Lesson 3
Effects of noise in analog communications

1 Key concepts

Before doing these exercises it is important to review and understand the following concepts:

- General diagram of an analog communication system with noise.
- Concept of signal to noise ratio. Difference between pre-detection and post-detection.
- Concept of threshold effect, and when it is necessary to take it into account.

2 Basic problems

This first part includes problems mostly extracted from the bibliography aimed at practicing basic calculations needed for the rest of the lesson.

Problem 3.1

[Carlson2010] A DSB signal plus noise is demodulated by synchronous detection. Find $(S/N)_D$ in dB given that the received power is $20\mathrm{nW}$, the message has a bandwidth of $5\mathrm{MHz}$ and the channel introduces a noise with $N_0 = 4 \cdot 10^{-20}$ W/Hz.

Results for problem

$\left(\frac{S}{N}\right)_D = 50 \text{ dB}$

Problem 3.2

[Carlson2010] A DSB signal plus noise is demodulated by a product detector with phase error ϕ. Take the local oscillator signal to be $2\cos(\omega_c t + \phi)$ and show that $(S/N)_D = \gamma \cos^2(\phi)$.

Results for problem

Problem 3.3

[Carlson2010] An AM system with envelope detection is operating at the threshold point. Find the power gain in dB needed at the transmitter to get up the $(S/N)_D$ to $40\mathrm{dB}$ with full-load tone modulation.

Results for problem

$\mathrm{G} = 32\mathrm{dB}$

Problem 3.4

[Carlson2010] An FM signal plus noise has a received power of 1nW, $W_x = 500\text{kHz}$, $S_{xn} = 0.1$, $|x(t)|_{max} = 4\text{V}$, $\omega_d = 500\text{kHz/V}$ and $N_0 = 4 \cdot 10^{-20}\text{W/Hz}$. Find $(S/N)_D$ in dB for FM detection and for deemphasized FM detection with $B_{de} = 5\text{kHz}$.

Results for problem

- FM: $(S/N)_D = 53.8dB$
- deemphasized FM: $(S/N)_D = 89dB$

Problem 3.5

[Carlson2010] An analog communication system has $S_x = 1/2$, $W_x = 10\text{kHz}$, $N_0 = 10^{-15}\text{W/Hz}$, and transmission loss 100dB. Calculate the transmitted power needed to get $(S/N)_D = 40\text{dB}$ when the modulation is:

a) AM with $m = 1$

b) AM with $M = 0.5$

c) FM with $D = 1$

d) FM with $D = 5$

e) FM with $D = 10$

Results for problem

a) $P_T = 3\text{kW}$
b) $P_T = 9\text{kW}$
c) $P_T = 667\text{W}$
d) $P_T = 26.7\text{W}$
e) $P_T = 24\text{W}$

3 Aditional problems

These problems are slightly more ellaborated than the previous ones, in many cases extracted from old exams.

Problem 3.6

A transmitter can perform either DSB or AM modulations. The post-detection signal-to-noise ratio has to be at least $20dB$ in order to meet the required quality specifications.
When DSB modulation is used, we get a maximum range of $15km$ with a transmission power of $1W$. The link attenuation (in dB) is proportional to the distance between transmitter and receiver.
Answer:

a) Calculate the power of the signal at the output on the transmitter when DSB modulation is used, and the desired range is $20km$.

b) Repeat a. when AM at 80% is used.

c) Calculate the envelope peak power values at the output of the transmitter under the conditions mentioned in a. and b., respectively.

Data:

- $\frac{N_0}{2} = 2 \cdot 10^{-9} W/Hz$
- $W_x = 2\pi \cdot 5krad/s$
- $|x(t)|_{max} = 1V$
- $S_x = 0.5W$
- $\alpha_t[dB] = A \cdot d[km]$, with A a constant.

Results for problem

a) $S_{T_{DBL}} = 7.96W$
b) $S_{T_{AM}} = 32.69W$
c) $PEP_{DBL} = 15.92W$, $PEP_{AM} = 80.25W$

Problem 3.7

A normalized signal with $5kHz$ bandwidth and average power $0.5W$ is transmitted by an AM system. The transmitter power is $660W$. At the receiver side, located $40km$ away, we get a post-detection signal-to-noise ratio of $33dB$.
We know that the noise power spectral density is $N_0/2 = 5 \cdot 10^{-13} W/Hz$, and that the propagation attenation is given by $A[dB] = 40 + 20 \cdot log(d[km])$. Answer:

a) Calculate the modulation index m.

b) Calculate the amount of power needed to transmit the carrier, and the amount of power devoted to the transmission of each sideband.

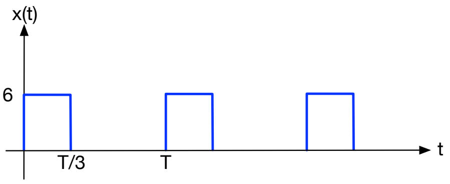
Results for problem

a) $m = 0.8$

b) $P_p = 500W$, $P_{BL} = 80W$

Problem 3.8

The signal $x(t)$ in the figure goes through a lowpass filter with bandwidth $10kHz$. We assume that the signal power is not affected, and that the DC component is removed.

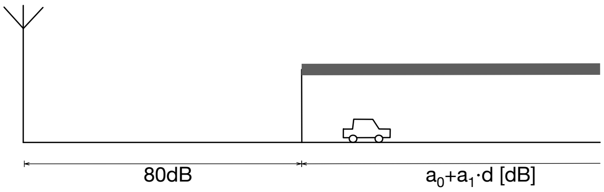


The resulting signal AM modulates a $125V$ carrier, with a modulation index of 0.7484. This signal is transmitted by a radio station. The transmission channel introduces 80 dB attenuation, and Gaussian white noise whose power spectral density is $N_0/2 = 10^{-11}W/Hz$.
A vehicle is tuned to this radio station, and it enters a tunnel where the propagation attenuation is no longer constant and varies according to:

$$A[dB] = a_0 + a_1 \cdot d$$

where:

- $a_0 = 10dB$
- $a_1 = \frac{1}{3}dB/m$
- d: Distance (m)



a) Calculate the average power of the modulated signal.

b) Calculate the demodulation signal-to-noise ratio at the tunnel entrance.

c) Calculate the distance where the signal will no longer be audible, if the demodulation threshold is $10dB$.

d) Repeat c. when the demodulator employed is an envelope detector, assuming that the pre-detection threshold is $13dB$.

Results for problem

a) $S_T = 10kW$
b) $\left(\frac{S}{N}\right)_D = 20.38dB$

c) $d = 11.94m$
d) $d' = 2.94m$

Problem 3.9

A communication system is such that the signal arriving at the receiver side has a power $100dB$ below the emitted power at the transmitter side, and that the noise power spectral density is $N_0/2 = 5 \cdot 10^{-15} W/Hz$. A post-detection signal-to-noise ratio of $40dB$ is required for a correct reception.
The maximum voltage value of the message is $2V$, its mean power is $1W$, and its bandwidth, $10kHz$. A pre-detection threshold signal-to-noise ratio of $10dB$ is required. Calculate the minimum transmitted power when using each one of the following modulation systems:

a) DSB modulation.

b) AM modulation with $m = 0.5$ and envelope detection.

c) FM modulation without de-emphasis ($f_d = 10^5 Hz/V$).

Results for problem

a) $S_T = 10kW$
b) $\gamma > \gamma_{th} \Rightarrow S_T = 170kW$
c) $\gamma < \gamma_{th} \Rightarrow S_T = S_{T_{th}} = 420W$

Problem 3.10

We want to transmit a signal with bandwidth $10kHz$ and normalized average power 0.5, by using an FM communication system. The post-detection signal-to-noise ratio has to be greater than $40dB$, but ensuring a minimum amount of output power consumption. The channel has $120kHz$ bandwidth and introduces white noise with power spectral density $N_0/2 = 0.5 \cdot 10^{-8} W/Hz$. This channel is characterized by an attenuation of $40dB$. Indicate the amount of power needed to trasmit the signal.

Results for problem

- $S_T = 416.66W$

Problem 3.11

A radio transmission system uses a frequency modulator (FM) with modulator constant $f_d = 20kHz/V$, and carrier signal $c(t) = 5 \cdot cos(2\pi \cdot 5 \cdot 10^7 t)$. Answer:

a) Calculate the modulation index D and the approximate bandwidth when the modulating signal is $x_1(t) = cos(2\pi \cdot 500t)$.

b) Repeat a. when the modulating signal is $x_2(t) = cos(2\pi \cdot 10^4 t)$.

In order to improve the demodulation signal-to-noise ratio, a pre-emphasis and de-emphasis filters are introduced, with cut-off frequency $1430Hz$. The system is tested with the modulating signal $x(t) = x_1(t) + x_2(t)$.

c) Determine the improvement in demodulation signal-to-noise ratio.

Results for problem

a) $D_1 = 40$, $B_T = 41kHz$
b) $D_2 = 2$, $B_T = 80kHz$
c) $\Delta \left(\frac{S}{N}\right)_D = 12.14dB$

Problem 3.12

A signal with bandwidth $5kHz$, whose maximum voltage value is $2V$ and with average power $0.5W$ is transmitted using FM through an AWGN channel. The channel has a noise power spectral density of $N_0/2 = 3 \cdot 10^{-14} W/Hz$. The receiver power is $50nW$. The maximum deviation of frequency is $20kHz$. Calculate the post-detection signal-to-noise ratio.

Results for problem

- $\left(\frac{S}{N}\right)_D = 30dB$

Problem 3.13

A signal with bandwidth $10kHz$ and normalized average power 0.5 is transmitted using FM through an AWGN channel. The channel has a noise power spectral density of $N_0/2 = 5 \cdot 10^{-10} W/Hz$ and attenuates the signal $40dB$. In order to meet the required quality specifications, a threshold γ_{th} of 100 is required for a pre-detection threshold signal-to-noise ratio of $10dB$.

a) Calculate the minimum possible power transmitted to the channel (S_T).

b) Find the demodulation (post-detection) signal-to-noise ratio.

In order to improve the demodulation signal-to-noise ratio, pre-emphasis and de-emphasis filters are introduced. The $3dB$ cutoff frequency of the de-emphasis filter, $B_{de} = 1kHz$.

c) Determine the improvement in dB of the demodulation (post-detection) signal-to-noise ratio.

Results for problem

a) $S_{T_{min}} = 10dBW$
b) $\left(\frac{S}{N}\right)_D = 31.3dB$
c) $\Delta\left(\frac{S}{N}\right)_D = 15.23dB$

References

[Carlson2010] A. Bruce Carlson and Paul B. Crilly. Communication Systems: An Introduction to Signals and Noise in Electrical Communication, 5th Ed. McGraw-Hill, 2010.