Lesson 5
Baseband digital transmission

1 Key concepts

Before doing these exercises it is important to review and understand the following concepts:

- General scheme of a baseband communications system, including the scheme of the receiver.
- PAM modulation. Calculation of the power spectral density of a PAM signal.
- Line codes. Types of line codes and differences among them.
- Intersymbol Interference (ISI). Concept of Nyquist ideal filter. Raised cosine filter.
- Eye diagram. How it is built.

2 Basic problems

This first part includes problems mostly extracted from the bibliography aimed at practicing basic calculations needed for the rest of the lesson.

Problem 5.1

A baseband quaternary transmission system transmits the following signal:

$$x(t) = \sum_{n=-\infty}^{\infty} a_n \cdot h(t - nT)$$

a) Which is the orthonormal basis that can be used for this modulation?

b) Represent the constellation

c) How could be calculate the probability of error?

Results for problem

a) The pulse $h(t)$, normalized if necessary.

b) A 4-ary unidimensional constellation

c) With the equation for a M-ary unidimensional constellation.

Problem 5.2

[Carlson2010] A computer generates 16-bits binary words, at 20000 words per second.

a) Calculate the bandwidth necessary for transmitting the output as a PAM binary signal.

b) Calculate M so the output can be transmitted as an M-ary signal in a channel with $B = 60kHz$.

Results for problem

a) $B_T \geq 160kHz$
b) $M = 8$

Problem 5.3

A baseband transmission system transmits the following signal:

$$x(t) = \sum_{n=-\infty}^{\infty} a_n \cdot h(t - nT)$$

where T is the symbol period, a_n is an equiprobable, uncorrelated sequence whose maximum value is A and $h(t)$ is a normalized squared pulse with length T_x.
Calculate the power spectral density and the bandwidth of the transmitted signal for the following cases:

a) NRZ Polar sequence.

b) RZ Polar sequence with $T_x = T/2$.

c) NRZ Unipolar sequence.

Results for problem

a) $S_x(\omega) = \frac{1}{T} \cdot \frac{1}{T} \frac{4\sin^2\left(\omega \frac{T}{2}\right)}{\omega^2} \cdot A^2$
$B = \frac{2\pi}{T}$

b) $S_x(\omega) = \frac{1}{T} \cdot \frac{2}{T} \frac{4\sin^2\left(\omega \frac{T}{4}\right)}{\omega^2} \cdot A^2$
$B = \frac{4\pi}{T}$

c) $S_x(\omega) = \frac{1}{T} \cdot \frac{4\sin^2\left(\omega \frac{T}{2}\right)}{\omega^2} \cdot \frac{A^2}{4} + \frac{A^2\pi}{2}\delta(\omega)$
$B = \frac{2\pi}{T}$

Problem 5.4

A baseband transmission system transmits the following signal:

$$x(t) = \sum_{n=-\infty}^{\infty} a_n \cdot h(t - nT)$$

where T is the symbol period, a_n is an equiprobable, uncorrelated sequence whose maximum value is A and $h(t)$ is a raised cosine pulse with roll-off factor α.
Calculate the power spectral density and the bandwidth of the transmitted signal assuming polar coding.

Results for problem

$S_x(\omega) = \frac{1}{T} \cdot |H(\omega)|^2 \cdot A^2$, with $H(\omega)$ being the frequency response of the raised cosine filter.
$B = \frac{1+\alpha}{2T}$

Problem 5.5

Consider a binary sequence b_n, from which we form the symbols $a_n = b_n - b_{n-1}$. b_n are uncorrelated and equiprobable random variables, taking values 1 and 0.
Calculate the power spectral density of the transmitted signal, in the case that the transmitter filter is:

$$h_T(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t < T \\ 0 & c.c. \end{cases}$$

Results for problem

$S_x(\omega) = \frac{4}{T^2\omega^2} sen^4\left(\frac{\omega T}{2}\right)$

Problem 5.6

[Carlson2010] Calculate the $\left(\frac{S}{N}\right)_R$ such that a binary unipolar system with white Gaussian noise has a $P_e = 0.001$.
What is the error probability of a polar system with the same $\left(\frac{S}{N}\right)_R$?

Results for problem

a) $\rho = \frac{E_s}{N_0/2} = 19,22$
   $P_e = Q(4,38) < 8.54 \cdot 10^{-6}$

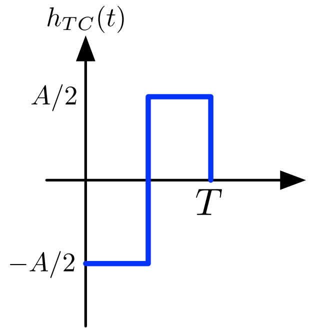
Problem 5.7

[Haykin2001] Consider the signal $h_{TC}(t)$ in the figure:

a) Determine the impulse response of the matched filter to that signal, and draw it in function of the time.

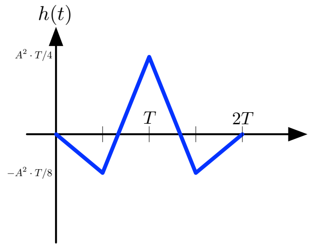
b) Draw the waveform of the global impulse response $h(t)$.

Results for problem



a) $h_R(t) = h_{TC}(T - t)$

b)

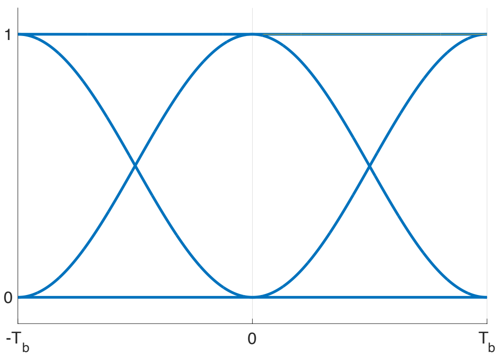


Problem 5.8

[Carlson2010] The following PAM signal is received: $r(t) = \sum_{n=-\infty}^{\infty} b_n h(t - nT)$.
Draw and determine its eye pattern, without distortion, considering the following unipolar data sequence: 1011100010.

$$h(t) = cos^2\left(\frac{2\pi}{4T_b}t\right)\prod\left(\frac{t}{2T_b}\right)$$

Results for problem



Problem 5.9

A computer generates pulses with $R_b = 1Mbps$, for their transmission through a noisy channel, with power spectral density $N_0/2 = 2 \cdot 10^{-20} W/Hz$. The error rate cannot be over 1 bit per hour.

a) Determine the noise power of the channel and the error probability of the system.

b) Suppose that the transmitter includes now a transfer characteristic using a raised cosine spectrum, with an excess of bandwidth of 75%, and the systems is PAM quaternary. Calculate the new bandwidth necessary for the transmission.

Results for problem

a) $P_N = 2 \cdot 10^{-14} W$, $P_b = 2.7 \cdot 10^{-10}$

b) $B = 437.5 kHz$

3 Aditional problems

These problems are slightly more ellaborated than the previous ones, in many cases extracted from old exams.

Problem 5.10

A communication system transmits the following signal:

$$s(t) = \sum_{n=-\infty}^{\infty} b_n \cdot h_T(t - nT)$$

where b_n represents a sequence of discrete random variables, independent and identically distributed, which take the values ± 1 with the same probability. The transmitted pulse is $h(t) = \frac{1}{\sqrt{T}} \cdot \prod\left(\frac{t-T/2}{T}\right)$, the channel impulse response is $h_c(t) = \delta(t)$ and the receiver filter $h_R(t)$ is matched to $h_T(t)$.

a) Determine the wave form of the global impulse response $h(t)$.

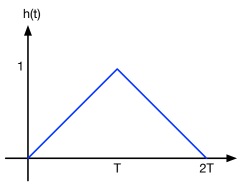
b) Draw the eye diagram of the receiver filter output, before sampling.

c) Repeat a) for a channel with impulse response $h_c(t) = \delta(t) - 0.5 \cdot \delta(t - T)$.

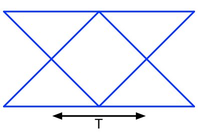
d) Calculate the values of the samples at the receiver filter output, and the value of the inter-symbol interference in each of them.

Note: $\prod(t) = \begin{cases} 1 & -0.5 \le t < 0.5 \\ 0 & c.c. \end{cases}$

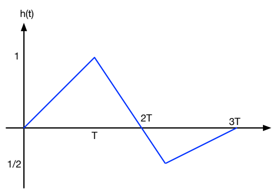
Results for problem



Apartado (a)



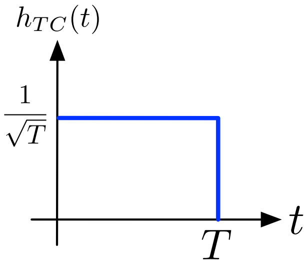
Apartado (b)



Apartado (c)

Problem 5.11

[Exam2012] Consider a binary digital communication system that transmits an NRZ polar code with the pulse shape $h_{TC}(t)$ in the figure.



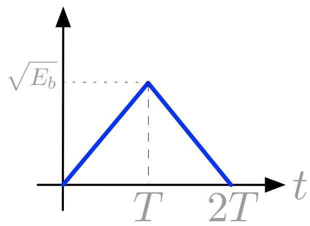
a) Determine the impulse response of the matched filter and plot the output of the matched filter as a function of time when a 1 is received and when a 0 is received.

b) Compute the mean probability of detection error in an additive white Gaussian noise channel, whose noise power spectral density is $N_0/2$ W/Hz.

Data: Mean bit energy $E_b = 4\,\mathrm{pJ}$, $N_0 = 3.6 \cdot 10^{-13}\,\mathrm{W/Hz}$.

Results for problem

a)



b) $P_e = 1.3 \cdot 10^{-6}$

Problem 5.12

[Exam2012] An electronic device generates 30000 binary words per second, which are transmitted as a unipolar NRZ PAM-binary signal with $10\,\mathrm{mV}$ pulse level at the input of the channel, whose bandwidth is $120\,\mathrm{kHz}$, which attenuates the signal by $10\,\mathrm{dB}$ and introduces Gaussian white noise of power spectral density $N_0/2 = 10^{-10}\,\mathrm{W/Hz}$.

a) Maximum number of bits, n, that each binary word has. All generated words are assumed to have the same number of bits.

b) Average symbol energy at the channel output, considering that the symbols are equiprobable.

If for economic reasons the channel bandwidth is reduced to $60\,\mathrm{kHz}$.

c) Determine the value of M so that the output can be transmitted as an M-ary signal.

If for design reasons a transfer characteristic with raised cosine spectrum is included in the transmitter, with an excess of bandwidth of 95% and the output is transmitted as an 8-ary signal.

d) Determine the new bandwidth needed for transmission

Results for problem

a) $n = 8$
b) $E_s = 2.083 \cdot 10^{-11}\,\mathrm{J}$
c) $M = 4$
d) $B_T = 78\,\mathrm{kHz}$

References

[Carlson2010] A. Bruce Carlson and Paul B. Crilly. Communication Systems: An Introduction to Signals and Noise in Electrical Communication, 5th Ed. McGraw-Hill, 2010.

[Haykin2001] Simon Haykin. Communication Systems, 4th Ed. John Wiley and Sons, 2001.