Exercises. Lesson 2
Analog modulations

Problem 2.1

A zero-mean periodic signal $x(t)$, with bandwidth $5kHz$, amplitude $4V$ and normalized average power 0.5, DSB modulates a $1MHz$ carrier. The result is a signal with average power $400W$. Determine:

a. Carrier amplitude.

b. Average power of the lower sideband.

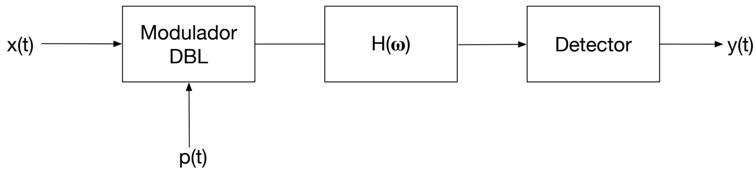
c. Outline of the detector needed to recover the signal x(t), and the value of its main parameters

RESULT:

1. $A_c = 10V$
2. $P_{LSB} = 200W$
3. Synchronous detector $f_{LO} = 1MHz$, $A_{LO} = \frac{2}{A_c}$ and $f_{lp} = 5kHz$.

Problem 2.2

The signal $x(t) = cos(2\pi \cdot 10 \cdot 10^3 t) + 4 \cdot cos(2\pi \cdot 15 \cdot 10^3 t) + cos(2\pi \cdot 20 \cdot 10^3 t)$ DSB modulates the carrier $p(t) = 2 \cdot cos(2\pi \cdot 10^5 t)$ and passes through a filter with frequency response $H(\omega)$ before reaching the detector, as it can be observed in the figure



with

$$H(\omega) = \begin{cases} 0 & |\omega| < 200\pi krad/s \\ 1 & |\omega| \geq 200\pi krad/s \end{cases}$$

a. Find the signal obtained at the output of the detector, when using an envelope detector of $K_D = 1$ with DC suppression.

b. b) Determine the detector needed to obtain a detected signal equal to the modulating signal. Please specify all necessary parameters.

RESULT:

1. $y(t) = 2 \cdot cos(2\pi \cdot 5 \cdot 10^3 t)$
2. Synchronous detector with $f_{LO} = 2 \cdot cos(\omega_c \cdot t)$ and a low pass filter with cut-off frequency of at least $20kHz$.

Problem 2.3

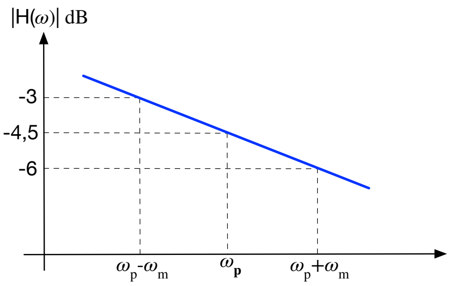
A transmitter has an average nominal power of $30W$ and a peak envelope power of $60W$. Determine:

a. The power in a sideband when the signal $x(t) = cos(\omega_m t)$ modulates the carrier given by $p(t) = A_p \cdot cos(\omega_p t)$, and the value of A_p in the following cases:

   (a) AM modulation when the modulation index is 80%.

   (b) DSB modulation

b. Considering the first case for a), and knowing that the channel presents a non-uniform attenuation as depicted in the figure: Obtain the signal detected in the following cases:



   (a) Envelope detector

   (b) Synchornous detector

NOTE: Assume in both cases that a DC suppressor is present.
RESULT:

1. (a) $A_p = 6.08V$, $P_{BL} = 2.96W$
   (b) $A_p = 10.95V$, $P_{BL} = 15W$
2. (a) $y_D(t) = k_D \cdot [A(t) - < A(t) >]$, con:
      - $A(t) = x_i(t) \cdot \left[1 + 0.5 \cdot \left(\frac{x_q(t)}{x_i(t)}\right)^2\right]$
      - $x_i(t) = 0.6 \cdot A_p + 0.48 \cdot A_p \cdot cos(\omega_m t)$
      - $x_q(t) = -0.08 \cdot A_p \cdot sen(\omega_m t)$
   (b) $y_D(t) = \frac{A_{OL}}{2} \cdot A_p \cdot 0.48 \cdot cos(\omega_m t)$

Problem 2.4

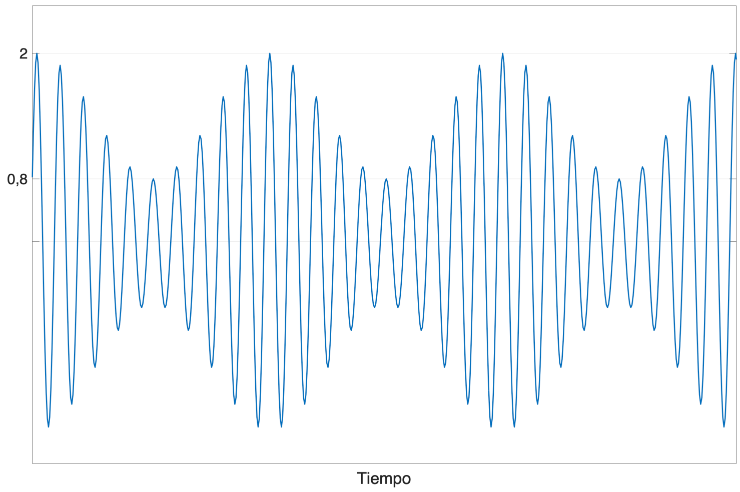
A $10kHz$ signal $x(t)$ modulates a $100kHz$ carrier and the result, as observed using an oscilloscope, is presented in the figure. Determine:

a. Modulation used.

b. Modulation index.

c. Carrier's power and Modulating signal's normalized power. Recovered signal when using a synchronous detector tuned to $100kHz$ and with an amplitude of $1V$.

d. Recovered signal when using an envelope detector.



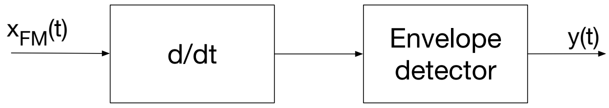
NOTE: It can be assumed that $K_D = 1$.

RESULT:

1. AM
2. $m \approx 0.43$
3. $P_p = 0.98W$, $S_{xn} = 0.5$
4. $y_D(t) = 0.3 \cdot cos(2\pi \cdot 10^4 t)$
5. $y_D(t) = 0.6 \cdot cos(2\pi \cdot 10^4 t)$

Problem 2.5

Considering that $x_{FM}(t)$ is the signal obtained when FM modulating a signal $x(t)$ with the carrier $p(t) = A_p \cdot cos(\omega_p t)$. Determine the condition needed to recover the signal $x(t)$ if the system outlined in the figure is used.



RESULT:

$\omega_p - \omega_\Delta \cdot |x(t)|_{max} > 0$

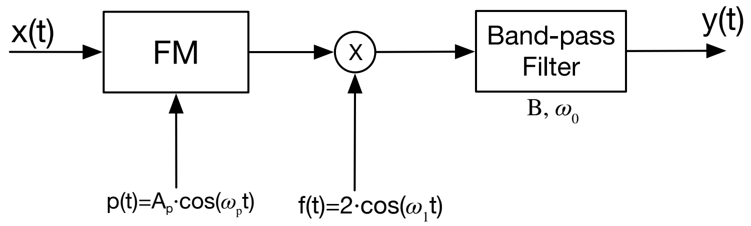
Problem 2.6

The outline presented in the figure shows a FM modulator followed by a frequency converter and a band-pass filter (used to adapt the modulated signal to a suitable transmission frequency band). In order to set the system's parameters, a test tone $x(t)$ is used. Determine:

a. Modulation index D, and modulated signal's bandwidth.

b. Value of the filter's bandwidth, B, and the filter's central frequency, ω_0, considering that a frequency band that is above ω_1 has been assigned for our transmission.

c. Average power of the output $y(t)$ as a function of A_p considering that the filter attenuates the signal a 10%.



DATA:

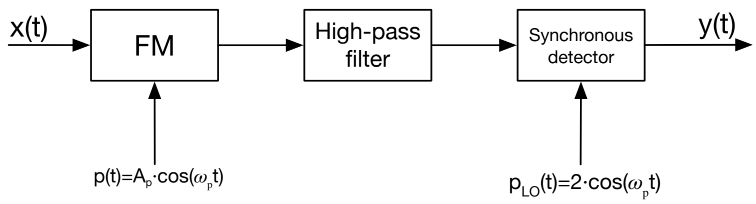
- $x(t) = cos(\omega_m t)$ $[V]$
- $\omega_m = 2\pi \cdot 4krad/s$
- $\omega_p = 2\pi \cdot 400krad/s$
- $\omega_1 = 2\pi \cdot 2Mrad/s$
- $\omega_d = 2\pi \cdot 16krad/s \cdot V$

RESULT:

1. $D = 4$, $B_T = 2\pi \cdot 48krad/s$
2. $\omega_0 = \omega_1 + \omega_p$, $B \geq B_T$
3. $P_y = 0.81 \cdot \frac{A_p^2}{2}$

Problem 2.7

The signal $x(t) = cos(\omega_1 t) + cos(\omega_2 t)$ FM modulates the carrier $p(t) = A_p \cdot cos(\omega_p t)$. The modulated signal goes through a high pass filter with cutoff frequency $2\pi \cdot 350krad/s$, whose output signal is fed to a synchronous detector where the local oscillator is adjusted to the carrier frequency, following the expression given by $p_{OL}(t)$ (see Data and Figure).



Calculate the output signal $y(t)$ as a function of A_p.
DATOS:

- $\omega_1 = 2\pi \cdot 64krad/s$