

Exercises. Lesson 6

Bandpass digital transmission

Problem 6.1

Calculate the expected value of the number of error bits during a day for the coherent BPSK receiver described below, under continuous operation. The data rate is 5000 bits/s . The input digital signals are $s_1(t) = A \cdot \cos(\omega_p t)$ and $s_2(t) = -A \cdot \cos(\omega_p t)$, where $A = 1 \text{ mV}$, and the unilateral noise power spectral density is $N_0 = 10^{-11} \text{ W/Hz}$.

RESULT:

2338 bits

Problem 6.2

A coherent BPSK system operating continuously produces errors at an average rate of 100 errors per day. The data rate is 1000 bits/s . The unilateral noise power spectral density is $N_0 = 10^{-10} \text{ W/Hz}$.

a) b)

Un sistema BPSK coherente que opera continuamente comete errores a razón de 100 errores al día como promedio. La velocidad de los datos es de 1000 bits/s . La densidad espectral unilateral de potencia de ruido es $N_0 = 10^{-10} \text{ W/Hz}$.

- If the system is ergodic, which is the average error probability?
- If the average received power is adjusted to 10^{-6} W , would this value be enough to keep the error probability calculated in a)?

RESULT:

- $1.16 \cdot 10^{-6}$
- No

Problem 6.3

The signal component of a coherent PSK system is defined by the expression

$$s(t) = A_c k \sin(\omega_p t) \pm A_c \sqrt{1 - k^2} \cos(\omega_p t)$$

where $0 \leq t < T_b$, and the plus sign corresponds to the 1 symbol, and the minus sign corresponds to the 0 one. The first term on the right hand side of the equation represents a carrier component, included to improve the synchronization between transmitter and receiver. Solve this:

- Plot the constellation of the signals described; what can be said about this diagram?
- Show that, in presence of zero-mean additive white Gaussian noise with power spectral density $N_0/2$, the average error probability is

$$P_e = Q \left(\sqrt{\frac{2E_b}{N_0} (1 - k^2)} \right)$$

with $E_b = \frac{1}{2} A_c^2 T_b$

- c. Assume that 10% of the transmitted power is located in the carrier component. Determine the value of E_b/N_0 required to obtain an error probability of 10^{-4} .
- d. Compare this E_b/N_0 value with the one required in a conventional PSK system with the same error probability.

RESULT:

- 1. PSK constellation
- 2. Demonstration
- 3. $\frac{E_b}{N_0} = 8.02$
- 4. $\frac{E_b}{N_0} = 7.22$

Problem 6.4

We want to compare two data transmission bandpass systems. One of them employs 16-PSK, the other, 16-QAM. Both systems have to provide an average symbol error probability of 10^{-3} . Compare the signal-to-noise requirements of said systems.

RESULT:

$$\Delta \left(\frac{E_s}{N_0} \right) = 3.68dB$$

Problem 6.5

If the performance criterion of a system is the bit error probability, which one of the following modulation schemes would be chosen to operate in an AWGN channel? Show the calculations.

- a. Coherent binary orthogonal FSK with $E_b/N_0 = 13dB$.
- b. Coherent binary PSK with $E_b/N_0 = 8dB$.

RESULT:

Coherent binary orthogonal FSK