

Lesson 3

Effects of noise in analog communications

1 Key concepts

Before doing these exercises it is important to review and understand the following concepts:

- General diagram of an analog communication system with noise.
- Concept of signal to noise ratio. Difference between pre-detection and post-detection.
- Concept of threshold effect, and when it is necessary to take it into account.

2 Basic problems

This first part includes problems mostly extracted from the bibliography aimed at practicing basic calculations needed for the rest of the lesson.

Problem 3.1

[Carlson2010] A DSB signal plus noise is demodulated by synchronous detection. Find $(S/N)_D$ in dB given that the received power is 20nW, the message has a bandwidth of 5MHz and the channel introduces a noise with $N_0 = 4 \cdot 10^{-20}$ W/Hz.

Results for problem

$$\left(\frac{S}{N}\right)_D = 50 \text{ dB}$$

Problem 3.2

[Carlson2010] A DSB signal plus noise is demodulated by a product detector with phase error ϕ . Take the local oscillator signal to be $2\cos(\omega_c t + \phi)$ and show that $(S/N)_D = \gamma \cos^2(\phi)$.

Results for problem

Problem 3.3

[Carlson2010] An AM system with envelope detection is operating at the threshold point. Find the power gain in dB needed at the transmitter to get up the $(S/N)_D$ to 40dB with full-load tone modulation.

Results for problem

$$G = 32\text{dB}$$

Problem 3.4

[Carlson2010] An FM signal plus noise has a received power of 1nW , $S_{xn} = 0.1$, $|x(t)|_{\max} = 4\text{V}$, $\omega_d = 500\text{kHz/V}$ and $N_0 = 4 \cdot 10^{-20}\text{W/Hz}$. Find $(S/N)_D$ in dB for FM detection and for deemphasized FM detection with $B_{de} = 5\text{kHz}$.

Results for problem

- FM: $(S/N)_D = 53.8\text{dB}$
- deemphasized FM: $(S/N)_D = 69\text{dB}$

Problem 3.5

[Carlson2010] An analog communication system has $S_x = 1/2$, $W_x = 10\text{kHz}$, $N_0 = 10^{-15}\text{W/Hz}$, and transmission loss 100dB . Calculate the transmitted power needed to get $(S/N)_D = 40\text{dB}$ when the modulation is:

- AM with $m = 1$
- AM with $M = 0.5$
- FM with $D = 1$
- FM with $D = 5$
- FM with $D = 10$

Results for problem

- $P_T = 3\text{kW}$
- $P_T = 9\text{kW}$
- $P_T = 667\text{W}$
- $P_T = 26.7\text{W}$
- $P_T = 24\text{W}$

3 Additional problems

These problems are slightly more elaborated than the previous ones, in many cases extracted from old exams.

Problem 3.6

A transmitter can perform either DSB or AM modulations. The post-detection signal-to-noise ratio has to be at least $20dB$ in order to meet the required quality specifications.

When DSB modulation is used, we get a maximum range of $15km$ with a transmission power of $1W$. The link attenuation (in dB) is proportional to the distance between transmitter and receiver.

Answer:

- Calculate the power of the signal at the output on the transmitter when DSB modulation is used, and the desired range is $20km$.
- Repeat a. when AM at 80% is used.
- Calculate the envelope peak power values at the output of the transmitter under the conditions mentioned in a. and b., respectively.

Data:

- $\frac{N_0}{2} = 2 \cdot 10^{-9} W/Hz$
- $W_x = 2\pi \cdot 5krad/s$
- $|x(t)|_{max} = 1V$
- $S_x = 0.5W$
- $\alpha_t[dB] = A \cdot d[km]$, with A a constant.

Results for problem

- $S_{T_{DBL}} = 7.96W$
- $S_{T_{AM}} = 32.69W$
- $PEP_{DBL} = 15.92W, PEP_{AM} = 80.25W$

Problem 3.7

A normalized signal with $5kHz$ bandwidth and average power $0.5W$ is transmitted by an AM system. The transmitter power is $660W$. At the receiver side, located $40km$ away, we get a post-detection signal-to-noise ratio of $33dB$.

We know that the noise power spectral density is $N_0/2 = 5 \cdot 10^{-13}W/Hz$, and that the propagation attenuation is given by $A[dB] = 40 + 20 \cdot \log(d[km])$. Answer:

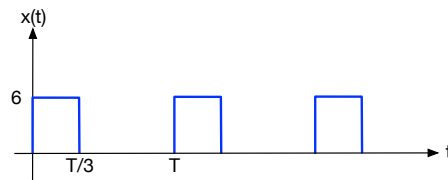
- Calculate the modulation index m .
- Calculate the amount of power needed to transmit the carrier, and the amount of power devoted to the transmission of each sideband.

Results for problem

- $m = 0.8$
- $P_p = 500W, P_{BL} = 80W$

Problem 3.8

The signal $x(t)$ in the figure goes through a lowpass filter with bandwidth $10kHz$. We assume that the signal power is not affected, and that the DC component is removed.



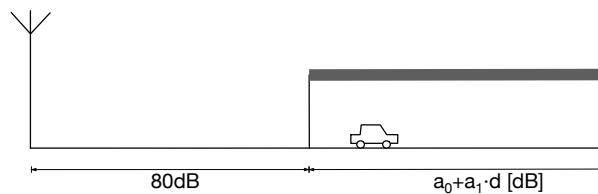
The resulting signal AM modulates a $125V$ carrier, with a modulation index of 0.7484 . This signal is transmitted by a radio station. The transmission channel introduces 80 dB attenuation, and Gaussian white noise whose power spectral density is $N_0/2 = 10^{-11}W/Hz$.

A vehicle is tuned to this radio station, and it enters a tunnel where the propagation attenuation is no longer constant and varies according to:

$$A[dB] = a_0 + a_1 \cdot d$$

where:

- $a_0 = 10dB$
- $a_1 = \frac{1}{3}dB/m$
- d : Distance (m)



- Calculate the average power of the modulated signal.
- Calculate the demodulation signal-to-noise ratio at the tunnel entrance.
- Calculate the distance where the signal will no longer be audible, if the demodulation threshold is $10dB$.
- Repeat c. when the demodulator employed is an envelope detector, assuming that the pre-detection threshold is $13dB$.

Results for problem

- $S_T = 10kW$
- $\left(\frac{S}{N}\right)_D = 20.38dB$

c) $d = 11.94m$

d) $d' = 2.94m$

Problem 3.9

A communication system is such that the signal arriving at the receiver side has a power $100dB$ below the emitted power at the transmitter side, and that the noise power spectral density is $N_0/2 = 5 \cdot 10^{-15}W/Hz$. A post-detection signal-to-noise ratio of $40dB$ is required for a correct reception.

The maximum voltage value of the message is $2V$, its mean power is $1W$, and its bandwidth, $10kHz$. A pre-detection threshold signal-to-noise ratio of $10dB$ is required. Calculate the minimum transmitted power when using each one of the following modulation systems:

- a) DSB modulation.
- b) AM modulation with $m = 0.5$ and envelope detection.
- c) FM modulation without de-emphasis ($f_d = 10^5 Hz/V$).

Results for problem

a) $S_T = 10kW$

b) $\gamma > \gamma_{th} \Rightarrow S_T = 170kW$

c) $\gamma < \gamma_{th} \Rightarrow S_T = S_{T_{th}} = 420W$

Problem 3.10

We want to transmit a signal with bandwidth $10kHz$ and normalized average power 0.5 , by using an FM communication system. The post-detection signal-to-noise ratio has to be greater than $40dB$, but ensuring a minimum amount of output power consumption. The channel has $120kHz$ bandwidth and introduces white noise with power spectral density $N_0/2 = 0.5 \cdot 10^{-8}W/Hz$. This channel is characterized by an attenuation of $40dB$. Indicate the amount of power needed to transmit the signal.

Results for problem

- $S_T = 416.66W$

Problem 3.11

A radio transmission system uses a frequency modulator (FM) with modulator constant $f_d = 20\text{kHz}/V$, and carrier signal $c(t) = 5 \cdot \cos(2\pi \cdot 5 \cdot 10^7 t)$. Answer:

- Calculate the modulation index D and the approximate bandwidth when the modulating signal is $x_1(t) = \cos(2\pi \cdot 500t)$.
- Repeat a. when the modulating signal is $x_2(t) = \cos(2\pi \cdot 10^4 t)$.

In order to improve the demodulation signal-to-noise ratio, a pre-emphasis and de-emphasis filters are introduced, with cut-off frequency 1430Hz . The system is tested with the modulating signal $x(t) = x_1(t) + x_2(t)$.

- Determine the improvement in demodulation signal-to-noise ratio.

Results for problem

- $D_1 = 40, B_T = 41\text{kHz}$
- $D_2 = 2, B_T = 80\text{kHz}$
- $\Delta \left(\frac{S}{N} \right)_D = 12.14\text{dB}$

Problem 3.12

A signal with bandwidth 5kHz , whose maximum voltage value is $2V$ and with average power $0.5W$ is transmitted using FM through an AWGN channel. The channel has a noise power spectral density of $N_0/2 = 3 \cdot 10^{-14} W/\text{Hz}$. The receiver power is 50nW . The maximum deviation of frequency is 20kHz . Calculate the post-detection signal-to-noise ratio.

Results for problem

- $\left(\frac{S}{N} \right)_D = 30\text{dB}$

Problem 3.13

A signal with bandwidth $10kHz$ and normalized average power 0.5 is transmitted using FM through an AWGN channel. The channel has a noise power spectral density of $N_0/2 = 5 \cdot 10^{-10} W/Hz$ and attenuates the signal $40dB$. In order to meet the required quality specifications, a threshold γ_{th} of 100 is required for a pre-detection threshold signal-to-noise ratio of $10dB$.

- a) Calculate the minimum possible power transmitted to the channel (S_T).
- b) Find the demodulation (post-detection) signal-to-noise ratio.

In order to improve the demodulation signal-to-noise ratio, pre-emphasis and de-emphasis filters are introduced. The $3dB$ cutoff frequency of the de-emphasis filter, $B_{de} = 1kHz$.

- c) Determine the improvement in dB of the demodulation (post-detection) signal-to-noise ratio.

Results for problem

- a) $S_{T_{min}} = 10dBW$
- b) $\left(\frac{S}{N}\right)_D = 31.3dB$
- c) $\Delta \left(\frac{S}{N}\right)_D = 15.23dB$

References

[Carlson2010] A. Bruce Carlson and Paul B. Crilly. Communication Systems: An Introduction to Signals and Noise in Electrical Communication, 5th Ed. McGraw-Hill, 2010.