# Electromagnetic Compatibility (EMC)

**Topic 5**

**Non-Ideal Components and NOISE**

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# Introduction

- The non-ideal performance and characteristics of various components will limit their ability to supress or improve the EMC advantages they can provide
- As an EMC engineer, you need to be aware of such non-ideal characteristics so that you can factor for the deviations expected
- We will focus on the frequency ranges applicable to government regulations (i.e. 150KHz to 30 MHz for conducted emissions and 30 MHz-40GHz for radiated emissions)
- Always try to measure and characterize the component behavior within the bands of interest and assess the deviations and consider them in your calculations and models
- We will consider effects of wires, component leads, resistors, capacitors, inductors, ferromagnetic cores, Ferrite beads, common-mode chokes, electromechanical devices (DC motors, stepper motors, AC motors, solenoids), and mechanical switches
- Finally, we will consider noise sources and noise calculations and metrics

# (1) Wires

- At high frequencies, the wire inductance is more important that its resistance.
- Wires are composed of several strands of solid wires of radii $\boldsymbol{r_{ws}}$ that are placed parallel to each other for flexibility. Thus a good approximation for the resistance of a group of wires or its inductance is calculated by dividing a single wire resistance/inductance by the number of wires
- The radius of a bundle (group) of wires is called gauge. American Wire Standard (AWG) is the most commonly used for specifying wire types (i.e. radii). https://www.powerstream.com/Wire_Size.htm
- Stranded wires are expressed as (number x gauge) in tables.
- The conductivity and permeability of metals are important when calculating their inductances and resistances

**Wire Gauges (AWG) and Wire Diameters**

| | Wire Diameter (mils) | |
|---|---|---|
| Wire Gauge | Solid | Stranded |
| 4/0 | 460.1 | 522.0 (427 × 23) |
| | | 522.0 (259 × 21) |
| 3/0 | 409.6 | 464.0 (427 × 24) |
| | | 464.0 (259 × 23) |
| 2/0 | 364.8 | 414.0 (259 × 23) |
| | | 414.0 (133 × 20) |
| 1/0 | 324.9 | 368.0 (259 × 24) |
| | | 368.0 (133 × 21) |
| 1 | 289.3 | 328.0 (2109 × 34) |
| | | 328.0 (817 × 30) |
| 2 | 257.6 | 292.0 (2646 × 36) |
| | | 292.0 (665 × 30) |
| 4 | 204.3 | 232.0 (1666 × 36) |
| 6 | 162.0 | 184.0 (1050 × 36) |
| | | 184.0 (259 × 30) |
| 8 | 128.5 | 147.0 (655 × 36) |
| 10 | 101.9 | 116.0 (105 × 30) |
| | | 115.0 (37 × 26) |
| 12 | 80.0 | 95.0 (165 × 34) |
| | | 96.0 (7 × 20) |
| 14 | 64.1 | 73.0 (105 × 30) |
| | | 73.0 (41 × 30) |
| | | 73.0 (7 × 22) |
| 16 | 50.8 | 59.0 (105 × 36) |
| | | 59.0 (26 × 30) |
| | | 60.0 (7 × 24) |
| 18 | 40.3 | 47.0 (65 × 36) |
| | | 49.0 (19 × 30) |
| | | 47.0 (16 × 30) |
| | | 48.0 (7 × 26) |

- Recall that, $R = \dfrac{l}{\sigma \pi r_w^2}$ , $\delta = \dfrac{1}{\sqrt{\pi f \sigma \mu_0}}$

$$r_{lf} = R = \frac{1}{\sigma \pi r_w^2} \quad (\Omega/\text{m}) \quad , \quad r_{hf} \approx r_{lf} \frac{r_w}{2\delta} \quad (\Omega/\text{m})$$

$$L_{i,DC} = \frac{\mu_0}{8\pi} \text{ (H/m)} \quad , \quad L_{i,hf} = L_{i,DC} \frac{2\delta}{r_w} \text{ (H/m)} \quad \text{[internal L]}$$

- Currents require a return path. The magnetic flux external to each wire contributes to the total flux penetrating the area between them. The external inductance (parallel wires) is,

$$L_e = \frac{(\psi_m / l)}{I} = \frac{\mu_0}{\pi} \ln\left(\frac{s}{r_w}\right) \quad \text{(H/m)}$$

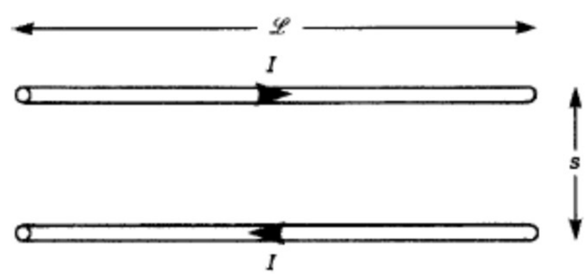
$$L_{loop} = (2L_i + L_e) \quad \text{(H/m)}$$

The values for l and c assume that the lines are separated with at least $(s/r_w)>5$ to avoid proximity effects (i.e. uniform currents)

- The capacitance of the wires is given by, $c = \dfrac{(Q/L)}{V} = \dfrac{\varepsilon_0 \pi}{\ln\left(\frac{s}{r_w}\right)}$ (F/m)

- The impedance of external inductor increases faster with frequency as compared to the internal one (recall the inverse frequency dependence of internal inductance). $Z_{l_{loop}} = j\omega(2L_i + L_e)$

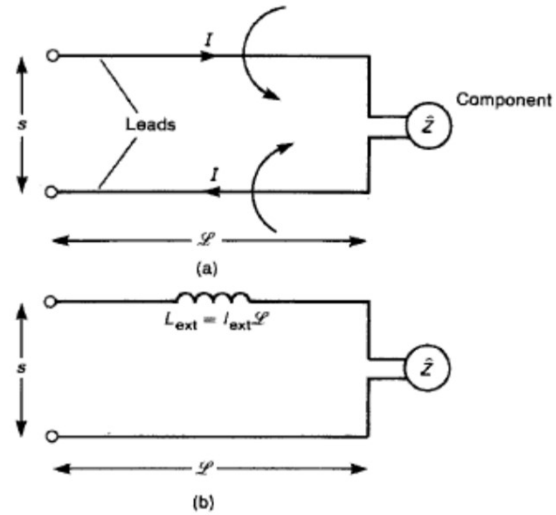
| Conductor | $\sigma_r$ | $\mu_r$ |
|---|---|---|
| Silver | 1.05 | 1 |
| Copper—annealed | 1.00 | 1 |
| Gold | 0.70 | 1 |
| Aluminum | 0.61 | 1 |
| Brass | 0.26 | 1 |
| Nickel | 0.20 | 600 |
| Bronze | 0.18 | 1 |
| Tin | 0.15 | 1 |
| Steel (SAE 1045) | 0.10 | 1000 |
| Lead | 0.08 | 1 |
| Monel | 0.04 | 1 |
| Stainless steel (430) | 0.02 | 500 |
| Zinc | 0.32 | 1 |
| Iron | 0.17 | 1000 |
| Beryllium | 0.10 | 1 |
| Mu-metal (at 1 kHz) | 0.03 | 30,000 |
| Permalloy (at 1 kHz) | 0.03 | 80,000 |

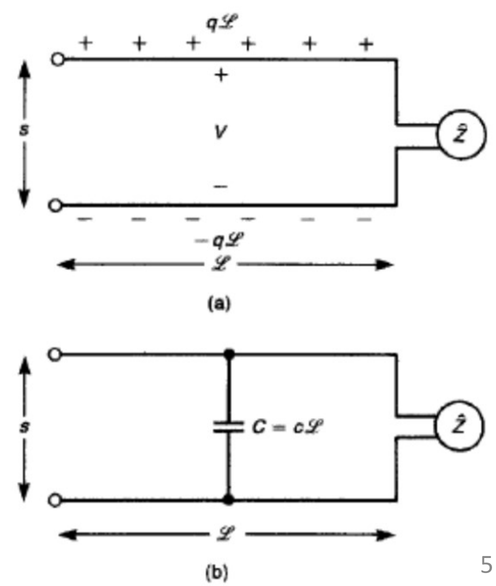


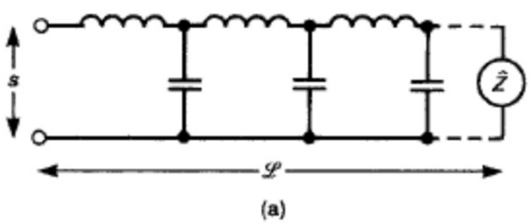

In reality, the presence of the plastic isolation around the wires deviates their responses. Be careful to check the measured responses.

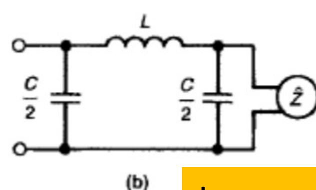
# (2) Component leads

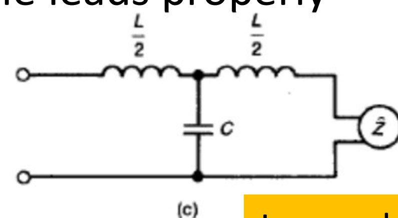
- The length of the component leads has negative effect at higher frequency due to the excess inductance and capacitance they will introduce
- Their effect must be included in the circuit model of that component to have accurate modeling.
- Several circuit models can be considered and should be investigated to take into account the effects of the leads properly

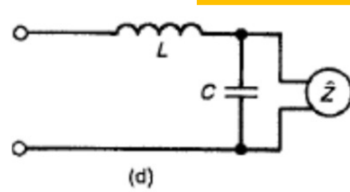


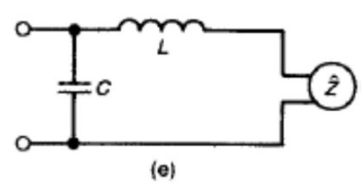





Distributed model

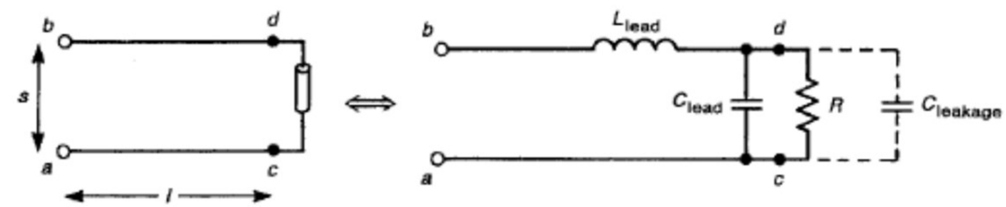

Lumped pi

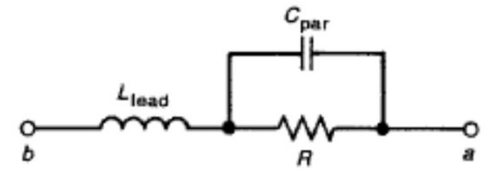

Lumped T


Lumped-Backward Γ


Lumped Γ

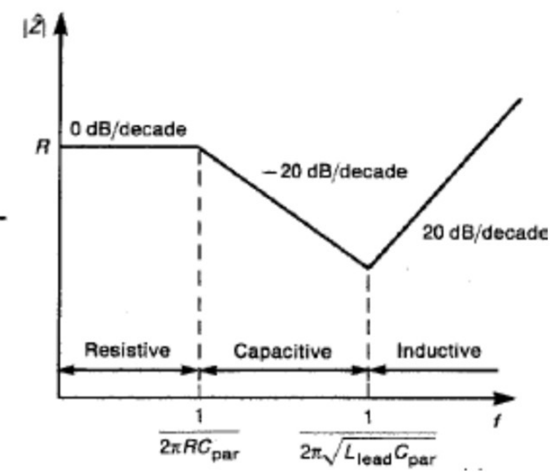
# (3) Resistors

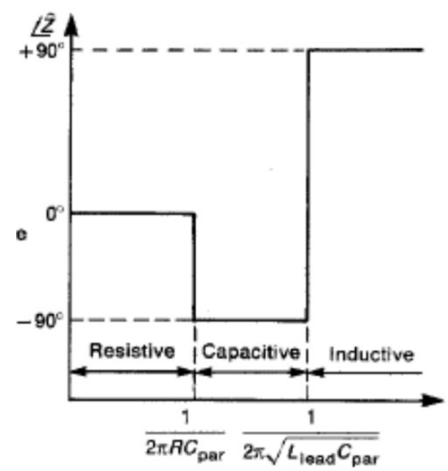



- Three major types; (1) carbon composition (2) wire wound, and (3) thin film
- Thin film resistors are more controlled in terms of resistance precise values and have lower inductive effects
- A non-ideal resistor with the lead effects is represented as →
- The parasitic capacitance is $C_{par} = C_{lead} + C_{leakage}$
- The impedance of this model is

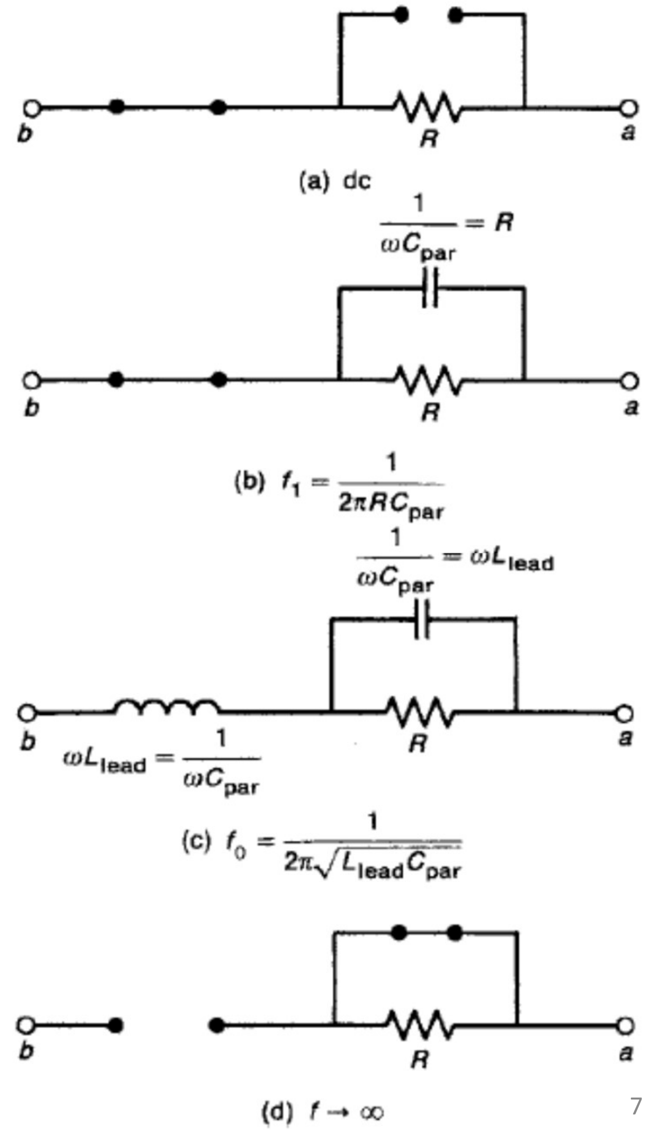
$$Z_{ab}(j\omega) = L_{lead}\frac{\left(\frac{1}{L_{lead}C_{par}}\right) - \omega^2 + \left(\frac{j\omega}{RC_{par}}\right)}{j\omega + \left(\frac{1}{RC_{par}}\right)}$$




- Note the break-point frequencies in the Bode plot.

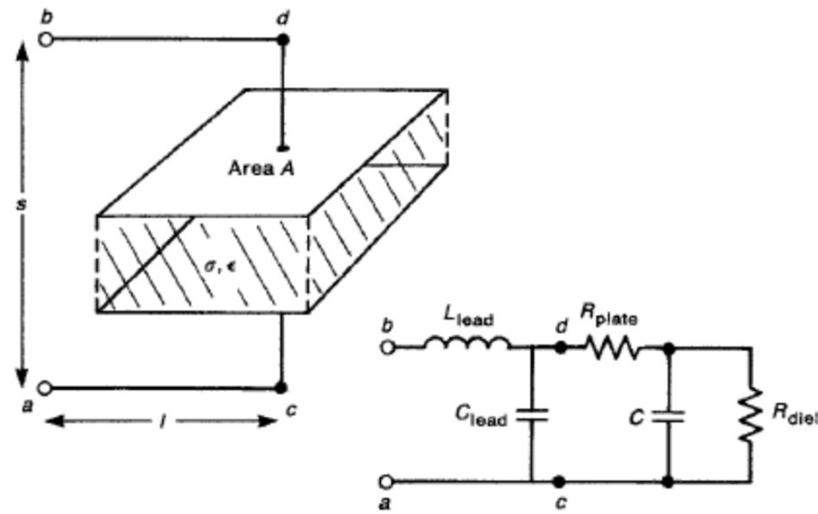
- Note some special cases for the impedance of a practical resistor
- Recall, that a Capacitor is open for DC and an Inductor is short at DC
- At very high frequencies, $Z_C \rightarrow 0$ (short), and $Z_L \rightarrow \infty$

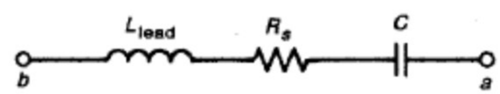


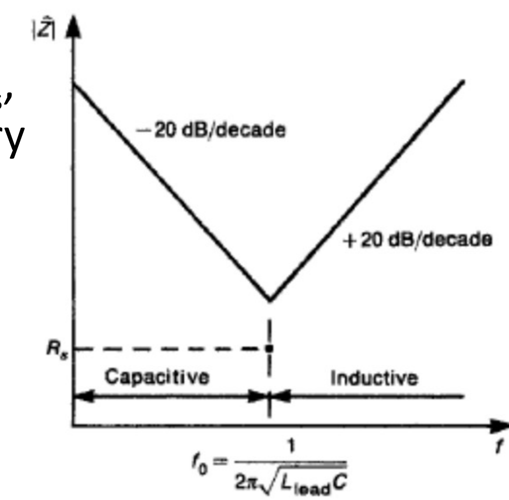
# (4) Capacitors

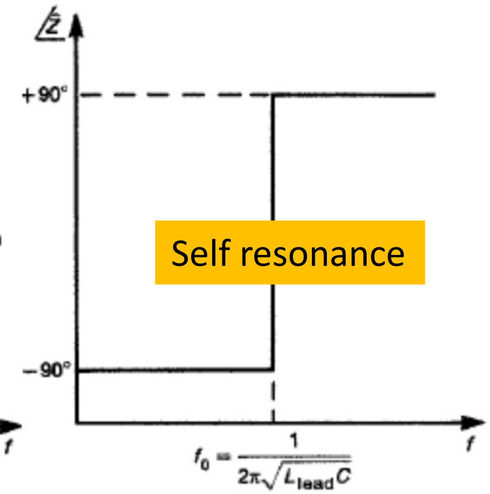
- Large caps are found in small package Tantalum electrolytic type (1 – 1000µF). Ceramic caps give lower capacitance values (1µF – 5pF)
- Ceramic caps are used for radiated emission suppressions, while electrolytic ones are used for conducted emission suppression and bulk charge storage on PCBs.
- A practical model of a capacitor is given by →
- $R_{plate}$ is also the equivalent series resistance (ESR) or $R_s$, and $R_{diel}$ is the dielectric resistance which is usually very large and can be neglected
- The impedance of a real capacitor is

$$Z_{ab}(j\omega) = L_{lead}\frac{\left(\frac{1}{L_{lead}C}\right) - \omega^2 + \left(\frac{j\omega R_s}{L_{lead}}\right)}{j\omega}$$



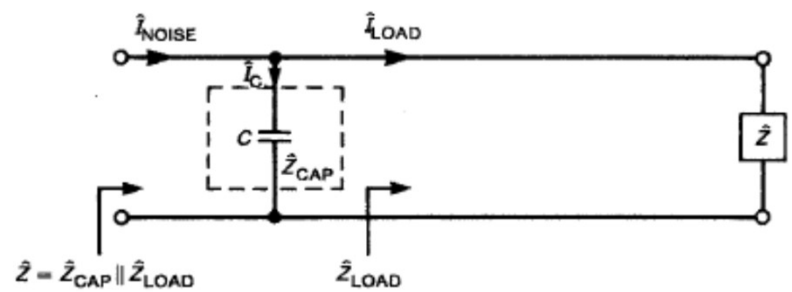



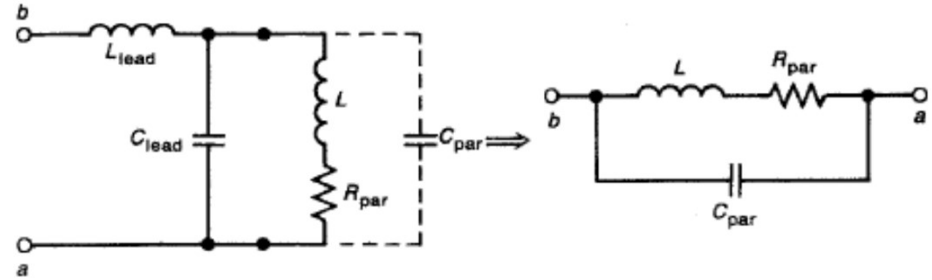



- When using caps to provide a low impedance path to shunt noise currents, then the frequency of the currents should be well below the self-resonance frequency of that capacitor value
- Always use multiple capacitors rather than using one large capacitor
- When using a capacitor to divert high frequency signals, be aware of ringing effects that might occur due to the resonance between the capacitor and the inductance of the cable of the PCB trace. Usually we use RC packs (low pass filters) to have the two effects (i.e. avoiding ringing and supressing high frequency signals)
- Note that when adding a cap, we will need to make sure that $Z_c < Z_{load}$, otherwise it will not be effective.

$$\hat{I}_C = \frac{\hat{Z}_{load}}{\hat{Z}_{load} + \hat{Z}_C} \hat{I}_{noise}$$

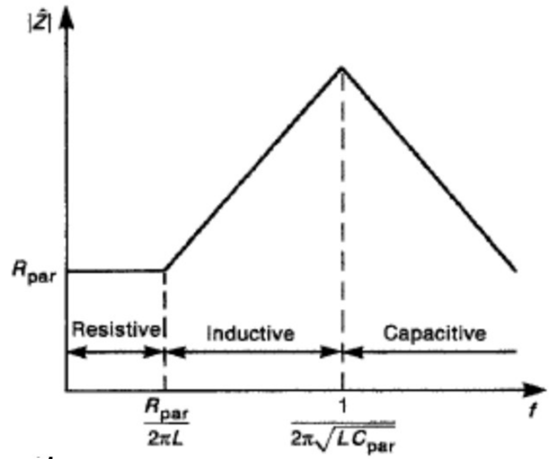


# (5) Inductors

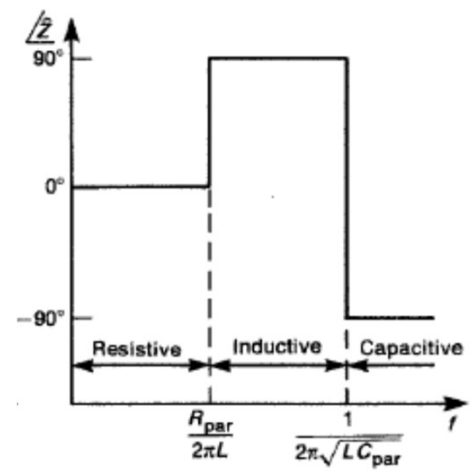



- For practical Inductors, we have a model that considers all parasitic effects as the one shown
- The lead inductance and capacitance can be neglected (leads far away and short).
- The impedance becomes,

$$Z_L(j\omega) = R_{par}\frac{1+\left(\dfrac{j\omega L}{R_{par}}\right)}{1-\omega^2 LC_{par} + j\omega R_{par}C_{par}}$$




- Inductors are places in series with wires and PCB traces to block noise currents. This will be effective if the inductor impedance is larger than the original series impedance looking at the load at the frequency of interest.
- Series inductances are most effective with low impedance loads.

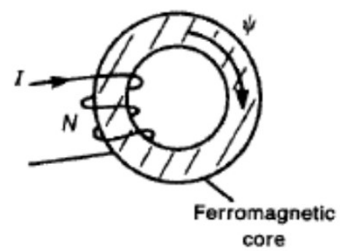


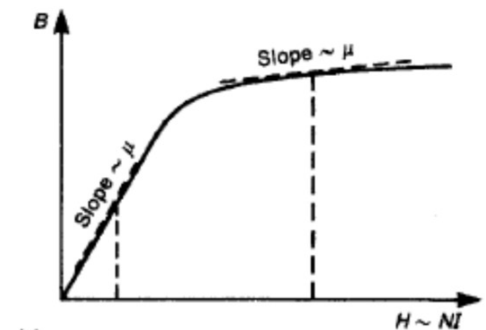
# (6) Ferromagnetic Materials

- Properties of Ferromagnetic materials that are of use in EMC,
  - Saturation
  - Frequency response
  - Ability to concentrate magnetic flux
- High relative permeability values, measured at low frequencies (i.e. 1KHz) and low current.
- Consider the classical Toroid,

$$L = \mu_r \mu_0 \frac{N^2 A}{l} \quad , \quad \oint \mathbf{H} \bullet d\mathbf{L} = NI \quad , \quad \mathbf{B} = \mu \mathbf{H}$$

$$\psi = \int_S \mathbf{B} \bullet d\mathbf{s} = BA \quad , \quad \mu = \frac{\Delta B}{\Delta H}$$



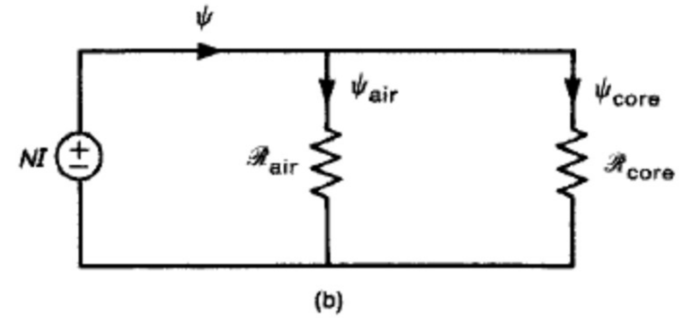



(a)

- As **current increases**, point moves upward the curve (slope decreases), thus **permeability decreases**. And L decreases, this is called **saturation** (reduction of μ due to current increase)

- Magnetic fields concentrate in high-permeability materials
- The ratio of how much flux remains in the core of the material and how much leaks depends on the ***reluctance ($R_m$)*** of the core. It is defined as,

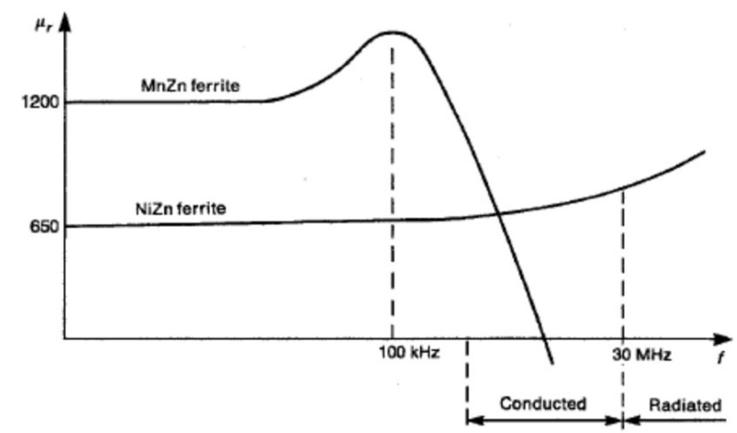
$$R_m = \frac{l}{\mu A} = \frac{\overbrace{NI}^{magnetomotive-force}}{\psi}$$



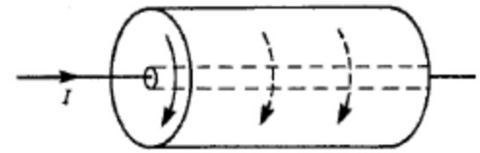

- We can make an analogy between magnetic circuits and electrical ones. By current division, the portion of the flux that remains in core is,

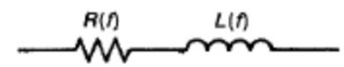
$$\psi_{core} = \frac{R_{air}}{R_{air} + R_{core}}\psi \quad , \quad R_{core} << R_{air} \quad (high \quad \mu \text{ values})$$

- Thus, majority of flux stays in the core (lower reluctance path), and we will utilize this property in the future to reduce leakage of magnetic flux.
- Make sure that the permeability values stays high at higher frequencies so that the effectiveness of focusing the flux is maintained.



# (7) Ferrite Beads




- Ferrite material are non-conductive ceramic materials that differ from ferromagnetic ones like iron. Ferrites have lower Eddy current losses at frequencies up to 100s MHz. Thus they are attractive from EMC standpoint. i.e. supress high-frequency and keep low frequency.
- A current passing through a wire generate magnetic flux. The flux passes through the bead material producing internal inductance.
- Ferrite beads are typically used in low impedance circuits such as those for power supplies.
- They can be used as filters (LPF) to damp ringing in fast-rise time circuits. They will dissipate noise as heat (due to high resistance)!
- Multi-turn beads can provide higher impedances at 100’s MHz (i.e. single hole ~ 100$\Omega$ , while 2.5 turns provides 850$\Omega$ @ 100 MHz)
- Like other ferromagnetic materials, Ferrites can saturate when high low-frequency current passes through them

$$L_{bead} = \mu_r \mu_0 \overbrace{K}^{\text{depends on bead dimensions}}$$

$$\mu_{r-bead} = \mu_r^{'}(f) - j\mu_r^{''}(f)$$

$$thus,$$

$$j\omega L_{bead} = \underbrace{\omega \mu_r^{''}(f)\mu_0 K}_{R(f)} + \underbrace{j\omega \mu_r^{'}(f)\mu_0 K}_{L(f)}$$

# (8) Common-Mode Chokes

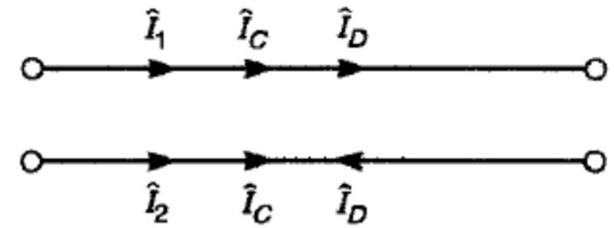
- One of the most important topics affecting radiated emissions is **common-mode (CM)** and **differential-mode (DM) currents**
- We can decompose the current on a two wire T-line as

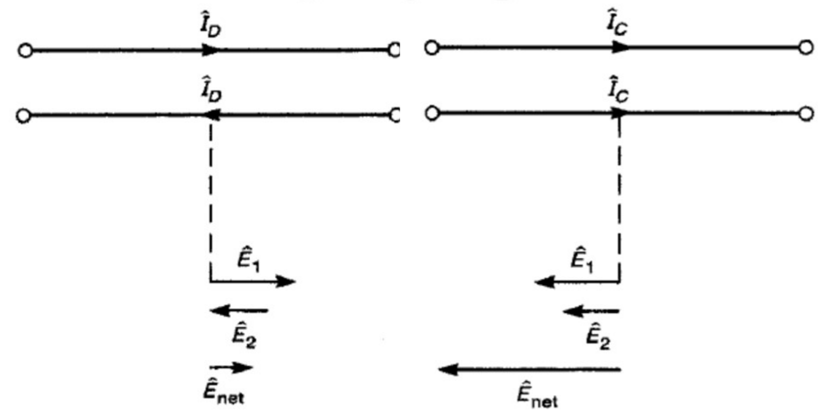
$$\hat{I}_1 = \hat{I}_C + \hat{I}_D$$
$$\hat{I}_2 = \hat{I}_C - \hat{I}_D$$

- Thus, we can write,

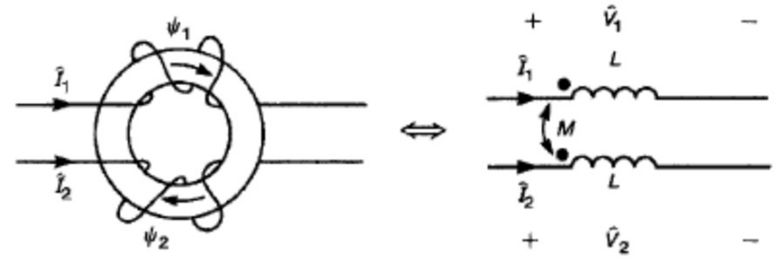
$$\hat{I}_D = \frac{1}{2}\left(\hat{I}_1 - \hat{I}_2\right)$$
$$\hat{I}_C = \frac{1}{2}\left(\hat{I}_1 + \hat{I}_2\right)$$

- DM currents are equal in magnitude but opposite is direction, these are the desired (functional) currents. CM currents are equal in magnitude and direction. These are un-wanted currents that will radiate (antenna-mode currents).
- The generated E-fields from DM currents are negligible (almost non-existent) while those from CM ones reinforce one another and radiate.
- To reduce CM currents, we use **CM chokes**. We wind the two wire lines around a ferromagnetic core as shown to create a CM choke. Note the winding directions




(a) (b)



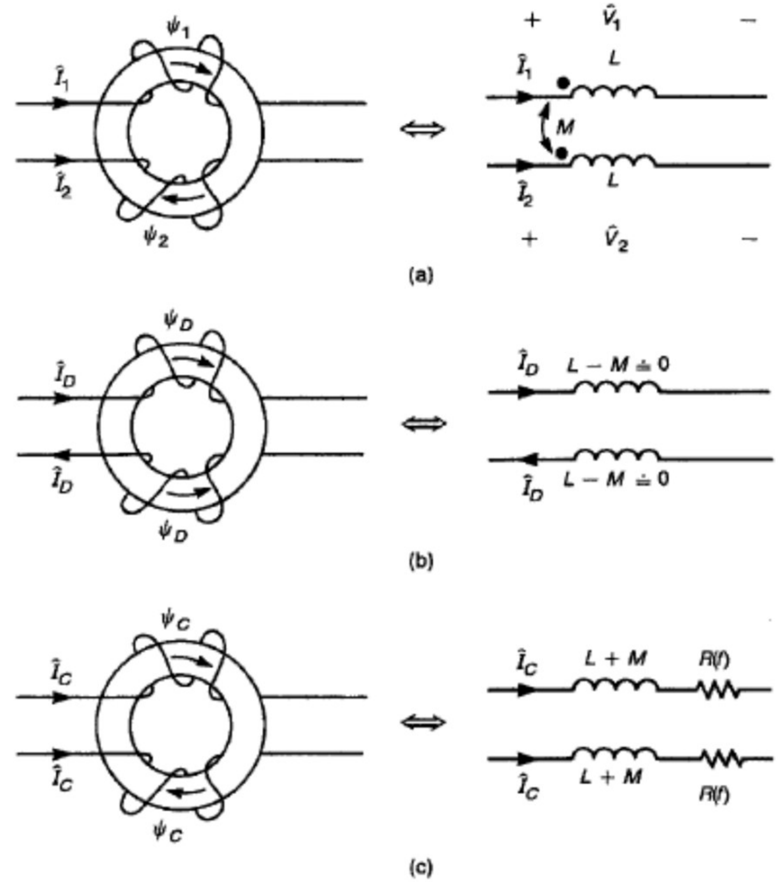
- Assume that the core windings are identical, i.e. L1=L2=L
- The impedance for **one-winding**, $\hat{Z}_1 = \frac{\hat{V}_1}{\hat{I}_1} = \frac{j\omega\left(L\hat{I}_1 + M\hat{I}_2\right)}{\hat{I}_1}$
- Consider CM currents, where $\hat{I}_1 = \hat{I}_C$ and $\hat{I}_2 = \hat{I}_C$ then,

$$\hat{Z}_{CM} = j\omega\left(L + M\right)$$

- For DM currents, $\hat{I}_1 = \hat{I}_D$ and $\hat{I}_2 = -\hat{I}_D$ then,

$$\hat{Z}_{DM} = j\omega\left(L - M\right)$$

- If the windings are symmetric (DM fluxes subtract, while CM fluxes add), and all flux remains in the core, then L=M, and ZDM = 0!
- Thus, in the ideal case where L=M, DM currents are not affected by the CM-choke, while CM currents see 2L in series with the two wires (thus, being blocked at high frequencies)
- The existence of the ferrite core will introduce a frequency dependant resistance R(f) that will dissipate the energy of the CM currents as well.

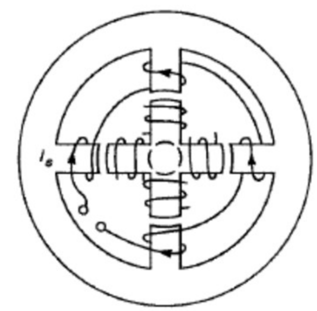


# (9) Electromechanical Devices

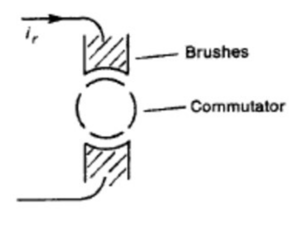
- Electromechanical devices such as DC motors, stepper motors, AC motors, and solenoids translate energy into mechanical motion and can generate EMC problems because of discharges or arcing.

(a) DC motors:

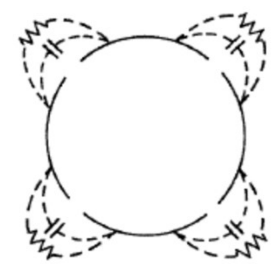
- Based on North-pole, South-pole attraction/repulsion.
- Consists of a stationary winding on the stator along with coils attached to the rotor. DC current is passed to create the magnetic poles.
- Carbon brushes make contact with the rotor segments and provide a means of alternating the current and magnetic fields of the rotor poles using a DC current from the source
- As current to the rotor coils connects and disconnects from the DC source, arcing at the brushes is created. This is a high frequency spectral content current that yields radiated emissions (200MHz – 1 GHz)
- To supress this arcing, resistors (R) and capacitors (C) can be placed across commutator segments Fig (c). Sometimes, small inductors (L) are inserted in the DC leads to block these noise currents.



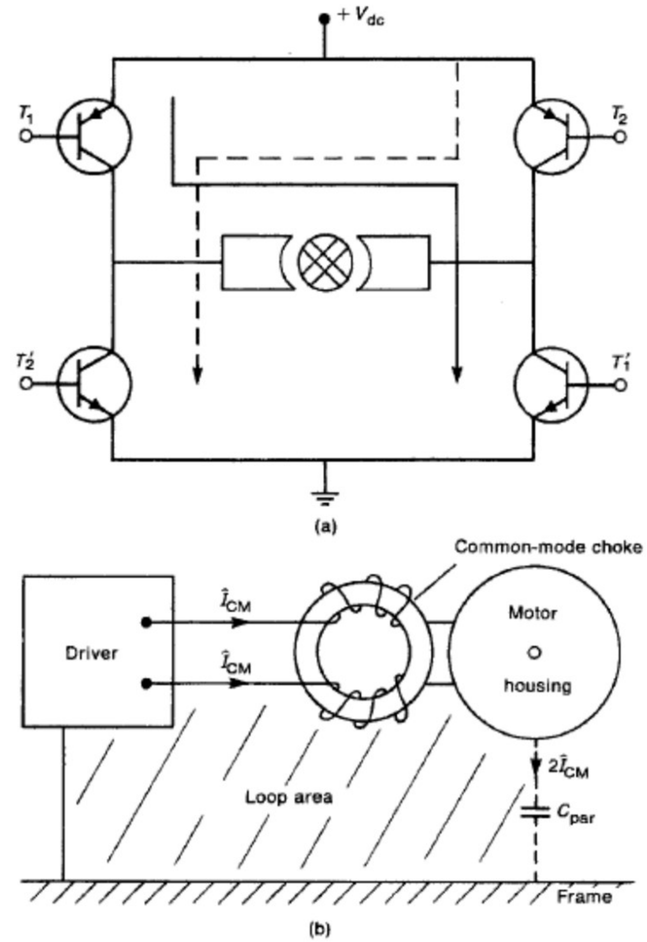
(a)



(b)


(c)

- Another source of high frequency noise is the motor drive circuit, the H-drive circuit.
- Two current directions (paths), T1-T1’ and T2-T2’. For thermal cooling of the motor, its housing is connected to the metallic frame of the product acting as a heatsink. This produces a large capacitance Cpar between the motor housing and the frame, thus creating a path for CM currents to pass through the connecting wires from the rotor to the stator via the capacitance of the windings and eventually to the frame via Cpar.
- The fast switching of the driver circuit creates fast rise-time current spikes with high frequency noise. These contribute to the radiation mechanism. The loop area created by the leads and their return current contributes to the severity of this radiated/conducted emission (the larger the area the worst).
- To block and reduce CM currents, insert a CM choke in the driver leads.



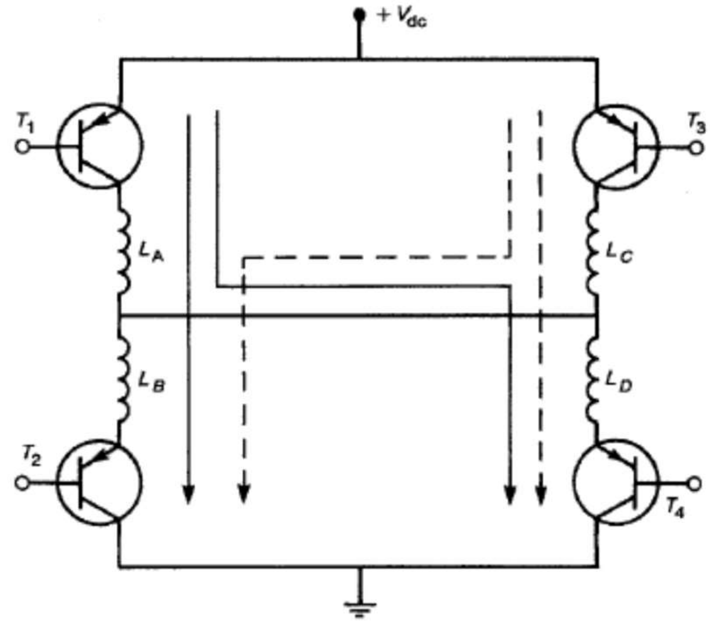
## (b) Stepper motors:

- Two types:
  - (1) Permanent magnet (PM)
  - (2) Variable Reluctance (VR)
- Both types apply DC currents to stator to produce magnetic poles
- Stator and rotor are segmented into large number of poles around their peripheries to provide fine steps

→No arcing issues

- CM currents within the driving circuit wires and the frame are the main issues in these motors

→CM chokes are used at the driver wires to block CM currents



A typical driver circuit for a stepper motor.

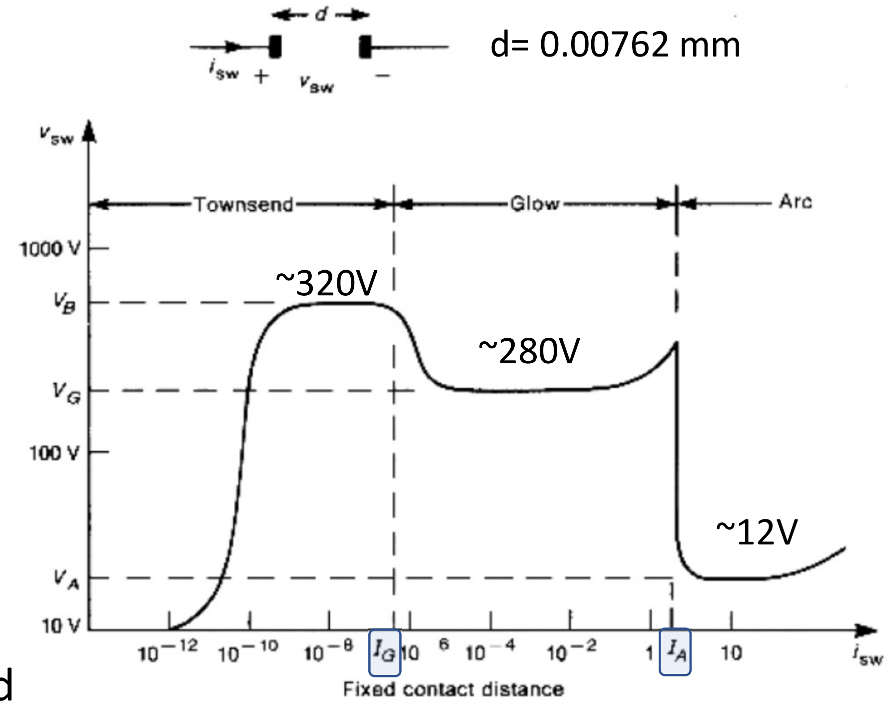
## (c)-(d) AC motors/Solenoids:

- AC motors are seldom used in positioning, but rather used to provide constant speeds and drive small cooling fans
- Major issue for EMC in these motors/solenoids is the closely spaced inductors of the rotor and stator for the AC motor (or the energized inductor and the housing in case of solenoid) that yield large parasitic capacitance between them. This will ease the coupling of CM currents from the AC power source to the product frame and vice versa.

→ CM chokes are used at the leads of such motors to block CM currents

# (10) Mechanical Switches

- Activation/deactivation of such switches can cause arcing, and thus high frequency noise
- The I-V curve for a typical switch is shown. 3 regions are identified:
    (1) Townsend discharge region ($V_B$) [breakdown], contact separation
    (2) Glow discharge region ($V_G$)
    (3) Arc discharge region ($V_A$), contact material
- As the voltage between contacts increases, the E-field accelerates electrons causing them to strike molecules and free more electrons, creating more free electrons. Several phenomena occurs (Avalanche, vaporization of metal contact, etc)
- The breakdown voltage depends on the gas (K1, K2), contact separation (d) and pressure (p) [Paschen formula]
- This is referred to as a **long** arc.




$$V_B = \frac{K_1 pd}{K_2 + \ln pd}$$

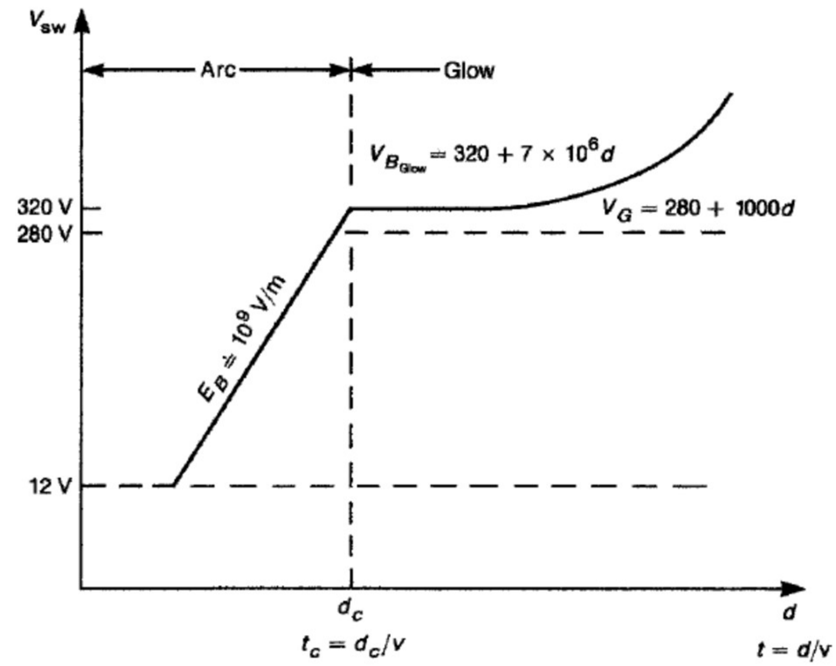
- For **smaller** contact spacings, Arcs can be formed before breakdown of gas.
- A glow discharge will form if the contact voltage exceeds the breakdown voltage as given by,

$$V_{B,glow} = 320 + 7 \times 10^{6} d$$

- If the current available from the external circuit exceeds the minimum glow discharge sustaining current $I_G$, a glow discharge will form, and the contact voltage will drop to,

$$V_G = 280 + 1000d$$

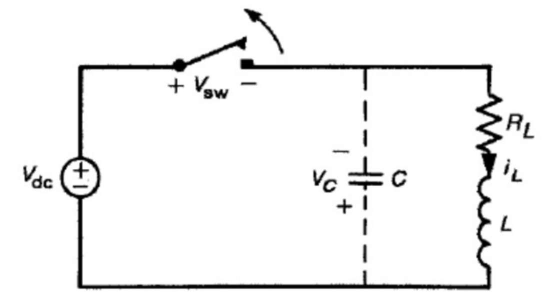
- In order to sustain a glow (arc) discharge, the voltage across and current through the contact available from the external circuit should exceed $V_G$ and $I_G$ ($V_A$ and $I_A$).
- The glow discharge is characterized by large voltage and low current, where the arc discharge is characterized by low voltage and large current

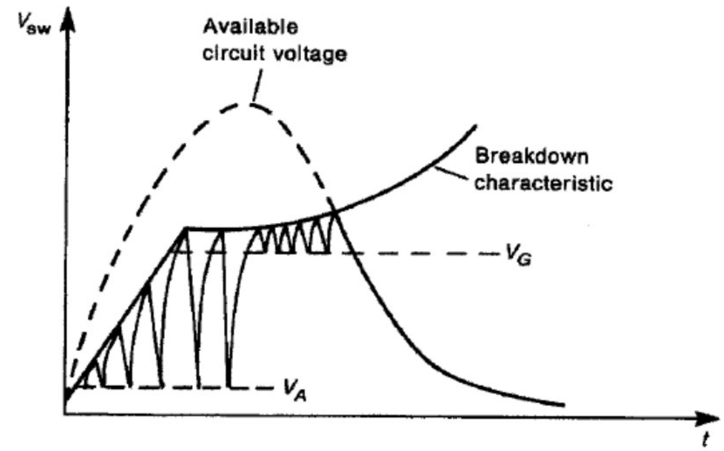


- Switches are often used to interrupt inductive loads (motors or solenoids)
- An interesting phenomena is observed because of such switching called ***Showering Arc***.
- An inevitable capacitance is present in parallel with the load
- When switch is closed, the Steady-State current $I_L = V_{DC}/R_L$ passes through the inductor. When the switch is open, the inductor tries to keep this current (recall, current in inductors does not change abruptly). The current is pushed into the capacitor and thus charging it.
- The voltage across the switch will become,

$$v_{sw}(t) = v_c(t) + V_{DC}$$

- Thus the switch rating might be exceeded because of the capacitor build up, which may exceed its breakdown voltage, and thus creates a short arc and its voltage drops to $V_A$.
- The capacitor discharges through the switch. If the switch current exceeds the minimum arc-sustaining current, the arc is sustained. If not, the arc is extinguished, and the capacitor begins to re-charge
- A sequence of rising and falling voltages across the contacts, called ***Showering Arc***.



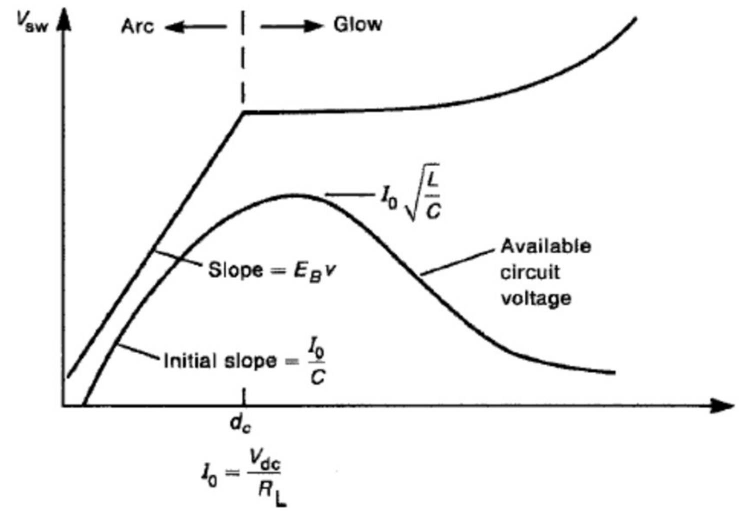



- Showering arcs have significant spectral content and cause EMC problems.
- The wiring carrying these currents can severely radiate
- Two methods are generally employed to prevent the formation of an arc:
  - (1) Prevent the switch voltage from exceeding the glow breakdown of the switch (i.e. 320V) [prevents arc forming]
  - (2) Ensure that the arc current is below the minimum sustaining current [prevents sustaining the arc]
- In order the implement (1), Note the contact breakdown voltage profile shown against the available circuit voltage. It is shown that the initial rise of contact voltage should be kept below the slope of the breakdown curve (top)
- The peak value of the available circuit voltage curve is $I_0\sqrt{\frac{L}{C}}$

When neglecting $R_L$ and assuming all energy stored in inductor $\frac{1}{2}LI_0^2$

Is transferred to capacitor $\frac{1}{2}CV_{peak}^2$

- In addition, the discharge waveform is non-oscillatory (overdamped) if $\sqrt{\frac{L}{C}} < \frac{1}{2}R_L$



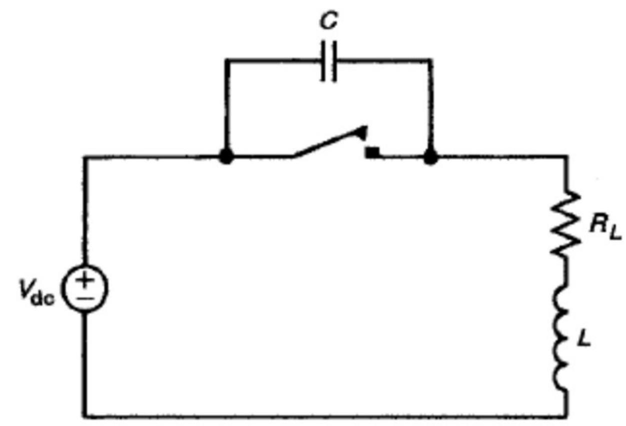
- Thus, to prevent initiation of an arc, two conditions should be satisfied (typical $E_B = 10^8$ V/m, and switch velocity v=0.01 m/s):

$$(1) \quad E_B v > \frac{V_{dc}}{R_L C} = \frac{I_0}{C}$$

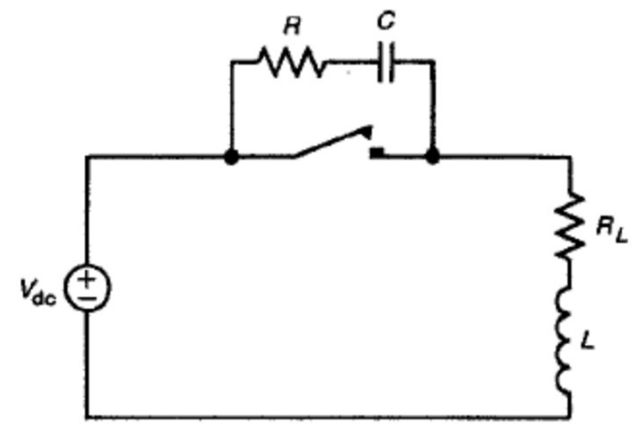
$$(2) \quad I_0\sqrt{\frac{L}{C}} = \frac{V_{dc}}{R_L}\sqrt{\frac{L}{C}} < V_{B,gas} \cong 320V$$

- This can be implemented by placing a large capacitor in parallel with the switch (or inductor) to increase the net capacitance and thus reducing the peak available circuit voltage and reducing the initial rise of the available circuit voltage (a).
- A major drawback of this configuration is that contact damage might occur due to large capacitor charging current.
- To overcome this, add a current limiting resistor in series with the large capacitor (b). The value of R should provide less than $I_A$ to prevent arcing and should be between,

$$\frac{V_{dc}}{I_{A,\min}} < R < R_L$$



(a)



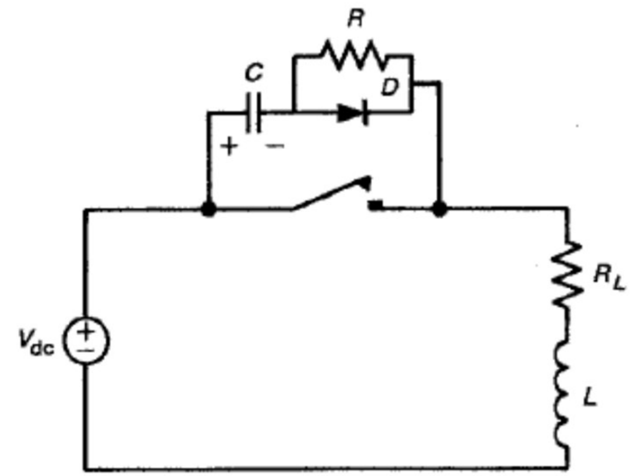
(b)

- The capacitor is chosen based on the two criteria mentioned,
  (1) The initial rate of the voltage rise of the available circuit , $I_0/C$ < 1V/µs to avoid arcing
  (2) the peak available voltage $I_0\sqrt{L/C}$ < 320 V to avoid gas breakdown.
- This leads to the following conditions for choosing C:

$$(1)\quad C \geq \left(\frac{1}{320} I_0\right)^2 L$$

$$(2)\quad C \geq I_0 \times 10^{-6}$$

- A slightly better but expensive network is the one with a diode. When switch closes, R limits discharge current. When switch opens, the diode shorts out the resistor and the capacitor diverts the load current .

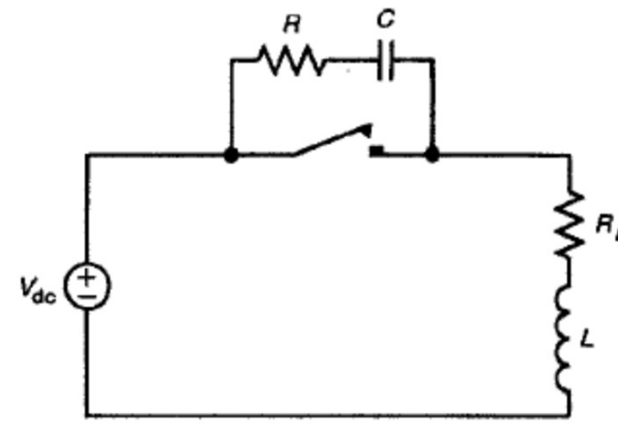


(c)


More references to check if interested more:
[R1] C. R. Paul, Analysis of Linear circuits, McGraw Hill, 1989
[R2] H. Ott, Noise Reduction Techniques in Electronic Systems, Wiley, 1988

# EX. 5.1

- For the R-C switch protection network shown, suppose that $V_{dc}$ = 50V, $R_L$ = 500 $\Omega$, L = 10 mH, $I_A$ = 0.25A, and the switch closing/opening velocity is 0.01 m/s. Determine R and C such that the contacts will be protected.




(b)

**<u>Sol.</u>**

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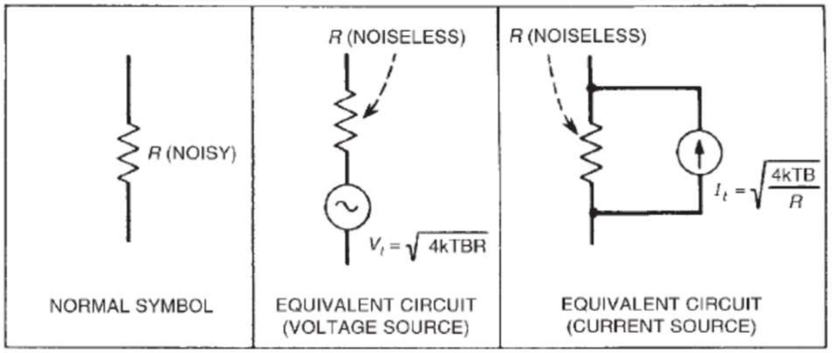
# Intrinsic Noise Sources

- Three intrinsic noise sources:
  - Thermal Noise
  - Shot Noise
  - Contact Noise

**(1) Thermal Noise**:



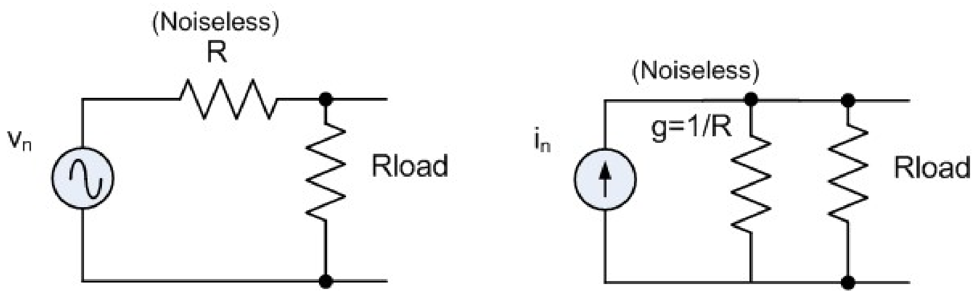

- Due to thermal agitation of electrons within a resistor
- Also called Johnson noise
- Open-circuit RMS noise voltage produced is $V_t = \sqrt{4kTBR}$
- k: Boltzmann's constant (1.38 x $10^{-23}$ J/K), T: temperature in kelvins, B: bandwidth in Hz and R: is resistance in ohms
- To reduce thermal noise, minimize resistance and bandwidth of the system
- The RMS thermal noise current is

$$I_t = \sqrt{\frac{4kTB}{R}}$$

- Electric circuit elements can produce thermal noise only if they dissipate energy (i.e. reactances do not produce thermal noise).

- The open circuit noise voltage variance (mean square value) due to thermal fluctuations is well approximated by:

$$v_n^2 = 4 \underbrace{k}_{1.38x10^{-23} J/K} \underbrace{T}_{Temperature(Kelvins)} R \underbrace{B}_{Bandwidth}$$

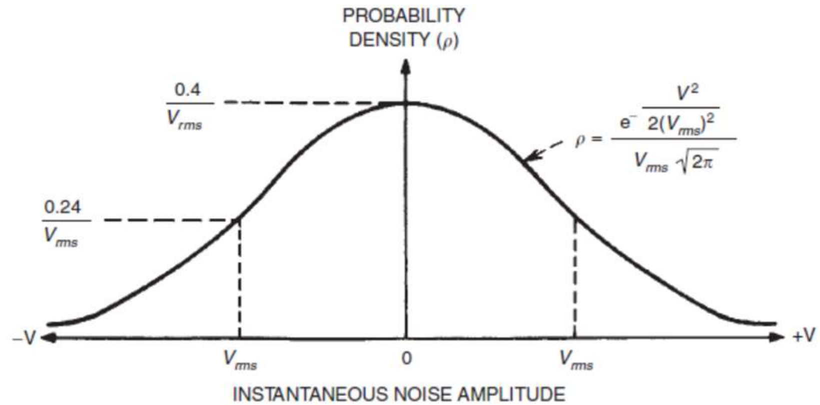



- Maximum power transfer occurs when R=Rload. Then, output power is

$$\underbrace{P_o}_{available \text{ noise power}} = \frac{v_o^2}{R} = \frac{(v_o/2)^2}{R} = \frac{v_n^2}{4R} = kTB \rightarrow \text{ independent of load resistance!}$$

- Note that the thermal noise generated by any arbitrary connection of passive elements is equal to the thermal noise generated by a resistance equal to the real part of the equivalent network impedance.

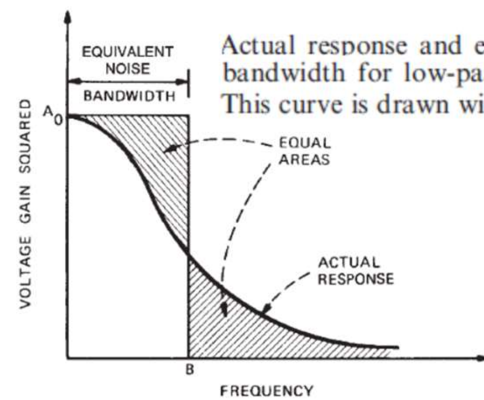
- The **instantaneous amplitude** of thermal noise has a Gaussian, or normal, distribution, with zero mean and RMS value as indicated before
- The probability of having an instantaneous voltage between any two values is equal to the integration of the probability density function (pdf) between them
- The crest factor of a waveform is defined as the ratio between the peak to the RMS value.



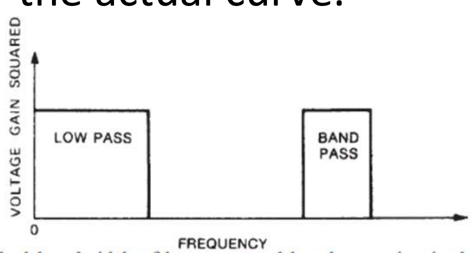

**Crest Factors for Thermal Noise**

| Percent of Time Peak Exceeded | Crest Factor (peak/rms) |
|---|---|
| 1.0 | 2.6 |
| 0.1 | 3.3 |
| 0.01 | 3.9 |
| 0.001 | 4.4 |
| 0.0001 | 4.9 |

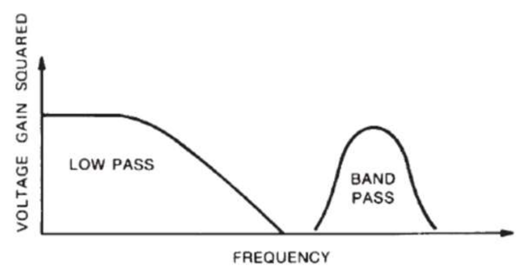
- The **equivalent noise bandwidth** (B) is the voltage-gain-squared bandwidth of the system or circuit being considered.
- For thermal (white) noise, the condition of equal noise power within the specified bandwidth is satisfied if the area under the equivalent noise bandwidth curve is made equal to the area under the actual curve.




Actual response and equivalent noise bandwidth for low-pass circuit. This curve is drawn with a linear scale.



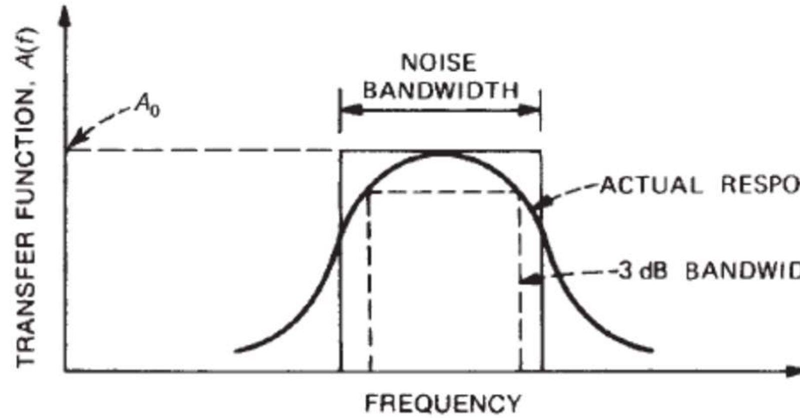

Ideal bandwidth of low-pass and band-pass circuit elements.




Actual bandwidth of low-pass and band-pass circuit elements.

- The equivalent noise bandwidth is calculated using the transfer function of the network A(f), as

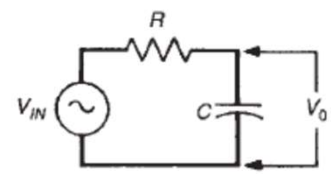
$$B = \frac{1}{|A_0|^2} \int_0^{\infty} |A(f)|^2 \, df$$



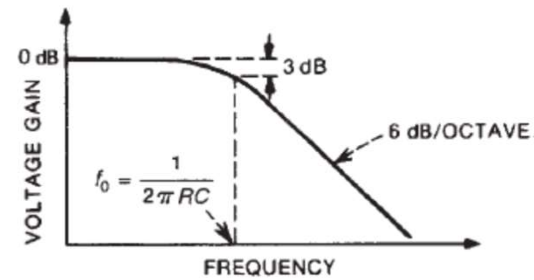

- **Ex. 5.2**

Calculate the equivalent noise bandwidth for the simple R-C circuit shown with a voltage gain given by

$$A(f) = \frac{f_0}{jf + f_0}$$




SCHEMATIC




TRANSFER FUNCTION

<u>**Sol.**</u>

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**Ratio of the Noise Bandwidth B to the 3-db Bandwidth $f_0$**

| Number of Poles | $B/f_0$ | High-Frequency Rolloff (dB per octave) |
|---|---|---|
| 1 | 1.57 | 6 |
| 2 | 1.22 | 12 |
| 3 | 1.15 | 18 |
| 4 | 1.13 | 24 |
| 5 | 1.11 | 30 |

- As the number of poles exceeds 3, we can approximate the effective noise BW as the 3dB BW.

(2) **Shot Noise**:

- associated with current flow across a potential barrier (in semiconductors)
- Caused by fluctuations of current around an average value that results from the random emission of electrons (or holes)
- RMS noise current is,

$$I_{sh} = \sqrt{2qI_{dc}B}$$

$$or$$

$$\frac{I_{sh}}{\sqrt{B}} = \sqrt{2qI_{dc}} = 5.66\times10^{-10}\sqrt{I_{dc}}$$

(3) **Contact Noise**:

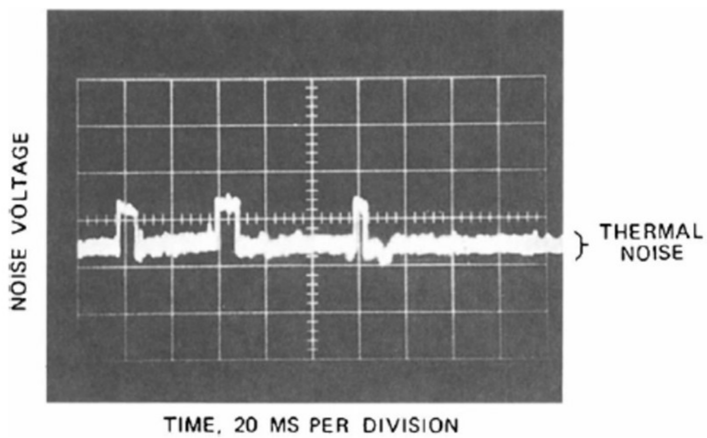
- Caused by fluctuating conductivity from an imperfect contact between two materials. When two conductors are joined together, as in switches and relay contacts
- Also referred to as “excess noise”, “flicker noise”, “pink noise”, “1/f noise” or “low frequency noise”
- Directly proportional to the value of the direct current flowing, and given by
- Most important noise source at low-frequencies

$$\frac{I_f}{\sqrt{\underbrace{B}_{BW \text{ centered at (f)}}}} \approx \frac{\overbrace{K}^{material}\ I_{dc}}{\sqrt{f}}$$

- Pink noise is bandlimited white noise. Its characteristics are similar to that of white noise that has been passed through a filter with a 3 dB per octave rolloff. Pink noise has equal noise power per octave, while white noise will have 3dB rise per octave
- For example, between 2 KHz and 4 KHz (1 octave), pink noise will be similar to that within 20 KHz and 40 KHz (1 octave). But, white noise would have 3dB increase per octave noise power in 20-40 KHz as compared to 2-4 KHz.

(4) **Popcorn Noise**:

- also called burst noise, might appear in defected ICs
- Has $1/f^n$ characteristic, typical n is 2



# Addition of noise voltages

- Noise sources are not correlated, the total sum is the sum of the individual powers

$$V_{total}^2 = V_1^2 + V_2^2 + V_3^2$$

$$V_{total} = \sqrt{V_1^2 + V_2^2 + V_3^2}$$

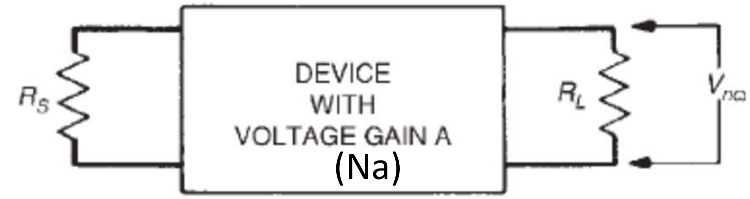
- Noise measurements are usually made at the output terminal of a circuit because:
  (1) The output noise is larger and easier to measure
  (2) It avoids the possibility of the noise meter upsetting the shielding, grounding or balancing of the input of the circuit
- Noise meters should
  (1) Respond to noise power
  (2) Should have a crest factor of 4 or more
  (3) Its bandwidth should be at least 10 times the noise bandwidth of the circuit being measured

**Charateristics of Meters Used to Measure White Noise**

**(voltmeters)**

| Type of Meter | Correction Factor | Remarks |
|---|---|---|
| True rms | None | Meter bandwidth greater than ten times noise bandwidth and meter crest factor 3 or grater. |
| RMS calibrated average responding | Multiply reading by 1.13 or add 1.1 dB | Meter bandwidth grater than ten times noise bandwidth, and meter crest factor 3 or grater. Read below one-half sclae to avoid clipping peaks. |
| RMS calibrated peak responding | Do not use | |
| Oscilloscope | RMS ≈ 1/8 peak-to-peak value | Waveshape can be observed to be sure it is random noise and not pickup. Ignore occasional, emtreme peaks. |

# Active Device Noise




- The noise factor (F) is a quantity that compares the noise performance of a device to that of an ideal device. Usually performed at 290K (room temperature).
- It can be written as: $$F = \frac{\text{Noise power output of actual device } (\text{P}_{\text{no}})}{\text{Power output due to source noise}}$$
- An equivalent definition of F is the input to output signal-to-noise ratio (SNR). SNR ratios should be power ratios unless Rs=RL, then we can express as voltage squared or current squared or power ratios. $$F = \frac{SNR_i}{SNR_o} = \frac{S_i / N_i}{S_o / N_o} = \frac{S_i / N_i}{AS_i / N_o} = \frac{AN_i + N_a}{AN_i} = 1 + \frac{N_a}{AN_i}$$
- All noise factor measurements must be taken with a resistive source. Thus, the input noise voltage is: $$V_t = \sqrt{4kTBR_s}$$
- The output noise due to the input only, is $AV_t$, while total output noise is $V_{no}$

$$F = \frac{(V_{no})^2 / R_L}{(AV_t)^2 / R_L} = \frac{(V_{no})^2}{(AV_t)^2}$$

- From the last equation, we observe that

$$F = \frac{(V_{no})^2}{(AV_t)^2} = \frac{(V_{no})^2}{4kTBR_s A^2}$$

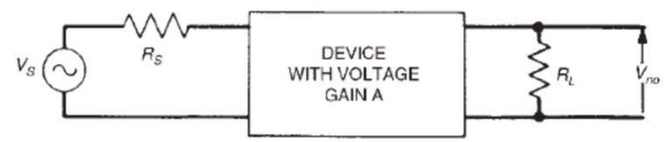
  (1) F is independent of RL
  (2) F is dependent on Rs
  (3) If device is noiseless, F=1

- When expressed in decibels, noise factor is called Noise Figure (NF)

$$\boxed{NF = 10\log F}$$

- The concept of noise factor has three limitations:
  (1) Increasing Rs will decrease F while increasing the total noise in the circuit
  (2) If reactive sources are used, F is meaningless because source noise is zero
  (3) F can yield inaccurate values if the device noise is a small portion of the source thermal noise (i.e low noise FETs)

- Measurement of the noise factor
  (1) Single Frequency method



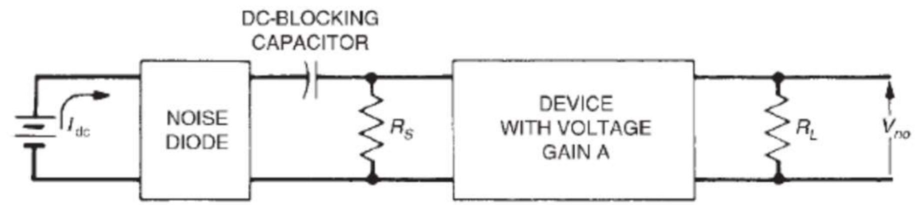

  - A single tone source (oscillator) is used as shown in figure.
  - First, source is OFF, and output RMS voltage Vno is measured. $V_{no} = \sqrt{(AV_t)^2 + (device - noise)^2}$
  - Then, source is ON, until output power doubles
  - Need to know noise BW of device for this method

$$(AV_s)^2 + (V_{no})^2 = 2(V_{no})^2$$

$$\rightarrow AV_s = V_{no}$$

$$and \quad F = \left(\frac{V_s}{V_t}\right)^2$$

(2) Noise Diode method




- Diode as a white noise source
- Shot noise in the Diode is: $I_{sh} = \sqrt{3.2\times10^{-19} I_{dc} B}$
- Using Thevenin's theorem, Ish parallel with Rs becomes Vsh in series with Rs

$$V_{sh} = I_{sh} R_s$$

- First, Diode current is zero, and measure output noise Vno

$$V_{no} = \sqrt{(AV_t)^2 + (device - noise)^2}$$

- Diode current is on and increased until output noise power doubles, then

$$(AV_{sh})^2 + (V_{no})^2 = 2(V_{no})^2$$
$$\rightarrow AV_{sh} = V_{no} = AI_{sh}R_s$$
$$and \quad F = \left(\frac{I_{sh}R_s}{V_t}\right)^2 = 20 I_{dc} R_s$$

- Note that this method only depends on the current (DC) and the source resistance. Neither the bandwidth nor the gain needs to be known.
- This is a easy method to use

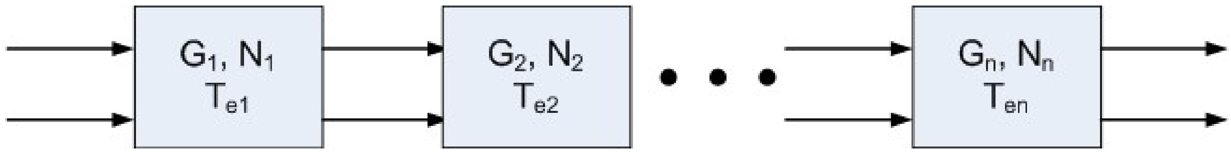
# Calculating SNR and Input noise voltage from F

- We have seen that $\boxed{F = \frac{(V_{no})^2}{(AV_t)^2} = \frac{(V_{no})^2}{4kTBR_s A^2}}$
- Rearranging, we get $\boxed{V_{no} = A\sqrt{4kTBR_s F}}$
- We can write, $\frac{S_o}{N_o} = \frac{P_{signal}}{P_{noise}} = \left(\frac{AV_s}{V_{no}}\right)^2 = \frac{(V_s)^2}{4kTBR_s F}$
- We can define the total equivalent noise voltage as $\boxed{V_{nt} = \frac{V_{no}}{A} = \sqrt{4kTBR_s F}}$
- For optimum noise performance, we want to minimize Vnt. Minimizing Vnt is equivalent to maximizing the SNR given that the signal is constant
- Let the device noise be Vnd, then $V_{nt} = \sqrt{(V_t)^2 + (V_{nd})^2}$
- and $V_{nd} = \sqrt{(V_{nt})^2 - (V_t)^2} = \sqrt{4kTBR_s (F-1)}$

# Noise Voltage and Current Models

- A better approach to represent the noise characteristics of devices and circuits
- Overcomes the limitations of the noise factor approach
- **We will not dig into it … Check the references (the book by Ott) for more information**

# Noise factor of cascaded stages




- For a cascaded network, the noise factor can be found using:

$$F = 1 + (F_1 - 1) + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \cdots + \frac{(F_n - 1)}{G_1 G_2 \cdots G_{n-1}}$$

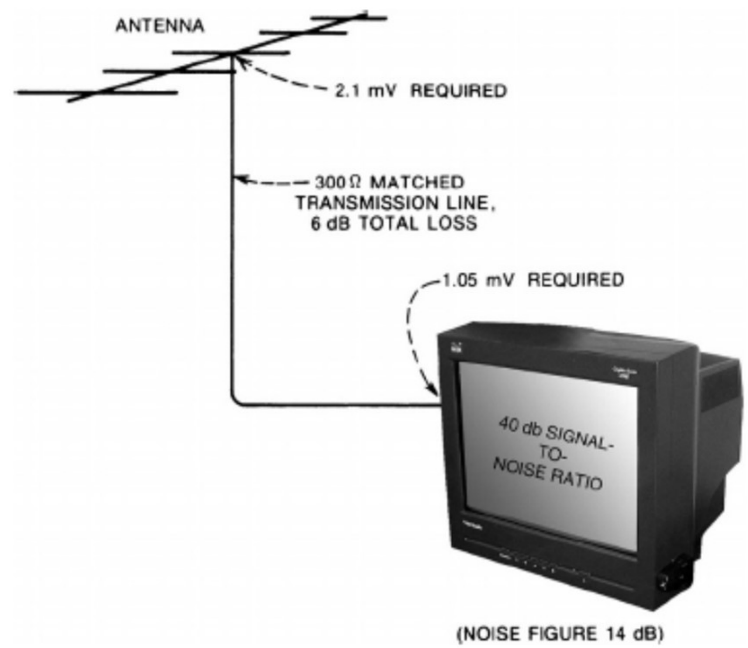
- The noise temperature can be described by the effective input temperature of the 2-port network (used in space applications, or extra-terrestrial links). Te is the equivalent temperature of a source impedance into a perfect (noise-free) device that would produce the same added noise:

$$T_e = \underbrace{T_0}_{290K} (F - 1), \qquad T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots + \frac{T_{en}}{G_1 G_2 \cdots G_{n-1}}$$

$$F = 1 + \frac{N_a / G_a}{N_{in}} = 1 + \frac{k T_e B}{k T_0 B}$$

# Ex. 5.3

The figure shows an antenna connected to a TV set by a section of 300 $\Omega$ matched transmission line. If the transmission line has 6 dB of insertion loss, and the TV set has a noise figure of 14dB, what signal voltage is required at the antenna terminal for a 40 dB SNR at the terminals of the TV set? Consider a 4MHz bandwidth.




**Sol.**

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# Next Time

- Cabling …