Trigonometric relations

$\cos(\alpha \pm \beta) = \cos(\alpha) \cdot \cos(\beta) \mp \sin(\alpha) \cdot \sin(\beta)$
$\sin(\alpha \pm \beta) = \sin(\alpha) \cdot \cos(\beta) \pm \cos(\alpha) \cdot \sin(\beta)$
$\sin(\alpha) \cdot \sin(\beta) = \frac{1}{2} \cdot [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos(\alpha) \cdot \cos(\beta) = \frac{1}{2} \cdot [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin(\alpha) \cdot \cos(\beta) = \frac{1}{2} \cdot [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$
$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$
$\sin(2\alpha) = 2 \cdot \sin(\alpha) \cdot \cos(\alpha)$
$\sin^2(\alpha) = \frac{1}{2} \cdot (1 - \cos(2\alpha))$
$\cos^2(\alpha) = \frac{1}{2} \cdot (1 + \cos(2\alpha))$

Normal probability density function	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Fourier transform pairs

$\mathbf{x(t)}$	$\mathbf{X}(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos(\omega_0 t)$	$\pi\,[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin(\omega_0 t)$	$\frac{\pi}{j}\,[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
1	$2\pi\delta(\omega)$
$\prod\left(\frac{t}{2T_1}\right) = \begin{cases} 1 & \lvert t\rvert < T_1 \\ 0 & \lvert t\rvert > T_1 \end{cases}$	$2T_1 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right) = \frac{2\sin(\omega T_1)}{\omega}$
$\frac{W}{\pi} \operatorname{sinc}\left(\frac{Wt}{\pi}\right) = \frac{\sin(Wt)}{\pi t}$	$X(\omega) = \begin{cases} 1 & \lvert\omega\rvert < W \\ 0 & \lvert\omega\rvert > W \end{cases}$
$\delta(t)$	1
$\delta(t - t_0)$	$e^{-j\omega t_0}$

Properties of the Fourier transform

Signal	**Fourier transform**
$x(t)$	$X(\omega)$
$y(t)$	$Y(\omega)$
$ax(t) + by(t)$	$aX(\omega) + bY(\omega)$
$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
$e^{-j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{A}\right)$
$x(t) * y(t))$	$X(\omega) \cdot Y(\omega)$
$x(t) \cdot y(t))$	$\frac{1}{2\pi} X(\omega) * Y(\omega)$

LINEAR MODULATIONS

	AM	**DSB**
$x_c(t)$	$A_c \cdot [1 + m x_n(t)] \cdot \cos(\omega_c t)$	$A_c \cdot x(t) \cdot \cos(\omega_c t)$
P_m	$\frac{A_c^2}{2} + \frac{m^2 A_c^2}{2} S_{xn} = P_c + 2P_{BL}$	$\frac{A_c^2}{2} S_x = 2P_{BL}$
PEP	$\frac{1}{2} A_c^2 (1+m)^2$	$\frac{1}{2} [A_c \cdot \lvert x(t) \rvert_{max}]^2$
B_T	$2 \cdot W_x$	

ANGLE MODULATIONS

	FM Modulation
Signal	$x(t) = A_p \cdot \cos\left(\omega_p t + \omega_d \int^t x(\lambda) d\lambda\right)$
Max. phase deviation	$D = \frac{\omega_d \lvert x(t) \rvert_{max}}{W_x}$
Bandwidth	$B_T \approx 2(D+a)W_x$ $a = \begin{cases} 2 & 2 \leq D \leq 10 \\ 1 & c.c. \end{cases}$

NOISE IN LINEAR MODULATIONS

Noise in linear modulations		**Noise after demodulation (Synchronous det.)**	
Baseband signal		AM Modulation	DSB Modulation
$\gamma = \frac{P_R}{N_0 W_x}$	$\left(\frac{S}{N}\right)_R = \frac{W_x}{B_T} \cdot \gamma$	$\left(\frac{S}{N}\right)_D = \frac{m^2 S_{xn}}{1 + m^2 S_{xn}} \cdot \gamma$	$\left(\frac{S}{N}\right)_D = \gamma$

Noise in AM (envelope detector)

$\left(\frac{S}{N}\right)_D = \frac{m^2 S_{xn}}{1 + m^2 S_{xn}} \gamma$ if $\left(\frac{S}{N}\right)_R \geq \left(\frac{S}{N}\right)_{RTh}$	No signal if $\left(\frac{S}{N}\right)_R < \left(\frac{S}{N}\right)_{RTh}$

Noise in angle modulations

	FM	**Deemphasis FM** ($B_{de} \ll W_x$)
$\left(\frac{S}{N}\right)_D$	$3D^2 S_{xn} \gamma$	$\left(\frac{\omega_d}{B_{de}}\right)^2 S_x \gamma$

One-dimensional M-ary system	**Union bound**	**Simplified union bound**
$P_e = \frac{2(M-1)}{M} Q\left(\frac{d}{\sqrt{2N_0}}\right)$	$P_e \leq \frac{1}{M} \sum_{i=1}^{M} \sum_{\substack{k=1 \\ k \neq i}}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right)$	$P_e \leq (M-1) \cdot Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

Pass-band modulations

PSK	$M=2$	$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
	$M \geq 4$	$P_e = 2\cdot Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$
FSK	$M=2$	$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
	$M \geq 4$	$P_e = (M-1)\cdot Q\left(\sqrt{\frac{E_s}{N_0}}\right)$
QAM	$log_2(M)$ par	$P_e = 4\left(1-\frac{1}{\sqrt{M}}\right)\cdot Q\left(\sqrt{\frac{3\cdot E_s}{(M-1)\cdot N_0}}\right)$

	Gray coding (ASK, QAM, PSK)	FSK
Bit error probability	$P_b = \frac{1}{log_2(M)}P_e$	$P_b = \frac{2^{log_2(M)-1}}{2^{log_2(M)}-1}P_e$

Raised cosine filter, $0 \leq \alpha \leq 1$

$$H(\omega) = \begin{cases} 1 & |\omega| \leq \pi\frac{1-\alpha}{T} \\ \frac{1}{2}\left[1+\cos\left(\frac{T}{2\alpha}\cdot\left(|\omega|-\pi\frac{1-\alpha}{T}\right)\right)\right] & \pi\frac{1-\alpha}{T} \leq |\omega| \leq \pi\frac{1+\alpha}{T} \\ 0 & c.c. \end{cases}$$

PAM power spectral density (baseband)

$x(t) = \sum_{n=-\infty}^{\infty} a_n h(t-nT)$	$S_x(\omega) = \frac{1}{T}\lvert H(\omega)\rvert^2 S_a(\omega)$	$S_a(\omega) = \sum_{m=-\infty}^{\infty} R_a[m]\cdot e^{-j\omega mT}$

Bandwidth (Hz) for pass-band modulations

Modulation	Nominal values*	Optimal values**
	B	B
M-PSK and M-QAM	$\frac{2R_b}{log_2(M)} = \frac{2}{T}$	$\frac{R_b}{log_2(M)} = \frac{1}{T}$
M-FSK	$\frac{(M+3)R_b}{2\cdot log_2(M)} = \frac{(M+3)}{2\cdot T}$	$\frac{(M+1)R_b}{2\cdot log_2(M)} = \frac{(M+1)}{2\cdot T}$
	$^{*} \Rightarrow h(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t < T \\ 0 & \text{c.c.} \end{cases}$	$^{**} \Rightarrow h(t) = \frac{\sin\left(\frac{\pi}{T}t\right)}{\frac{\pi}{T}t}$

Q(x) values

x	Q(x)	x	Q(x)	x	Q(x)	x	Q(x)
0,0	5,000000e-01	1,8	3,593032e-02	3,6	1,591086e-04	5,4	3,332043e-08
0,1	4,601722e-01	1,9	2,871656e-02	3,7	1,077997e-04	5,5	1,898956e-08
0,2	4,207403e-01	2,0	2,275013e-02	3,8	7,234806e-05	5,6	1,071760e-08
0,3	3,820886e-01	2,1	1,786442e-02	3,9	4,809633e-05	5,7	5,990378e-09
0,4	3,445783e-01	2,2	1,390345e-02	4,0	3,167124e-05	5,8	3,315742e-09
0,5	3,085375e-01	2,3	1,072411e-02	4,1	2,065752e-05	5,9	1,817507e-09
0,6	2,742531e-01	2,4	8,197534e-03	4,2	1,334576e-05	6,0	9,865876e-10
0,7	2,419637e-01	2,5	6,209665e-03	4,3	8,539898e-06	6,1	5,303426e-10
0,8	2,118554e-01	2,6	4,661189e-03	4,4	5,412542e-06	6,2	2,823161e-10
0,9	1,840601e-01	2,7	3,466973e-03	4,5	3,397673e-06	6,3	1,488226e-10
1,0	1,586553e-01	2,8	2,555131e-03	4,6	2,112456e-06	6,4	7,768843e-11
1,1	1,356661e-01	2,9	1,865812e-03	4,7	1,300809e-06	6,5	4,016001e-11
1,2	1,150697e-01	3,0	1,349898e-03	4,8	7,933274e-07	6,6	2,055790e-11
1,3	9,680049e-02	3,1	9,676035e-04	4,9	4,791830e-07	6,7	1,042099e-11
1,4	8,075666e-02	3,2	6,871378e-04	5,0	2,866516e-07	6,8	5,230951e-12
1,5	6,680720e-02	3,3	4,834242e-04	5,1	1,698268e-07	6,9	2,600125e-12
1,6	5,479929e-02	3,4	3,369291e-04	5,2	9,964437e-08	7,0	1,279813e-12
1,7	4,456546e-02	3,5	2,326291e-04	5,3	5,790128e-08		