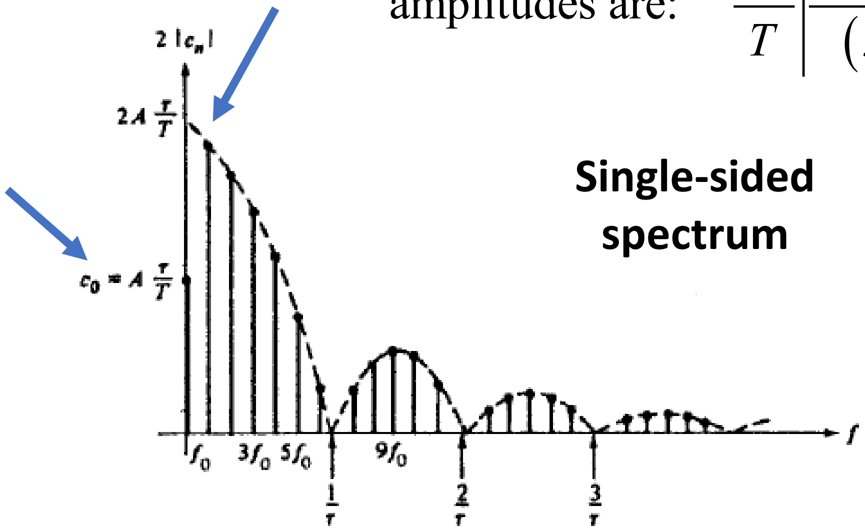
(b)

$$zero @ \frac{n}{\tau}$$

$$\rightarrow when \quad \pi\tau f = m\pi$$



(c)



Single-sided spectrum

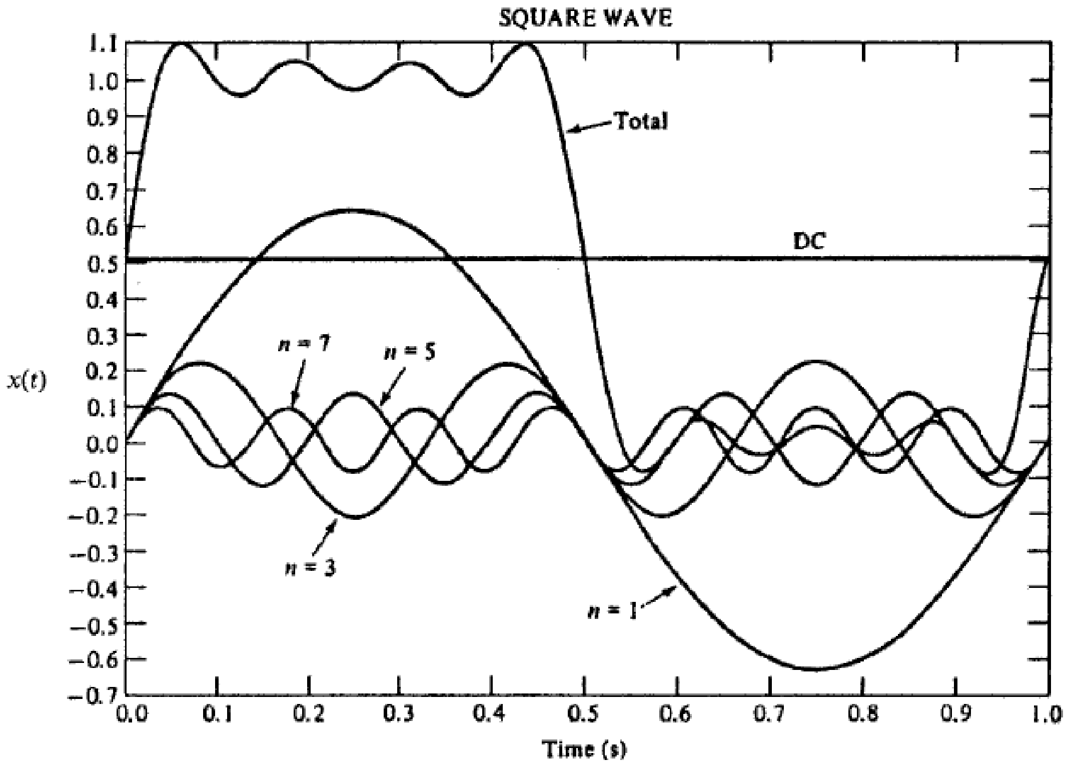
- Calculated coefficient values for D = 0.5 (50%) duty cycle,

$$|c_n| = \frac{A\tau}{T}\left|\frac{\sin\left(\frac{n\pi\tau}{T}\right)}{\left(\frac{n\pi\tau}{T}\right)}\right| = \frac{A}{2}\left|\frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)}\right| = \frac{A}{n\pi} \quad , \quad \text{n = 1, 3, 5, ...}$$

$$= 0 \quad , \text{n = 2, 4, 6, ...}$$

$$|c_0| = \frac{A}{2}$$

$$\angle c_n = \angle\left(-\frac{n\pi}{2}\right) + \angle\sin\left(-\frac{n\pi}{2}\right) = -90^o \quad , \quad \text{n = 1, 3, 5, ...}$$



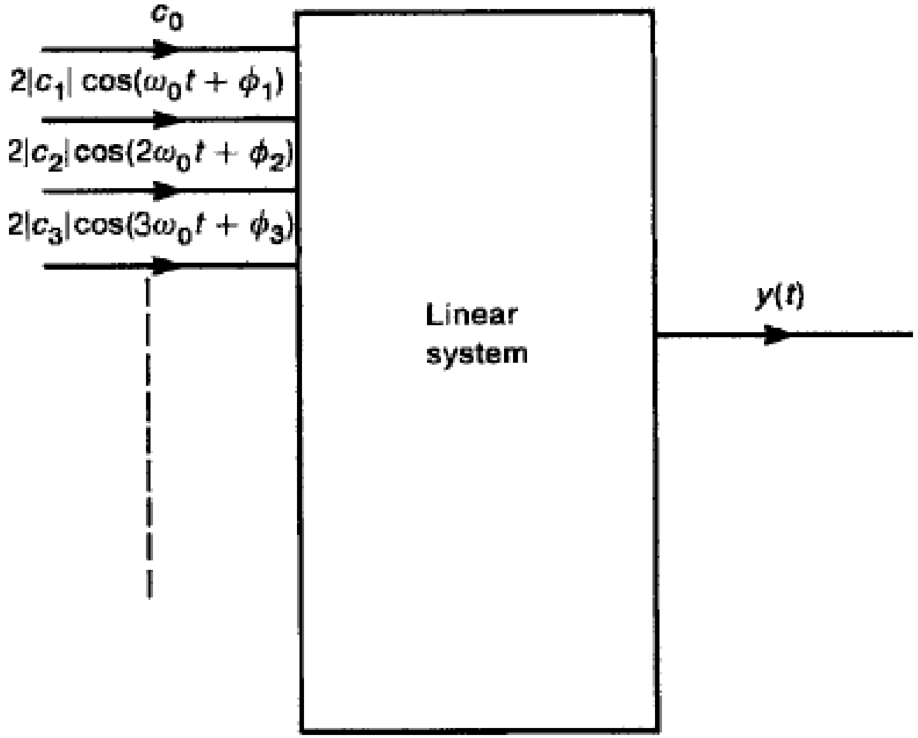
- Hence, the complex FS for the square wave with D = 50% is

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}\cos(\omega_0 t - 90^o) + \frac{2A}{3\pi}\cos(3\omega_0 t - 90^o) + \ldots$$

or

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}\sin(\omega_0 t) + \frac{2A}{3\pi}\sin(3\omega_0 t) + \ldots \quad \blacksquare$$

Linear Systems



- In a linear system (sinusoidal steady state), the impulse response of the system is used in frequency domain to scale the input signal and yield the output signal,

$$Y\angle\theta_y = H(j\omega)X\angle\theta_x$$

$$Y = |H(j\omega)|X$$

$$\theta_y = \angle H(j\omega) + \theta_x$$

- Suppose,

$$x(t) = c_0 + \sum_{n=1}^{\infty} 2|c_n|\cos(n\omega_0 t + \angle c_n)$$

then

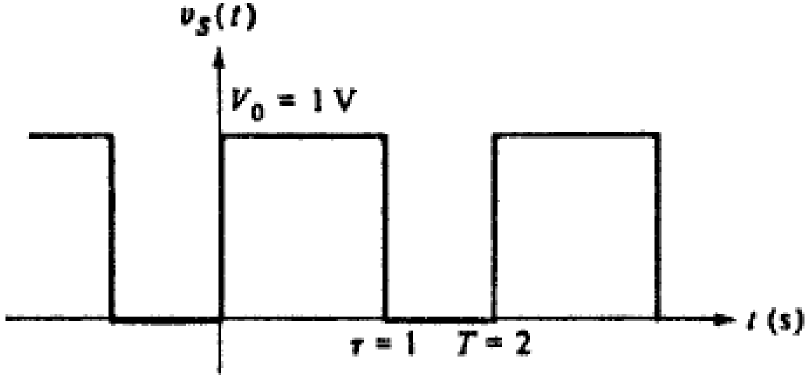
$$y(t) = c_0 H(0) + \sum_{n=1}^{\infty} 2|c_n||H(jn\omega_0)|\cos(n\omega_0 t + \angle c_n + \angle H(jn\omega_0))$$

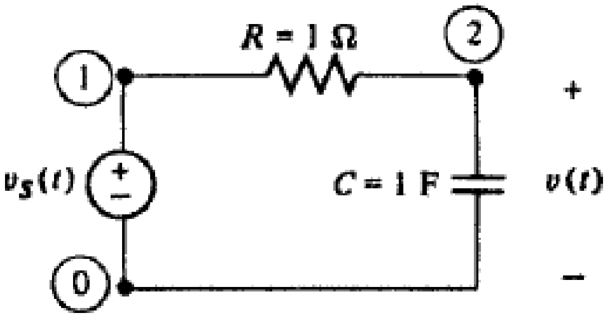
Example 2

- Consider the application of a 1V square wave to a lowpass filter (at node 1) as shown in the Figure. The output is the voltage across the capacitor v(t). Determine the output waveform (node 2) by summing the components of the FS.

Sol.

On the BOARD!

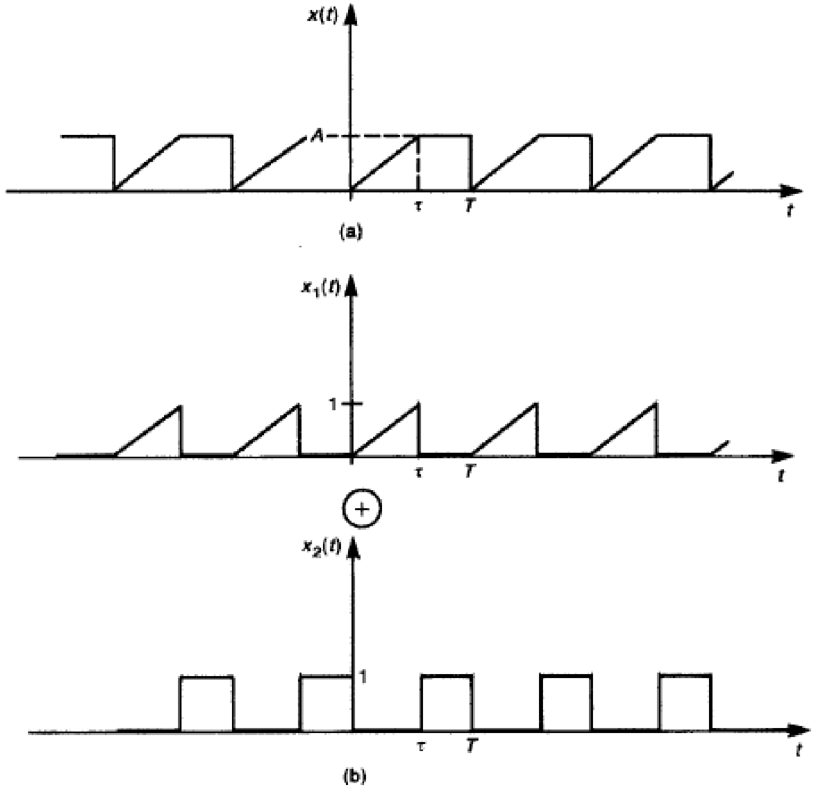




Useful properties for computing the FS

- **Linearity**: the linear combination of 2 or more waveforms

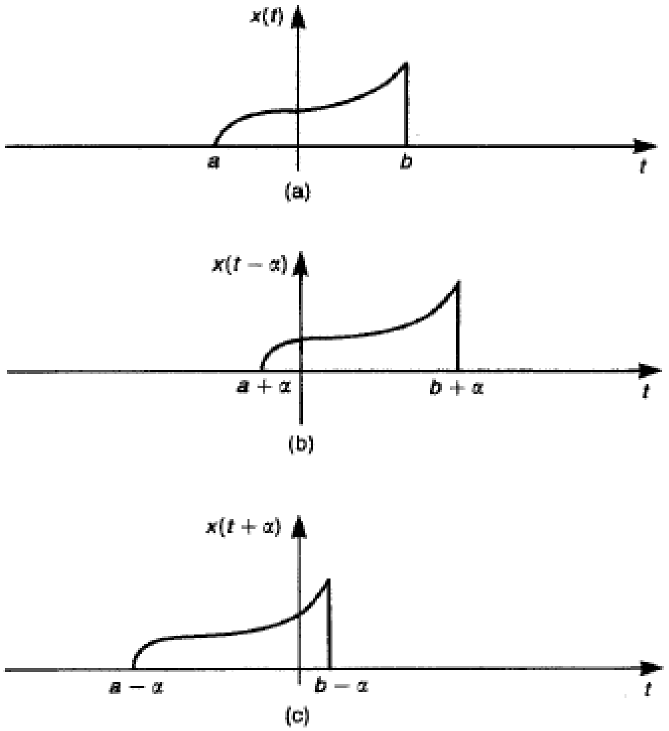
$$x(t) = x_1(t) + x_2(t) = \sum_{-\infty}^{\infty} c_{1n} e^{jn\omega_0 t} + \sum_{-\infty}^{\infty} c_{2n} e^{jn\omega_0 t} = \sum_{-\infty}^{\infty} \left(c_{1n} + c_{2n}\right) e^{jn\omega_0 t}$$



- **Time shifting**:

$$x(t-\alpha) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 (t-\alpha)}$$

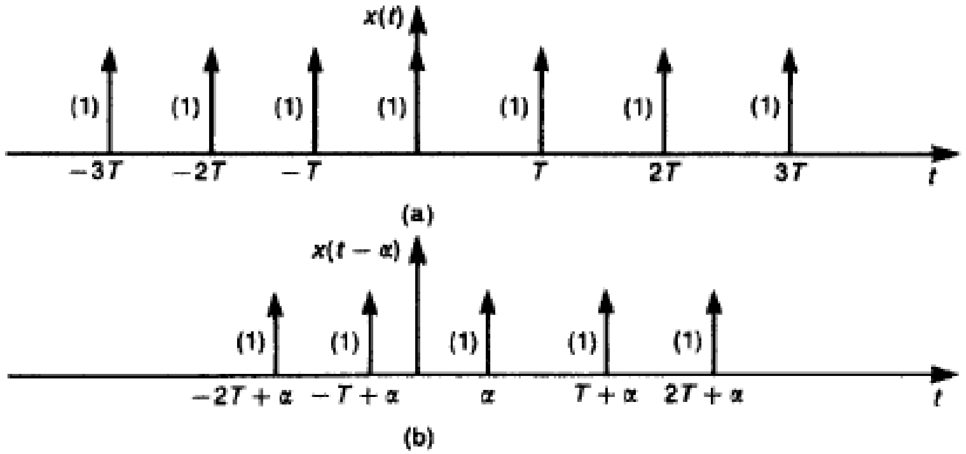
$$x(t-\alpha) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t} e^{jn\omega_0 (-\alpha)}$$



- The **unit impulse function,**

$$\delta(t)=\begin{cases}0 & \text{for } t<0\\ 0 & \text{for } t>0\\ \int_{0-}^{0+}\delta(t)dt=1\end{cases} \quad , \quad x(t)=\delta\left(t\pm kT\right) \quad , \text{ k=0, } \pm 1, \ \pm 2, \ldots$$

$$c_n=\frac{1}{T}\int_0^T \delta(t)e^{-jn\omega_0 t}dt=\frac{1}{T}$$



- **Derivatives,**

$$x(t)=\sum_{-\infty}^{\infty}c_n e^{jn\omega_0 t}$$

$$\frac{d^k x(t)}{dt^k}=\sum_{-\infty}^{\infty}c_n^{(k)}e^{jn\omega_0 t}$$

$$c_n=\frac{1}{\left(jn\omega_0\right)^k}\boxed{c_n^{(k)}}$$ The nth expansion coefficient for the kth derivative of x(t)

$$\frac{d^k x(t)}{dt^k}=\sum_{-\infty}^{\infty}\underbrace{\left(jn\omega_0\right)^k c_n}_{c_n^{(k)}} e^{jn\omega_0 t}$$

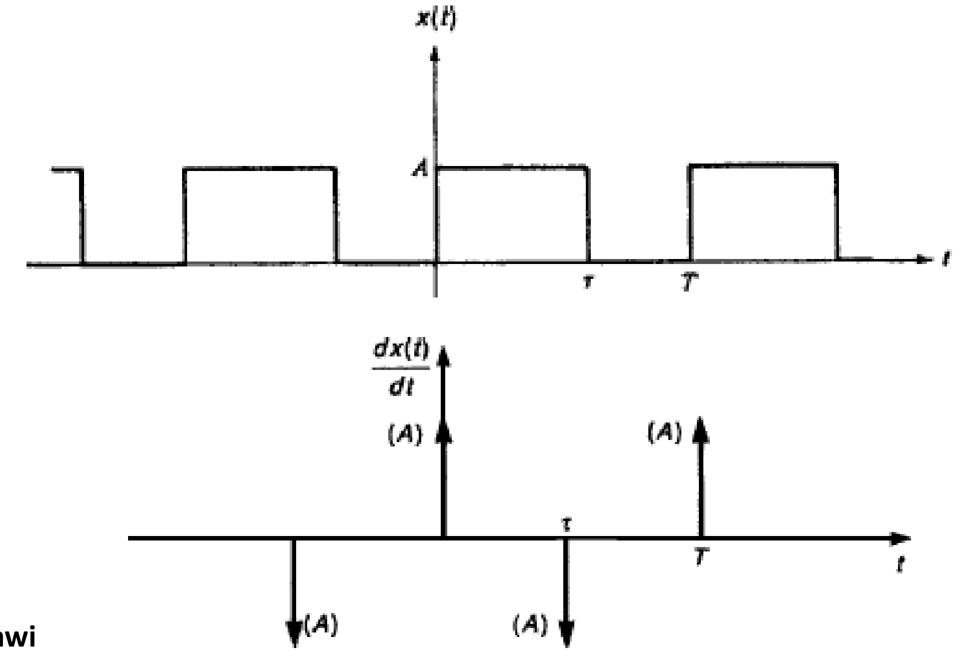
- For piecewise linear functions, it is easier to repeatedly differentiate the function until the occurrence of the first impulse function before we compute the FS coefficients.
- If the differentiated function includes other parts in addition to the impulse function, write the results as the sum of the two.
- Use the properties mentioned before for the calculation of FS

Example 3:

- Determine the Fourier coefficients for the square wave in Example 1.

<u>Sol:</u>

- The derivative of such a waveform is shown below it.
- The sums of the expansion coefficients,



$$c_n^{(1)} = \underbrace{A\frac{1}{T}}_{impulse} - \underbrace{A\frac{1}{T}e^{-jn\omega_0\tau}}_{shifted}$$

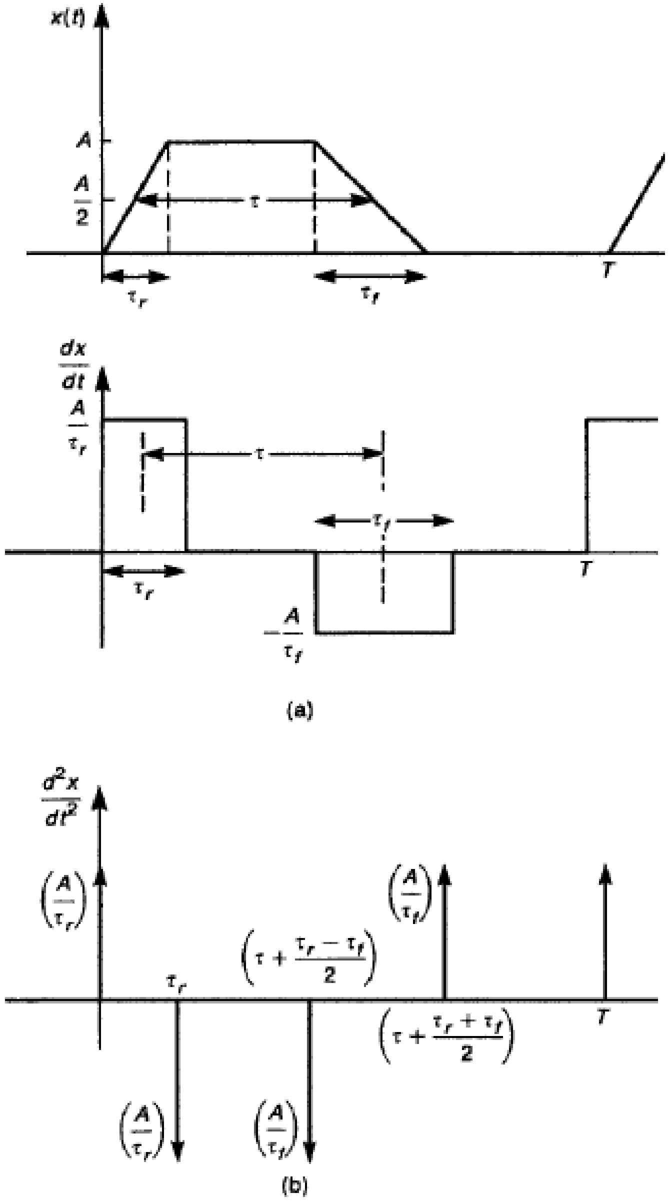
$$= \frac{A}{T}\left(1 - e^{-jn\omega_0\tau}\right) = \frac{A}{T}e^{-jn\omega_0\frac{\tau}{2}}\left(e^{jn\omega_0\frac{\tau}{2}} - e^{-jn\omega_0\frac{\tau}{2}}\right)$$

$$= jn\omega_0\frac{A\tau}{T}e^{-jn\omega_0\frac{\tau}{2}}\left(\frac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}}\right)$$

$$\rightarrow \boxed{c_n = \frac{1}{jn\omega_0}c_n^{(1)} = \frac{A\tau}{T}e^{-jn\omega_0\frac{\tau}{2}}\left(\frac{\sin\left(n\omega_0\frac{\tau}{2}\right)}{n\omega_0\frac{\tau}{2}}\right)}$$

Spectrum of Trapezoidal waveforms (clock)

- Each clock pulse will have a **rise-time (t_r)**, an **amplitude (A)**, a **pulse-width (τ)** and a **fall-time (t_f)**
- Rise and fall times are defined between **0.1A** and **0.9A**, the 10% and 90% amplitude levels
- The key parameters that contribute to the high frequency spectral content of the waveform are the rise and fall times of the pulse
- We will rely on the properties we discussed to come up with the FS of such a clock signal
- We will use the derivative property to have the impulses first. Note the location and amplitudes for each derivative step.



$x(t)$

$\frac{dx}{dt}$

$\frac{d^2x}{dt^2}$

- We have 4 components (impulses), make sure to incorporate the 1/T factor from property of **unit impulse function** and value of c_n

$$c_n^{(2)} = \frac{1}{T}\frac{A}{\tau_r} - \frac{1}{T}\frac{A}{\tau_r}e^{-jn\omega_0\tau_r} - \frac{1}{T}\frac{A}{\tau_f}e^{-jn\omega_0\left[\tau+\frac{(\tau_r-\tau_f)}{2}\right]} + \frac{1}{T}\frac{A}{\tau_f}e^{-jn\omega_0\left[\tau+\frac{(\tau_r+\tau_f)}{2}\right]}$$

$$c_n^{(2)} = \frac{A}{T}\left[\frac{1}{\tau_r}e^{-jn\omega_0\frac{\tau_r}{2}}\left(e^{jn\omega_0\frac{\tau_r}{2}} - e^{-jn\omega_0\frac{\tau_r}{2}}\right) - \frac{1}{\tau_f}e^{-jn\omega_0\frac{\tau_r}{2}}e^{-jn\omega_0\tau}\left(e^{jn\omega_0\frac{\tau_f}{2}} - e^{-jn\omega_0\frac{\tau_f}{2}}\right)\right]$$

$$c_n^{(2)} = j\frac{A}{2\pi n}(n\omega_0)^2 e^{-jn\omega_0\frac{(\tau+\tau_r)}{2}}\left[\frac{\sin\left(\frac{1}{2}n\omega_0\tau_r\right)}{\frac{1}{2}n\omega_0\tau_r}e^{jn\omega_0\frac{\tau}{2}} - \frac{\sin\left(\frac{1}{2}n\omega_0\tau_f\right)}{\frac{1}{2}n\omega_0\tau_f}e^{-jn\omega_0\frac{\tau}{2}}\right]$$

$$then\ , \quad c_n = \frac{1}{(jn\omega_0)^2}c_n^{(2)} = -\frac{c_n^{(2)}}{(n\omega_0)^2} \qquad (\mathrm{n} \neq 0)$$

$$c_n = -j\frac{A}{2\pi n}e^{-jn\omega_0\frac{(\tau+\tau_r)}{2}}\left[\frac{\sin\left(\frac{1}{2}n\omega_0\tau_r\right)}{\frac{1}{2}n\omega_0\tau_r}e^{jn\omega_0\frac{\tau}{2}} - \frac{\sin\left(\frac{1}{2}n\omega_0\tau_f\right)}{\frac{1}{2}n\omega_0\tau_f}e^{-jn\omega_0\frac{\tau}{2}}\right]$$

- A useful special case would be when $\tau_r = \tau_f$ and thus, c_n becomes,

$$c_n = \frac{A\tau}{T} e^{-jn\omega_0 \frac{(\tau+\tau_r)}{2}} \left[\frac{\sin\left(\frac{1}{2}n\omega_0\tau\right)}{\frac{1}{2}n\omega_0\tau} \frac{\sin\left(\frac{1}{2}n\omega_0\tau_r\right)}{\frac{1}{2}n\omega_0\tau_r} \right], \quad (\tau_f = \tau_r)$$

and the single sided spectrum becomes,

$$x(t) = c_0 + \sum_{n=1}^{\infty} \left|c_n^+\right| \cos\left(n\omega_0 t + \angle c_n\right), \quad (\tau_f = \tau_r)$$

$$\left|c_n^+\right| = 2\left|c_n\right| = \frac{2A\tau}{T} \left[\frac{\sin\left(n\pi\frac{\tau}{T}\right)}{n\pi\frac{\tau}{T}} \frac{\sin\left(n\pi\frac{\tau_r}{T}\right)}{n\pi\frac{\tau_r}{T}} \right], \quad (\tau_f = \tau_r)$$

$$\angle c_n = \pm n\pi \frac{(\tau+\tau_r)}{T}, \quad (\tau_f = \tau_r)$$

$$c_0 = \frac{A\tau}{T}$$

- Several important points,
    - Theoretically, there are no even harmonics when we have 50% duty cycle. But, in reality, clock signals do not have exactly 50% duty cycle, thus even harmonics can exist.
    - The levels of even harmonics will be much smaller than odd ones the closer we get to 50% duty cycle.
    - Repeated measurements might be difficult if the cause of the problem is an even harmonic due to the slight deviation from 50% duty cycle, so this should be carefully tested (i.e. duty cycle variations)

Spectral bounds for Trapezoidal waveforms

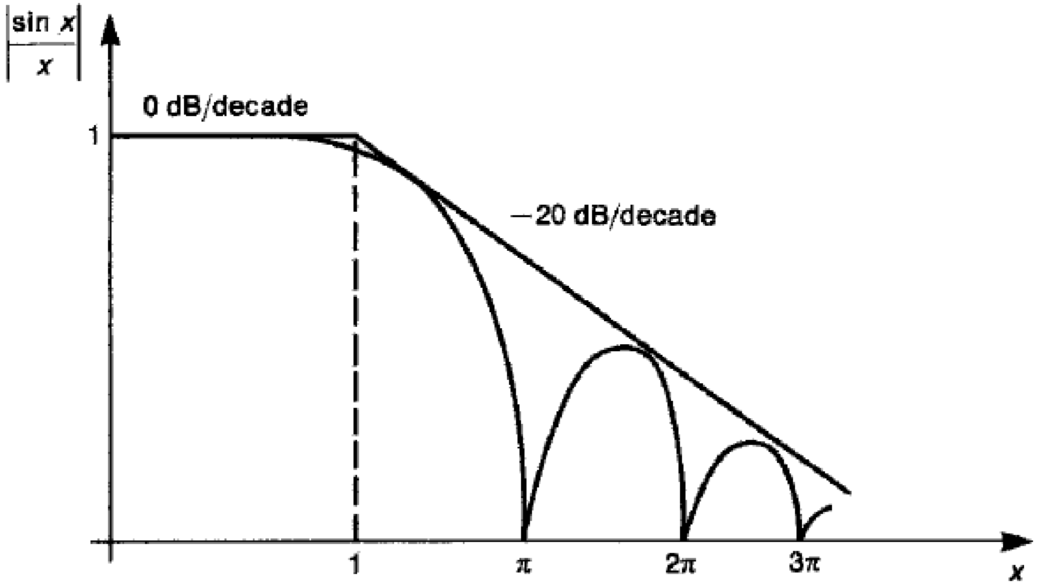
- Upper Bounds on the magnitude spectrum give approximate worst-case levels of the spectral components as a function of pulse wide, rise and fall times.
- Approximations allow us to have asymptotes that bound the levels
- Recall that,

$$\left|\frac{\sin x}{x}\right| \le \begin{cases} 1 & \text{for small x} \\ \dfrac{1}{x} & \text{for large x} \end{cases}$$

in a Bode plot, for a square wave

$x = \pi\tau f$ where $f = \dfrac{n}{T}$ (from the FS)

$\rightarrow 1 = \pi\tau f \rightarrow f = \dfrac{1}{\pi\tau}$ (asymptote point 1)



$f = \dfrac{1}{\pi\tau}$

- For a trapezoidal (clock) signal,

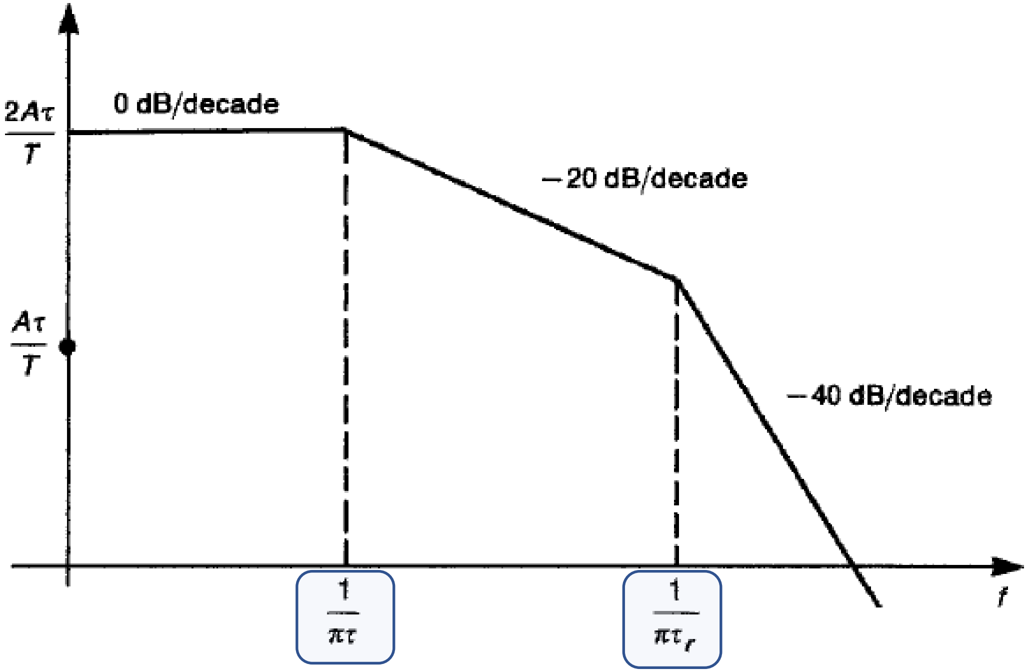
let $\mathbf{t_r} = \mathbf{t_f}$ (the approximation we made)

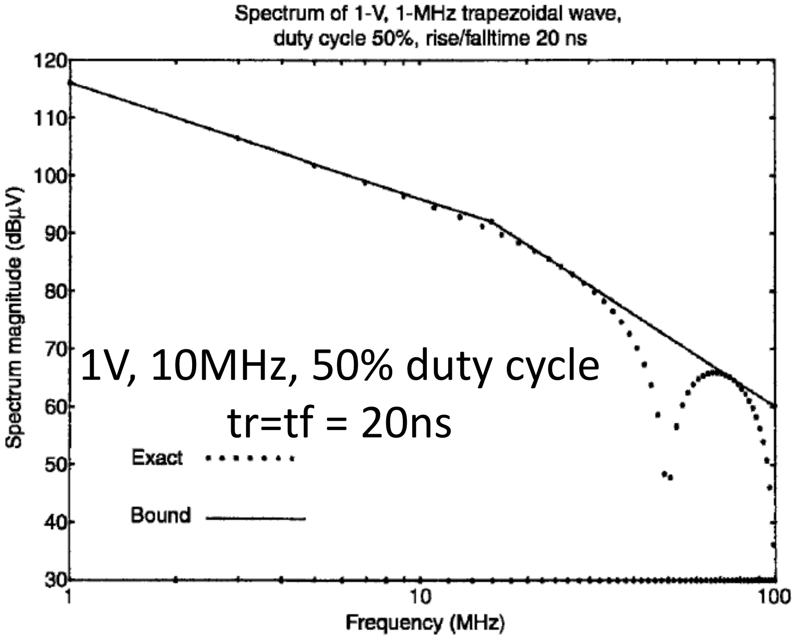
$$\left|c_n^+\right| = envelope = 2A\frac{\tau}{T}\left|\frac{\sin(\pi\tau f)}{\pi\tau f}\right|\left|\frac{\sin(\pi\tau_r f)}{\pi\tau_r f}\right|$$

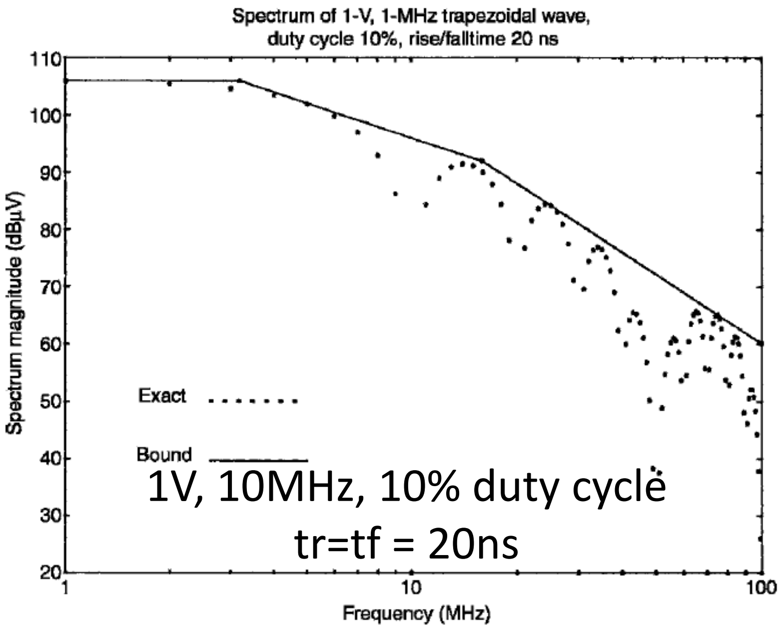
$in - dB$

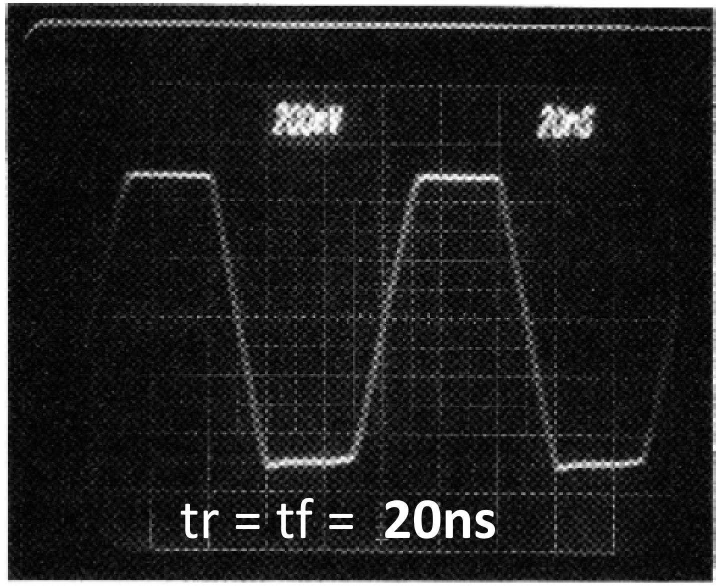
$$20\log_{10}(c_n^+) = 20\log_{10}\left(2A\frac{\tau}{T}\right) + 20\log_{10}\left|\frac{\sin(\pi\tau f)}{\pi\tau f}\right| + 20\log_{10}\left|\frac{\sin(\pi\tau_r f)}{\pi\tau_r f}\right|$$

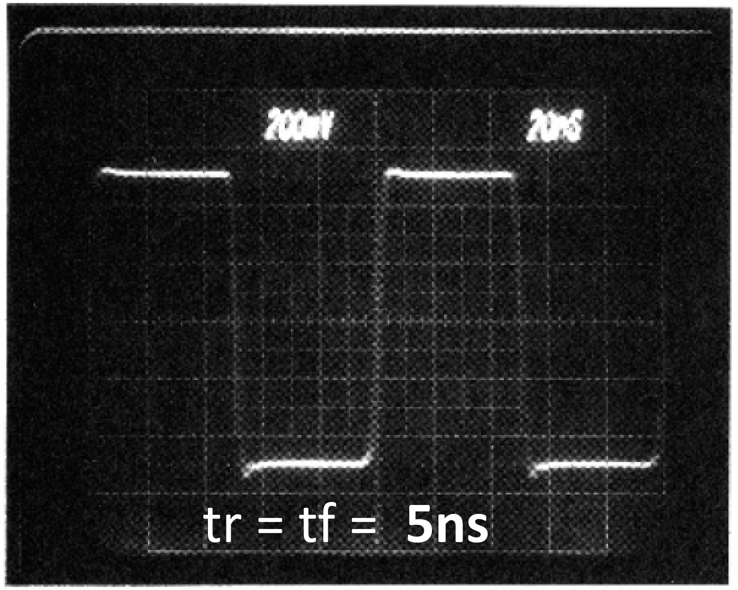
- First break point due to pulse width, second due to rise/fall times
- The high frequency content of a clock signal is due to its rise/fall times. To reduce this, increase the rise and fall time durations (slow the edges)

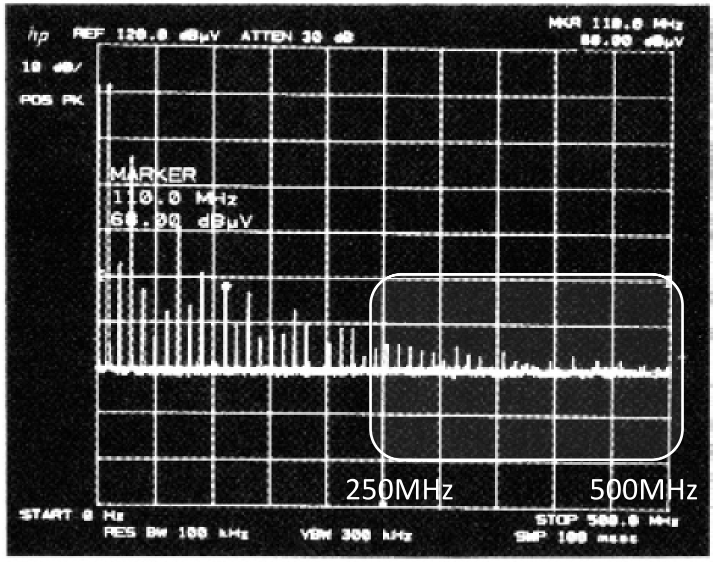


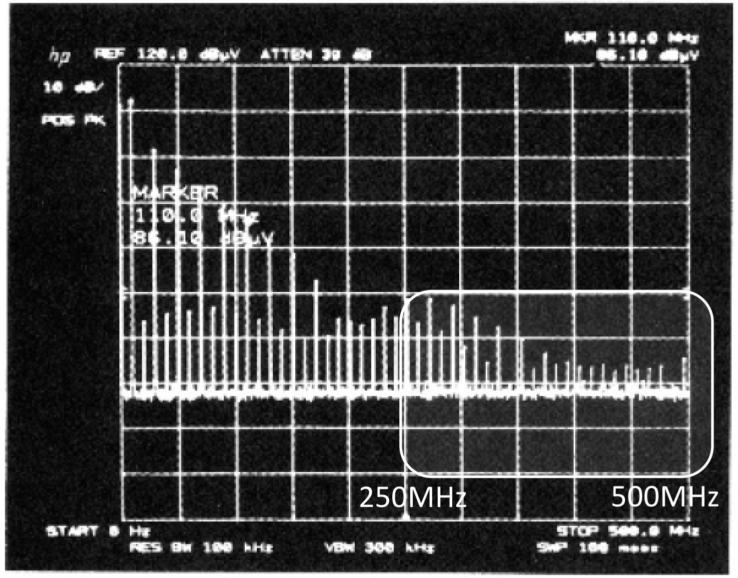












1V, 10MHz, 50% duty cycle

→ @ 11th harmonic (110MHz) level increased from 68 to 86.1 dBµV

- Some calculations to find the values of breakpoints
- In Fig(a), we have,

$$\log_{10} Y_2 - \log_{10} Y_1 = M\left(\log_{10} f_2 - \log_{10} f_1\right)$$

$$\log_{10} Y_2 = \log_{10} Y_1 + M \log_{10}\left(\frac{f_2}{f_1}\right)$$

- In Fig. (b) To find values other than the segment break points (i.e. f_1 and f_3) such as at f_2 and f_4,

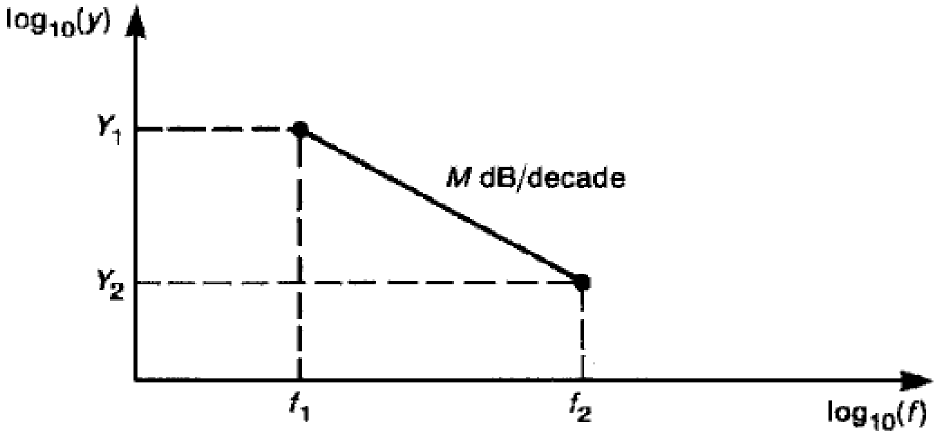
$$\Delta_1 = -20\log_{10}\left(\frac{f_2}{f_1}\right) \quad , \quad \Delta_2 = -20\log_{10}\left(\frac{f_3}{f_1}\right) \quad ,$$

$$\Delta_3 = -40\log_{10}\left(\frac{f_4}{f_3}\right)$$

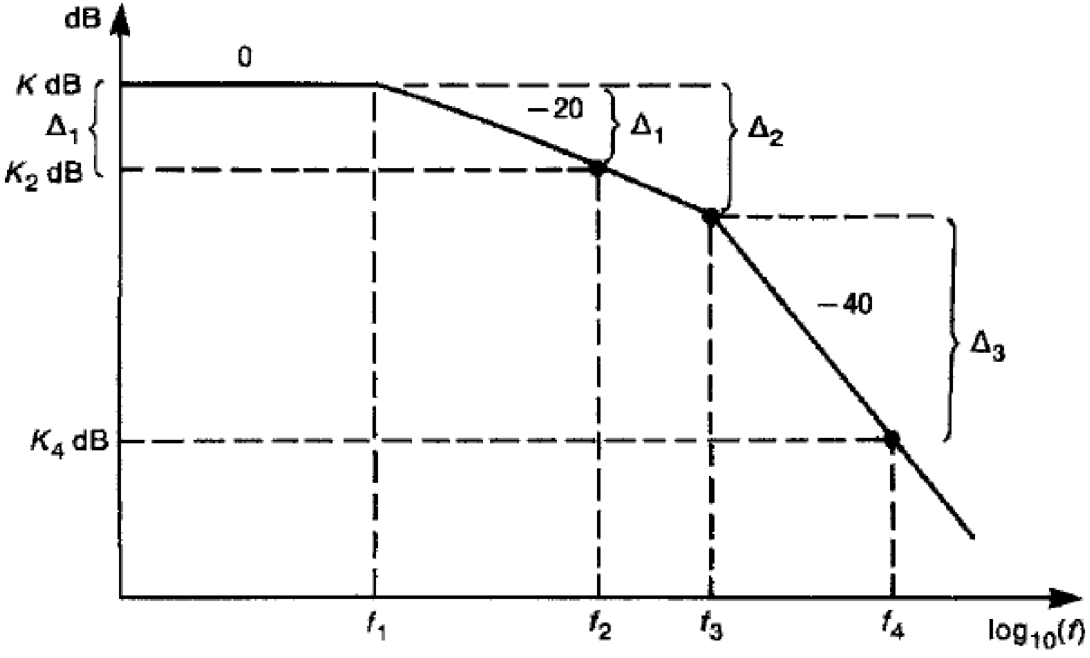
thus,

$$K_2(dB) = K(dB) + \Delta_1$$

$$K_4(dB) = K(dB) + \Delta_2 + \Delta_3$$



(a)



(b)

Example 4

For a 1V, 10MHz, 50% Duty cycle trapezoidal waveform, determine the level at 110MHz, for 20ns rise/fall times and for the 5ns rise/fall time case.

Sol.

On the BOARD!

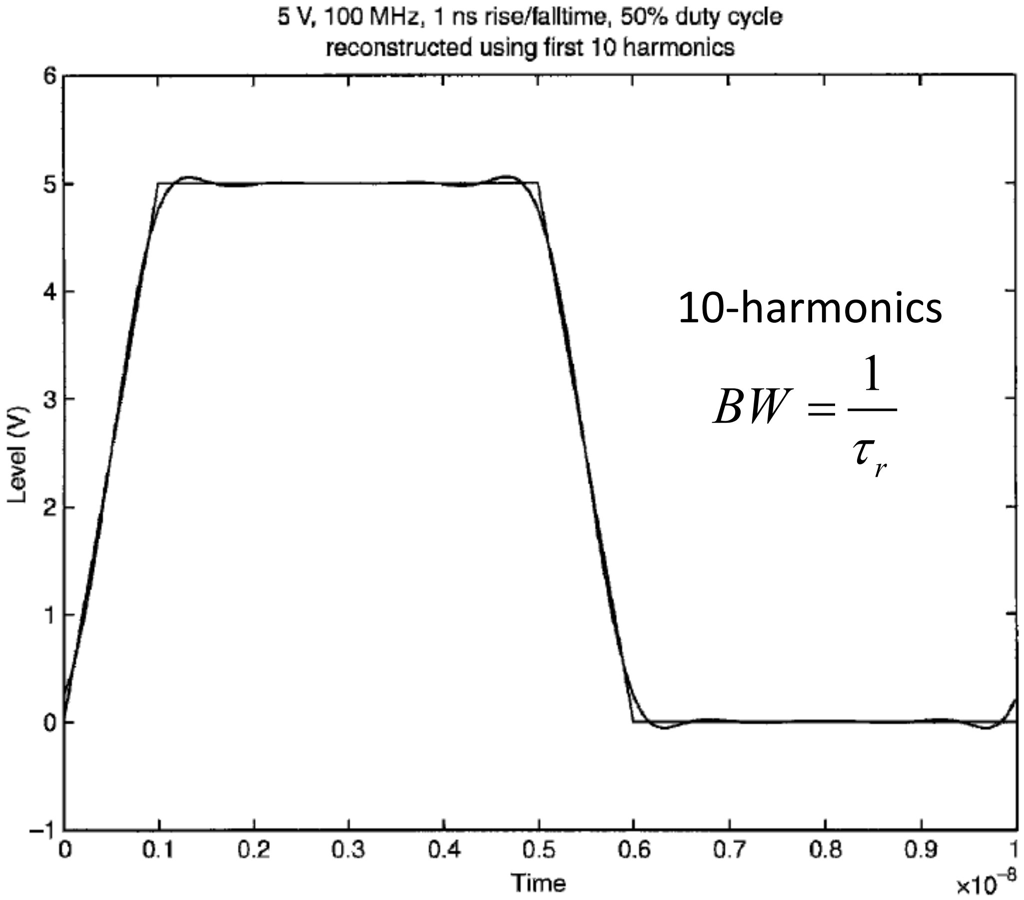
Bandwidth of Digital waveforms

- Mathematically, we need infinite number of harmonics (FS) to reconstruct exactly a waveform or a function.
- In reality, we need to limit this number and still get a very close replica of the original waveform. So, how many harmonics should we keep?
- A good approximation that minimizes the error between the exact and approximate coefficient numbers (thus limiting the bandwidth of the spectrum), is choosing 3 times the second break frequency of a trapezoidal waveform, i.e.,

$$BW = 3f_2 = \frac{3}{\pi\tau_r} \approx \frac{1}{\tau_r}$$

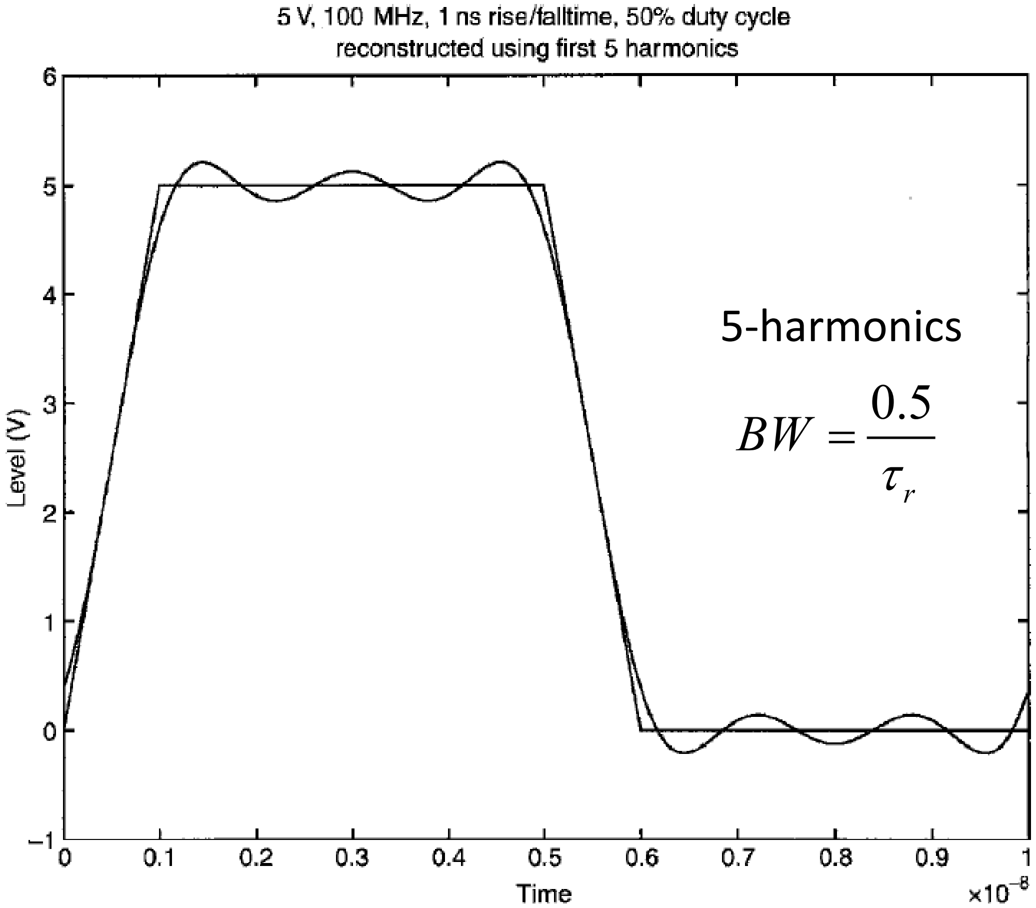
- This is also the value of f that will give the first NULL in the sinc function of the FS (see the expression for $\left|c_n^+\right|$)
- Eliminating higher frequency components is like passing the signal through a low-pass filter (LPF)

- Consider a 5V, 100MHz, 1ns rise/falltime, 50% duty –cycle waveform



10-harmonics

$$BW = \frac{1}{\tau_r}$$



5-harmonics

$$BW = \frac{0.5}{\tau_r}$$

- For the previous trapezoidal waveform, with 50% duct-cycle and t_r=t_f, the average power is,

$$P_{av} = \frac{1}{T}\int_0^T x^2(t)dt = V^2\left[\frac{1}{2} - \frac{1}{3}\frac{\tau_r}{T}\right] \quad \text{(W)}$$

if V=5V, $\tau_r = 1\ ns$, $T = 10\ ns$, then

$$P_{av} = 11.667\ \ (W)$$

- From the FS coefficients, the first 10-harmonics amplitudes are,

$$\left.\begin{aligned} c_0 &= 2.5 \\ c_1^+ &= 3.131 \\ c_3^+ &= 0.9108 \\ c_5^+ &= 0.4053 \\ c_7^+ &= 0.1673 \\ c_9^+ &= 0.0387 \end{aligned}\right\} P_{av} = c_0^2 + \frac{1}{2}c_1^2 + \ldots + \frac{1}{2}c_9^2 = 11.663\text{(W)} \ \rightarrow 99.97\% \text{ of power is in 10-harmonics}$$

Other effects on the waveform spectrum

- **Repetition rate and Duty cycle**:
  - Changing the duty-cycle (D) will change the location of the first frequency break point
  - Also, this will affect the level of the initial level as it is directly affected by D
  - Specifically,

$$D = \frac{\tau}{T}$$

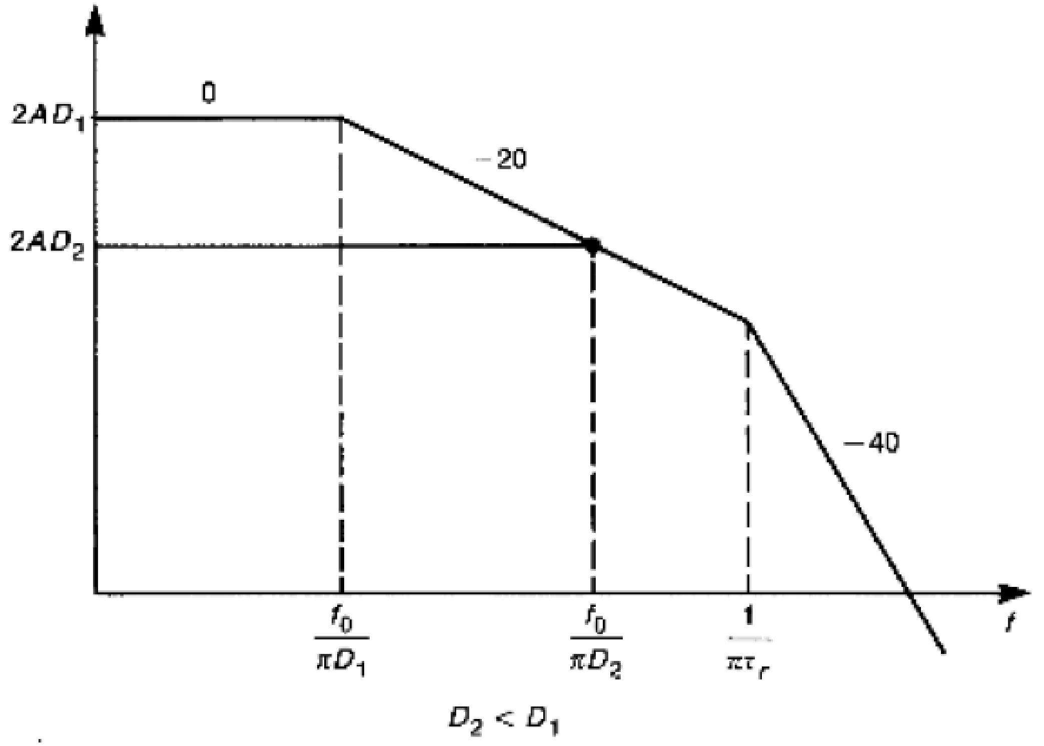
from the FS coefficients

$$\left|c_n^+\right| = 2AD\left|\frac{\sin(n\pi D)}{n\pi D}\right|\left|\frac{\sin(n\pi\tau_r f_0)}{n\pi\tau_r f_0}\right|$$

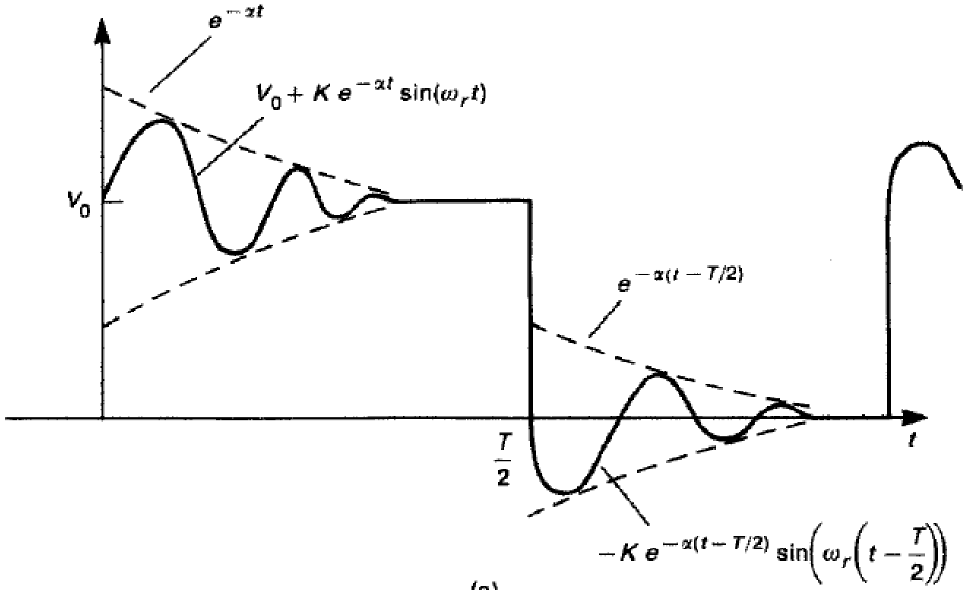
$$c_0 = AD$$

first frequency break point is $\dfrac{1}{\pi\tau} = \dfrac{1}{\pi DT} = \dfrac{f_0}{\pi D}$

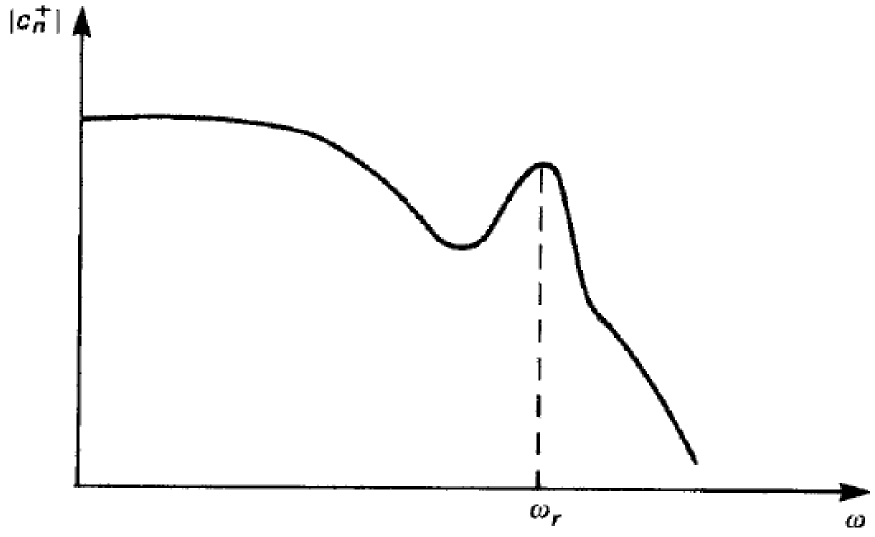
  - Reducing D reduces the low-frequency content



- **Effect of ringing (overshoot and undershoot)**:
  - Due to inductances and capacitances of PCB lands and wires, as well as impedance mismatches (filtered), ringing can occur when the signal changes levels (i.e. 0→1 or 1 → 0)
  - This ringing has a damping coefficient (α) and a frequency (f_r) $$Ke^{-\alpha t}\sin(\omega_r t)$$
  - This ringing will enhance the spectral contents of certain regions within the signal spectrum as shown in Fig(b), thus degrading EMC compliance
  - The ringing can be reduced via the use of **series resistors** (to damp it), the use of **ferrite beads** and **proper termination** of transmission lines
  - Note that we can find the spectrum of the waveform with ringing via decomposing it to (a) a square wave, (b) a damped sinusoid and (c) a shifter damped sinusoid (by 0.5T)



(a)

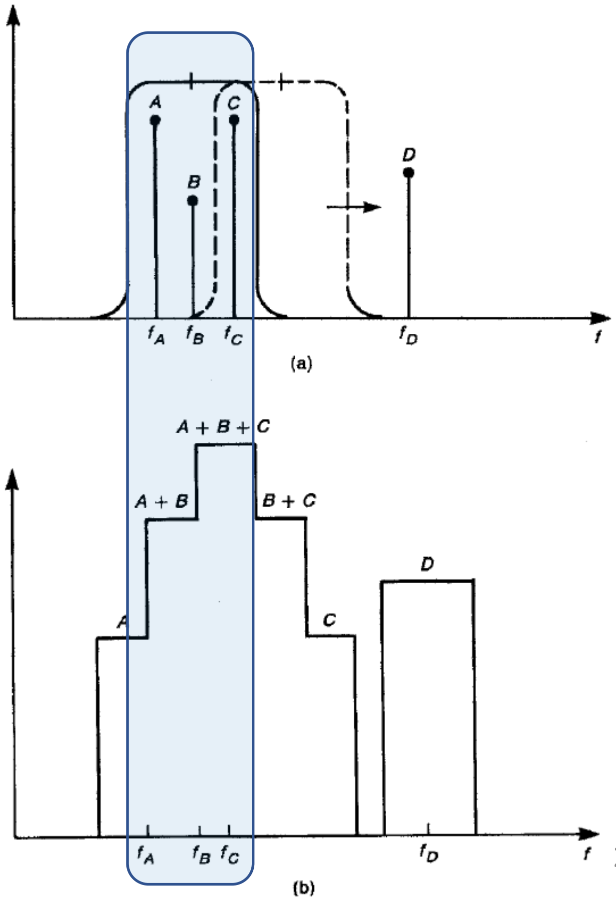


(b)

$$c_n = c_{n(square-wave)} + \frac{1}{T}\int_0^{T/2} Ke^{-\alpha t}\sin(\omega_r t)e^{-jn\omega_0 t}dt - e^{-jn\omega_0\frac{T}{2}}\left[\frac{1}{T}\int_0^{T/2} Ke^{-\alpha t}\sin(\omega_r t)e^{-jn\omega_0 t}dt\right]$$

Spectrum Analyzers (SA)

- Spectrum analyzers (SA) are instruments that display the magnitude spectrum for signals. Simply put, they are basically radio receivers with swept bandpass filters (BPF). In reality they are superheterodyne receivers where the incoming signal is mixed with a sweeping oscillator
- The **bandwidth of the filter** is very important parameter for the operation of a SA.
- The BW is defined by the reduction of the maximum signal at the center frequency to **6dB (6dB BW)**
- ***The displayed level at the center frequency of the BW will be the sum of the spectral levels that fall within that BW at that time***
- **Choose the smallest BW possible** to pick up proper frequency components



- The FCC and CISPR32 regulations have already **set the minimum BW** to be used for measurements. These values are shown in the tables.
- We should attempt to choose clock and data repetition rates such that none of the harmonics of any signal in the system will be closer than the measurement BW of the SA.
- *One way to determine whether two or more harmonics are adding with the BW of the SA is to narrow the BW of the receiver.* If when the BW is reduced, the level of the spectrum is not changed then we are assured that no two signals are added within the larger BW.

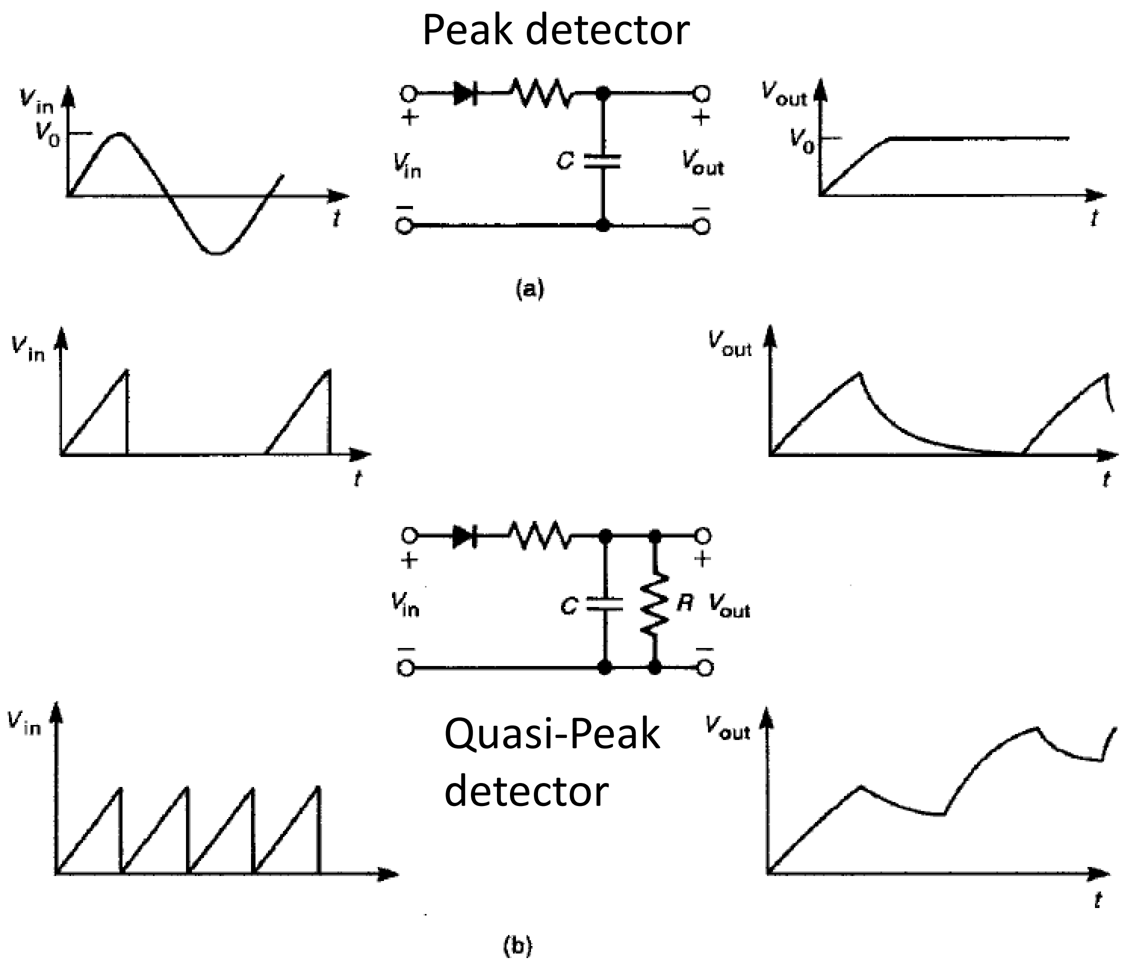
FCC Minimum Spectrum Analyzer Bandwidths (6 dB)

Radiated emissions:	30 MHz–1 GHz	120 kHz
Radiated emissions:	>1 GHz	1 MHz
Conducted emissions:	150 kHz–30 MHz	9 kHz

CISPR 22 Minimum Spectrum Analyzer Bandwidths (6 dB)

Radiated emissions:	30 MHz–1 GHz	120 kHz
Conducted emissions:	150 kHz–30 MHz	9 kHz

- **Peak vs. Quasi-Peak vs. Average**
  - We have been assuming the SA measurements in Peak mode, that is maximum RMS values of the sinusoidal harmonic (see peak detector circuit).
  - Regulatory agencies specify measurements in Quasi-peak mode (using quasi-peak detector).
  - In the case of infrequent occurring signals, the output levels from a quasi-peak detector will be smaller than a peak detector. Such infrequent signals might not affect the operation or create significant noise levels that degrade signal quality. So, if the quasi-peak levels are not satisfied, for sure the peak levels are not satisfied either.
  - The FCC and CISPR22 conducted emission limits are given in **quasi-peak (QP)** and in **average (AV)** levels. The average levels are obtained with an average detector. The average detector is a 1 Hz LPF placed after the usual envelope detector which passes signals with amplitude durations of 1sec or more.



Fourier Transform for non-periodic signals

- For non-periodic signals, the frequency domain representation is given by the Fourier Transform (FT).

$$F\left\{x(t)\right\} = X\left(j\omega\right) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

and

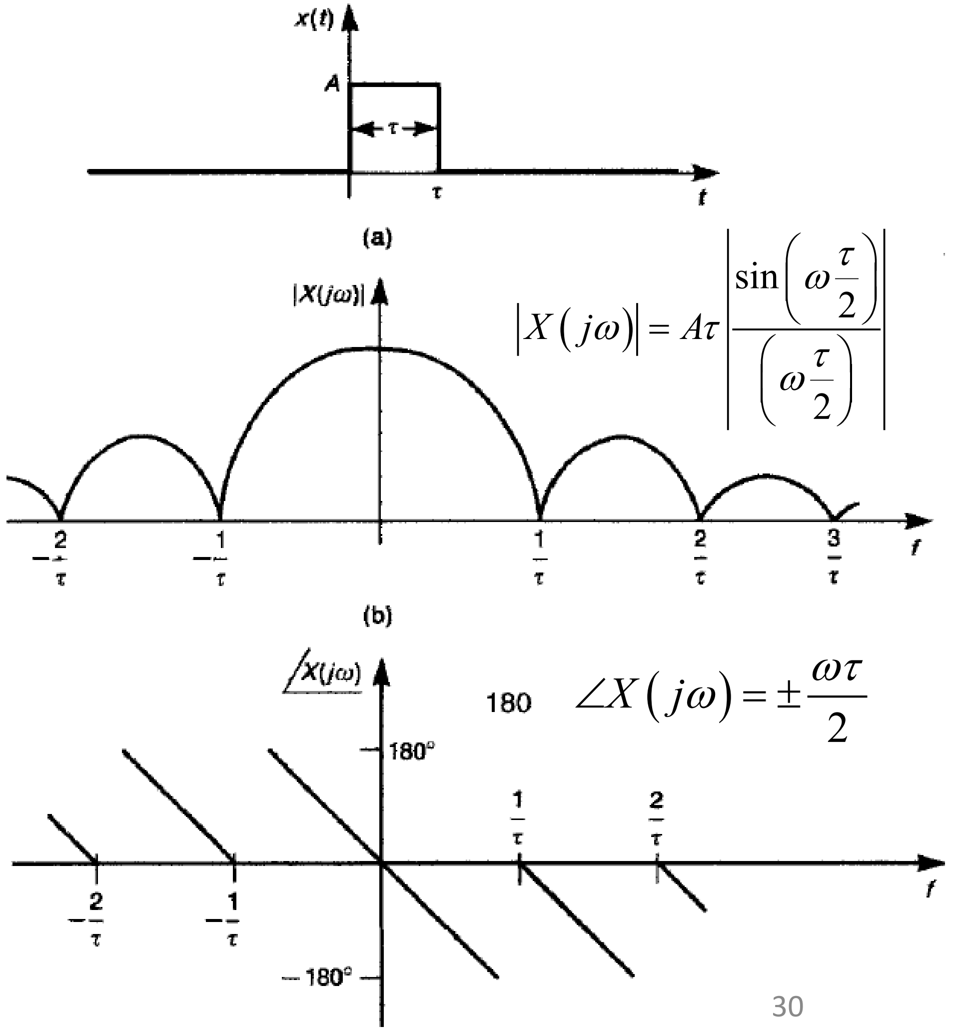
$$F^{-1}\left(X\left(j\omega\right)\right) = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X\left(j\omega\right)e^{j\omega t}d\omega$$

- For a single pulse, the FT becomes,

$$X\left(j\omega\right) = \int_{0}^{\tau} Ae^{-j\omega t}dt$$

$$= -\frac{A}{j\omega}\left[e^{-j\omega\tau}-1\right] = -\frac{A}{j\omega}e^{-j\omega\frac{\tau}{2}}\left[e^{-j\omega\frac{\tau}{2}}-e^{j\omega\frac{\tau}{2}}\right]$$

$$= A\tau\frac{\sin\left(\omega\frac{\tau}{2}\right)}{\left(\omega\frac{\tau}{2}\right)}e^{-j\omega\frac{\tau}{2}}$$



- If we know the FT of a single pulse $X(j\omega)$, we can directly obtain the coefficients of the complex-exponential FS of a periodic train of such pulses by replacing ω in $X(j\omega)$ with $n\omega_0$ and dividing the result by T, i.e.

$$c_n = \frac{1}{T} X\left(jn\omega_0 \right)$$

- For Random signals whose time behavior is described statistically (i.e. digital waveforms) we define the Autocorrelation function $R_x(t)$ as

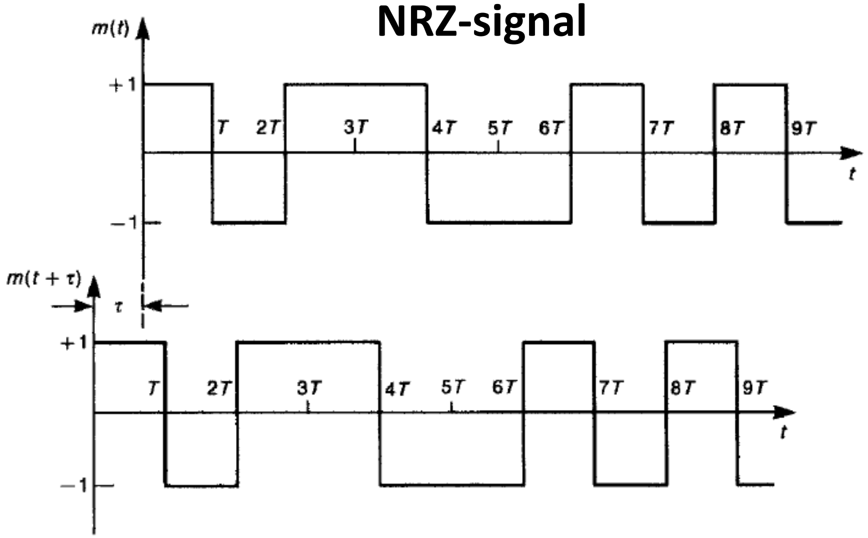
$$R_x\left(\tau\right) = \overline{x(t)x(t+\tau)}$$

- A non-return to zero (NRZ) signal is a waveform that does not return to zero in a state transition.
- The Spectrum of a random signal is given by the power spectral density (PSD), which is found from the FT of the Autocorrelation function of the signal.

- The Autocorrelation function of m(t) (shown on the side) is given by,

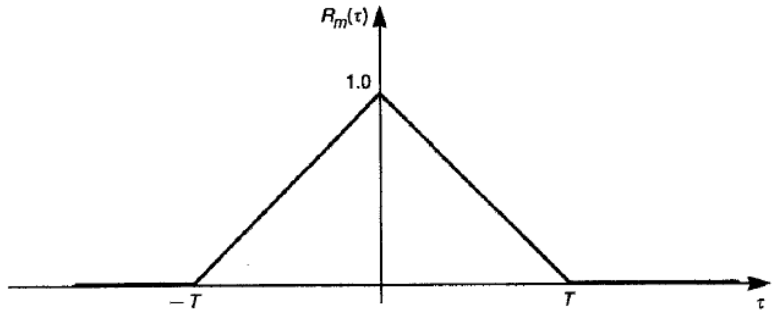
$$R_m(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} & , \text{ for } |\tau| < T \\ 0 & , \text{ for } |\tau| > T \end{cases}$$

NRZ-signal



- The PSD of m(t), is

$$G_m(f) = \int_{-\infty}^{\infty} R_m(\tau) e^{-j\omega\tau} d\tau \quad \left(W / Hz\right)$$

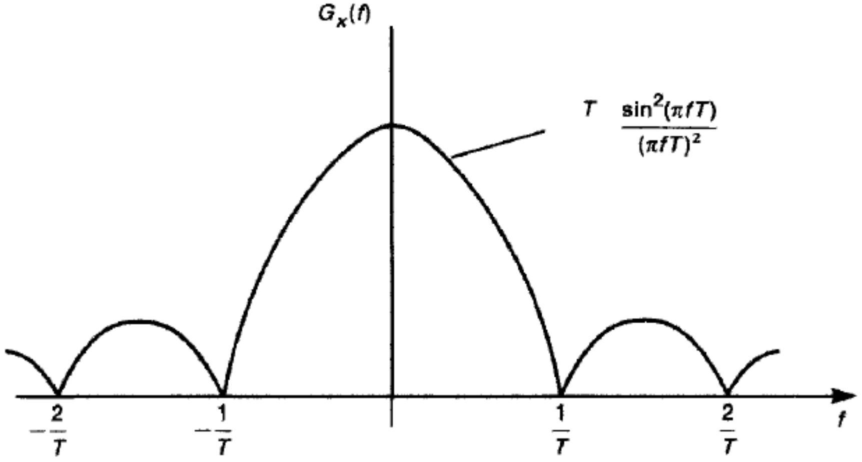


(b)

- Observe the nulls in the PSD at the inverse of T (the bit rate), and similarity with a pulse train spectrum

- The average power is,

$$P_{av} = \int_{-\infty}^{\infty} G_m(f) df \quad \left(W\right)$$



LTSPICE Notes

- SPICE can calculate the FS coefficients (magnitude and phase) and plot them
- We need to add the .FOUR command along with a transient analysis type .TRAN

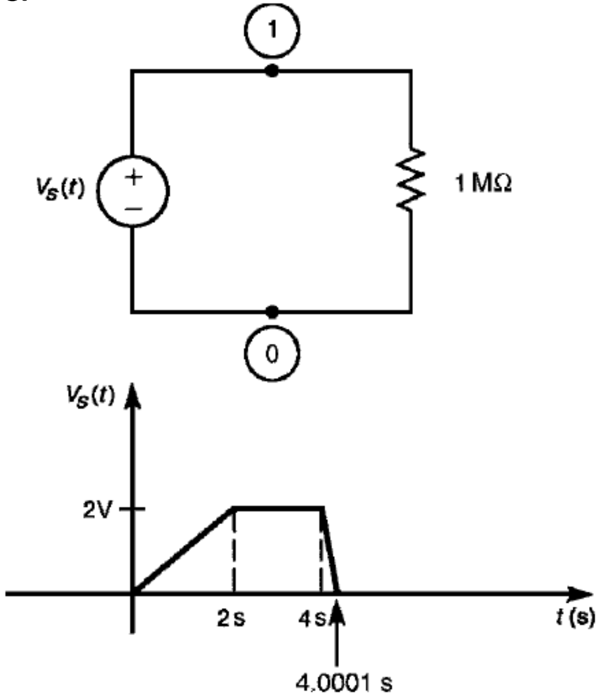
.FOUR   f0   [output variables]

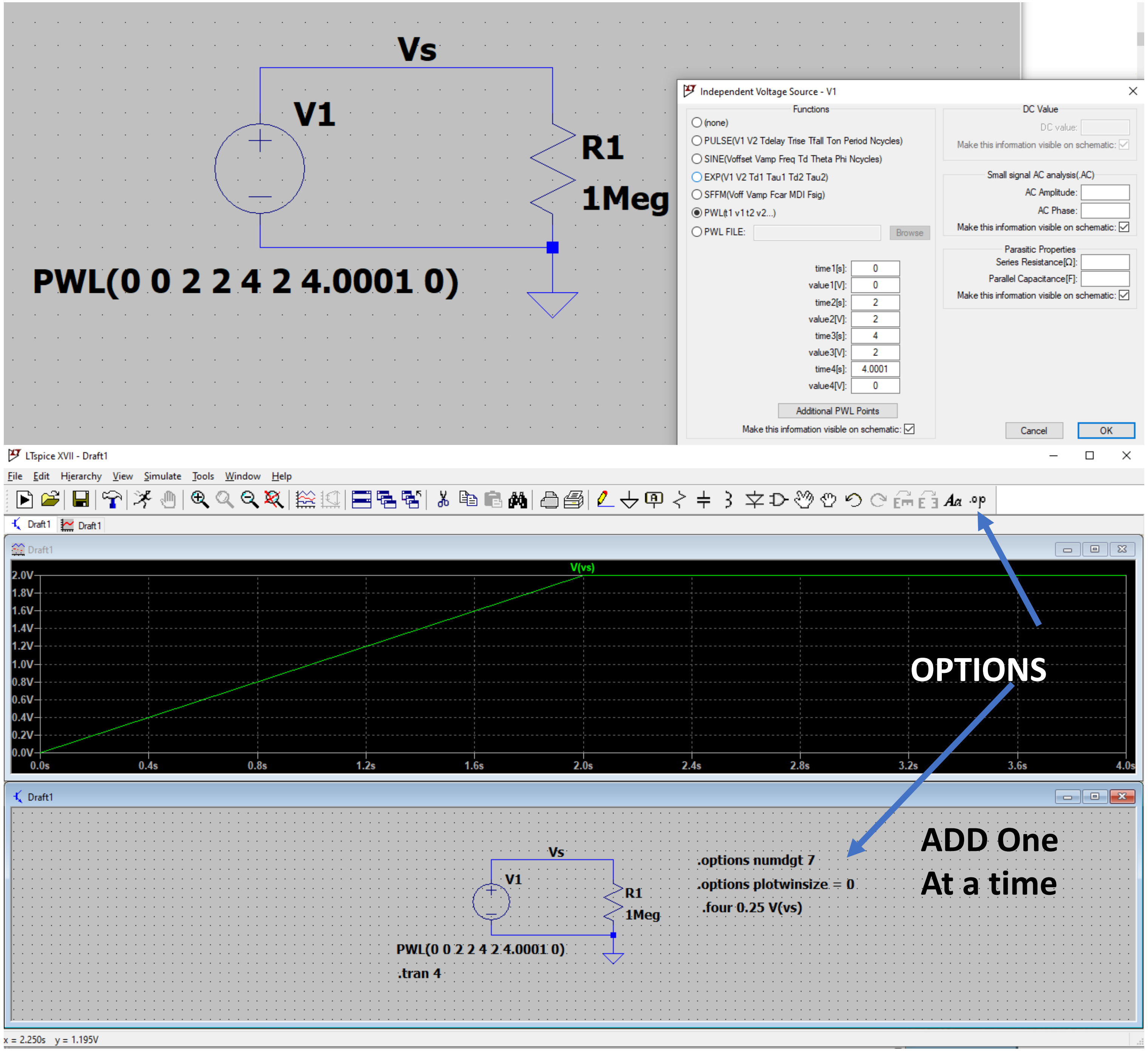
f0 is the fundamental frequency of waveform

[output variables] are the voltages or currents to be analyzed

- Example:

```
VS 1 0 PWL(0 0 2 2 4 2 4.0001 0)
R 1 0 1E6
.TRAN 0.0001 4
.FOUR 0.25 V(1)
.PROBE
.END
```



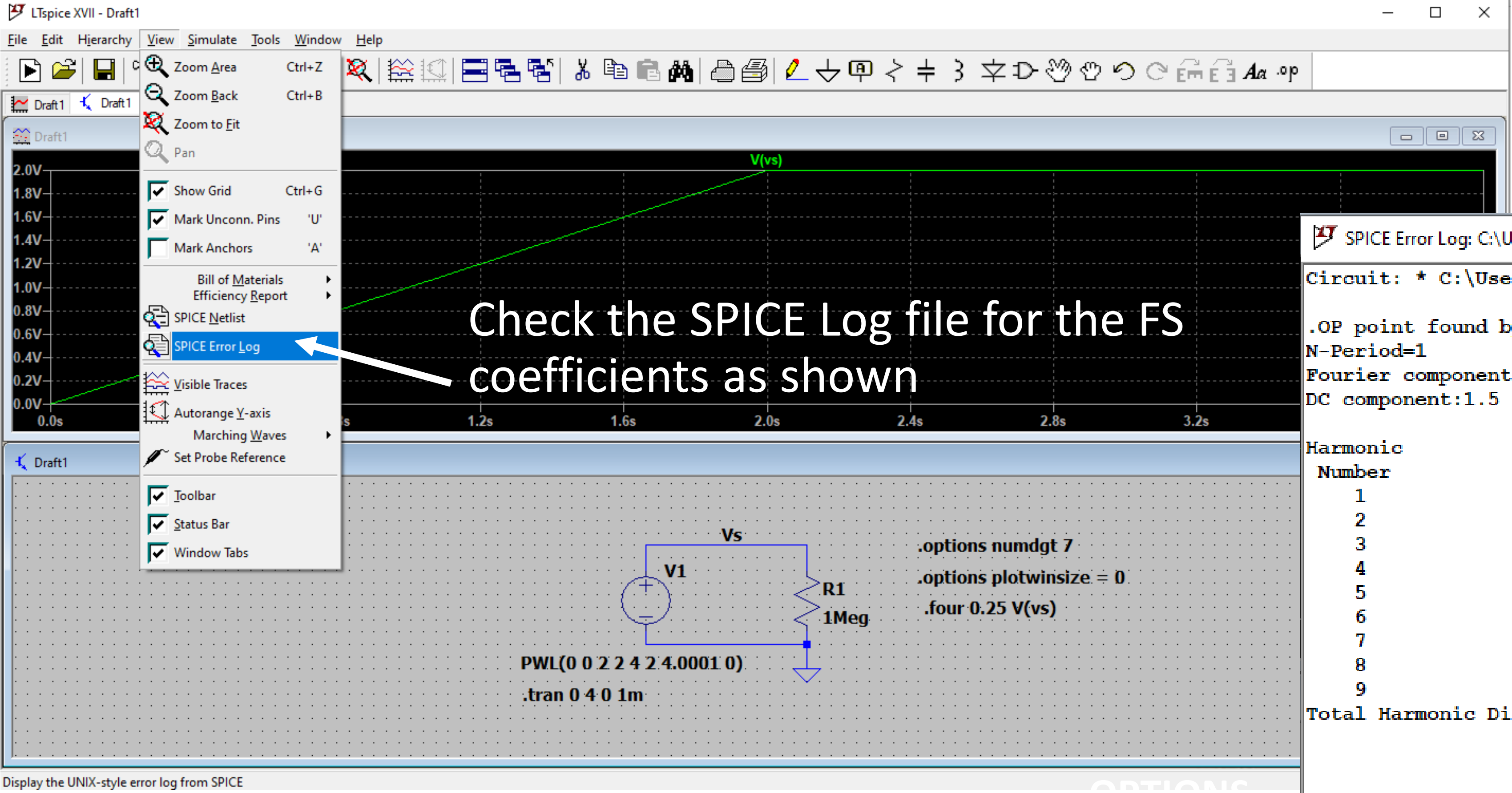


- From options icon, add the commands:

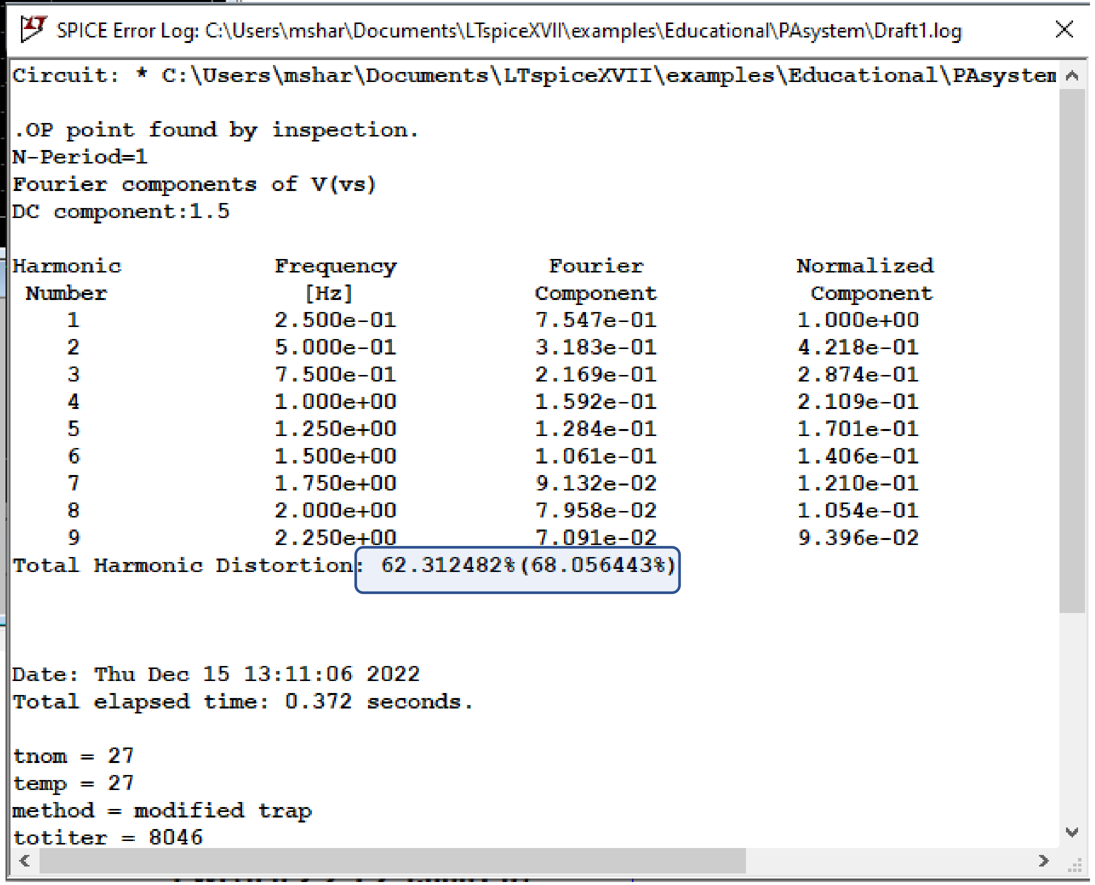
.options numdgt = 7 (double precision)

.options plotwinsize =0 (disable compression)

.options four 0.25 v(vs) (FS coefficients)



Check the SPICE Log file for the FS coefficients as shown


SPICE Error Log: C:\Users\mshar\Documents\LTspiceXVII\examples\Educational\PAsystem\Draft1.log

Circuit: * C:\Users\mshar\Documents\LTspiceXVII\examples\Educational\PAsystem

.OP point found by inspection.
N-Period=1
Fourier components of V(vs)
DC component:1.5

Harmonic Number	Frequency [Hz]	Fourier Component	Normalized Component
1	2.500e-01	7.547e-01	1.000e+00
2	5.000e-01	3.183e-01	4.218e-01
3	7.500e-01	2.169e-01	2.874e-01
4	1.000e+00	1.592e-01	2.109e-01
5	1.250e+00	1.284e-01	1.701e-01
6	1.500e+00	1.061e-01	1.406e-01
7	1.750e+00	9.132e-02	1.210e-01
8	2.000e+00	7.958e-02	1.054e-01
9	2.250e+00	7.091e-02	9.396e-02

Total Harmonic Distortion: 62.312482%(68.056443%)

Date: Thu Dec 15 13:11:06 2022
Total elapsed time: 0.372 seconds.

tnom = 27
temp = 27
method = modified trap
totiter = 8046

- The Total Harmonic Distortion is important and shows the effect of harmonics on the purity of the fundamental. For a sine wave, THD=0.
- Using .four, the THD is automatically calculated for you.
- Check this useful video:

https://www.youtube.com/watch?v=YPO3DEVzk90

$$THD = \frac{\sqrt{V_{2-RMS}^2 + V_{3-RMS}^2 + V_{4-RMS}^2 + V_{5-RMS}^2 + \cdots}}{V_{1-RMS}}$$

Next …

- Transmission lines and Signal integrity…