

## Exercises. Lesson 6

### Bandpass digital transmission

#### Problem 6.1

Calculate the expected value of the number of error bits during a day for the coherent BPSK receiver described below, under continuous operation. The data rate is  $5000 \text{ bits/s}$ . The input digital signals are  $s_1(t) = A \cdot \cos(\omega_p t)$  and  $s_2(t) = -A \cdot \cos(\omega_p t)$ , where  $A = 1 \text{ mV}$ , and the unilateral noise power spectral density is  $N_0 = 10^{-11} \text{ W/Hz}$ .

RESULTS FOR PROBLEM

2338 bits

#### Problem 6.2

A coherent BPSK system operating continuously produces errors at an average rate of 100 errors per day. The data rate is  $1000 \text{ bits/s}$ . The unilateral noise power spectral density is  $N_0 = 10^{-10} \text{ W/Hz}$ .

- If the system is ergodic, which is the average error probability?
- If the average received power is adjusted to  $10^{-6} \text{ W}$ , would this value be enough to keep the error probability calculated in a)?

RESULTS FOR PROBLEM

- $1.16 \cdot 10^{-6}$
- No

### Problem 6.3

The signal component of a coherent PSK system is defined by the expression

$$s(t) = A_c k \sin(\omega_p t) \pm A_c \sqrt{1 - k^2} \cos(\omega_p t)$$

where  $0 \leq t < T_b$ , and the plus sign corresponds to the 1 symbol, and the minus sign corresponds to the 0 one. The first term on the right hand side of the equation represents a carrier component, included to improve the synchronization between transmitter and receiver. Solve this:

- Plot the constellation of the signals described; what can be said about this diagram?
- Show that, in presence of zero-mean additive white Gaussian noise with power spectral density  $N_0/2$ , the average error probability is

$$P_e = Q \left( \sqrt{\frac{2E_b}{N_0} (1 - k^2)} \right)$$

with  $E_b = \frac{1}{2} A_c^2 T_b$

- Assume that 10% of the transmitted power is located in the carrier component. Determine the value of  $E_b/N_0$  required to obtain an error probability of  $10^{-4}$ .
- Compare this  $E_b/N_0$  value with the one required in a conventional PSK system with the same error probability.

#### RESULTS FOR PROBLEM

- PSK constellation
- Demonstration
- $\frac{E_b}{N_0} = 8.02$
- $\frac{E_b}{N_0} = 7.22$

### Problem 6.4

We want to compare two data transmission bandpass systems. One of them employs 16-PSK, the other, 16-QAM. Both systems have to provide an average symbol error probability of  $10^{-3}$ . Compare the signal-to-noise requirements of said systems. RESULTS FOR PROBLEM

$$\Delta \left( \frac{E_s}{N_0} \right) = 3.68 dB$$

## Problem 6.5

If the performance criterion of a system is the bit error probability, which one of the following modulation schemes would be chosen to operate in an AWGN channel? Show the calculations.

- Coherent binary orthogonal FSK with  $E_b/N_0 = 13dB$ .
- Coherent binary PSK with  $E_b/N_0 = 8dB$ .

### RESULTS FOR PROBLEM

Coherent binary orthogonal FSK