Exercises. Lesson 5
Baseband digital transmission

Problem 5.1

A computer generates 16-bits binary words, at 20000 words per second.

a. Calculate the bandwidth necessary for transmitting the output as a PAM binary signal.

b. Calculate M so the output can be transmitted as an M-ary signal in a channel with $B = 60kHz$.

Result:

1. $B_T \geq 160kHz$
2. $M = 8$

Problem 5.2

Consider a binary sequence a b_n from which we form the symbols $a_n = b_n + b_{n-1}$. Coefficients b_n stand for uncorrelated random variables, which take values $+1$ and -1, with zero mean and variance 1.
Calculate the power spectral density of the transmitted signal.

Result:

1. $S_x(\omega) = \frac{4}{T}|H(\omega)|^2 cos^2\left(\omega\frac{T}{2}\right)$

Problem 5.3

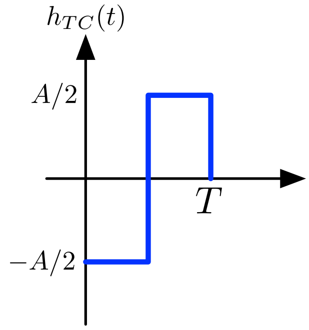
Calculate the $\left(\frac{S}{N}\right)_R$ such that a binary unipolar system with white Gaussian noise has a $P_e = 0.001$.
What is the error probability of a polar system with the same $\left(\frac{S}{N}\right)_R$?

Result:

1. $\rho = \frac{E_s}{N_0/2} = 19,22$
   $P_e = Q(4,38) < 8.54 \cdot 10^{-6}$

Problem 5.4

Consider the signal $h_{TC}(t)$ in the figure:

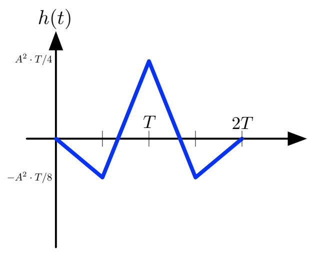


a. Determine the impulse response of the matched filter to that signal, and draw it in function of the time.

b. Draw the waveform of the global impulse response $h(t)$.

RESULT:

1. $h_R(t) = h_{TC}(T - t)$
2.

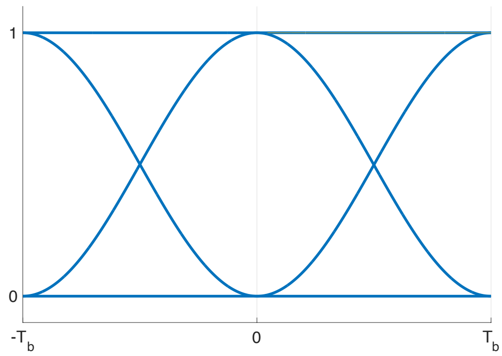


Problem 5.5

The following PAM signal is received: $r(t) = \sum_{n=-\infty}^{\infty} b_n h(t - nT)$.
Draw and determine its eye pattern, without distortion, considering the following unipolar data sequence: 1011100010.

$$h(t) = cos^2\left(\frac{2\pi}{4T_b}t\right)\prod\left(\frac{t}{2T_b}\right)$$

Result:



Problem 5.6

A computer generates pulses with $R_b = 1 Mbps$, for their transmission through a noisy channel, with power spectral density $N_0/2 = 2 \cdot 10^{-20} W/Hz$. The error rate cannot be over 1 bit per hour.

a. Determine the noise power of the channel and the error probability of the system.

b. Suppose that the transmitter includes now a transfer characteristic using a raised cosine spectrum, with an excess of bandwidth of 75%, and the systems is PAM NRZ quaternary. Calculate the new bandwidth necessary for the transmission.

RESULT:

1. $P_N = 2 \cdot 10^{-14} W$, $P_b = 2.7 \cdot 10^{-10}$
2. $B = 437.5 kHz$

Problem 5.7

A base-band transmission system receives the following signal:

$$r(t) = \sum_{n=-\infty}^{\infty} a_n \cdot h(t - nT)$$

where T stands for the symbol interval, an is a polar sequence equiprobable and uncorrelated, with values $-A$ or $+A$, and $h_T(t)$ corresponds to a pulse form in raised cosine, with roll-off factor α.

a. Calculate the power spectral density of the signal.

b. Obtain the bandwidth.

RESULT:

1. $S_r(\omega) = \frac{1}{T} \cdot |H(\omega)|^2 \cdot A^2$ with $H(\omega)$ the frequency response of the raised cosine filter.
2. $B = \pi \frac{1+\alpha}{T}$

Problem 5.8

Consider a binary sequence b_n, from which we form the symbols $a_n = b_n - b_{n-1}$, b_n are uncorrelated and equiprobable random variables, taking values 1 and 0.
Calculate the power spectral density of the transmitted signal, in the case that the transmitter filter is:

$$h_T(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t < T \\ 0 & c.c. \end{cases}$$

RESULT:

$S_x(\omega) = \frac{4}{T^2\omega^2} sen^4\left(\frac{\omega T}{2}\right)$

Problem 5.9

A communication system transmits the following signal:

$$s(t) = \sum_{n=-\infty}^{\infty} b_n \cdot h_T(t - nT)$$

where b_n represents a sequence of discrete random variables, independent and identically distributed, which take the values ± 1 with the same probability. The transmitted pulse is $h(t) = \frac{1}{\sqrt{T}} \cdot \prod \left(\frac{t-T/2}{T} \right)$, the channel impulse response is $h_c(t) = \delta(t)$ and the receiver filter $h_R(t)$ is matched to $h_T(t)$. Se pide:
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a. Determine the wave form of the global impulse response $h(t)$.

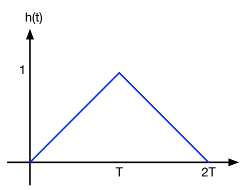
b. Draw the eye diagram of the receiver filter output, before sampling.

c. Repeat a) for a channel with impulse response $h_c(t) = \delta(t) - 0.5 \cdot \delta(t - T)$.

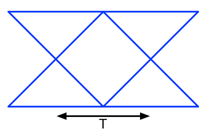
d. Calculate the values of the samples at the receiver filter output, and the value of the inter-symbol interference in each of them.

Note: $\prod(t) = \begin{cases} 1 & -0.5 \leq t < 0.5 \\ 0 & c.c. \end{cases}$

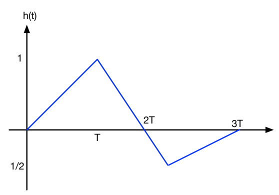
Result:



Apartado (a)



Apartado (b)



Apartado (c)