REFEREE REPORT: ALGEBRAIC AND SYMPLECTIC CURVES OF DEGREE 8

The article is a detailed study of a class of singular plane curves, where there is the potential to find differences between the symplectic and algebro-geometric categories. Although the author does not answer all of the questions one might like to learn about these curves, the article is a good record of the current progress and a strong display of many techniques that could be applied. The author has incredible expertise in these techniques, so it is quite valuable to see them in action on this interesting problem.

Because this article may be of significant interest to mathematicians from different areas (e.g. symplectic geometry, algebraic geometry, singularity theory), and because the author has so much expertise to share with this audience, it may be valuable to add a bit more details and exposition in some of the proofs and explanations. Below I will list some places where the author may be able to make such additions, as well as some minor typos/corrections.

(1) p. 1 At the end of the first paragraph it says symplectic curves "can be constructed by suitable deformations of ... algebraic plane curves". While "suitable deformations" might be sufficiently vague for this to not be incorrect, it might be better to say that symplectic curves can be *locally* constructed by algebraic plane curves, or to say that symplectic curves can be globally constructed via braid monodromy/braid factorization.

(2) p. 2 It might be useful to state how the equation $v^4+4(1+u)v^3+18uv^2-27u^2=0$ is related to the symmetric equation for the deltoid (choice of affine coordinates). Additionally, in Figure 1 on the right, is there a strand missing? The curve has degree 4, but the figure makes it seem like a generic line intersects the curve in 3 points. An additional strand would also be more consistent with the result of Proposition 1.2 which describes the braid monodromy in B_4.

(3) p. 3 Proposition 1.2. It could be helpful to describe which loops represent the generators $c_1,\dots,c_4,\ell_1,\ell_2,\ell_\infty$ for those who are not immediately familiar with such computations.

(4) p. 5 Proof of Corollary 3.2, the last sentence says "An easy computation gives the result." Is this a computer computation or a by hand computation? Is there a quick way to see this from (R1)-(R10)?

(5) It might be useful to briefly specify the definition/model of an $\mathbb{E}_6$ singularity (particularly in section 4)

(6) Section 5. There seem to be some facts about finite order projective automorphisms (classification?) that are assumed here that are not immediately obvious to the reader outside of the area. Could you briefly summarize the classification of types of projective involutions, and other automorphisms of finite order (in terms of fixed points)?

(7) Section 5. A bit more details as to how Bézout Theorem is being applied in the proofs of Lemma 5.1 and 5.2 would be helpful. Particularly since intersection numbers in orbifolds are a bit more nonstandard.

(8) The last sentence of the proof of Lemma 5.2 "It is clear that the curve contains two of them..." could use more specific explanation.

(9) In the proof of Lemma 5.3, the case where $n=4$, the author means to rule out the fixed points being singular. The justification for the order 2 fixed points is related to the fact that they are in a line of fixed points for ϕ^2, and ϕ^2 is an involution, but it seems by Lemma 5.1 that it is possible to have singular points in the line of fixed points of an involution. Could you explain this reasoning? Additional details for $n=5$ and $n=6$ would be helpful as well.

(10) Lemma 6.1 (not two of them in the same curve of ω-degree 1)–does this follow from Bezout?

(11) Section 6, paragraph before Remark 6.2. It is stated that the curve tangent to the cusps is $z=bxy$, but the cusps are placed at $[1:1:0]_\omega$ and $[a_1:1:1]_\omega$ which do not seem to satisfy this equation. Additionally, it is not clear why this curve can be fixed via automorphisms.

(12) Section 10. The author states that the Cremona transformation described corresponds to the 2-dimensional projective system of quintics having double points at six fixed points. Could you spell out this correspondence a bit more?

(13) We have noted some typos through annotations of the pdf of the article.