

## Lesson 3

### Effects of noise in analog communications

## 1 Key concepts

Before doing these exercises it is important to review and understand the following concepts:

- General diagram of an analog communication system with noise.
- Concept of signal to noise ratio. Difference between pre-detection and post-detection.
- Concept of threshold effect, and when it is necessary to take it into account.

## 2 Basic problems

This first part includes problems mostly extracted from the bibliography aimed at practicing basic calculations needed for the rest of the lesson.

### Problem 3.1

[Carlson2010] A DSB signal plus noise is demodulated by synchronous detection. Find  $(S/N)_D$  in dB given that the received power is 20nW, the message has a bandwidth of 5MHz and the channel introduces a noise with  $N_0 = 4 \cdot 10^{-20}$  W/Hz.

RESULTS FOR PROBLEM

$$\left(\frac{S}{N}\right)_D = 50 \text{ dB}$$

### Problem 3.2

[Carlson2010] A DSB signal plus noise is demodulated by a product detector with phase error  $\phi$ . Take the local oscillator signal to be  $2 \cos(\omega_c t + \phi)$  and show that  $(S/N)_D = \gamma \cos^2(\phi)$ .

RESULTS FOR PROBLEM

### Problem 3.3

[Carlson2010] An AM system with envelope detection is operating at the threshold point. Find the power gain in dB needed at the transmitter to get up the  $(S/N)_D$  to 40dB with full-load tone modulation.

RESULTS FOR PROBLEM

$$G = 32\text{dB}$$

### Problem 3.4

[Carlson2010] An FM signal plus noise has a received power of 1nW,  $S_{xn} = 0.1$ ,  $|x(t)|_{max} = 4\text{V}$ ,  $\omega_d = 500\text{kHz/V}$  and  $N_0 = 4 \cdot 10^{-20}\text{W/Hz}$ . Find  $(S/N)_D$  in dB for FM detection and for deemphasized FM detection with  $B_{de} = 5\text{kHz}$ .

## RESULTS FOR PROBLEM

- FM:  $(S/N)_D = 53.8dB$
- deemphasized FM:  $(S/N)_D = 69dB$

**Problem 3.5**

[Carlson2010] An analog communication system has  $S_x = 1/2$ ,  $W_x = 10\text{kHz}$ ,  $N_0 = 10^{-15}\text{W/Hz}$ , and transmission loss 100dB. Calculate the transmitted power needed to get  $(S/N)_D = 40\text{dB}$  when the modulation is:

- AM with  $m = 1$
- AM with  $M = 0.5$
- FM with  $D = 1$
- FM with  $D = 5$
- FM with  $D = 10$

## RESULTS FOR PROBLEM

- $P_T = 3\text{kW}$
- $P_T = 9\text{kW}$
- $P_T = 667\text{W}$
- $P_T = 26.7\text{W}$
- $P_T = 24\text{W}$

### 3 Additional problems

These problems are slightly more elaborated than the previous ones, in many cases extracted from old exams.

#### Problem 3.6

A transmitter can perform either DSB or AM modulations. The post-detection signal-to-noise ratio has to be at least  $20dB$  in order to meet the required quality specifications.

When DSB modulation is used, we get a maximum range of  $15km$  with a transmission power of  $1W$ . The link attenuation (in dB) is proportional to the distance between transmitter and receiver.

Answer:

- Calculate the power of the signal at the output on the transmitter when DSB modulation is used, and the desired range is  $20km$ .
- Repeat a. when AM at 80% is used.
- Calculate the envelope peak power values at the output of the transmitter under the conditions mentioned in a. and b., respectively.

DATA:

- $\frac{N_0}{2} = 2 \cdot 10^{-9} W/Hz$
- $W_x = 2\pi \cdot 5krad/s$
- $|x(t)|_{max} = 1V$
- $S_x = 0.5W$
- $\alpha_t[dB] = A \cdot d[km]$ , with  $A$  a constant.

RESULTS FOR PROBLEM

- $S_{T_{DBL}} = 7.96W$
- $S_{T_{AM}} = 32.69W$
- $PEP_{DBL} = 15.92W$ ,  $PEP_{AM} = 80.25W$

### Problem 3.7

A normalized signal with  $5kHz$  bandwidth and average power  $0.5W$  is transmitted by an AM system. The transmitter power is  $660W$ . At the receiver side, located  $40km$  away, we get a post-detection signal-to-noise ratio of  $33dB$ .

We know that the noise power spectral density is  $N_0/2 = 5 \cdot 10^{-13}W/Hz$ , and that the propagation attenuation is given by  $A[dB] = 40 + 20 \cdot \log(d[km])$ . Answer:

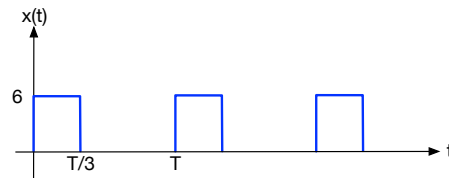
- a) Calculate the modulation index  $m$ .
- b) Calculate the amount of power needed to transmit the carrier, and the amount of power devoted to the transmission of each sideband.

#### RESULTS FOR PROBLEM

- a)  $m = 0.8$
- b)  $P_p = 500W$ ,  $P_{BL} = 80W$

### Problem 3.8

The signal  $x(t)$  in the figure goes through a lowpass filter with bandwidth  $10kHz$ . We assume that the signal power is not affected, and that the DC component is removed.



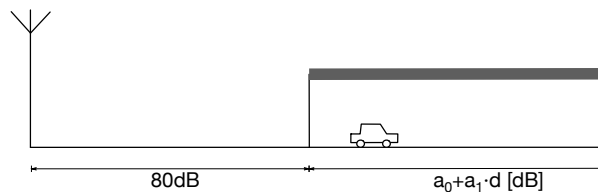
The resulting signal AM modulates a  $125V$  carrier, with a modulation index of  $0.7484$ . This signal is transmitted by a radio station. The transmission channel introduces  $80\text{ dB}$  attenuation, and Gaussian white noise whose power spectral density is  $N_0/2 = 10^{-11}W/Hz$ .

A vehicle is tuned to this radio station, and it enters a tunnel where the propagation attenuation is no longer constant and varies according to:

$$A[dB] = a_0 + a_1 \cdot d$$

where:

- $a_0 = 10dB$
- $a_1 = \frac{1}{3}dB/m$
- $d$ : Distance (m)



- a) Calculate the average power of the modulated signal.
- b) Calculate the demodulation signal-to-noise ratio at the tunnel entrance.
- c) Calculate the distance where the signal will no longer be audible, if the demodulation threshold is  $10dB$ .
- d) Repeat c. when the demodulator employed is an envelope detector, assuming that the pre-detection threshold is  $13dB$ .

### RESULTS FOR PROBLEM

- a)  $S_T = 10kW$
- b)  $\left(\frac{S}{N}\right)_D = 20.38dB$
- c)  $d = 11.94m$
- d)  $d' = 2.94m$

### Problem 3.9

A communication system is such that the signal arriving at the receiver side has a power  $100dB$  below the emitted power at the transmitter side, and that the noise power spectral density is  $N_0/2 = 5 \cdot 10^{-15} W/Hz$ . A post-detection signal-to-noise ratio of  $40dB$  is required for a correct reception.

The maximum voltage value of the message is  $2V$ , its mean power is  $1W$ , and its bandwidth,  $10kHz$ . A pre-detection threshold signal-to-noise ratio of  $10dB$  is required. Calculate the minimum transmitted power when using each one of the following modulation systems:

- a) DSB modulation.
- b) AM modulation with  $m = 0.5$  and envelope detection.
- c) FM modulation without de-emphasis ( $f_d = 10^5 Hz/V$ ).

#### RESULTS FOR PROBLEM

- a)  $S_T = 10kW$
- b)  $\gamma > \gamma_{th} \Rightarrow S_T = 170kW$
- c)  $\gamma < \gamma_{th} \Rightarrow S_T = S_{T_{th}} = 420W$

### Problem 3.10

We want to transmit a signal with bandwidth  $10kHz$  and normalized average power  $0.5$ , by using an FM communication system. The post-detection signal-to-noise ratio has to be greater than  $40dB$ , but ensuring a minimum amount of output power consumption. The channel has  $120kHz$  bandwidth and introduces white noise with power spectral density  $N_0/2 = 0.5 \cdot 10^{-8} W/Hz$ . This channel is characterized by an attenuation of  $40dB$ . Indicate the amount of power needed to transmit the signal.

#### RESULTS FOR PROBLEM

- $S_T = 416.66W$

### Problem 3.11

A radio transmission system uses a frequency modulator (FM) with modulator constant  $f_d = 20\text{kHz/V}$ , and carrier signal  $c(t) = 5 \cdot \cos(2\pi \cdot 5 \cdot 10^7 t)$ . Answer:

- Calculate the modulation index  $D$  and the approximate bandwidth when the modulating signal is  $x_1(t) = \cos(2\pi \cdot 500t)$ .
- Repeat a. when the modulating signal is  $x_2(t) = \cos(2\pi \cdot 10^4 t)$ .

In order to improve the demodulation signal-to-noise ratio, a pre-emphasis and de-emphasis filters are introduced, with cut-off frequency  $1430\text{Hz}$ . The system is tested with the modulating signal  $x(t) = x_1(t) + x_2(t)$ .

- Determine the improvement in demodulation signal-to-noise ratio.

#### RESULTS FOR PROBLEM

- $D_1 = 40$ ,  $B_T = 41\text{kHz}$
- $D_2 = 2$ ,  $B_T = 80\text{kHz}$
- $\Delta \left( \frac{S}{N} \right)_D = 12.14\text{dB}$

### Problem 3.12

A signal with bandwidth  $5\text{kHz}$ , whose maximum voltage value is  $2\text{V}$  and with average power  $0.5\text{W}$  is transmitted using FM through an AWGN channel. The channel has a noise power spectral density of  $N_0/2 = 3 \cdot 10^{-14}\text{W/Hz}$ . The receiver power is  $50\text{nW}$ . The maximum deviation of frequency is  $20\text{kHz}$ . Calculate the post-detection signal-to-noise ratio.

#### RESULTS FOR PROBLEM

- $\left( \frac{S}{N} \right)_D = 30\text{dB}$

### Problem 3.13

A signal with bandwidth  $10kHz$  and normalized average power 0.5 is transmitted using FM through an AWGN channel. The channel has a noise power spectral density of  $N_0/2 = 5 \cdot 10^{-10} W/Hz$  and attenuates the signal  $40dB$ . In order to meet the required quality specifications, a threshold  $\gamma_{th}$  of 100 is required for a pre-detection threshold signal-to-noise ratio of  $10dB$ .

- Calculate the minimum possible power transmitted to the channel ( $S_T$ ).
- Find the demodulation (post-detection) signal-to-noise ratio.

In order to improve the demodulation signal-to-noise ratio, pre-emphasis and de-emphasis filters are introduced. The  $3dB$  cutoff frequency of the de-emphasis filter,  $B_{de} = 1kHz$ .

- Determine the improvement in dB of the demodulation (post-detection) signal-to-noise ratio.

#### RESULTS FOR PROBLEM

- $S_{T_{min}} = 10dBW$
- $\left(\frac{S}{N}\right)_D = 31.3dB$
- $\Delta \left(\frac{S}{N}\right)_D = 15.23dB$

### References

[Carlson2010] A. Bruce Carlson and Paul B. Crilly. Communication Systems: An Introduction to Signals and Noise in Electrical Communication, 5th Ed. McGraw-Hill, 2010.