

Lesson 2

Analog modulations

1 Key concepts

Before doing these exercises it is important to review and understand the following concepts:

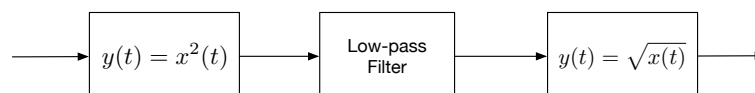
- Linear analog modulations: AM and DSB.
- Phasors. In-phase and quadrature components of a signal
- Detectors: synchronous and envelope detectors.
- Angular modulations. Bandwidth and interference. Preemphasis and deemphasis.
- Superheterodine receiver.

2 Basic problems

This first part includes problems mostly extracted from the bibliography aimed at practicing basic calculations needed for the rest of the lesson.

Problem 2.1

The AM signal $s(t) = A_p(1 + mx(t)) \cos(2\pi f_p t)$ is applied to the system shown in the figure.



Assuming that the message is normalized, its bandwidth is W , that $0 \leq m \leq 1$ and that $f_p > 2W$, show that the message can be obtained from the system output.

Results for problem

It's possible just selecting the cut-off frequency of the low-pass filter to be $2W$

Problem 2.2

A zero-mean periodic signal $x(t)$, with bandwidth $5kHz$, amplitude $4V$ and normalized average power 0.5 , DSB modulates a $1MHz$ carrier. The result is a signal with average power $400W$. Determine:

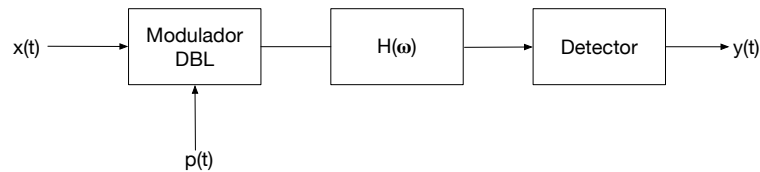
- Carrier amplitude.
- Average power of the lower sideband.
- Outline of the detector needed to recover the signal $x(t)$, and the value of its main parameters

Results for problem

- a) $A_c = 10V$
- b) $P_{LSB} = 200W$
- c) Synchronous detector $f_{LO} = 1MHz$, $A_{LO} = \frac{2}{A_c}$ and $f_{lp} = 5kHz$.

Problem 2.3

The signal $x(t) = \cos(2\pi \cdot 10 \cdot 10^3 t) + 4 \cdot \cos(2\pi \cdot 15 \cdot 10^3 t) + \cos(2\pi \cdot 20 \cdot 10^3 t)$ DSB modulates the carrier $p(t) = 2 \cdot \cos(2\pi \cdot 10^5 t)$ and passes through a filter with frequency response $H(\omega)$ before reaching the detector, as it can be observed in the figure



with

$$H(\omega) = \begin{cases} 0 & |\omega| < 200\pi krad/s \\ 1 & |\omega| \geq 200\pi krad/s \end{cases}$$

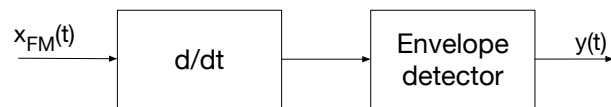
- a) Find the signal obtained at the output of the detector, when using an envelope detector of $K_D = 1$ with DC suppression.
- b) Determine the detector needed to obtain a detected signal equal to the modulating signal. Please specify all necessary parameters.

Results for problem

- a) $y(t) = 2 \cdot \cos(2\pi \cdot 5 \cdot 10^3 t)$
- b) Synchronous detector with $f_{LO} = 2 \cdot \cos(\omega_c \cdot t)$ and a low pass filter with cut-off frequency of at least $20kHz$.

Problem 2.4

Considering that $x_{FM}(t)$ is the signal obtained when FM modulating a signal $x(t)$ with the carrier $p(t) = A_p \cdot \cos(\omega_p t)$. Determine the condition needed to recover the signal $x(t)$ if the system outlined in the figure is used.



Results for problem

$$\omega_p - \omega_\Delta \cdot |x(t)|_{max} > 0$$

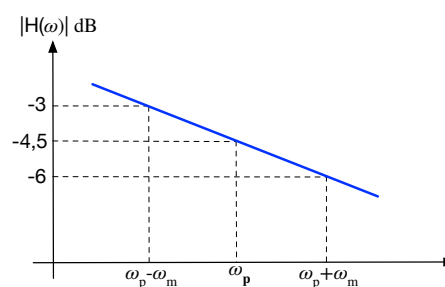
3 Additional problems

These problems are slightly more elaborated than the previous ones, in many cases extracted from old exams.

Problem 2.5

A transmitter has an average nominal power of $30W$ and a peak envelope power of $60W$. Determine:

- a) The power in a sideband when the signal $x(t) = \cos(\omega_m t)$ modulates the carrier given by $p(t) = A_p \cdot \cos(\omega_p t)$, and the value of A_p in the following cases:
 - (a) AM modulation when the modulation index is 80%.
 - (b) DSB modulation
- b) Considering the first case for a), and knowing that the channel presents a non-uniform attenuation as depicted in the figure: Obtain the signal detected in the following cases:



- (a) Envelope detector
- (b) Synchronous detector

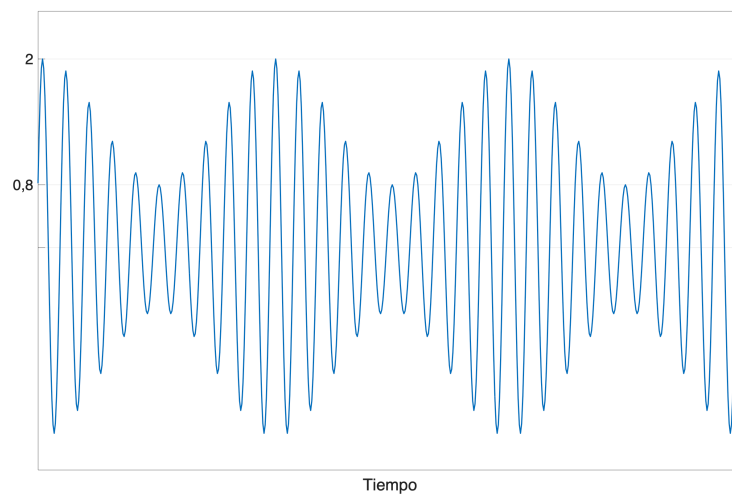
Note: Assume in both cases that a DC suppressor is present.

Results for problem

- a) (a) $A_p = 6.08V$, $P_{BL} = 2.96W$
- (b) $A_p = 10.95V$, $P_{BL} = 15W$
- b) (a) $y_D(t) = k_D \cdot [A(t) - \langle A(t) \rangle]$, con:
 - $A(t) = x_i(t) \cdot \left[1 + 0.5 \cdot \left(\frac{x_q(t)}{x_i(t)} \right)^2 \right]$
 - $x_i(t) = 0.6 \cdot A_p + 0.48 \cdot A_p \cdot \cos(\omega_m t)$
 - $x_q(t) = -0.08 \cdot A_p \cdot \sin(\omega_m t)$
- (b) $y_D(t) = \frac{A_{OL}}{2} \cdot A_p \cdot 0.48 \cdot \cos(\omega_m t)$

Problem 2.6

A 10kHz signal $x(t)$ modulates a 100kHz carrier and the result, as observed using an oscilloscope, is presented in the figure. Determine:



- Modulation used.
- Modulation index.
- Carrier's power and Modulating signal's normalized power. Recovered signal when using a synchronous detector tuned to 100kHz and with an amplitude of 1V .
- Recovered signal when using an envelope detector.

Note: It can be assumed that $K_D = 1$.

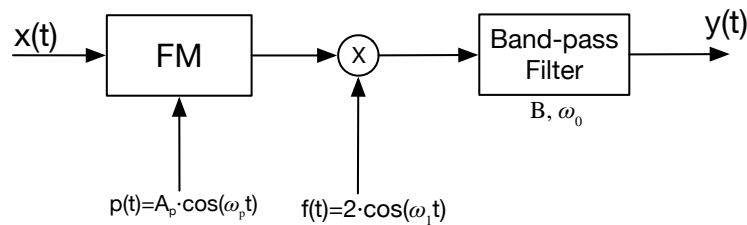
Results for problem

- AM
- $m \approx 0.43$
- $P_p = 0.98\text{W}$, $S_{xn} = 0.5$
- $y_D(t) = 0.3 \cdot \cos(2\pi \cdot 10^4 t)$
- $y_D(t) = 0.6 \cdot \cos(2\pi \cdot 10^4 t)$

Problem 2.7

The outline presented in the figure shows a FM modulator followed by a frequency converter and a band-pass filter (used to adapt the modulated signal to a suitable transmission frequency band). In order to set the system's parameters, a test tone $x(t)$ is used. Determine:

- Modulation index D , and modulated signal's bandwidth.
- Value of the filter's bandwidth, B , and the filter's central frequency, ω_0 , considering that a frequency band that is above ω_1 has been assigned for our transmission.
- Average power of the output $y(t)$ as a function of A_p considering that the filter attenuates the signal a 10%.



Data:

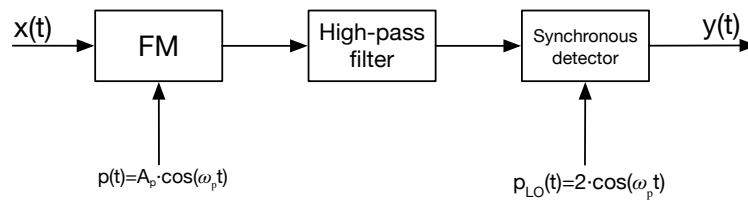
- $x(t) = \cos(\omega_m t)$ [V]
- $\omega_m = 2\pi \cdot 4 \text{krad/s}$
- $\omega_p = 2\pi \cdot 400 \text{krad/s}$
- $\omega_1 = 2\pi \cdot 2 \text{Mrad/s}$
- $\omega_d = 2\pi \cdot 16 \text{krad/s} \cdot V$

Results for problem

- $D = 4, B_T = 2\pi \cdot 48 \text{krad/s}$
- $\omega_0 = \omega_1 + \omega_p, B \geq B_T$
- $P_y = 0.81 \cdot \frac{A_p^2}{2}$

Problem 2.8

The signal $x(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$ FM modulates the carrier $p(t) = A_p \cdot \cos(\omega_p t)$. The modulated signal goes through a high pass filter with cutoff frequency $2\pi \cdot 350 \text{krad/s}$, whose output signal is fed to a synchronous detector where the local oscillator is adjusted to the carrier frequency, following the expression given by $p_{OL}(t)$ (see Data and Figure).



Calculate the output signal $y(t)$ as a function of A_p .

Datos:

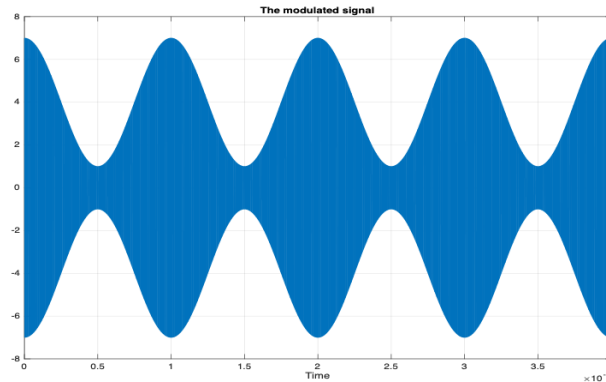
- $\omega_1 = 2\pi \cdot 64 \text{krad/s}$
- $\omega_2 = 2\pi \cdot 128 \text{krad/s}$
- $\omega_d = 2\pi \cdot 2 \text{krad/s} \cdot V$
- $\omega_p = 2\pi \cdot 400 \text{krad/s}$
- $p_{OL}(t) = 2 \cdot \cos(\omega_p t)$

Results for problem

$$y(t) = A_p \cdot \left[1 + \left(\frac{\omega_d}{2\omega_1} \right) \cos(\omega_1 t) + \left(\frac{\omega_d}{2\omega_2} \right) \cos(\omega_2 t) \right]$$

Problem 2.9

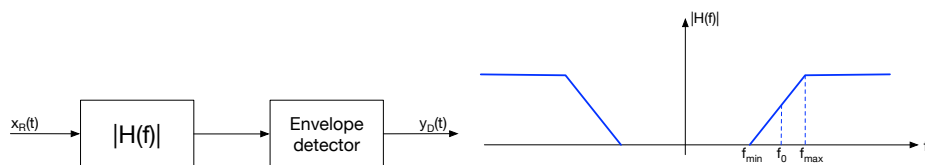
A given 1kHz frequency tone $x(t)$, with a 1V amplitude, DSB-AM modulates a 1MHz carrier $c(t)$. The following figure shows the modulated signal $y(t)$ characterized by a maximum value of 7V , and a minimum value of 1V .



- State the type of modulation and determine the modulation index as well as the carrier's amplitude (with its units).
- Plot the modulated signal, $y(t)$, spectrum showing in the plot the specific amplitude and frequency values.

At this point, we want to modulate the carrier $c(t)$ in frequency (FM) using the tone $x(t)$. The FM modulator has a frequency deviation of 15kHz .

- Plot, roughly, the time-domain signal obtained when the tone $x(t)$ modulates in frequency (FM) the same carrier $c(t)$. Please provide the envelope's amplitude range value.
- Is it possible to obtain the modulated signal using the following scheme?



Results for problem

- It is an AM modulation with $m = 0.75$ y $A_p = 4\text{V}$.
- $$Y(\omega) = 4\pi [\delta(\omega - \omega_p) + \delta(\omega + \omega_p)] + \left(\frac{3\pi}{2}\right) \cdot [\delta(\omega - \omega_p - \omega_m) + \delta(\omega - \omega_p + \omega_m) + \delta(\omega + \omega_p - \omega_m) + \delta(\omega + \omega_p + \omega_m)]$$
- $[-A_p, A_p]$
- Yes, it is possible.

References

[Haykin2001] Simon Haykin. Communication Systems, 4th Ed. John Wiley and Sons, 2001.