Lesson 2
Analog modulations

1 Key concepts

Before doing these exercises it is important to review and understand the following concepts:

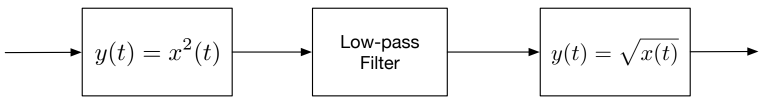
- Linear analog modulations: AM and DSB.
- Phasors. In-phase and quadrature components of a signal
- Detectors: synchronous and envelope detectors.
- Angular modulations. Bandwidth and interference. Preemphasis and deemphasis.
- Superheterodine receiver.

2 Basic problems

This first part includes problems mostly extracted from the bibliography aimed at practicing basic calculations needed for the rest of the lesson.

Problem 2.1

The AM signal $s(t) = A_p(1 + mx(t))\cos(2\pi f_p t)$ is applied to the system shown in the figure.



Assuming that the message is normalized, its bandwidth is W, that $0 \leq m \leq 1$ and that $f_p > 2W$, show that the message can be obtained from the system output.

RESULTS FOR PROBLEM

It's possible just selecting the cut-off frequency of the low-pass filter to be $2W$

Problem 2.2

A zero-mean periodic signal $x(t)$, with bandwidth $5kHz$, amplitude $4V$ and normalized average power 0.5, DSB modulates a $1MHz$ carrier. The result is a signal with average power $400W$. Determine:

a) Carrier amplitude.

b) Average power of the lower sideband.

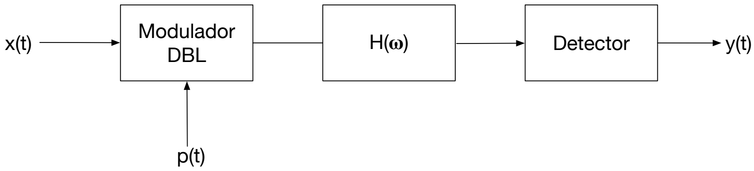
c) Outline of the detector needed to recover the signal x(t), and the value of its main parameters

RESULTS FOR PROBLEM

a) $A_c = 10V$
b) $P_{LSB} = 200W$
c) Synchronous detector $f_{LO} = 1MHz$, $A_{LO} = \frac{2}{A_c}$ and $f_{lp} = 5kHz$.

Problem 2.3

The signal $x(t) = cos(2\pi \cdot 10 \cdot 10^3 t) + 4 \cdot cos(2\pi \cdot 15 \cdot 10^3 t) + cos(2\pi \cdot 20 \cdot 10^3 t)$ DSB modulates the carrier $p(t) = 2 \cdot cos(2\pi \cdot 10^5 t)$ and passes through a filter with frequency response $H(\omega)$ before reaching the detector, as it can be observed in the figure



with

$$H(\omega) = \begin{cases} 0 & |\omega| < 200\pi krad/s \\ 1 & |\omega| \geq 200\pi krad/s \end{cases}$$

a) Find the signal obtained at the output of the detector, when using an envelope detector of $K_D = 1$ with DC suppression.

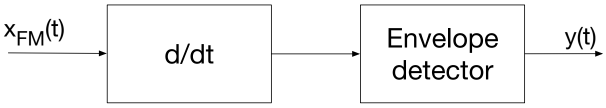
b) b) Determine the detector needed to obtain a detected signal equal to the modulating signal. Please specify all necessary parameters.

Results for problem

a) $y(t) = 2 \cdot cos(2\pi \cdot 5 \cdot 10^3 t)$
b) Synchronous detector with $f_{LO} = 2 \cdot cos(\omega_c \cdot t)$ and a low pass filter with cut-off frequency of at least $20kHz$.

Problem 2.4

Considering that $x_{FM}(t)$ is the signal obtained when FM modulating a signal $x(t)$ with the carrier $p(t) = A_p \cdot cos(\omega_p t)$. Determine the condition needed to recover the signal $x(t)$ if the system outlined in the figure is used.



Results for problem

$\omega_p - \omega_\Delta \cdot |x(t)|_{max} > 0$

3 Aditional problems

These problems are slightly more ellaborated than the previous ones, in many cases extracted from old exams.

Problem 2.5

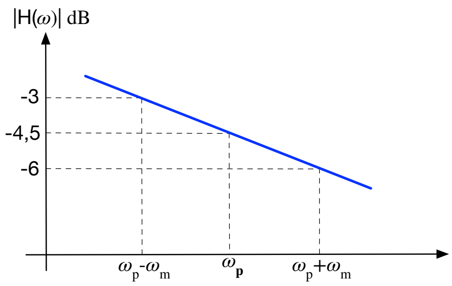
A transmitter has an average nominal power of $30W$ and a peak envelope power of $60W$. Determine:

a) The power in a sideband when the signal $x(t) = cos(\omega_m t)$ modulates the carrier given by $p(t) = A_p \cdot cos(\omega_p t)$, and the value of A_p in the following cases:

   (a) AM modulation when the modulation index is 80%.

   (b) DSB modulation

b) Considering the first case for a), and knowing that the channel presents a non-uniform attenuation as depicted in the figure: Obtain the signal detected in the following cases:



   (a) Envelope detector

   (b) Synchornous detector

NOTE: Assume in both cases that a DC suppressor is present.

RESULTS FOR PROBLEM

a) (a) $A_p = 6.08V$, $P_{BL} = 2.96W$

   (b) $A_p = 10.95V$, $P_{BL} = 15W$

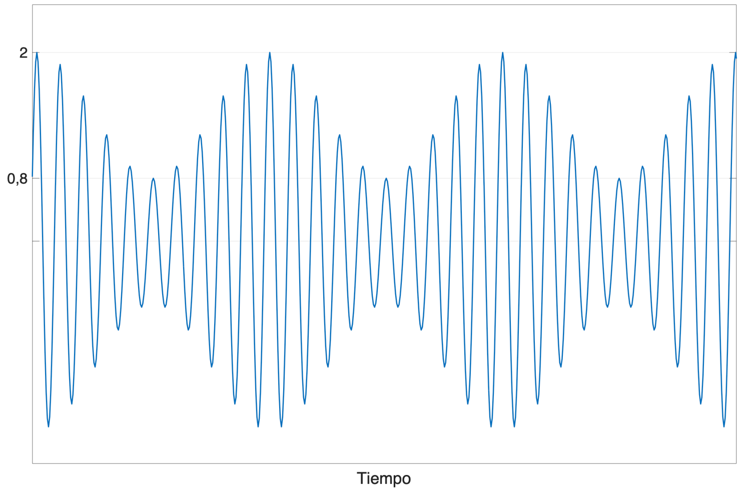
b) (a) $y_D(t) = k_D \cdot [A(t) - < A(t) >]$, con:

   - $A(t) = x_i(t) \cdot \left[1 + 0.5 \cdot \left(\frac{x_q(t)}{x_i(t)}\right)^2\right]$
   - $x_i(t) = 0.6 \cdot A_p + 0.48 \cdot A_p \cdot cos(\omega_m t)$
   - $x_q(t) = -0.08 \cdot A_p \cdot sen(\omega_m t)$

   (b) $y_D(t) = \frac{A_{OL}}{2} \cdot A_p \cdot 0.48 \cdot cos(\omega_m t)$

Problem 2.6

A $10kHz$ signal $x(t)$ modulates a $100kHz$ carrier and the result, as observed using an oscilloscope, is presented in the figure. Determine:



a) Modulation used.

b) Modulation index.

c) Carrier's power and Modulating signal's normalized power. Recovered signal when using a synchronous detector tuned to $100kHz$ and with an amplitude of $1V$.

d) Recovered signal when using an envelope detector.

NOTE: It can be assumed that $K_D = 1$.

RESULTS FOR PROBLEM

a) AM
b) $m \approx 0.43$
c) $P_p = 0.98W$, $S_{xn} = 0.5$
d) $y_D(t) = 0.3 \cdot cos(2\pi \cdot 10^4 t)$
e) $y_D(t) = 0.6 \cdot cos(2\pi \cdot 10^4 t)$

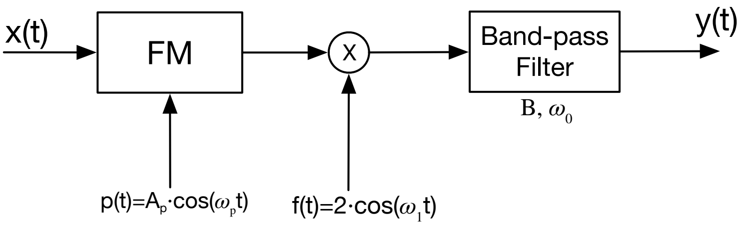
Problem 2.7

The outline presented in the figure shows a FM modulator followed by a frequency converter and a band-pass filter (used to adapt the modulated signal to a suitable transmission frequency band). In order to set the system's parameters, a test tone $x(t)$ is used. Determine:

a) Modulation index D, and modulated signal's bandwidth.

b) Value of the filter's bandwidth, B, and the filter's central frequency, ω_0, considering that a frequency band that is above ω_1 has been assigned for our transmission.

c) Average power of the output $y(t)$ as a function of A_p considering that the filter attenuates the signal a 10%.



Data:

- $x(t) = cos(\omega_m t)\ [V]$
- $\omega_m = 2\pi \cdot 4krad/s$
- $\omega_p = 2\pi \cdot 400krad/s$
- $\omega_1 = 2\pi \cdot 2Mrad/s$
- $\omega_d = 2\pi \cdot 16krad/s \cdot V$

Results for problem

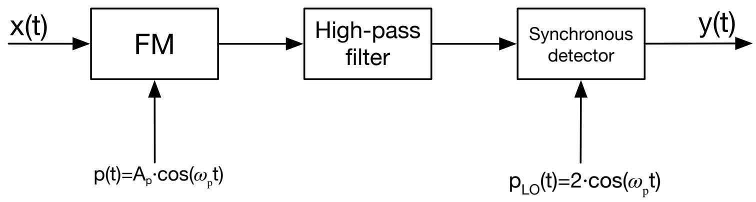
a) $D = 4,\ B_T = 2\pi \cdot 48krad/s$

b) $\omega_0 = \omega_1 + \omega_p,\ B \geq B_T$

c) $P_y = 0.81 \cdot \frac{A_p^2}{2}$

Problem 2.8

The signal $x(t) = cos(\omega_1 t) + cos(\omega_2 t)$ FM modulates the carrier $p(t) = A_p \cdot cos(\omega_p t)$. The modulated signal goes through a high pass filter with cutoff frequency $2\pi \cdot 350 krad/s$, whose output signal is fed to a synchronous detector where the local oscillator is adjusted to the carrier frequency, following the expression given by $p_{OL}(t)$ (see Data and Figure).



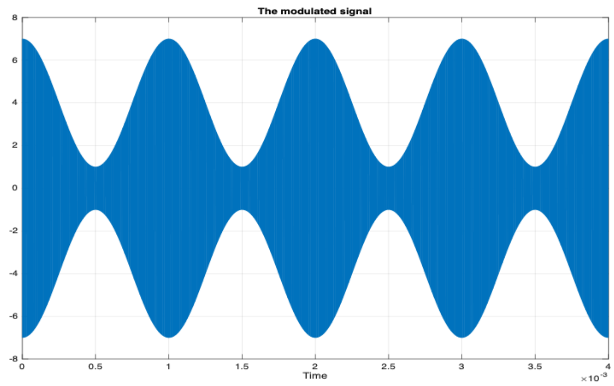
Calculate the output signal $y(t)$ as a function of A_p.
Datos:

- $\omega_1 = 2\pi \cdot 64 krad/s$
- $\omega_2 = 2\pi \cdot 128 krad/s$
- $\omega_d = 2\pi \cdot 2 krad/s \cdot V$
- $\omega_p = 2\pi \cdot 400 krad/s$
- $p_{OL}(t) = 2 \cdot cos(\omega_p t)$

Results for problem
$y(t) = A_p \cdot \left[1 + \left(\frac{\omega_d}{2\omega_1}\right) cos(\omega_1 t) + \left(\frac{\omega_d}{2\omega_2}\right) cos(\omega_2 t)\right]$

Problem 2.9

A given $1kHz$ frequency tone $x(t)$, with a $1V$ amplitude, DSB-AM modulates a $1MHz$ carrier $c(t)$. The following figure shows the modulated signal $y(t)$ characterized by a maximum value of $7V$, and a minimum value of $1V$.



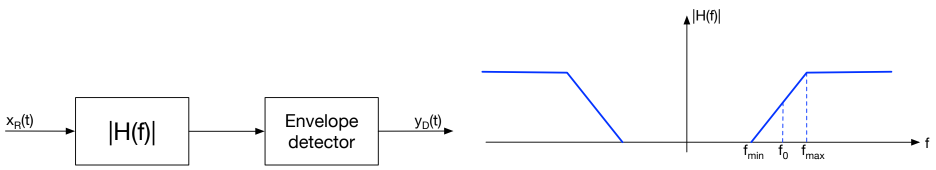
a) State the type of modulation and determine the modulation index as well as the carrier's amplitude (with its units).

b) Plot the modulated signal, $y(t)$, spectrum showing in the plot the specific amplitude and frequency values.

At this point, we want to modulate the carrier $c(t)$ in frequency (FM) using the tone $x(t)$. The FM modulator has a frequency deviation of $15kHz$.

c) Plot, roughly, the time-domain signal obtained when the tone x(t) modulates in frequency (FM) the same carrier $c(t)$. Please provide the envelope's amplitude range value.

d) Is it possible to obtain the modulated signal using the following scheme?



RESULTS FOR PROBLEM

a) It is an AM modulation with $m = 0.75$ y $A_p = 4V$.

b) $Y(\omega) = 4\pi\left[\delta(\omega-\omega_p)+\delta(\omega+\omega_p)\right]+$
$+\left(\frac{3\pi}{2}\right)\cdot\left[\delta(\omega-\omega_p-\omega_m)+\delta(\omega-\omega_p+\omega_m)+\delta(\omega+\omega_p-\omega_m)+\delta(\omega+\omega_p+\omega_m)\right]$

c) $[-A_p, A_p]$

d) Yes, it is possible.

References

[Haykin2001] Simon Haykin. Communication Systems, 4th Ed. John Wiley and Sons, 2001.