Actividad_2_Enrique_Corimayo

November 20, 2024

1 Caso de Estudio 1. Sistema de tres variables de estado

[1]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

[2]:
```python
# Definimos Parámetros de Motor
Laa = 5e-3    # Inductance (H)
J = 0.004     # Moment of inertia (kg*m^2)
Ra = 0.2      # Resistance (ohms)
Bm = 0.005    # Damping coefficient (N*m*s/rad)
Ki = 6.5e-5   # Torque constant (N*m/A)
Km = 0.055    # Back emf constant (V*s/rad)
```

[7]:
```python
# Definimos las matrices del sistema
A = np.array([
    [-Ra / Laa, -Km / Laa, 0],
    [Ki / J, -Bm / J, 0],
    [0, 1, 0]
])

B = np.array([
    [1 / Laa, 0],
    [0, -1 / J],
    [0, 0]
])

C = np.array([[0, 0, 1]])

A, B, C
```

[7]:
```
(array([[-4.000e+01, -1.100e+01,  0.000e+00],
        [ 1.625e-02, -1.250e+00,  0.000e+00],
        [ 0.000e+00,  1.000e+00,  0.000e+00]]),
 array([[ 200.,    0.],
        [   0., -250.],
        [   0.,    0.]]),
```

```
array([[0, 0, 1]]))
```

1.1 LQR

[8]:
```
# Matriz Q
Q = np.diag([0.1, 0.1, 0.1, 100000])

print(Q)
```

```
[[1.e-01 0.e+00 0.e+00 0.e+00]
 [0.e+00 1.e-01 0.e+00 0.e+00]
 [0.e+00 0.e+00 1.e-01 0.e+00]
 [0.e+00 0.e+00 0.e+00 1.e+05]]
```

[9]:
```
# Matriz R
R = np.array([[2000]])

# Ampliamos la matriz A para agregar un integrador
A_aug = np.block([
    [A, np.zeros((3, 1))],
    [-C, np.array([[0]])]
])

print("R:", R)
print("Aamp:", A_aug)
```

```
R: [[2000]]
Aamp: [[-4.000e+01 -1.100e+01  0.000e+00  0.000e+00]
 [ 1.625e-02 -1.250e+00  0.000e+00  0.000e+00]
 [ 0.000e+00  1.000e+00  0.000e+00  0.000e+00]
 [ 0.000e+00  0.000e+00 -1.000e+00  0.000e+00]]
```

[10]:
```
# Ampliamos la matriz B
B_aug = np.vstack([B[:, [0]], [0]])

print("Bamp:", B_aug)
```

```
Bamp: [[200.]
 [  0.]
 [  0.]
 [  0.]]
```

[11]:
```
C_aug = np.hstack((C, [[0]]))
```

[12]:
```
# Construcción del Hamiltoniano para el cálculo de la solución de Riccati
H_upper = np.hstack((A_aug, -B_aug @ np.linalg.inv(R) @ B_aug.T))
H_lower = np.hstack((-Q, -A_aug.T))
```

```python
H = np.vstack((H_upper, H_lower))
print("Hamilitoniano: ",H)
```

```
Hamilitoniano:  [[-4.000e+01 -1.100e+01  0.000e+00  0.000e+00 -2.000e+01
0.000e+00
   0.000e+00  0.000e+00]
 [ 1.625e-02 -1.250e+00  0.000e+00  0.000e+00  0.000e+00  0.000e+00
   0.000e+00  0.000e+00]
 [ 0.000e+00  1.000e+00  0.000e+00  0.000e+00  0.000e+00  0.000e+00
   0.000e+00  0.000e+00]
 [ 0.000e+00  0.000e+00 -1.000e+00  0.000e+00  0.000e+00  0.000e+00
   0.000e+00  0.000e+00]
 [-1.000e-01 -0.000e+00 -0.000e+00 -0.000e+00  4.000e+01 -1.625e-02
  -0.000e+00 -0.000e+00]
 [-0.000e+00 -1.000e-01 -0.000e+00 -0.000e+00  1.100e+01  1.250e+00
  -1.000e+00 -0.000e+00]
 [-0.000e+00 -0.000e+00 -1.000e-01 -0.000e+00 -0.000e+00 -0.000e+00
  -0.000e+00  1.000e+00]
 [-0.000e+00 -0.000e+00 -0.000e+00 -1.000e+05 -0.000e+00 -0.000e+00
  -0.000e+00 -0.000e+00]]
```

[13]:

```python
# Calcular valores y vectores propios del Hamiltoniano
eigvals, eigvecs = np.linalg.eig(H)
eigvecs = eigvecs[:, np.real(eigvals) < 0]  # Seleccionar solo valores propios␣
 ↪negativos
print("Eigenvalues: ",eigvals)
print("Eigenvectors: ",eigvecs)
```

```
Eigenvalues:  [-40.0203818 +0.j          40.0203818 +0.j
  -1.29986953+0.j          -0.43815013+0.49978542j
  -0.43815013-0.49978542j   0.43815013+0.49978542j
   0.43815013-0.49978542j   1.29986953+0.j        ]
Eigenvectors:  [[-9.99998861e-01+0.00000000e+00j -5.19633376e-05+0.00000000e+00j
  -8.26477034e-05-2.46039664e-05j -8.26477034e-05+2.46039664e-05j]
 [ 4.19133904e-04+0.00000000e+00j  1.69322693e-05+0.00000000e+00j
  -1.41949234e-06+3.81384699e-07j -1.41949234e-06-3.81384699e-07j]
 [-1.04730111e-05+0.00000000e+00j -1.30261299e-05+0.00000000e+00j
   1.83936849e-06+1.22767246e-06j  1.83936849e-06-1.22767246e-06j]
 [-2.61691935e-07+0.00000000e+00j -1.00211057e-05+0.00000000e+00j
   4.35409079e-07+3.29860353e-06j  4.35409079e-07-3.29860353e-06j]
 [-1.24961227e-03+0.00000000e+00j  9.12366491e-05+0.00000000e+00j
   1.63650687e-04+5.05244655e-05j  1.63650687e-04-5.05244655e-05j]
 [ 3.34476219e-04+0.00000000e+00j  2.32200495e-01+0.00000000e+00j
   4.09307521e-01+1.20848375e-01j  4.09307521e-01-1.20848375e-01j]
 [ 1.63129215e-05+0.00000000e+00j  5.93082875e-01+0.00000000e+00j
   7.53171101e-01+0.00000000e+00j  7.53171101e-01-0.00000000e+00j]
 [-6.53896647e-04+0.00000000e+00j -7.70931658e-01+0.00000000e+00j
```

```
-3.30001833e-01+3.76424056e-01j -3.30001833e-01-3.76424056e-01j]]
```

[14]:
```python
# Separar matrices para resolver Riccati
X1 = eigvecs[:4, :]
X2 = eigvecs[4:, :]
P = np.real(X2 @ np.linalg.inv(X1))
print("P", P)
```

```
P [[ 4.73275792e-02  1.14800133e+02  1.96430561e+02 -7.07106781e+01]
 [ 1.14800133e+02  2.86046756e+05  4.90611902e+05 -1.78175903e+05]
 [ 1.96430561e+02  4.90611902e+05  8.88254801e+05 -3.85849602e+05]
 [-7.07106781e+01 -1.78175903e+05 -3.85849602e+05  2.77794763e+05]]
```

[15]:
```python
# Ganancia del controlador LQR
K_lqr = np.linalg.inv(R) @ B_aug.T @ P
K_feedback = K_lqr[0, :-1]
K_integral = K_lqr[0, -1]
print("K_lqr", K_lqr)
print("K_feedback", K_feedback)
print("K_integral", K_integral)
```

```
K_lqr [[ 4.73275792e-03  1.14800133e+01  1.96430561e+01 -7.07106781e+00]]
K_feedback [4.73275792e-03 1.14800133e+01 1.96430561e+01]
K_integral -7.07106781186877
```

[16]:
```python
# Polos del sistema lazo cerrado lqr
poles,_ = np.linalg.eig(A_aug-B_aug*K_lqr)
poles
```

[16]:
```
array([-40.0203818 +0.j        ,  -1.29986953+0.j        ,
        -0.43815013+0.49978542j,  -0.43815013-0.49978542j])
```

[17]:
```python
# determinamos el tiempo del euler
# Polo mas rapido
lambda_val = np.min(poles)
lambda_val
```

[17]:
```
(-40.02038179564798+0j)
```

[18]:
```python
# tiempo de euler
tr = np.log(0.95)/lambda_val
tr
```

[18]:
```
(0.0012816792865561436-0j)
```

[19]:
```python
# Definimos el paso de simulación
h = 1e-4
simTime = 50  # Tiempo de Simulación
```

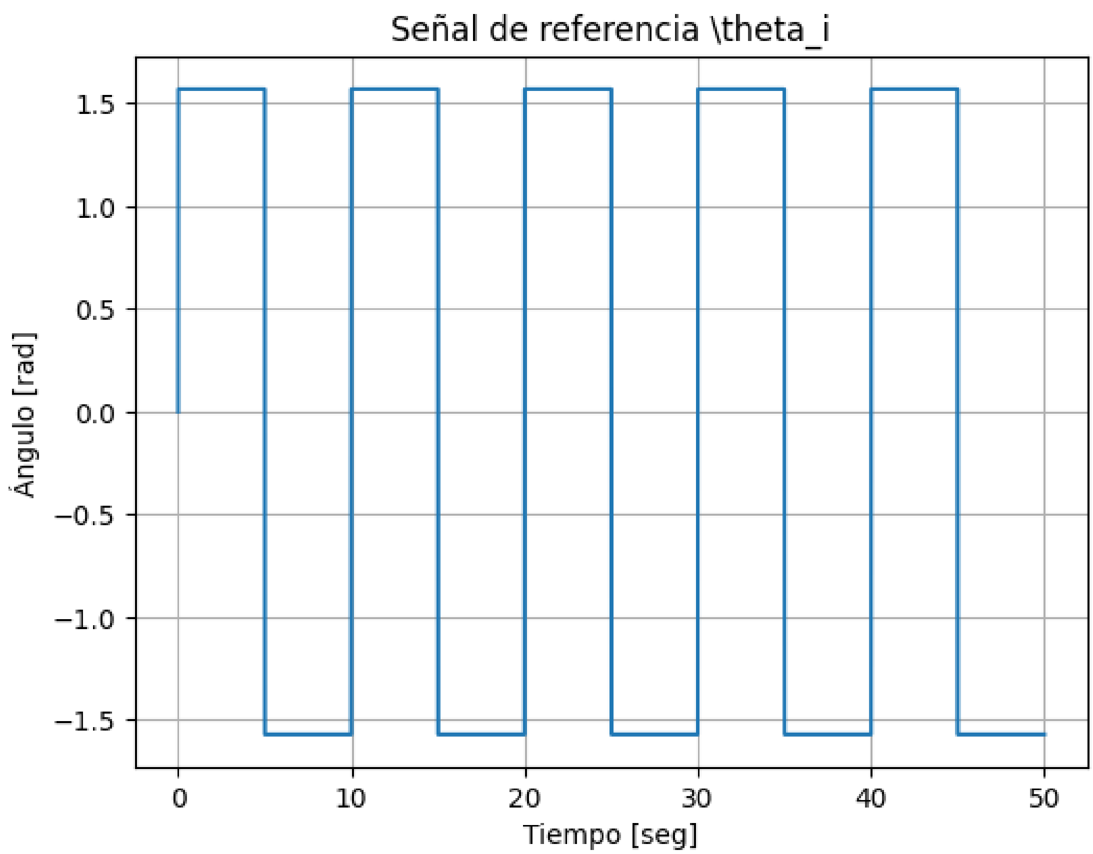
```
# Creamos el vector de tiempo
t = np.arange(0, simTime, h)

print(t)
```

```
[0.00000e+00 1.00000e-04 2.00000e-04 … 4.99997e+01 4.99998e+01
 4.99999e+01]
```

[20]:
```
# Generamos la referencia q cambia cada 5 segundos
reference = (np.pi / 2) * np.sign(np.sin(2 * np.pi * (1 / 10) * t))

# Printing or using the reference as needed
print(reference)
```

```
[ 0.          1.57079633  1.57079633 … -1.57079633 -1.57079633
 -1.57079633]
```

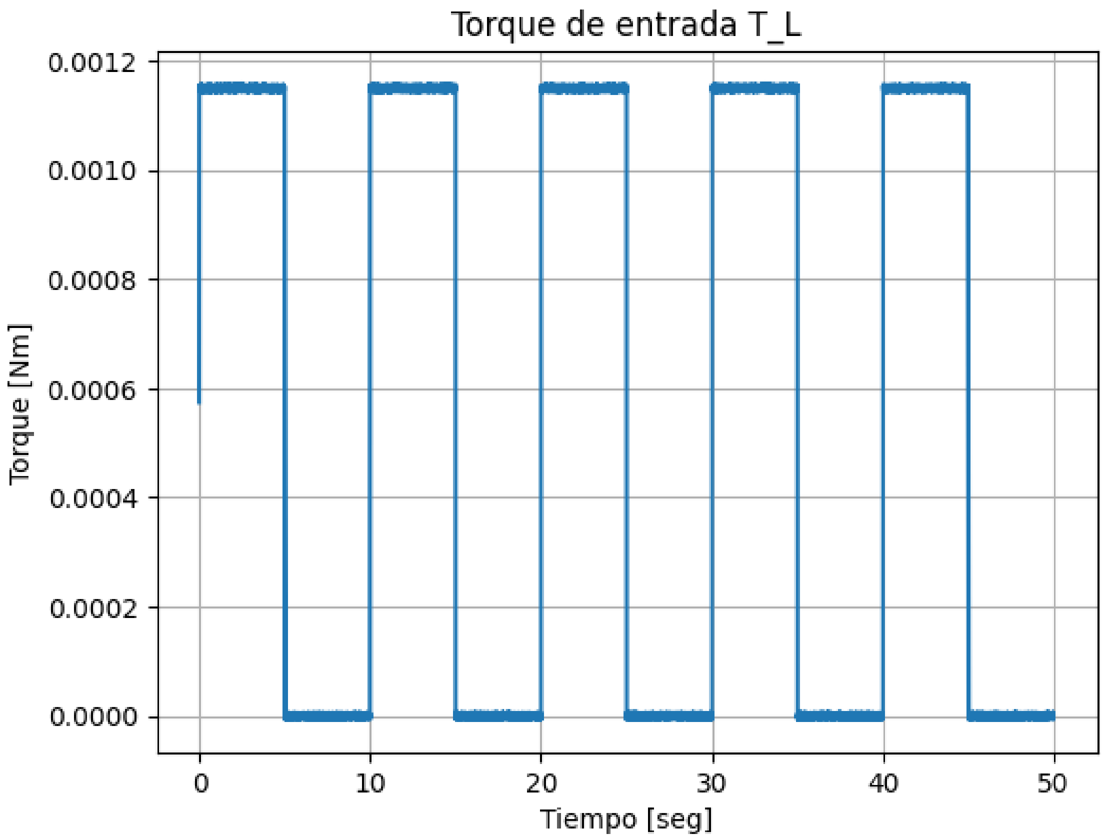
[21]:
```
# Graficamos la referencia
plt.plot(t, reference, linewidth=1.5)
plt.xlabel('Tiempo [seg]')
plt.ylabel('Ángulo [rad]')
plt.title('Señal de referencia \\theta_i')
plt.grid(True)
plt.show()
```



[22]:
```
# Generamos
torque = ((1.15e-3) / 2) * np.sign(np.sin(2 * np.pi * (1 / 10) * t)) + ((1.
    ↪15e-3) / 2)
# Add Gaussian noise with mean 0 and standard deviation 0.0001
noise = np.random.normal(0, 0.000002, size=torque.shape)
torque = torque + noise
# Printing or using the torque as needed
print(torque)
```

```
[ 5.73989032e-04  1.15054218e-03  1.15102704e-03 …  3.81096126e-07
  3.15733659e-06 -7.85982703e-07]
```

[23]:
```
# Plotting the torque signal
plt.plot(t, torque, linewidth=1.5)
plt.xlabel('Tiempo [seg]')
plt.ylabel('Torque [Nm]')
plt.title('Torque de entrada T_L')
plt.grid(True)
plt.show()
```



[24]:

```
# Inicializar variables
ia = np.zeros(len(t))
theta = np.zeros(len(t))
omega = np.zeros(len(t))
```

[25]:

```
# vector de estado y condiciones iniciales
stateVector = np.array([ia[0], omega[0], theta[0]])

print("State Vector:", stateVector)
```

```
State Vector: [0. 0. 0.]
```

[26]:

```
# Valores iniciales
xop = np.array([0, 0, 0]) # Punto de operación
x = np.array([ia[0], omega[0], theta[0]])  # Vector de estado
zeta = np.zeros(len(t))  # zeta
integ = np.zeros(len(t))
u = np.zeros(len(t))

zeta[0] = 0
```

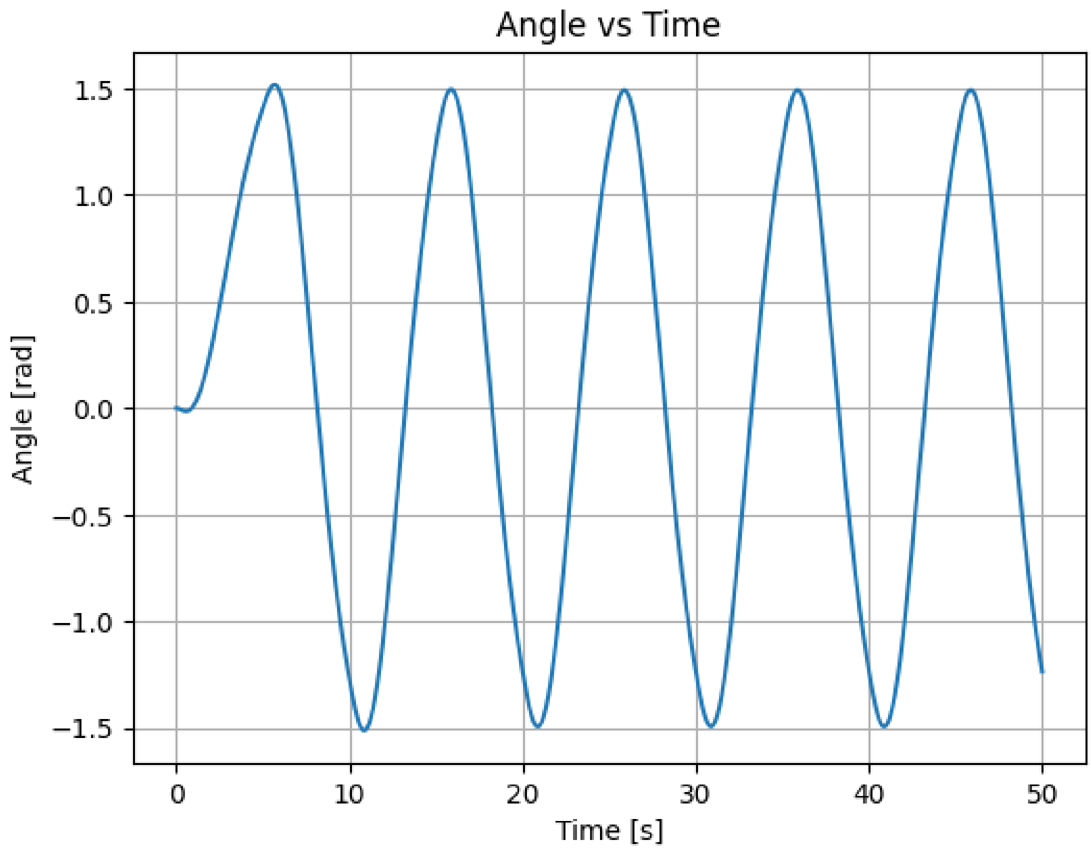
```
integ[0] = zeta[0]
```

1.1.1 Sin Observador

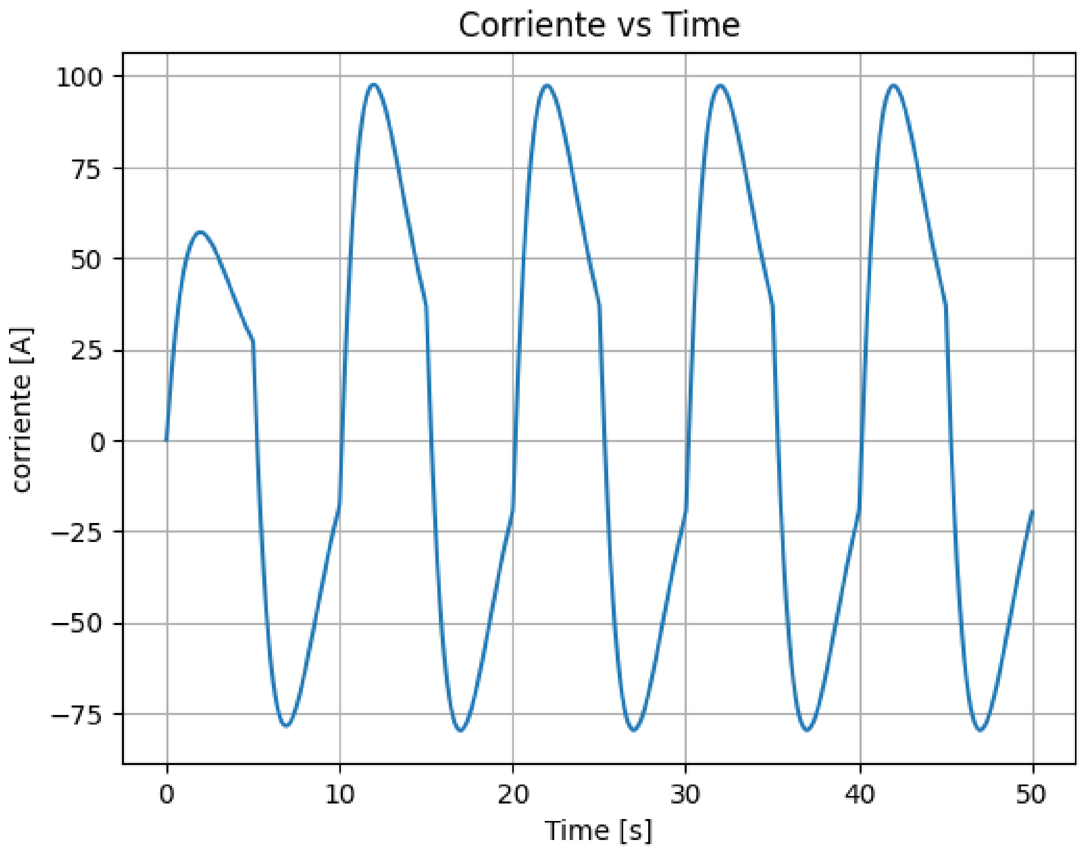
[27]:
```
# Simulación
for i in range(1, len(t)):
    zetaP = reference[i] - np.dot(C, stateVector)  # Error de referencia
    zeta[i] = integ[i - 1] + zetaP * h  # Integral de zeta
    u[i] = -np.dot(K_feedback, stateVector) - K_integral * zeta[i]  # Control␣
 ↪input
    ia[i] = x[0]
    omega[i] = x[1]
    theta[i] = x[2]

    # Derivadas de estado
    x1P = -Ra * x[0] / Laa - Km * x[1] / Laa + u[i] / Laa
    x2P = Ki * x[0] / J - Bm * x[1] / J - torque[i] / J
    x3P = x[1]

    # Actulizar estado
    xP = np.array([x1P, x2P, x3P])
    x = x + h * xP  # Euler
    stateVector = np.array([ia[i], omega[i], theta[i]])

    # Actualizar integrador
    integ[i] = zeta[i]
```

```
<ipython-input-27-89ba30c5f486>:4: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  zeta[i] = integ[i - 1] + zetaP * h  # Integral de zeta
```

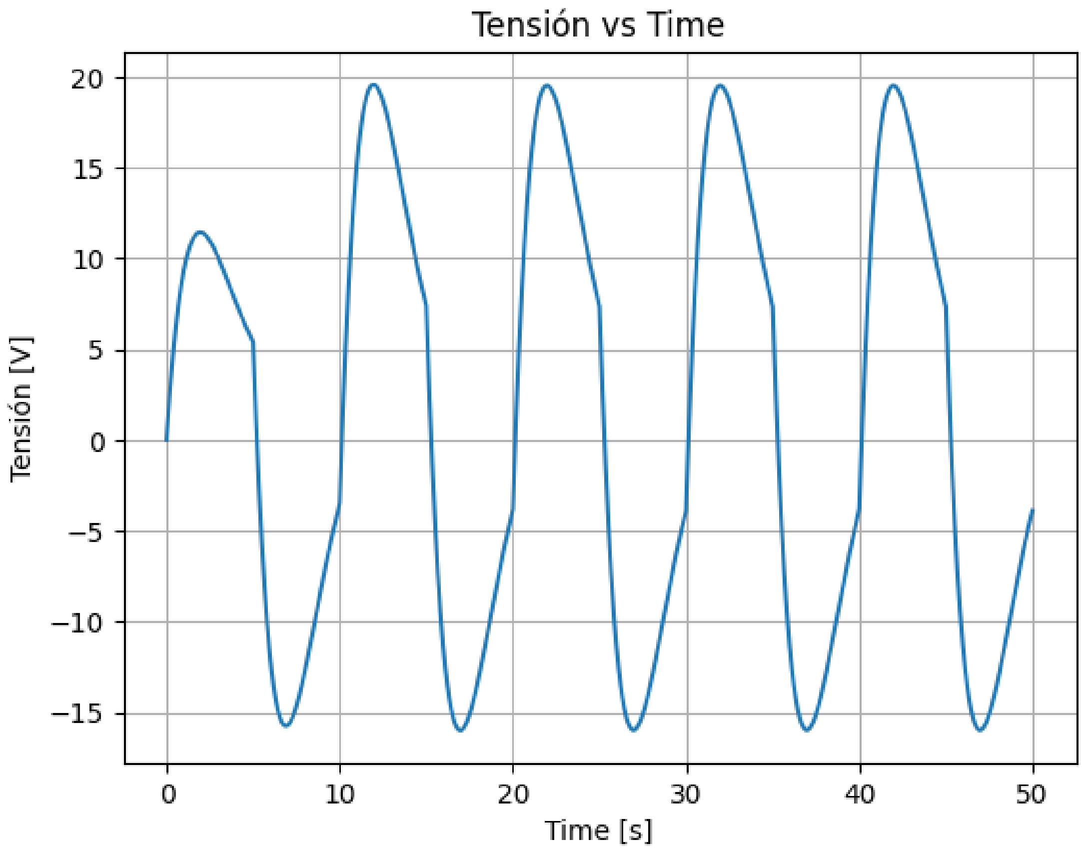
[28]:
```
plt.plot(t, theta)
plt.xlabel('Time [s]')
plt.ylabel('Angle [rad]')
plt.title('Angle vs Time')
plt.grid(True)
plt.show()
```



[29]:
```
plt.plot(t, ia)
plt.xlabel('Time [s]')
plt.ylabel('corriente [A]')
plt.title('Corriente vs Time')
plt.grid(True)
plt.show()
```



[30]:
```
plt.plot(t, u)
plt.xlabel('Time [s]')
plt.ylabel('Tensión [V]')
plt.title('Tensión vs Time')
plt.grid(True)
plt.show()
```



Observación Podemos observar que con el LQR la tensión de entrada es menor a 24V

1.1.2 Con Observador

[31]:
```
Ao = np.transpose(A)
Bo = np.transpose(C)
Co = np.transpose(B[:,[0]])
```

[32]:
```
Qo = np.diag([100, 0.01, 1000])
```

[33]:
```
Ro = np.array([[1]])
```

[34]:
```
# Construcción del Hamiltoniano para el cálculo de la solución de Riccati
H_upper_o = np.hstack((Ao, -Bo @ np.linalg.inv(Ro) @ Bo.T))
H_lower_o = np.hstack((-Qo, -Ao.T))
H_o = np.vstack((H_upper_o, H_lower_o))
print("Hamilitoniano: ",H_o)
```

```
Hamilitoniano:  [[-4.000e+01  1.625e-02  0.000e+00  0.000e+00  0.000e+00
0.000e+00]
```

```
 [-1.100e+01 -1.250e+00  1.000e+00  0.000e+00  0.000e+00  0.000e+00]
 [ 0.000e+00  0.000e+00  0.000e+00  0.000e+00  0.000e+00 -1.000e+00]
 [-1.000e+02 -0.000e+00 -0.000e+00  4.000e+01  1.100e+01 -0.000e+00]
 [-0.000e+00 -1.000e-02 -0.000e+00 -1.625e-02  1.250e+00 -0.000e+00]
 [-0.000e+00 -0.000e+00 -1.000e+03 -0.000e+00 -1.000e+00 -0.000e+00]]
```

[35]:

```python
# Calcular valores y vectores propios del Hamiltoniano
eigvals_o, eigvecs_o = np.linalg.eig(H_o)
eigvecs_o = eigvecs_o[:, np.real(eigvals_o) < 0]  # Seleccionar solo valores␣
↪propios negativos
print("Eigenvalues: ",eigvals_o)
print("Eigenvectors: ",eigvecs_o)
```

```
Eigenvalues:  [-39.99538655  39.99538655 -31.62277644  -1.25461745   1.25461745
  31.62277644]
Eigenvectors:  [[-6.15097649e-01 -2.02002481e-06 -4.19401435e-04]
 [-1.74629141e-01 -1.04136612e-03 -9.99991942e-01]
 [-5.75788680e-07  3.16069601e-02  3.99857695e-06]
 [-7.68868927e-01 -2.77150323e-06  4.78726690e-05]
 [-3.45260710e-04 -3.18156823e-07 -3.99228293e-03]
 [-2.30288908e-05  9.99499833e-01  5.01668442e-06]]
```

[36]:

```python
# Separar matrices para resolver Riccati
X1o = eigvecs_o[:3, :]
X2o = eigvecs_o[3:, :]
Po = np.real(X2o @ np.linalg.inv(X1o))
print("Po", Po)
```

```
Po [[ 1.25015735e+00 -5.72195175e-04 -2.66403040e-05]
 [-5.72195175e-04  3.99255556e-03  1.21441588e-04]
 [-2.66403040e-05  1.21441588e-04  3.16227804e+01]]
```

[37]:

```python
# Ganancia del controlador LQR
K_lqr_o = np.linalg.inv(Ro) @ Bo.T @ Po
K_feedback_o = K_lqr_o[0, :-1]
K_integral_o = K_lqr_o[0, -1]
print("K_lqr_o", K_lqr_o)
```

```
K_lqr_o [[-2.66403040e-05  1.21441588e-04  3.16227804e+01]]
```

[38]:

```python
obsStateVector = np.array([ia[0], omega[0], theta[0]])

xObs = np.array([0, 0, 0])
```

[39]:

```python
ia0 = np.zeros(len(t))
omega0 = np.zeros(len(t))
theta0 = np.zeros(len(t))
y0 = np.zeros(len(t))
```

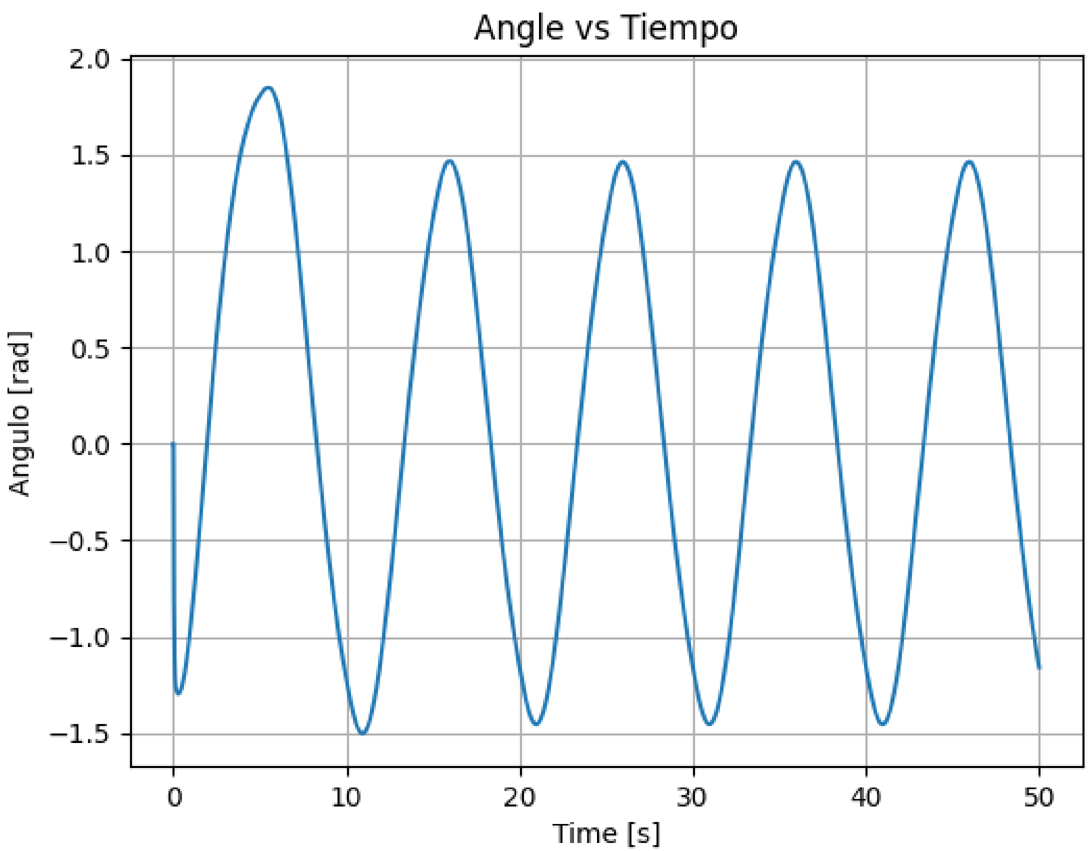
```python
y = np.zeros(len(t))
u = np.zeros(len(t))
zeta = np.zeros(len(t))

obsStateVector = np.array([ia[0], omega[0], theta[0]])
xObs = np.array([0, 0, 0])
```

[40]:
```python
for i in range(1, len(t)):
    zetaP = reference[i] - np.dot(C_aug[0,:-1], stateVector) - C_aug[0,-1] *␣
 ↪integ[i - 1]
    zeta[i] = integ[i - 1] + zetaP * h
    u[i] = -np.dot(K_feedback, obsStateVector) - K_integral * zeta[i]

    ia[i] = x[0]
    omega[i] = x[1]
    theta[i] = x[2]

    x1P = -Ra * x[0] / Laa - Km * x[1] / Laa + u[i] / Laa
    x2P = Ki * x[0] / J - Bm * x[1] / J - torque[i] / J
    x3P = x[1]
    xP = np.array([x1P, x2P, x3P])

    x = x + xP * h

    iaO[i] = xObs[0]
    omegaO[i] = xObs[1]
    thetaO[i] = xObs[2]

    # Salidad del Observador y del sistema
    yO[i] = np.dot(C, obsStateVector)  # Observador
    y[i] = np.dot(C_aug[0,:-1], stateVector) + C_aug[0,-1] * integ[i - 1]  #␣
 ↪Sistema

    # Actualizar estado del observador
    xTP = np.dot(A, xObs) + np.dot(B[:, 0], u[i]) + np.dot(K_lqr_o, (y[i] -␣
 ↪yO[i]))
    xObs = xObs + xTP[0] * h  # Observador

    # Actualizar vector de estados
    stateVector = np.array([ia[i], omega[i], theta[i]])
    integ[i] = zeta[i]
    obsStateVector = np.array([iaO[i], omegaO[i], thetaO[i]])
```

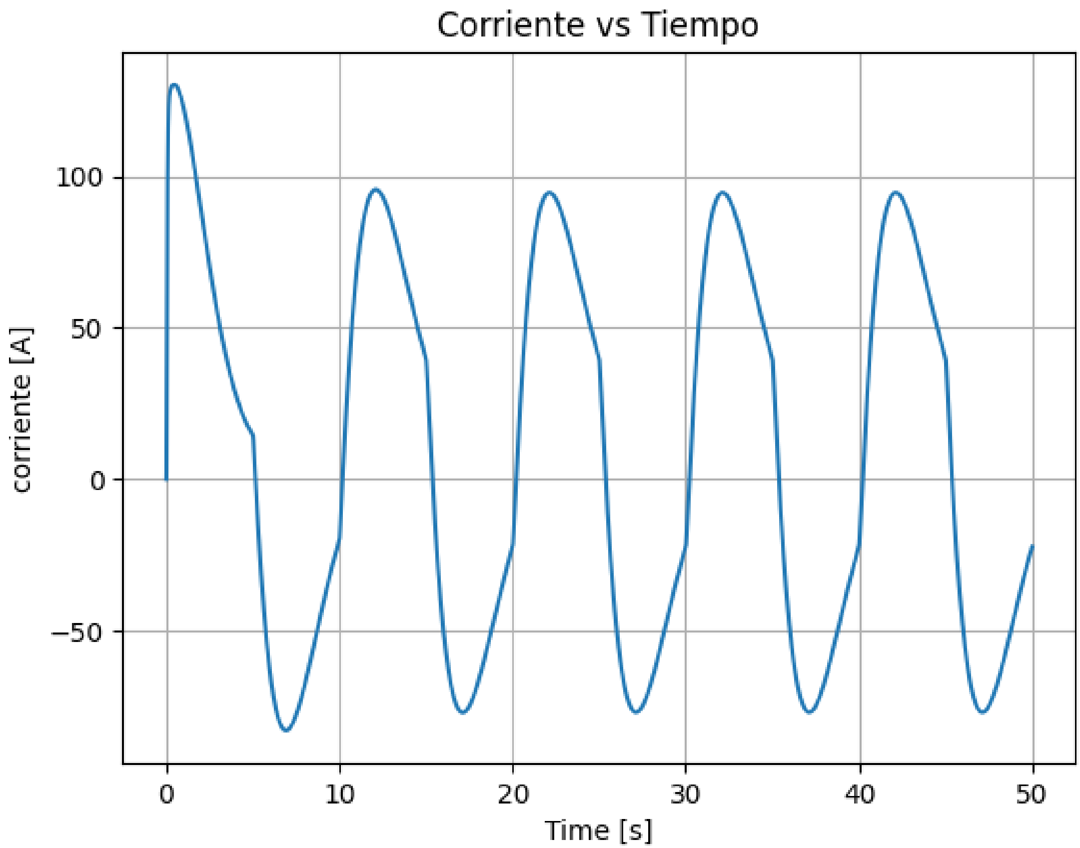
```
<ipython-input-40-569a03f3bf1a>:22: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
```

```
    y0[i] = np.dot(C, obsStateVector)  # Observador
```

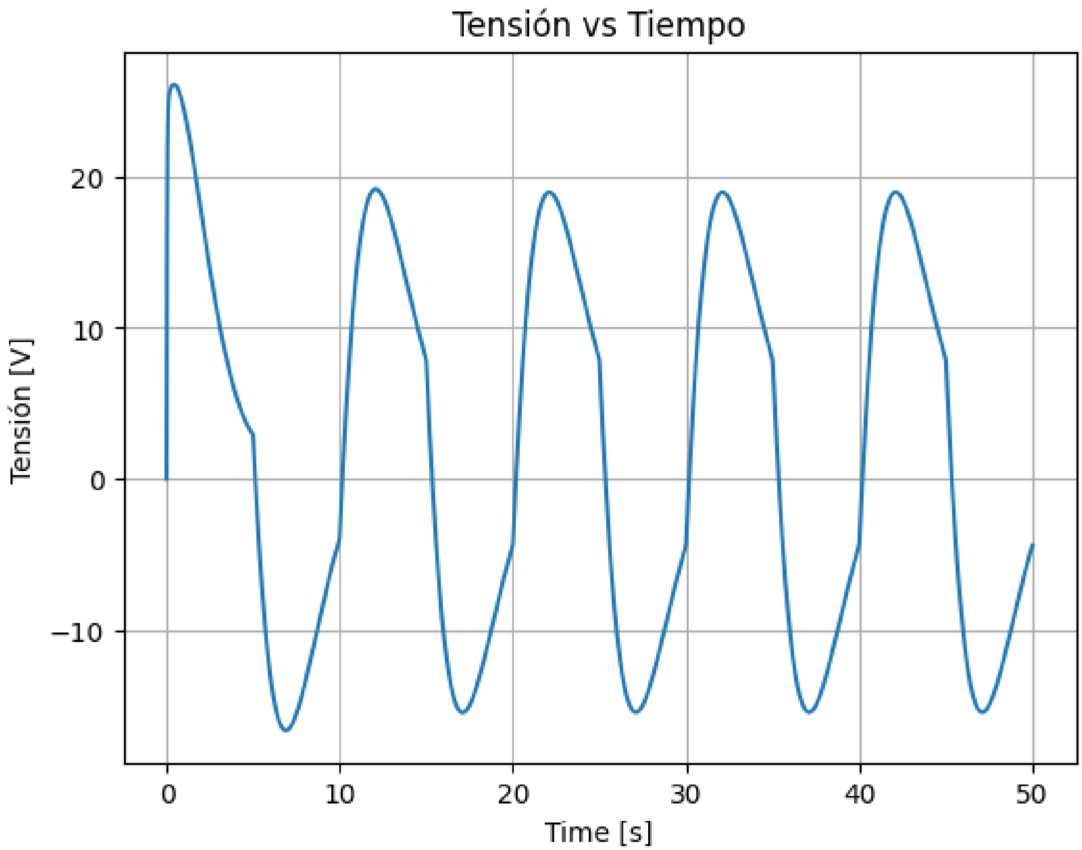
[44]:
```
plt.plot(t, theta0)
plt.xlabel('Time [s]')
plt.ylabel('Angulo [rad]')
plt.title('Angle vs Tiempo')
plt.grid(True)
plt.show()
```



[45]:
```
plt.plot(t, ia0)
plt.xlabel('Time [s]')
plt.ylabel('corriente [A]')
plt.title('Corriente vs Tiempo')
plt.grid(True)
plt.show()
```



[46]:
```
plt.plot(t, u)
plt.xlabel('Time [s]')
plt.ylabel('Tensión [V]')
plt.title('Tensión vs Tiempo')
plt.grid(True)
plt.show()
```



1.1.3 Actuador con no-linealidad

Aplicamos una no-linealidad en el actuador del tipo zona muerta

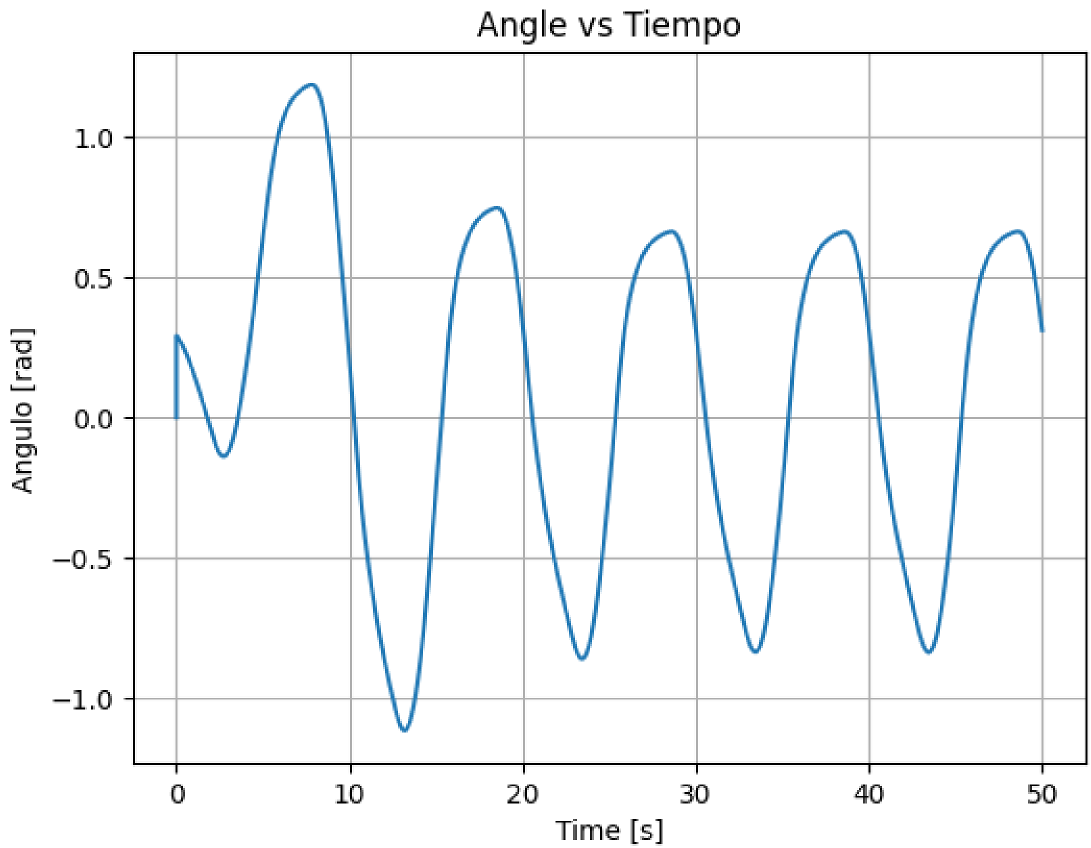
[59]:
```
dead_zone = 20
for i in range(1, len(t)):
    zetaP = reference[i] - np.dot(C_aug[0,:-1], stateVector) - C_aug[0,-1] *␣
 ↪integ[i - 1]
    zeta[i] = integ[i - 1] + zetaP * h
    u[i] = -np.dot(K_feedback, obsStateVector) - K_integral * zeta[i]
    if abs(u[i]) < dead_zone:
        u[i] = 0
    else:
        u[i] = np.sign(u[i]) * (abs(u[i]) - dead_zone)
    ia[i] = x[0]
    omega[i] = x[1]
    theta[i] = x[2]

    x1P = -Ra * x[0] / Laa - Km * x[1] / Laa + u[i] / Laa
    x2P = Ki * x[0] / J - Bm * x[1] / J - torque[i] / J
```

```
    x3P = x[1]
    xP = np.array([x1P, x2P, x3P])

    x = x + xP * h

    iaO[i] = xObs[0]
    omegaO[i] = xObs[1]
    thetaO[i] = xObs[2]

    # Salidad del Observador y del sistema
    yO[i] = np.dot(C, obsStateVector)  # Observador
    y[i] = np.dot(C_aug[0,:-1], stateVector) + C_aug[0,-1] * integ[i - 1]  #␣
↪Sistema

    # Actualizar estado del observador
    xTP = np.dot(A, xObs) + np.dot(B[:, 0], u[i]) + np.dot(K_lqr_o, (y[i] -␣
↪yO[i]))
    xObs = xObs + xTP[0] * h  # Observador

    # Actualizar vector de estados
    stateVector = np.array([ia[i], omega[i], theta[i]])
    integ[i] = zeta[i]
    obsStateVector = np.array([iaO[i], omegaO[i], thetaO[i]])
```

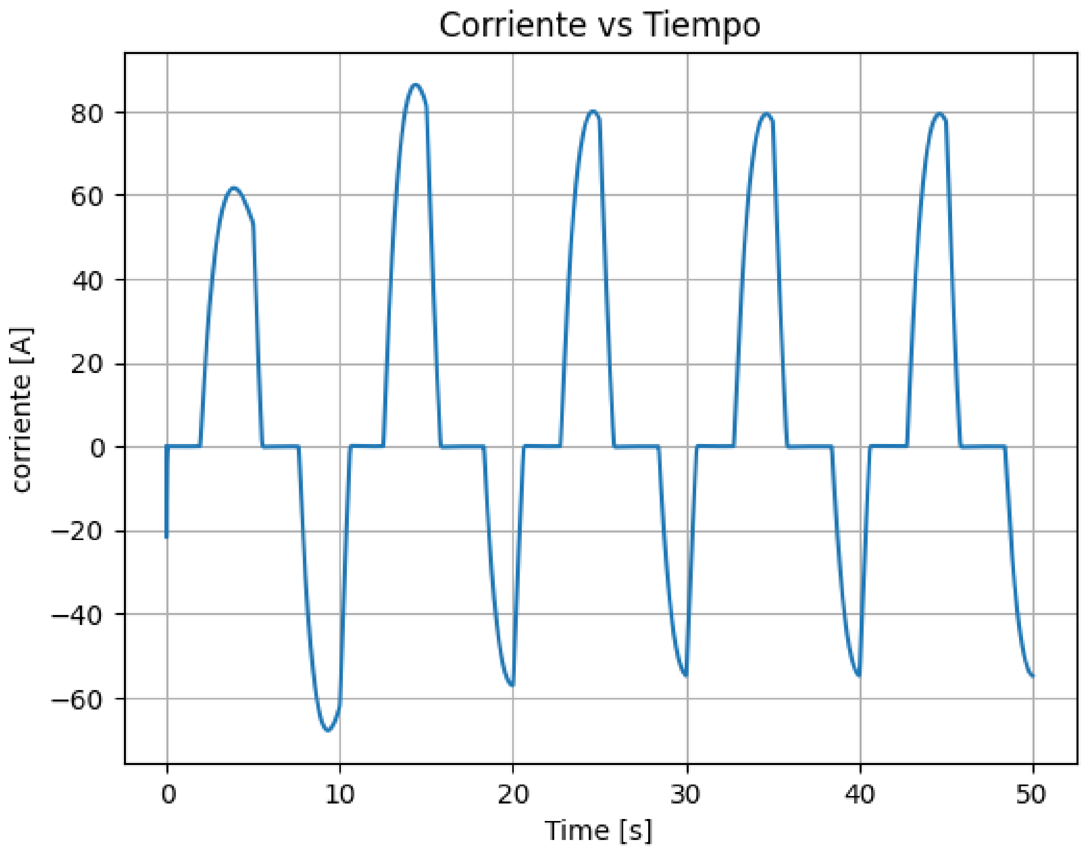
```
<ipython-input-59-48ea7f83a3e8>:26: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  yO[i] = np.dot(C, obsStateVector)  # Observador
```

[60]:

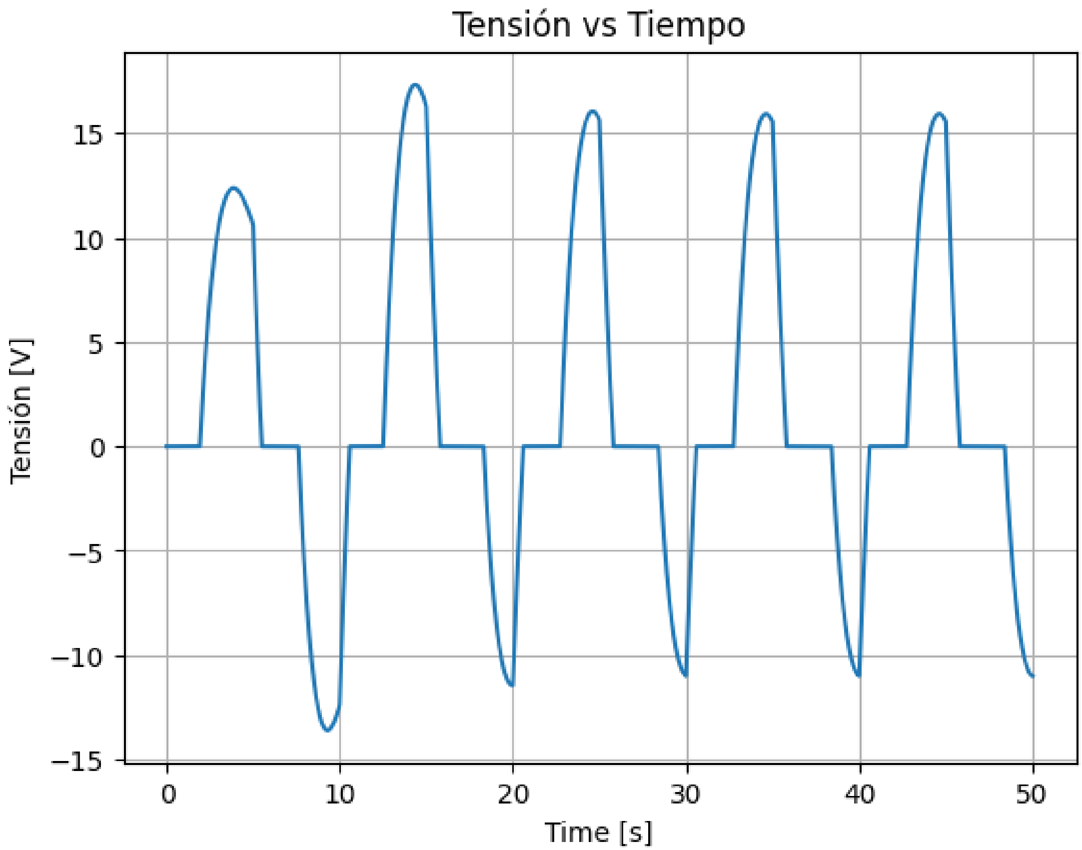
```
plt.plot(t, thetaO)
plt.xlabel('Time [s]')
plt.ylabel('Angulo [rad]')
plt.title('Angle vs Tiempo')
plt.grid(True)
plt.show()
```



[61]:
```
plt.plot(t, ia0)
plt.xlabel('Time [s]')
plt.ylabel('corriente [A]')
plt.title('Corriente vs Tiempo')
plt.grid(True)
plt.show()
```



[62]:
```
plt.plot(t, u)
plt.xlabel('Time [s]')
plt.ylabel('Tensión [V]')
plt.title('Tensión vs Tiempo')
plt.grid(True)
plt.show()
```



Con una no linealidad de 20V en la zona muerta del actuador el sistema tiene problemas para llegar a la referencia

Observación La tensión de entrada esta por debajo de los 24V

1.1.4 Conclusiones

Con el controlador LQR fuimos capaces de regular la tensión de entrada para que no sobrepasase el límite de 24V. Si lo comparamos con el PID de la actividad 1 este tiene mucho mas tensión al principio superando los 30V.

[79]:
```
# Clear all variables (manually in Python)
# Usually, you can use the following to clear the namespace
from IPython import get_ipython
get_ipython().magic('reset -sf')  # Clear all variables in an interactive␣
 ↪session

# Close all figures
import matplotlib.pyplot as plt
plt.close('all')
```

```python
# Clear the command window (not directly possible, but you can mimic it)
import os
os.system('cls' if os.name == 'nt' else 'clear')
```

[79]: 0

2 Caso de Estudio 2. Sistema lineal de cuatro variables de estado

2.0.1 Controlador Continuo Por Asinación de Polos

[80]:
```python
a = 0.07
b = 5
c = 150
w = 9
```

[81]:
```python
import numpy as np

A = np.array([
    [-a, a, 0, 0],
    [0, 0, 1, 0],
    [w**2, -w**2, 0, 0],
    [c, 0, 0, 0]
])
```

[82]:
```python
B = np.array([[0], [0], [b * w**2], [0]])
```

[83]:
```python
C = np.array([
    [0, 0, 0, 1],
    [0, 1, 0, 0]
])
```

[84]:
```python
D = np.array([[0]])
```

[85]:
```python
from scipy.signal import place_poles

# Definimos los polos de la consigna
p1 = -15 + 15j
p2 = -15 - 15j
p3 = -0.5 + 0.5j
p4 = -0.5 - 0.5j
poles = [p1, p2, p3, p4]

# Obtención de la matriz K usando places poles
result = place_poles(A, B, poles)
K = result.gain_matrix
```

```python
print("K =", K)
```

```
K = [[15.42104257  0.98107383  0.07637037  0.05291005]]
```

[86]:
```python
Ap = A - B @ K
```

[87]:
```python
C_row = C[0, :].reshape(1, -1)
```

[88]:
```python
C[0,:]
```

[88]:
```
array([0, 0, 0, 1])
```

[89]:
```python
G = -np.linalg.inv([C[0,:]@np.linalg.inv(A-B*K)@B])
```

[90]:
```python
G
```

[90]:
```
array([[0.05291005]])
```

[91]:
```python
# Parametros de inicialización
T = 100
h = 1e-4
t = np.arange(0, T, h)
ref = 100

# Variables de estado
alpha = np.zeros(len(t))
phi = np.zeros(len(t))
phiP = np.zeros(len(t))
high = np.zeros(len(t))
u = np.zeros(len(t))
uu = np.zeros(len(t))

# Condiciones Iniciales
alpha[0] = 0
phi[0] = 0
phiP[0] = 0
high[0] = 500
u[0] = 0
uu[0] = 0

# Vector de estado
stateVector = np.array([alpha[0], phi[0], phiP[0], high[0]])

# Punto de Operación
xOp = np.array([0, 0, 0, 0])
xOp = np.array([alpha[0], phi[0], phiP[0], high[0]])
```

```
# Condiciones Iniciales del observador
alphaO = np.zeros(len(t))
phiO = np.zeros(len(t))
phiPO = np.zeros(len(t))
highO = np.zeros(len(t))

alphaO[0] = 0
phiO[0] = 0
phiPO[0] = 0
highO[0] = 400

# variables de estado del observador
xObs = np.array([alphaO[0], phiO[0], phiPO[0], highO[0]])
obsStateVector = np.array([alphaO[0], phiO[0], phiPO[0], highO[0]])

# Zona muerta
deadZone = 0.1
```

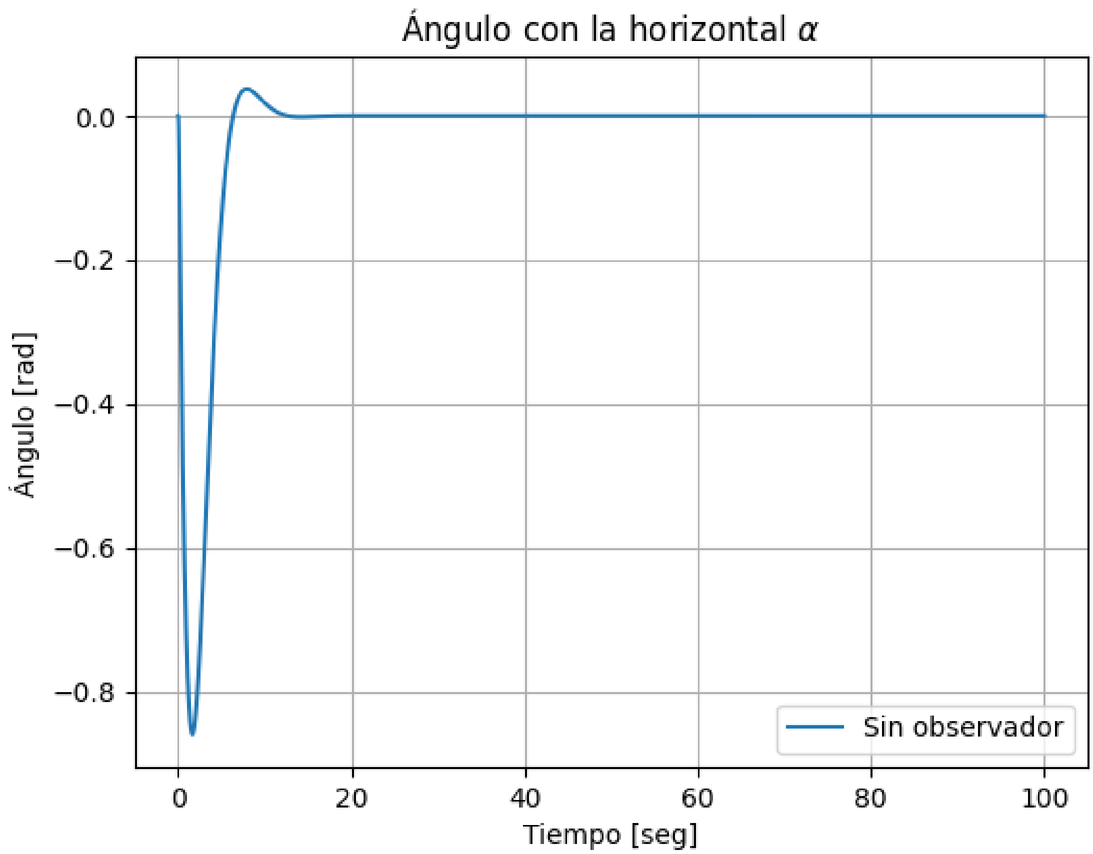
[92]:
```
uSO = np.zeros(len(t))
alphaSO = np.zeros(len(t))
phiSO = np.zeros(len(t))
phiPSO = np.zeros(len(t))
highSO = np.zeros(len(t))

uSO[0] = 0
xSO = np.array([alpha[0], phi[0], phiP[0], high[0]])
xSO=xSO.reshape(-1,1)
stateVectorSO = np.array([alpha[0], phi[0], phiP[0], high[0]])

# Simulación
for i in range(1, len(t)):
    uSO[i] = -K @ stateVectorSO + G * ref

    alphaSO[i] = xSO[0]
    phiSO[i] = xSO[1]
    phiPSO[i] = xSO[2]
    highSO[i] = xSO[3]

    xp_so = A @ xSO + B * uSO[i]
    xSO = xSO + h * xp_so

    stateVectorSO = np.array([alphaSO[i], phiSO[i], phiPSO[i], highSO[i]])

print("AlphaSO:", alphaSO)
print("PhiSO:", phiSO)
print("PhiPSO:", phiPSO)
```

```
print("HighSO:", highSO)
```

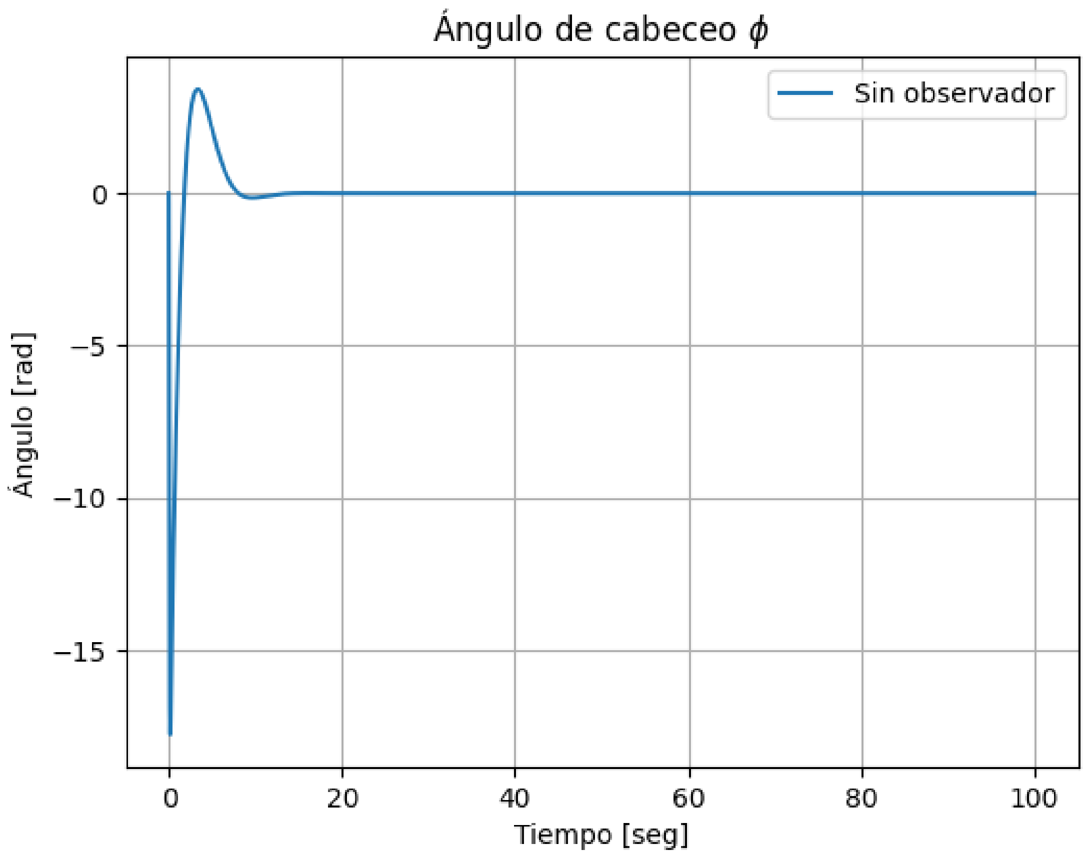
```
<ipython-input-92-96175e54fd1b>:18: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  uSO[i] = -K @ stateVectorSO + G * ref  # Use @ for matrix multiplication
<ipython-input-92-96175e54fd1b>:21: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  alphaSO[i] = xSO[0]
<ipython-input-92-96175e54fd1b>:22: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  phiSO[i] = xSO[1]
<ipython-input-92-96175e54fd1b>:23: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  phiPSO[i] = xSO[2]
<ipython-input-92-96175e54fd1b>:24: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  highSO[i] = xSO[3]
```

```
AlphaSO: [0.00000000e+00 0.00000000e+00 0.00000000e+00 … 2.83286497e-13
 2.83286497e-13 2.83286497e-13]
PhiSO: [0.00000000e+00 0.00000000e+00 0.00000000e+00 … 2.83288101e-13
 2.83288103e-13 2.83288105e-13]
PhiPSO: [ 0.00000000e+00  0.00000000e+00 -8.57142857e-01 …  1.99221824e-17
  1.99091863e-17  1.98961741e-17]
HighSO: [  0. 500. 500. … 100. 100. 100.]
```

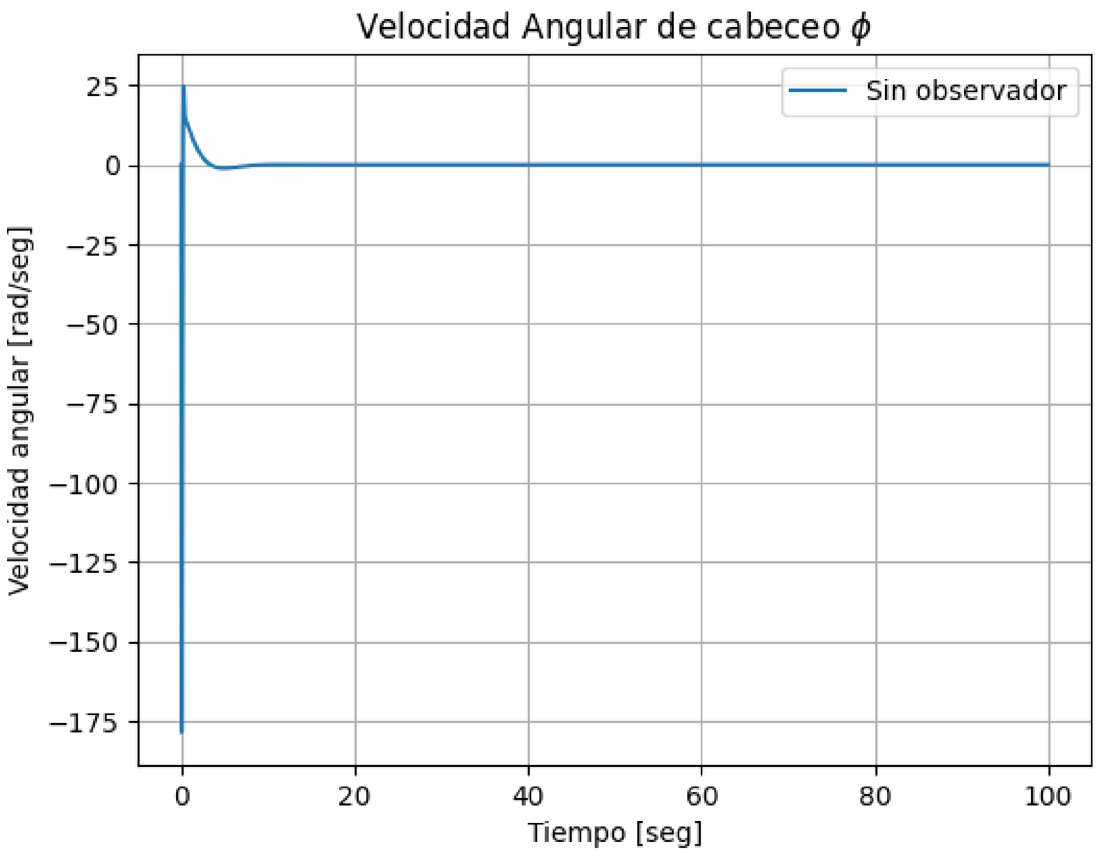
[93]:
```
plt.plot(t, alphaSO, linewidth=1.5, label='Sin observador')



plt.title(r'Ángulo con la horizontal $\alpha$')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Ángulo [rad]')
plt.grid(True)

plt.legend()

plt.show()
```



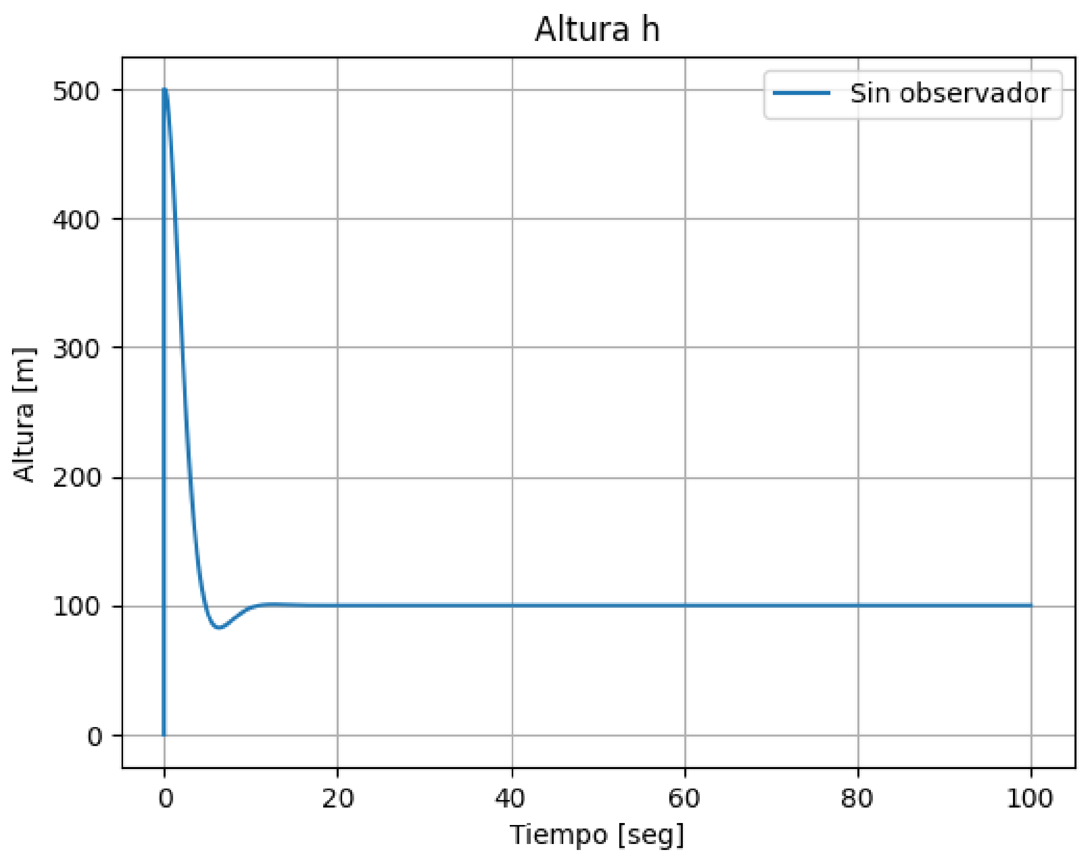
[94]:
```
plt.plot(t, phiSO, linewidth=1.5, label='Sin observador')

plt.title(r'Ángulo de cabeceo $\phi$')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Ángulo [rad]')
plt.grid(True)

plt.legend()

plt.show()
```



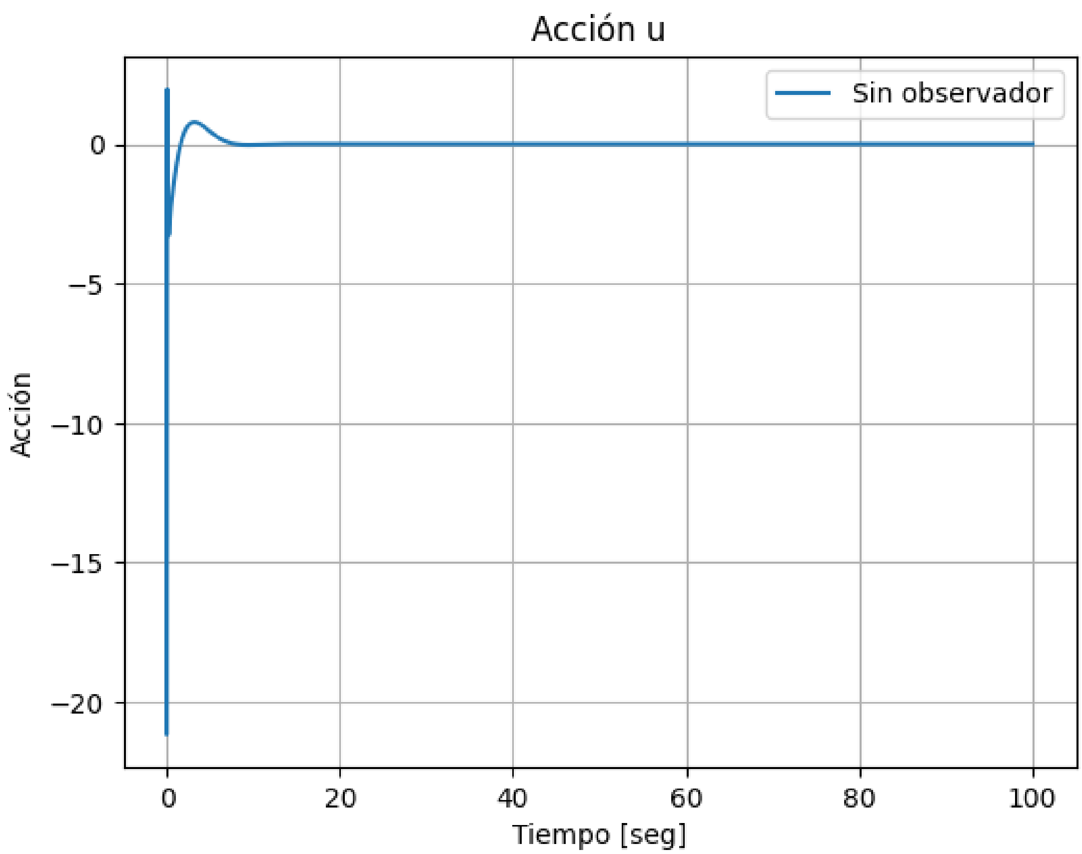
[95]:
```
plt.plot(t, phiPSO, linewidth=1.5, label='Sin observador')

plt.title(r'Velocidad Angular de cabeceo $\phi$')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Velocidad angular [rad/seg]')
plt.grid(True)

plt.legend()

plt.show()
```



[96]:
```
plt.plot(t, highSO, linewidth=1.5, label='Sin observador')

plt.title(r'Altura h')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Altura [m]')
plt.grid(True)

plt.legend()

plt.show()
```



[97]:
```
plt.plot(t, uSO, linewidth=1.5, label='Sin observador')

plt.title(r'Acción u')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Acción')
plt.grid(True)

plt.legend()

plt.show()
```



[98]:
```
# Clear all variables (manually in Python)
# Usually, you can use the following to clear the namespace
from IPython import get_ipython
get_ipython().magic('reset -sf')  # Clear all variables in an interactive␣
 ↪session

# Close all figures
import matplotlib.pyplot as plt
plt.close('all')

# Clear the command window (not directly possible, but you can mimic it)
import os
os.system('cls' if os.name == 'nt' else 'clear')
```

[98]: 0

2.0.2 Controlador Discreto LQR con Observador y Referencia distinta de Cero

[100]:
```
!pip install control
```

```
Collecting control
  Downloading control-0.10.1-py3-none-any.whl.metadata (7.6 kB)
Requirement already satisfied: numpy>=1.23 in /usr/local/lib/python3.10/dist-
packages (from control) (1.26.4)
Requirement already satisfied: scipy>=1.8 in /usr/local/lib/python3.10/dist-
packages (from control) (1.13.1)
Requirement already satisfied: matplotlib>=3.6 in
/usr/local/lib/python3.10/dist-packages (from control) (3.8.0)
Requirement already satisfied: contourpy>=1.0.1 in
/usr/local/lib/python3.10/dist-packages (from matplotlib>=3.6->control) (1.3.1)
Requirement already satisfied: cycler>=0.10 in /usr/local/lib/python3.10/dist-
packages (from matplotlib>=3.6->control) (0.12.1)
Requirement already satisfied: fonttools>=4.22.0 in
/usr/local/lib/python3.10/dist-packages (from matplotlib>=3.6->control) (4.54.1)
Requirement already satisfied: kiwisolver>=1.0.1 in
/usr/local/lib/python3.10/dist-packages (from matplotlib>=3.6->control) (1.4.7)
Requirement already satisfied: packaging>=20.0 in
/usr/local/lib/python3.10/dist-packages (from matplotlib>=3.6->control) (24.2)
Requirement already satisfied: pillow>=6.2.0 in /usr/local/lib/python3.10/dist-
packages (from matplotlib>=3.6->control) (11.0.0)
Requirement already satisfied: pyparsing>=2.3.1 in
/usr/local/lib/python3.10/dist-packages (from matplotlib>=3.6->control) (3.2.0)
Requirement already satisfied: python-dateutil>=2.7 in
/usr/local/lib/python3.10/dist-packages (from matplotlib>=3.6->control) (2.8.2)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.10/dist-
packages (from python-dateutil>=2.7->matplotlib>=3.6->control) (1.16.0)
Downloading control-0.10.1-py3-none-any.whl (549 kB)
                          549.6/549.6 kB
6.3 MB/s eta 0:00:00
Installing collected packages: control
Successfully installed control-0.10.1
```

[101]:
```
import control as ctl  # Import the control library instead of scipy.signal
from control import ss,c2d
import numpy as np
```

[107]:
```
Ts = 0.01  # Tiempo de muestreo
T = 25     # Tiempo Total
Kmax = int(T / Ts)  # Numero de pasos

a = 0.07
b = 5
c = 150
```

```
w = 9
```

[108]:
```
A = np.array([
    [-a, a, 0, 0],
    [0, 0, 1, 0],
    [w**2, -w**2, 0, 0],
    [c, 0, 0, 0]
])

B = np.array([[0], [0], [b * w**2], [0]])

C = np.array([
    [0, 0, 0, 1],
    [0, 1, 0, 0]
])

D = np.array([[0]])

sys1 = ss(A, B, C, [[0],[0]])

print(sys1)
```

```
<StateSpace>: sys[2]
Inputs (1): ['u[0]']
Outputs (2): ['y[0]', 'y[1]']
States (4): ['x[0]', 'x[1]', 'x[2]', 'x[3]']

A = [[-7.0e-02  7.0e-02  0.0e+00  0.0e+00]
     [ 0.0e+00  0.0e+00  1.0e+00  0.0e+00]
     [ 8.1e+01 -8.1e+01  0.0e+00  0.0e+00]
     [ 1.5e+02  0.0e+00  0.0e+00  0.0e+00]]

B = [[  0.]
     [  0.]
     [405.]
     [  0.]]

C = [[0. 0. 0. 1.]
     [0. 1. 0. 0.]]

D = [[0.]
     [0.]]
```

[109]:
```
# Convertimos a discreto usando un rententor de orden cero
dSys1 = c2d(sys1, Ts, method='zoh')
```

```python
print(dSys1)
```

```
<StateSpace>: sys[2]$sampled
Inputs (1): ['u[0]']
Outputs (2): ['y[0]', 'y[1]']
States (4): ['x[0]', 'x[1]', 'x[2]', 'x[3]']

A = [[ 9.99301189e-01  6.98810770e-04  3.49682228e-06  0.00000000e+00]
     [ 4.04632292e-03  9.95953677e-01  9.98650783e-03  0.00000000e+00]
     [ 8.08623891e-01 -8.08623891e-01  9.95953677e-01  0.00000000e+00]
     [ 1.49947548e+00  5.24523341e-04  1.74898534e-06  1.00000000e+00]]

B = [[4.72226043e-06]
     [2.02363369e-02]
     [4.04453567e+00]
     [1.77114873e-06]]

C = [[0. 0. 0. 1.]
     [0. 1. 0. 0.]]

D = [[0.]
     [0.]]

dt = 0.01
```

[110]:
```python
# Matrices en tiempo discreto
A = dSys1.A
B = dSys1.B

print("Discrete-time A matrix:\n", A)
print("Discrete-time B matrix:\n", B)
```

```
Discrete-time A matrix:
 [[ 9.99301189e-01  6.98810770e-04  3.49682228e-06  0.00000000e+00]
 [ 4.04632292e-03  9.95953677e-01  9.98650783e-03  0.00000000e+00]
 [ 8.08623891e-01 -8.08623891e-01  9.95953677e-01  0.00000000e+00]
 [ 1.49947548e+00  5.24523341e-04  1.74898534e-06  1.00000000e+00]]
Discrete-time B matrix:
 [[4.72226043e-06]
 [2.02363369e-02]
 [4.04453567e+00]
 [1.77114873e-06]]
```

[111]:
```python
Cref = C[0, :]

# Construcción de matriz ampliada
```

```
Aamp1 = np.block([
    [A, np.zeros((4, 1))],
    [-np.dot(Cref, A), np.eye(1)]
])

print("Cref:\n", Cref)
print("Aamp1:\n", Aamp1)
```

```
Cref:
 [0 0 0 1]
Aamp1:
 [[ 9.99301189e-01  6.98810770e-04  3.49682228e-06  0.00000000e+00
   0.00000000e+00]
 [ 4.04632292e-03  9.95953677e-01  9.98650783e-03  0.00000000e+00
   0.00000000e+00]
 [ 8.08623891e-01 -8.08623891e-01  9.95953677e-01  0.00000000e+00
   0.00000000e+00]
 [ 1.49947548e+00  5.24523341e-04  1.74898534e-06  1.00000000e+00
   0.00000000e+00]
 [-1.49947548e+00 -5.24523341e-04 -1.74898534e-06 -1.00000000e+00
   1.00000000e+00]]
```

[112]:
```
Bamp1 = np.vstack([B, -np.dot(Cref, B)])

print("Bamp1:\n", Bamp1)
```

```
Bamp1:
 [[ 4.72226043e-06]
 [ 2.02363369e-02]
 [ 4.04453567e+00]
 [ 1.77114873e-06]
 [-1.77114873e-06]]
```

[113]:
```
Q1 = np.diag([50000, 100000, 10000, 1, 1])

print("Q1:\n", Q1)
```

```
Q1:
 [[ 50000      0      0      0      0]
 [     0 100000      0      0      0]
 [     0      0  10000      0      0]
 [     0      0      0      1      0]
 [     0      0      0      0      1]]
```

[115]:
```
from control import dlqr

R1 = 250000000000
```

```python
# Matriz de Ganacia del LQR
K1, P1, _ = dlqr(Aamp1, Bamp1, Q1, R1)

print("K1:\n", K1)
```

```
K1:
 [[ 1.25030703e+00  2.33663798e-04  1.18057570e-03  1.82295663e-03
  -1.99521941e-06]]
```

[116]:
```python
Kp1 = K1[0, :4]  # Proporcional
print("Kp1:\n", Kp1)
```

```
Kp1:
 [1.25030703e+00 2.33663798e-04 1.18057570e-03 1.82295663e-03]
```

[117]:
```python
Kint1 = -K1[0, 4] # Integrador
print("Kint:\n", Kint1)
```

```
Kint:
 1.9952194120860476e-06
```

[118]:
```python
# Matrices Observador
Ao = A.T
Bo = C.T
Co = B.T

Qo = np.diag([0.001, 5, 0.5, 0.0001])
Ro = np.diag([80000, 1280000])

Ko, _, _ = dlqr(Ao, Bo, Qo, Ro)

Ko = Ko.T

print("Ko:\n", Ko)
```

```
Ko:
 [[ 1.13649950e-04  1.12402770e-06]
 [-2.50053620e-03  1.28292024e-03]
 [-3.31737875e-03 -1.13490298e-03]
 [ 1.86871478e-02 -1.52540115e-04]]
```

[119]:
```python
KMAX = 5000
```

[120]:
```python
t = np.arange(0, KMAX * Ts, Ts)
x = np.array([[0], [0], [0], [500]])
ve = np.zeros(KMAX)
u_k = np.zeros(KMAX)
```

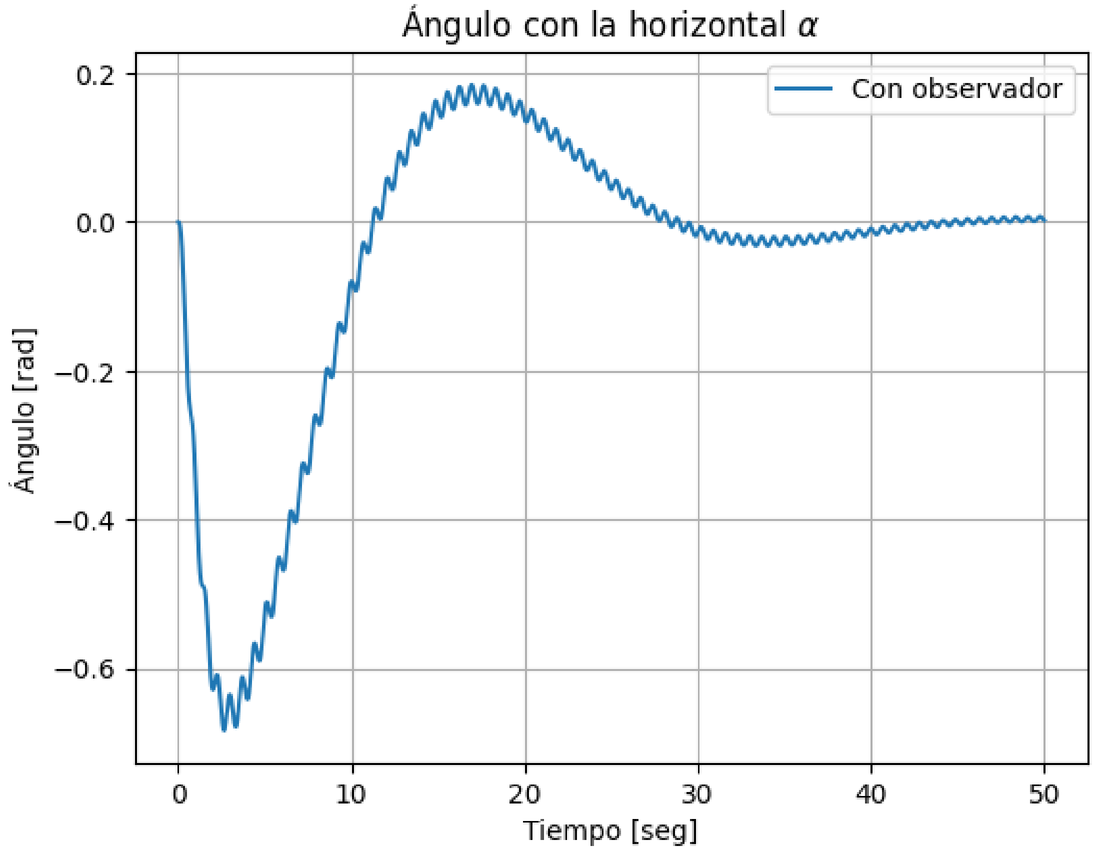
```python
xang = np.zeros((4, 1))
xang[3][0] = 400
alpha = np.zeros(KMAX)
phi = np.zeros(KMAX)
phip = np.zeros(KMAX)
high = np.zeros(KMAX)

ref = 100

dead_zone = 0

# Simulación

for ki in range(1, KMAX):
    ve[ki] = ve[ki - 1] + ref - Cref @ x
    u = -Kp1 @ xang + Kint1 * ve[ki]
    if abs(u)<dead_zone:
      u = np.sign(u)*(abs(u)-dead_zone)
    ys = Cref @ x  # This should be measured y
    x = A @ x + B * u
    alpha[ki] = x[0]
    phi[ki] = x[1]
    phip[ki] = x[2]
    high[ki] = x[3]
    u_k[ki] = u
    xang = A @ xang + B * u + Ko @ (ys - C @ xang)
```

```
<ipython-input-120-264ab19a527f>:19: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  ve[ki] = ve[ki - 1] + ref - Cref @ x
<ipython-input-120-264ab19a527f>:25: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  alpha[ki] = x[0]
<ipython-input-120-264ab19a527f>:26: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  phi[ki] = x[1]
<ipython-input-120-264ab19a527f>:27: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  phip[ki] = x[2]
```

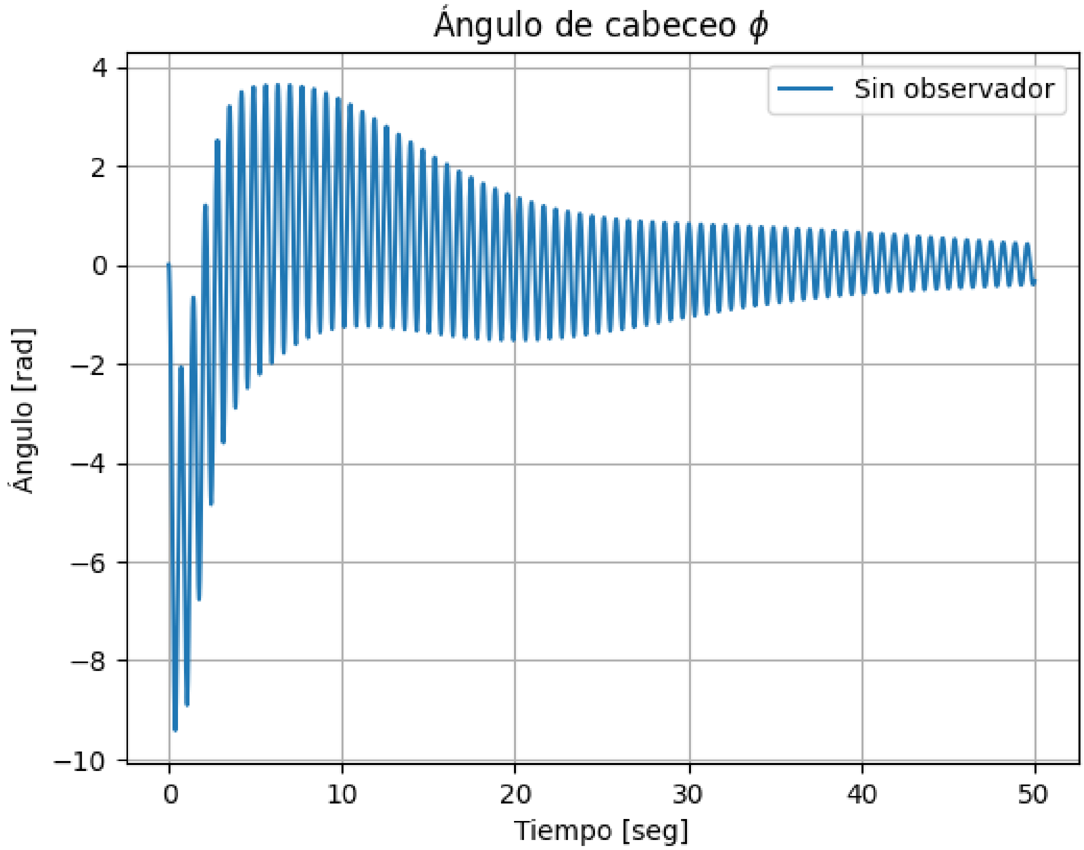
```
<ipython-input-120-264ab19a527f>:28: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  high[ki] = x[3]
<ipython-input-120-264ab19a527f>:29: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  u_k[ki] = u
```

[121]:

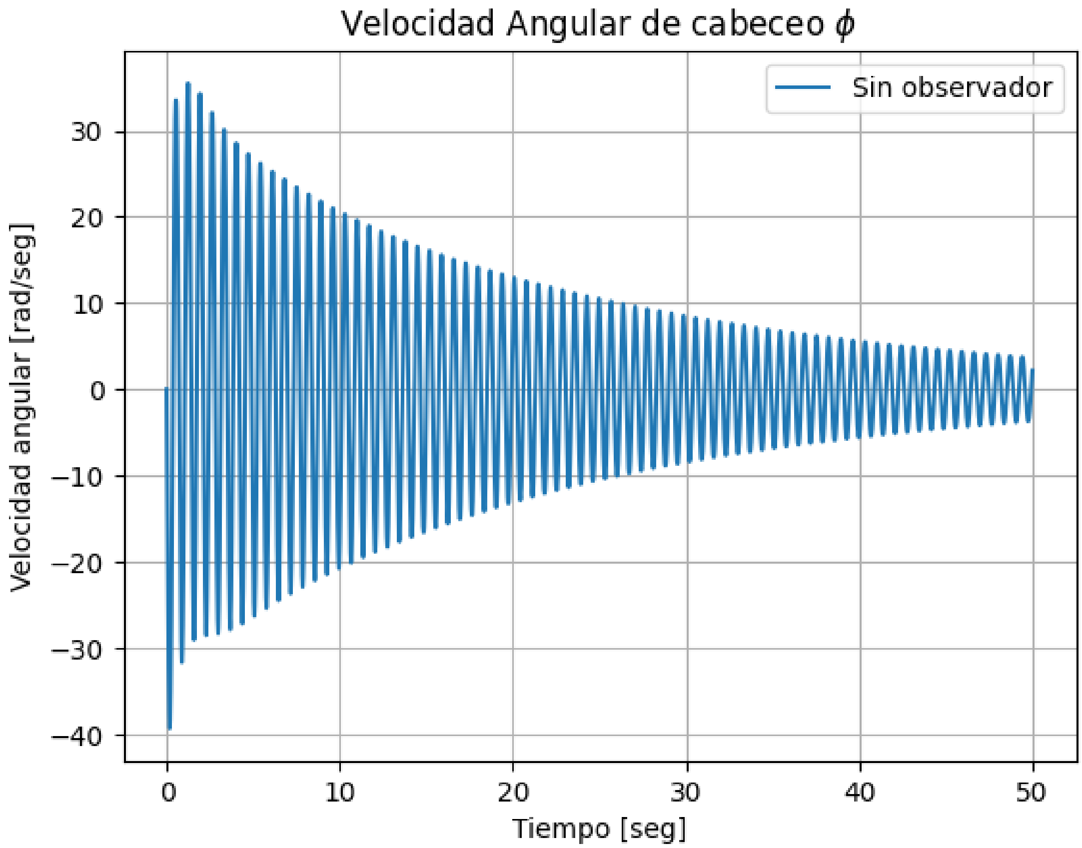
```
# Plotting the data
plt.plot(t, alpha, linewidth=1.5, label='Con observador')

# If you have another dataset for comparison (without observer)
# plt.plot(t, alpha_without_observer, label='Sin observador')  # Uncomment this␣
 ↪if you have data

# Add title, labels, and grid
plt.title(r'Ángulo con la horizontal $\alpha$')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Ángulo [rad]')
plt.grid(True)

# Add legend
plt.legend()

# Display the plot
plt.show()
```



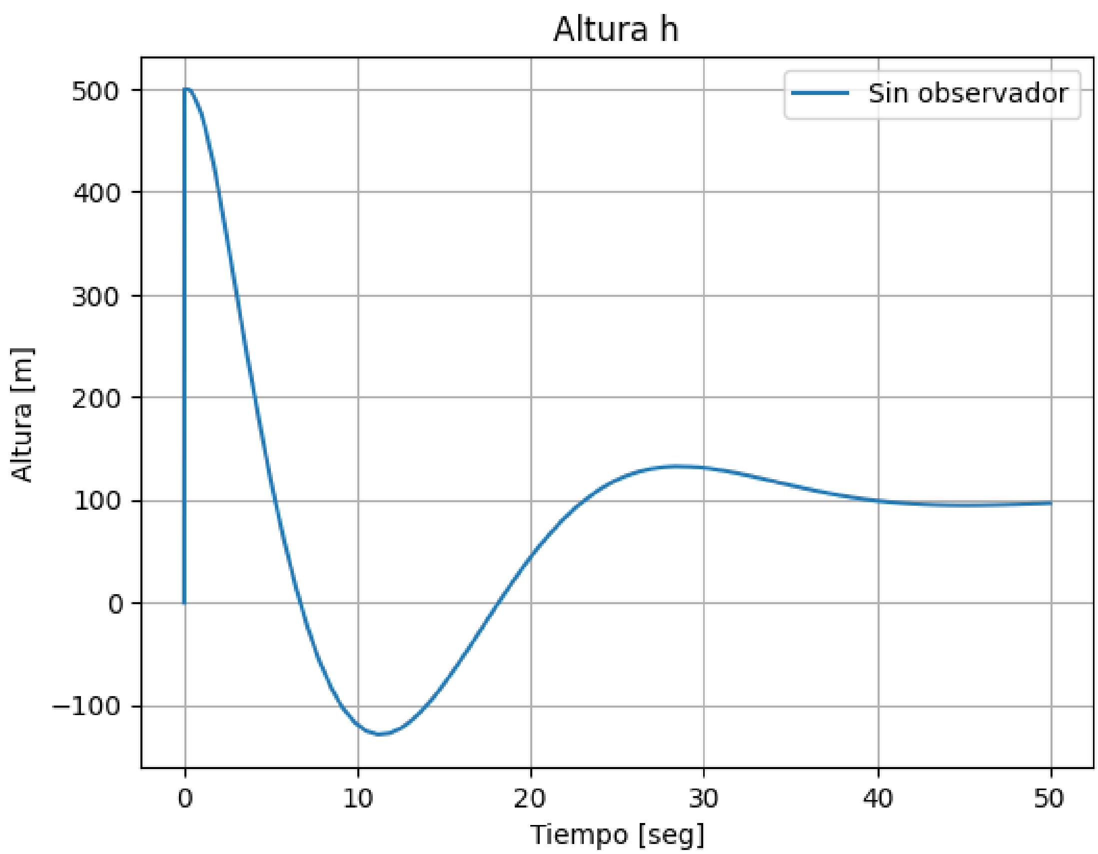
[122]:
```
plt.plot(t, phi, linewidth=1.5, label='Sin observador')

# Add title, labels, and grid
plt.title(r'Ángulo de cabeceo $\phi$')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Ángulo [rad]')
plt.grid(True)

# Add legend
plt.legend()

# Display the plot
plt.show()
```



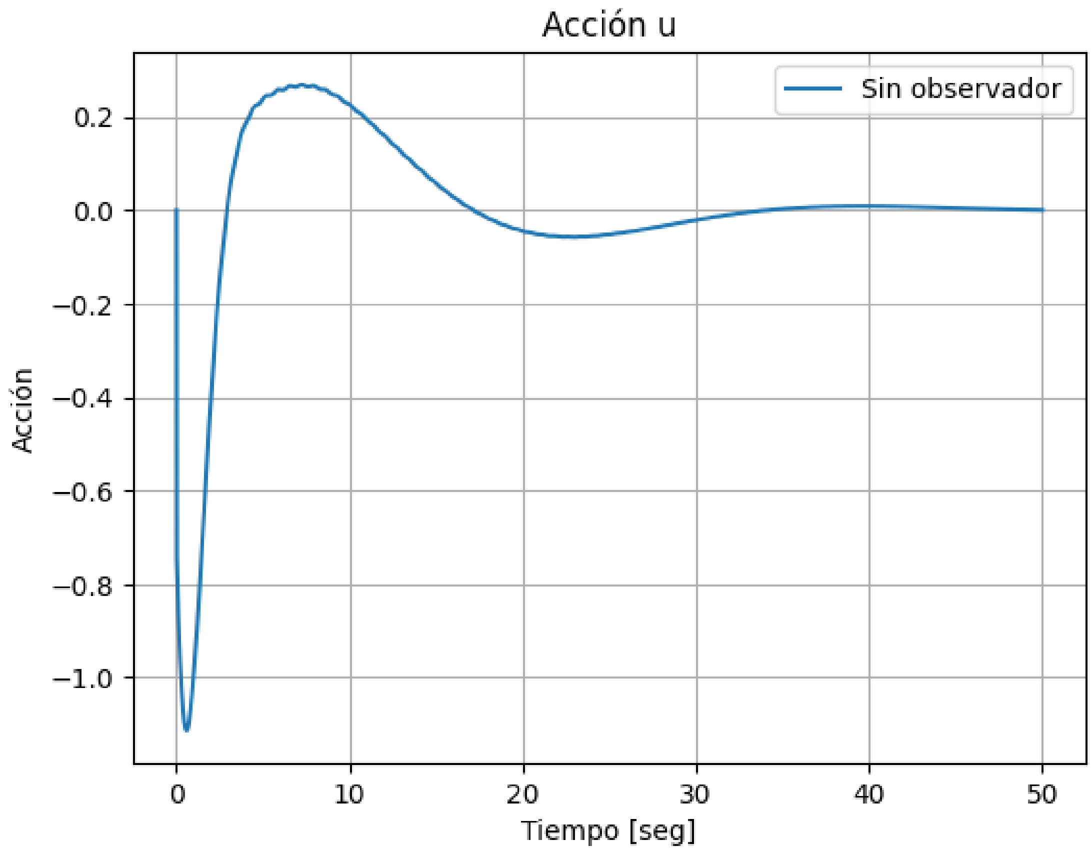
[123]:
```
plt.plot(t, phip, linewidth=1.5, label='Sin observador')

# Add title, labels, and grid
plt.title(r'Velocidad Angular de cabeceo $\phi$')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Velocidad angular [rad/seg]')
plt.grid(True)

# Add legend
plt.legend()

# Display the plot
plt.show()
```



[124]:
```
plt.plot(t, high, linewidth=1.5, label='Sin observador')

# Add title, labels, and grid
plt.title(r'Altura h')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Altura [m]')
plt.grid(True)

# Add legend
plt.legend()

# Display the plot
plt.show()
```



[125]:
```
plt.plot(t, u_k, linewidth=1.5, label='Sin observador')

# Add title, labels, and grid
plt.title(r'Acción u')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Acción')
plt.grid(True)

# Add legend
plt.legend()

# Display the plot
plt.show()
```



2.0.3 Actuador con no-linealidad

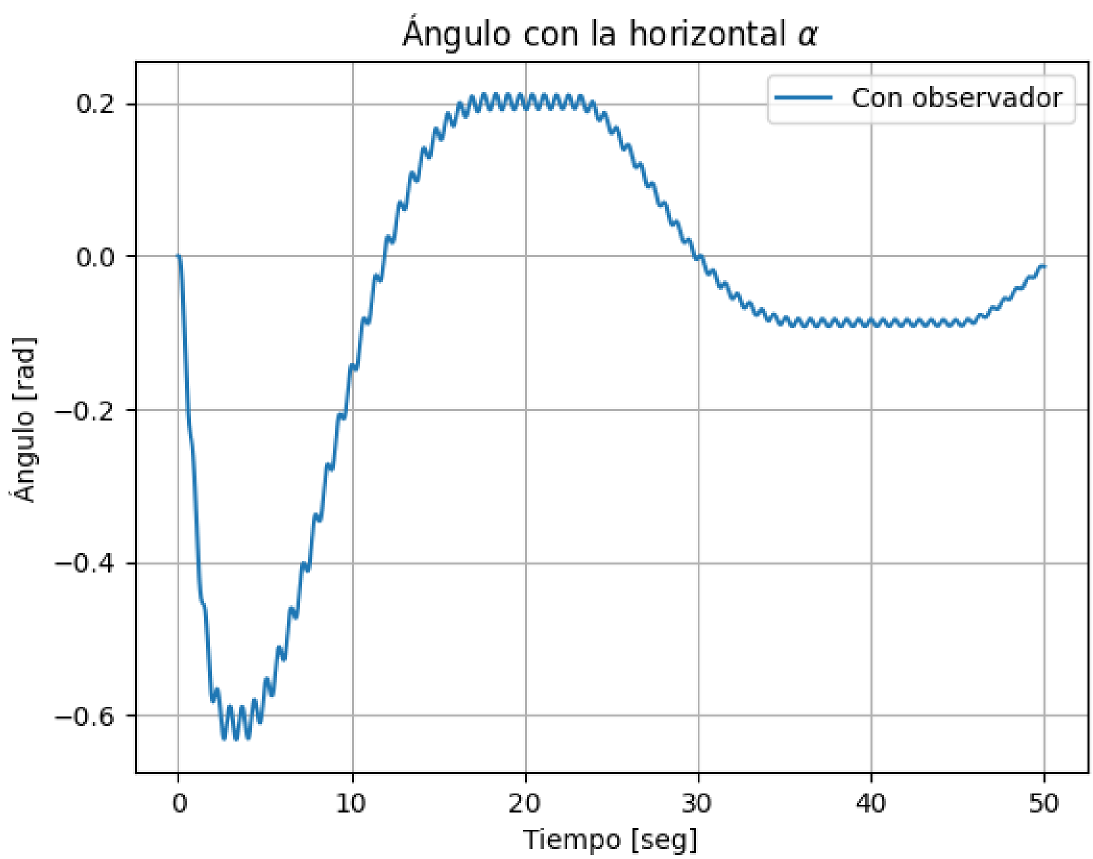
[153]:
```
t = np.arange(0, KMAX * Ts, Ts)
x = np.array([[0], [0], [0], [500]])
ve = np.zeros(KMAX)
u_k = np.zeros(KMAX)
xang = np.zeros((4, 1))
xang[3][0] = 400
alpha = np.zeros(KMAX)
phi = np.zeros(KMAX)
phip = np.zeros(KMAX)
high = np.zeros(KMAX)

ref = 100

dead_zone = 0.1

# Simulación

for ki in range(1, KMAX):
```

```
    ve[ki] = ve[ki - 1] + ref - Cref @ x
    u = -Kp1 @ xang + Kint1 * ve[ki]
    if abs(u)<dead_zone:
      u = 0
    else:
      u = np.sign(u)*(abs(u)-dead_zone)
    ys = Cref @ x  # This should be measured y
    x = A @ x + B * u
    alpha[ki] = x[0]
    phi[ki] = x[1]
    phip[ki] = x[2]
    high[ki] = x[3]
    u_k[ki] = u
    xang = A @ xang + B * u + Ko @ (ys - C @ xang)
```

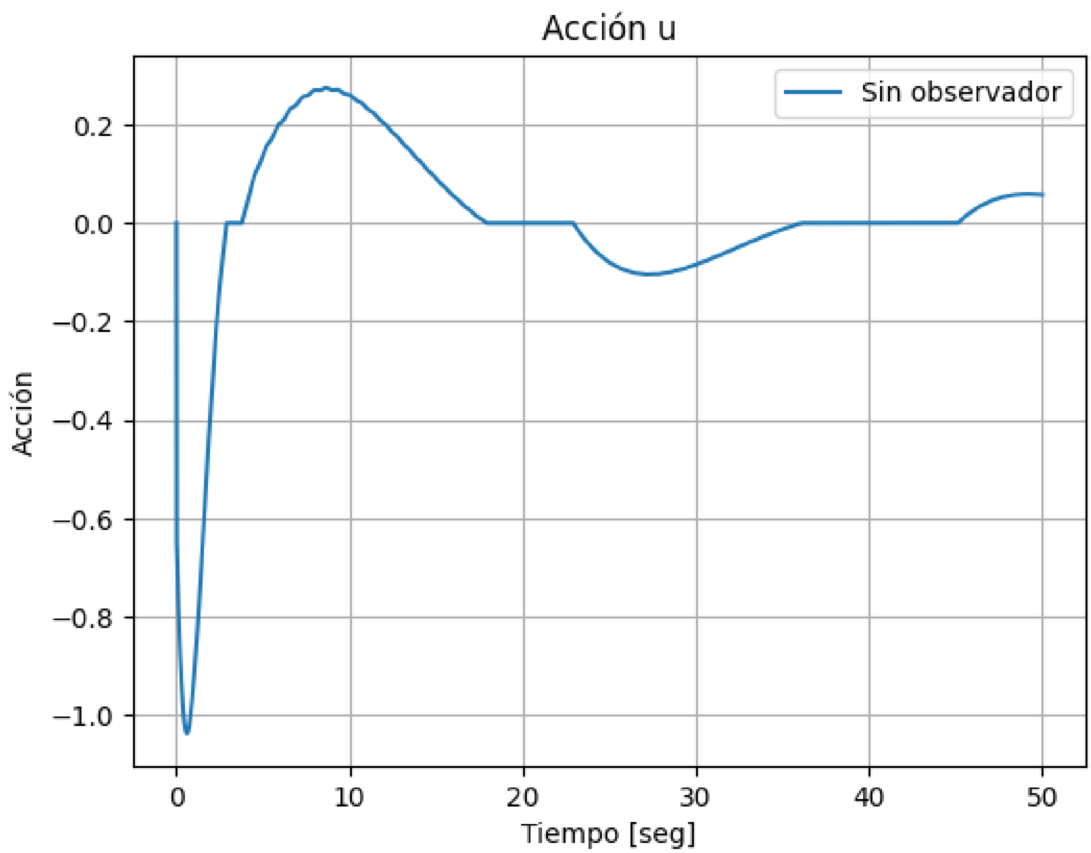
```
<ipython-input-153-bb52d65fbcf5>:19: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  ve[ki] = ve[ki - 1] + ref - Cref @ x
<ipython-input-153-bb52d65fbcf5>:27: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  alpha[ki] = x[0]
<ipython-input-153-bb52d65fbcf5>:28: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  phi[ki] = x[1]
<ipython-input-153-bb52d65fbcf5>:29: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  phip[ki] = x[2]
<ipython-input-153-bb52d65fbcf5>:30: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  high[ki] = x[3]
<ipython-input-153-bb52d65fbcf5>:31: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  u_k[ki] = u
```

[154]:

```python
# Plotting the data
plt.plot(t, alpha, linewidth=1.5, label='Con observador')

# If you have another dataset for comparison (without observer)
# plt.plot(t, alpha_without_observer, label='Sin observador')  # Uncomment this␣
 ↪if you have data

# Add title, labels, and grid
plt.title(r'Ángulo con la horizontal $\alpha$')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Ángulo [rad]')
plt.grid(True)

# Add legend
plt.legend()

# Display the plot
plt.show()
```



[155]:
```python
plt.plot(t, u_k, linewidth=1.5, label='Sin observador')

# Add title, labels, and grid
plt.title(r'Acción u')
plt.xlabel('Tiempo [seg]')
plt.ylabel('Acción')
plt.grid(True)

# Add legend
plt.legend()

# Display the plot
plt.show()
```



2.0.4 Conclusiones

Implementando el controlador discreto del tipo LQR se logró mantener dentro de la unidad el valor de la acción de control u y del angulo horizontal α. Se debe sequir ajustando los valores de Q y R para que el angulo de cabeceo de menor a la unidad. Por otro lado si aplicamos una no-linealidad en el actuador encontramos el limite en 0.1, apartir de este valor el sistema no logra llegar a la referencia

[156]:
```
# Clear all variables (manually in Python)
# Usually, you can use the following to clear the namespace
from IPython import get_ipython
get_ipython().magic('reset -sf')  # Clear all variables in an interactive␣
 ↪session

# Close all figures
import matplotlib.pyplot as plt
plt.close('all')

# Clear the command window (not directly possible, but you can mimic it)
import os
os.system('cls' if os.name == 'nt' else 'clear')
```

[156]:
```
0
```

3 Caso de Estudio 3. Sistema no lineal de cuatro variables de estado

3.0.1 Controlador LQR en tiempo discreto con observador y referencia distinta de cero

[157]:
```
m = 0.1  # Mass of the pendulum (kg)
F = 0.1  # Force (N)
l = 1.6  # Length of the pendulum (m)
g = 9.8  # Acceleration due to gravity (m/s^2)
M = 1.5  # Mass of the cart (kg)
```

[158]:
```
import numpy as np

#Versión linealizada en el equilibrio inestable. Sontag Pp 104.
Ac = np.array([
    [0, 1, 0, 0],
    [0, -F/M, -m*g/M, 0],
    [0, 0, 0, 1],
    [0, -F/(l*M), -g*(m+M)/(l*M), 0]
])

print(Ac)
```

```
[[ 0.          1.          0.          0.        ]
 [ 0.         -0.06666667 -0.65333333  0.        ]
 [ 0.          0.          0.          1.        ]
 [ 0.         -0.04166667 -6.53333333  0.        ]]
```

[159]:
```python
Bc = np.array([[0],
               [1/M],
               [0],
               [1/(l*M)]])

Cc = np.array([
    [1, 0, 0, 0],
    [0, 0, 1, 0]
])

Dc = np.array([[0],[0]])

print("Bc:\n", Bc)
print("Cc:\n", Cc)
print("Dc:\n", Dc)
```

```
Bc:
 [[0.        ]
 [0.66666667]
 [0.        ]
 [0.41666667]]
Cc:
 [[1 0 0 0]
 [0 0 1 0]]
Dc:
 [[0]
 [0]]
```

[161]:
```python
Ts = 1e-2
T = 25
Kmax = int(T / Ts)  # Numero de Pasos

print("Ts:", Ts)
print("T:", T)
print("Kmax:", Kmax)
```

```
Ts: 0.01
T: 25
Kmax: 2500
```

[162]:
```python
from control import ss,c2d

sys1 = ss(Ac, Bc, Cc, Dc)

print(sys1)
```

```
<StateSpace>: sys[4]
Inputs (1): ['u[0]']
```

```
Outputs (2): ['y[0]', 'y[1]']
States (4): ['x[0]', 'x[1]', 'x[2]', 'x[3]']

A = [[ 0.          1.          0.          0.        ]
     [ 0.         -0.06666667 -0.65333333  0.        ]
     [ 0.          0.          0.          1.        ]
     [ 0.         -0.04166667 -6.53333333  0.        ]]

B = [[0.        ]
     [0.66666667]
     [0.        ]
     [0.41666667]]

C = [[1. 0. 0. 0.]
     [0. 0. 1. 0.]]

D = [[0.]
     [0.]]
```

[163]:

```python
# Convertimos el sistema a tiempo discreto
dSys1 = c2d(sys1, Ts, method='zoh')

print(dSys1)
```

```
<StateSpace>: sys[4]$sampled
Inputs (1): ['u[0]']
Outputs (2): ['y[0]', 'y[1]']
States (4): ['x[0]', 'x[1]', 'x[2]', 'x[3]']

A = [[ 1.00000000e+00  9.99666742e-03 -3.26576304e-05 -1.08867187e-07]
     [ 0.00000000e+00  9.99333560e-01 -6.53044478e-03 -3.26576304e-05]
     [ 0.00000000e+00 -2.08275704e-06  9.99673356e-01  9.99891116e-03]
     [ 0.00000000e+00 -4.16482448e-04 -6.53248588e-02  9.99673356e-01]]

B = [[3.33258138e-05]
     [6.66439958e-03]
     [2.08275704e-05]
     [4.16482448e-03]]

C = [[1. 0. 0. 0.]
     [0. 0. 1. 0.]]

D = [[0.]
     [0.]]

dt = 0.01
```

[164]:
```
# Matrices en tiempo discreto
A = dSys1.A
B = dSys1.B

print("Discrete-time A matrix:\n", A)
print("Discrete-time B matrix:\n", B)
```

```
Discrete-time A matrix:
 [[ 1.00000000e+00  9.99666742e-03 -3.26576304e-05 -1.08867187e-07]
 [ 0.00000000e+00  9.99333560e-01 -6.53044478e-03 -3.26576304e-05]
 [ 0.00000000e+00 -2.08275704e-06  9.99673356e-01  9.99891116e-03]
 [ 0.00000000e+00 -4.16482448e-04 -6.53248588e-02  9.99673356e-01]]
Discrete-time B matrix:
 [[3.33258138e-05]
 [6.66439958e-03]
 [2.08275704e-05]
 [4.16482448e-03]]
```

[165]:
```
Cref = Cc[0, :]

Aamp1 = np.block([
    [A, np.zeros((4, 1))],
    [-np.dot(Cref, A), np.eye(1)]
])

print("Cref:\n", Cref)
print("Aamp1:\n", Aamp1)
```

```
Cref:
 [1 0 0 0]
Aamp1:
 [[ 1.00000000e+00  9.99666742e-03 -3.26576304e-05 -1.08867187e-07
   0.00000000e+00]
 [ 0.00000000e+00  9.99333560e-01 -6.53044478e-03 -3.26576304e-05
   0.00000000e+00]
 [ 0.00000000e+00 -2.08275704e-06  9.99673356e-01  9.99891116e-03
   0.00000000e+00]
 [ 0.00000000e+00 -4.16482448e-04 -6.53248588e-02  9.99673356e-01
   0.00000000e+00]
 [-1.00000000e+00 -9.99666742e-03  3.26576304e-05  1.08867187e-07
   1.00000000e+00]]
```

[166]:
```
Bamp1 = np.vstack([B, -np.dot(Cref, B)])

print("Bamp1:\n", Bamp1)
```

```
Bamp1:
 [[ 3.33258138e-05]
```

```
 [ 6.66439958e-03]
 [ 2.08275704e-05]
 [ 4.16482448e-03]
 [-3.33258138e-05]]
```

[167]:
```python
Q1 = np.diag([0.1, 1, 1, 1e-4, 1e-4])

print("Q1:\n", Q1)
```

```
Q1:
 [[1.e-01 0.e+00 0.e+00 0.e+00 0.e+00]
 [0.e+00 1.e+00 0.e+00 0.e+00 0.e+00]
 [0.e+00 0.e+00 1.e+00 0.e+00 0.e+00]
 [0.e+00 0.e+00 0.e+00 1.e-04 0.e+00]
 [0.e+00 0.e+00 0.e+00 0.e+00 1.e-04]]
```

[169]:
```python
from control import dlqr

R1 = 0.05
# calculo de la matriz de ganancia del lqr
K1, P1, _ = dlqr(Aamp1, Bamp1, Q1, R1)

print("K1:\n", K1)
```

```
K1:
 [[ 7.97315188  6.99811521  3.48833679 -0.08883985 -0.04368156]]
```

[170]:
```python
Kp1 = K1[0, :4]  # Ganancia proporcional

print("Kp1:\n", Kp1)
```

```
Kp1:
 [ 7.97315188  6.99811521  3.48833679 -0.08883985]
```

[171]:
```python
Kint1 = -K1[0, 4] # Ganancia del integrador
Kint1
```

[171]:
```
0.04368156192915914
```

[172]:
```python
m1 = 10 * m

# Definimos la matriz A para el actuador en la segunda condicion cuando la masa␣
↪es 10*m
Ac2 = np.array([
    [0, 1, 0, 0],
    [0, -F/M, -m1*g/M, 0],
    [0, 0, 0, 1],
    [0, -F/(l*M), -g*(m1+M)/(l*M), 0]
```

```
])

print("Ac2:\n", Ac2)
```

```
Ac2:
 [[  0.           1.           0.           0.        ]
 [  0.          -0.06666667  -6.53333333   0.        ]
 [  0.           0.           0.           1.        ]
 [  0.          -0.04166667 -10.20833333   0.        ]]
```

[173]:
```
sys2 = ss(Ac2, Bc, Cc, Dc)

print(sys2)
```

```
<StateSpace>: sys[6]
Inputs (1): ['u[0]']
Outputs (2): ['y[0]', 'y[1]']
States (4): ['x[0]', 'x[1]', 'x[2]', 'x[3]']

A = [[  0.           1.           0.           0.        ]
     [  0.          -0.06666667  -6.53333333   0.        ]
     [  0.           0.           0.           1.        ]
     [  0.          -0.04166667 -10.20833333   0.        ]]

B = [[0.        ]
     [0.66666667]
     [0.        ]
     [0.41666667]]

C = [[1. 0. 0. 0.]
     [0. 0. 1. 0.]]

D = [[0.]
     [0.]]
```

[174]:
```
dSys2 = c2d(sys2, Ts, method='zoh')

print(dSys2)
```

```
<StateSpace>: sys[6]$sampled
Inputs (1): ['u[0]']
Outputs (2): ['y[0]', 'y[1]']
States (4): ['x[0]', 'x[1]', 'x[2]', 'x[3]']

A = [[ 1.00000000e+00  9.99666752e-03 -3.26566303e-04 -1.08865186e-06]
     [ 0.00000000e+00  9.99333601e-01 -6.53004478e-02 -3.26566303e-04]
     [ 0.00000000e+00 -2.08269326e-06  9.99489672e-01  9.99829881e-03]
```

```
 [ 0.00000000e+00 -4.16456938e-04 -1.02052360e-01  9.99489672e-01]]

B = [[3.33247932e-05]
     [6.66399141e-03]
     [2.08269326e-05]
     [4.16456938e-03]]

C = [[1. 0. 0. 0.]
     [0. 0. 1. 0.]]

D = [[0.]
     [0.]]

dt = 0.01
```

[175]:
```
import numpy as np
from control import ss, c2d, dlqr,lqr
A2 = dSys2.A
B2 = dSys2.B

Cref = Cc[0, :]

Aamp2 = np.block([
    [A2, np.zeros((4, 1))],
    [-np.dot(Cref, A2), np.eye(1)]
])

Q2 = np.diag([0.1, 1, 1, 1e-4, 1e-4])
R2 = 0.05

K2, P2, _ = dlqr(Aamp2, Bamp1, Q2, R2)

Kp2 = K2[0, :4]

Kint2 = -K2[0, 4]

print("K2:\n", K2)
print("Kp2:\n", Kp2)
print("Kint2:", Kint2)
```

```
K2:
 [[ 8.39724121  7.7828109  -0.47307662 -2.0307997  -0.04374543]]
Kp2:
 [ 8.39724121  7.7828109  -0.47307662 -2.0307997 ]
Kint2: 0.04374543043349237
```

[176]:
```python
# Matrices del observador
Ao = A.T  # Transpose of A
Bo = Cc.T  # Transpose of Cc
Co = B.T  # Transpose of B
Qo = np.diag([0.001, 5, 0.5, 0.0001])
Ro = np.diag([80, 10000])

Ko, _, _ = dlqr(Ao, Bo, Qo, Ro)

Ko = Ko.T

print("Ko:\n", Ko)
```

```
Ko:
 [[ 7.05370879e-02 -1.08525551e-04]
 [ 2.39714091e-01 -1.06543847e-03]
 [-1.19592528e-02  6.85613825e-03]
 [ 3.05646867e-02 -5.11816519e-04]]
```

[177]:
```python
phi = np.array([np.pi])  # phi(1) = pi
x = np.array([0, 0, phi[0], 0])  # x = [0; 0; phi(1); 0]
x = x.reshape(-1,1)

phiPP = np.array([0])  # phiPP(1) = 0
h = Ts / 20

num_inner_loop_steps = int(Ts / h) * Kmax
delta = np.zeros(num_inner_loop_steps + 1)
deltaP = np.zeros(num_inner_loop_steps + 1)
phi = np.zeros(num_inner_loop_steps + 1)
omega = np.zeros(num_inner_loop_steps + 1)

delta[0] = 0
deltaP[0] = 0
phi[0] = np.pi
omega[0] = 0
i = 1
deltaRef = 10
bool_val = 0  #
v = np.array([0])  # v(1) = 0
xHat = np.array([0, 0, np.pi, 0])  # xHat = [0; 0; pi; 0]
xHat = xHat.reshape(-1,1)
xOp = np.array([0, 0, np.pi, 0])  # xOp = [0 0 pi 0]'
xOp = xOp.reshape(-1,1)
reference = np.array([10])  # reference(1) = 10

print("x:", x)
```

```
print("delta:", delta)
print("phi:", phi)
print("omega:", omega)
print("reference:", reference)
```

```
x: [[0.        ]
 [0.        ]
 [3.14159265]
 [0.        ]]
delta: [0. 0. 0. … 0. 0. 0.]
phi: [3.14159265 0.         0.         … 0.         0.         0.        ]
omega: [0. 0. 0. … 0. 0. 0.]
reference: [10]
```

[178]:
```
import numpy as np

# Controladores iniciales
K = Kp1
KI = Kint1
i = 0
bool_val = 0

v = np.zeros(Kmax + 1)
u1 = np.zeros(Kmax)
u = np.zeros(Kmax*int(Ts/h)+1)
reference = np.zeros(Kmax*int(Ts/h) + 1)
# Funcional de costo LQR
Jn = [0]
Qloop = Q1
Rloop = R1

# Funcion de Lyapunov
initial_state_vector = np.vstack([x.reshape(-1, 1), 0])
V_L = [initial_state_vector.T @ P1 @ initial_state_vector]

V_L = [V_L[0].item()]

V_L = []
Ploop = P1
# Simulación
deadZone = 1

for index in range(Kmax):
    state_vector_prev = np.vstack([x, v[index]])
    quadratic_value = state_vector_prev.T @ Ploop @ state_vector_prev
    V_L.append(quadratic_value.item())
    yOut = Cc @ x
```

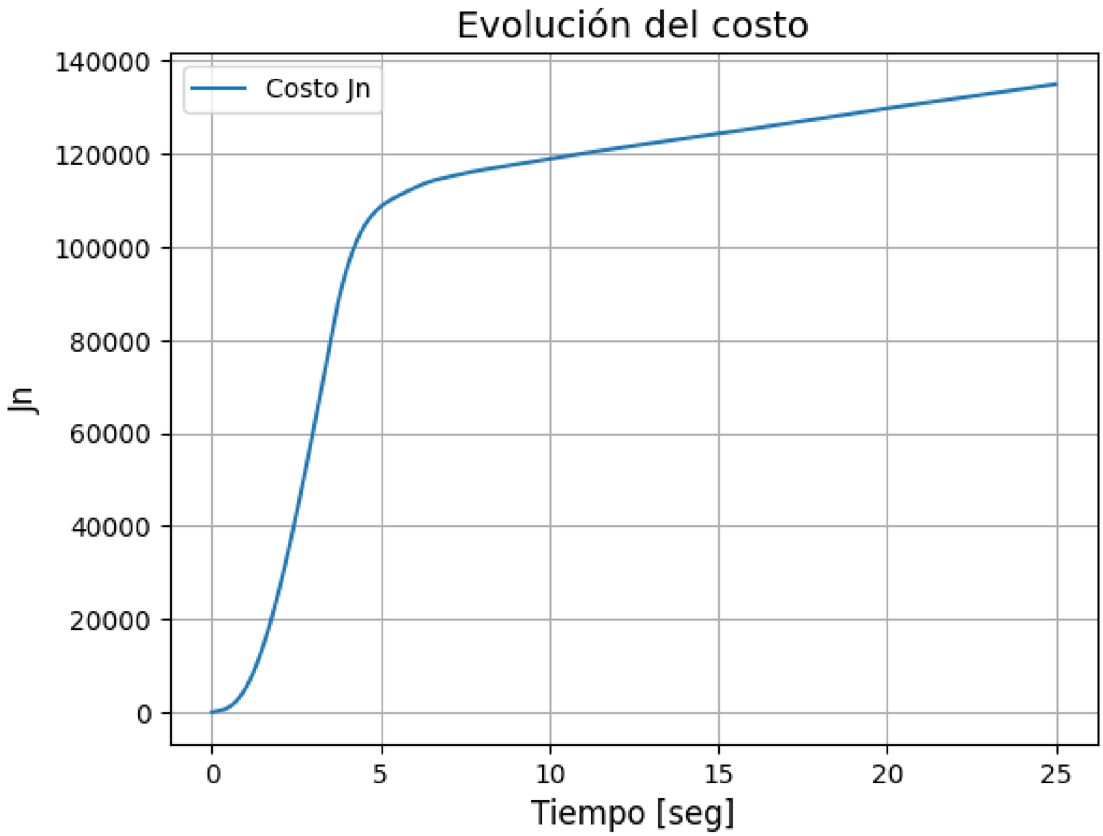
```
    yOutObs = Cc @ (xHat)
    v[index + 1] = v[index] + deltaRef - yOut[0]

    u1[index] = -K @ (xHat) + KI * v[index + 1]

    # Zona muerta
    if np.abs(u1[index]) < deadZone:
        u1[index] = 0
    else:
        u1[index] = np.sign(u1[index]) * (np.abs(u1[index]) - deadZone)

    # Loop interno para simular con Euler la evolución No-lineal del sistema
    for j in range(int(Ts / h)):
        u[i] = u1[index]
        p_pp = (1 / (M + m)) * (u[i] - m * l * phiPP * np.cos(phi[i]) + m * l *
↪omega[i]**2 * np.sin(phi[i]) - F * deltaP[i])
        phiPP = (1 / l) * (g * np.sin(phi[i]) - p_pp * np.cos(phi[i]))
        deltaP[i + 1] = deltaP[i] + h * p_pp
        delta[i + 1] = delta[i] + h * deltaP[i]
        omega[i + 1] = omega[i] + h * phiPP
        phi[i + 1] = phi[i] + h * omega[i]

        # Una vez que se llega a la disntancia de 10m cambiamos el controlador
↪y el valor de la masa
        if delta[i] >= 9.99:
            if bool_val == 0:
                deltaRef = 0
                m = m * 10
                bool_val = 1
                K = Kp2
                KI = Kint2
                Qloop = Q2
                Rloop = R2
                Ploop = P2
        i += 1
        reference[i] = deltaRef

    x = np.array([delta[i-1], deltaP[i-1], phi[i-1], omega[i-1]])
    x = x.reshape(-1,1)

    xHat = A @ xHat.reshape(-1,1) + B * u1[index] + Ko @ (yOut - yOutObs)

    state_vector = np.vstack([x, v[index+1]])

    quadratic_cost = state_vector.T @ Qloop @ state_vector + u1[index].T *
↪Rloop * u1[index]
    Jn.append(Jn[-1] + quadratic_cost[0][0])
```

```
u[i] = u1[index]

t = np.arange(0, T, h)
```

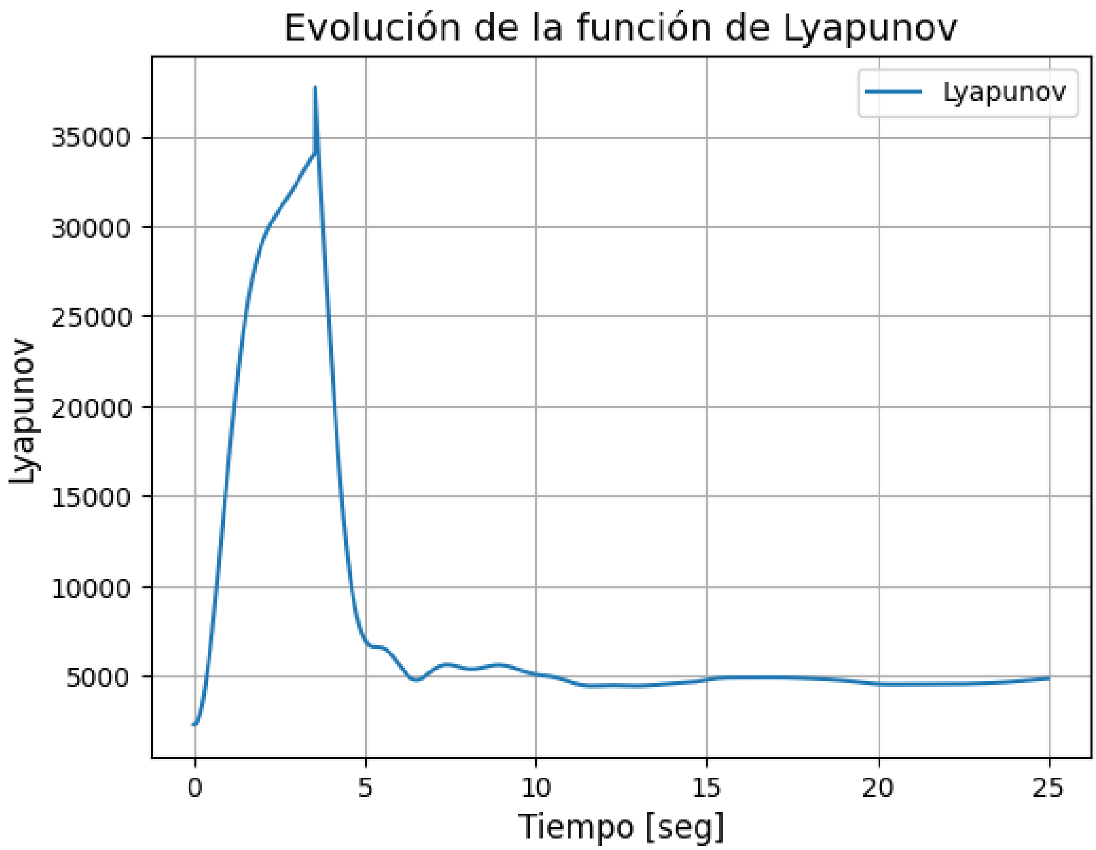
```
<ipython-input-178-3a21d5ee049b>:35: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  v[index + 1] = v[index] + deltaRef - yOut[0]
<ipython-input-178-3a21d5ee049b>:37: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  u1[index] = -K @ (xHat) + KI * v[index + 1]
<ipython-input-178-3a21d5ee049b>:50: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  deltaP[i + 1] = deltaP[i] + h * p_pp
<ipython-input-178-3a21d5ee049b>:52: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  omega[i + 1] = omega[i] + h * phiPP
```

[179]:
```
t_ = np.arange(0, T, Ts)
```

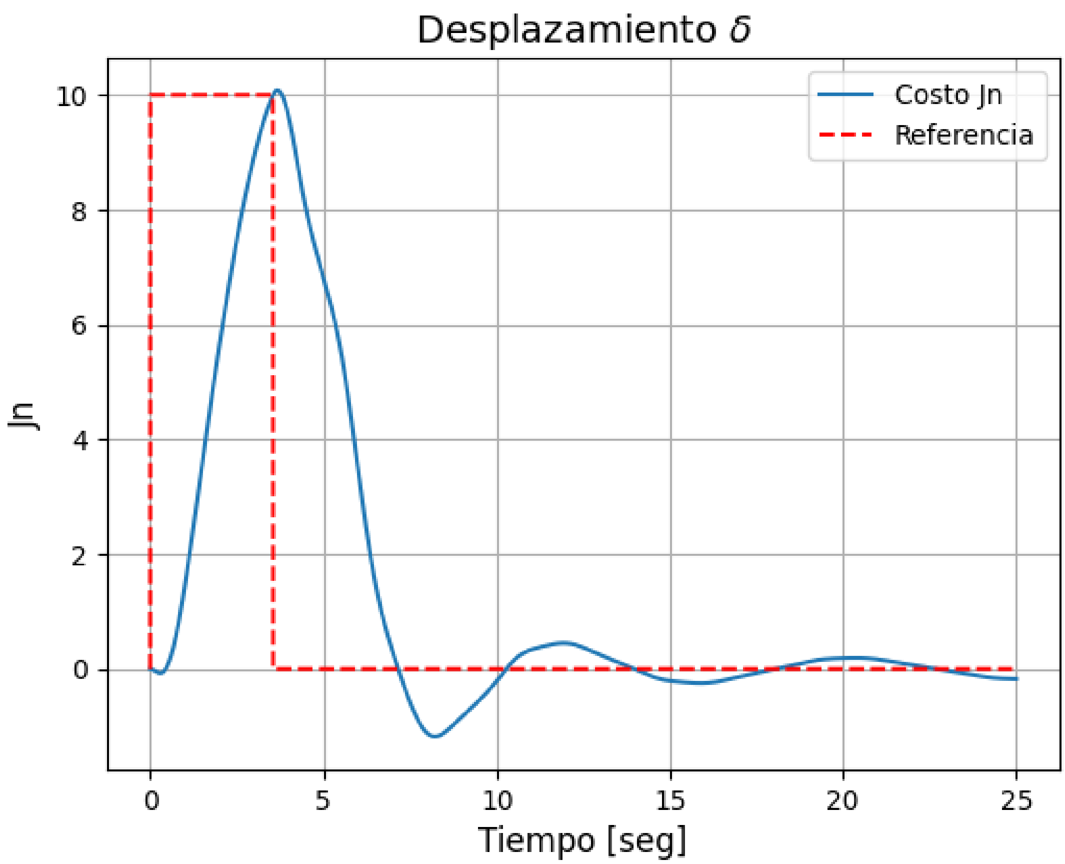
[191]:
```
plt.plot(t_, Jn[:len(t_)], linewidth=1.5, label="Costo Jn")

plt.grid(True)
plt.title('Evolución del costo', fontsize=14)
plt.xlabel('Tiempo [seg]', fontsize=12)
plt.ylabel('Jn', fontsize=12)

plt.legend()

plt.show()
```



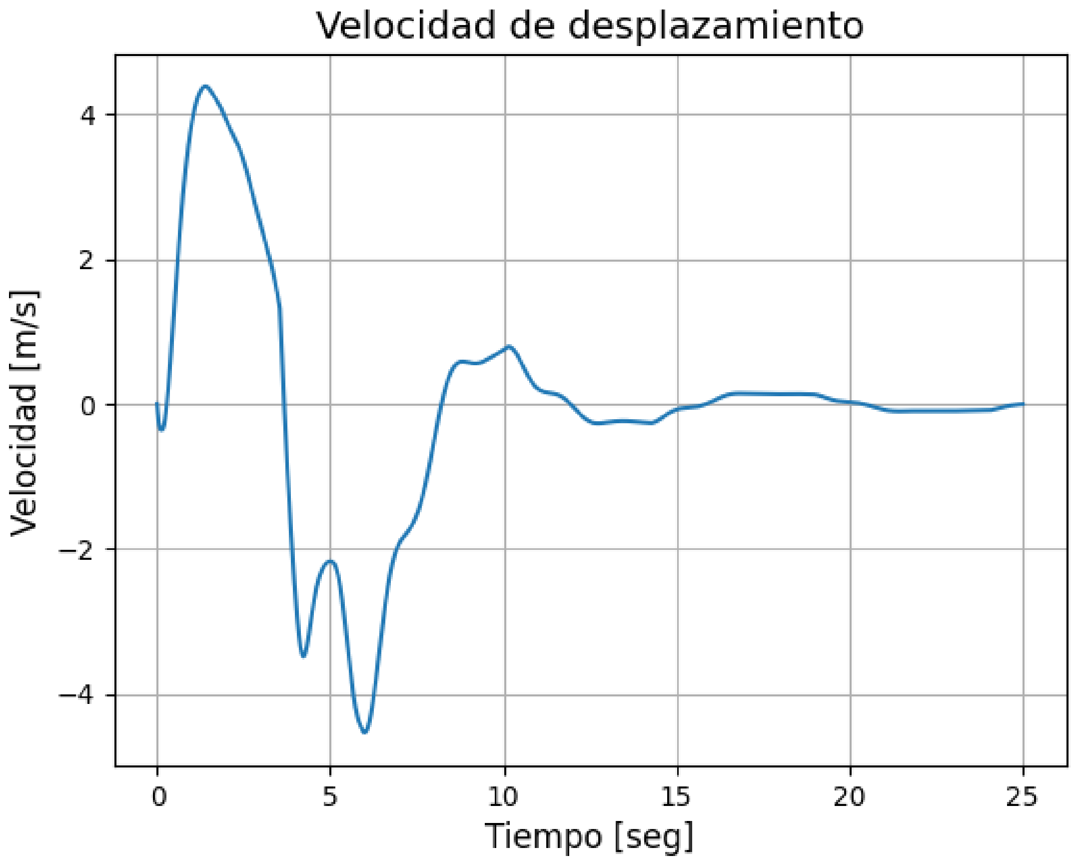
[192]:
```
plt.plot(t_, V_L[:len(t_)], linewidth=1.5, label="Lyapunov")

plt.grid(True)
plt.title('Evolución de la función de Lyapunov', fontsize=14)
plt.xlabel('Tiempo [seg]', fontsize=12)
plt.ylabel('Lyapunov', fontsize=12)

plt.legend()

plt.show()
```



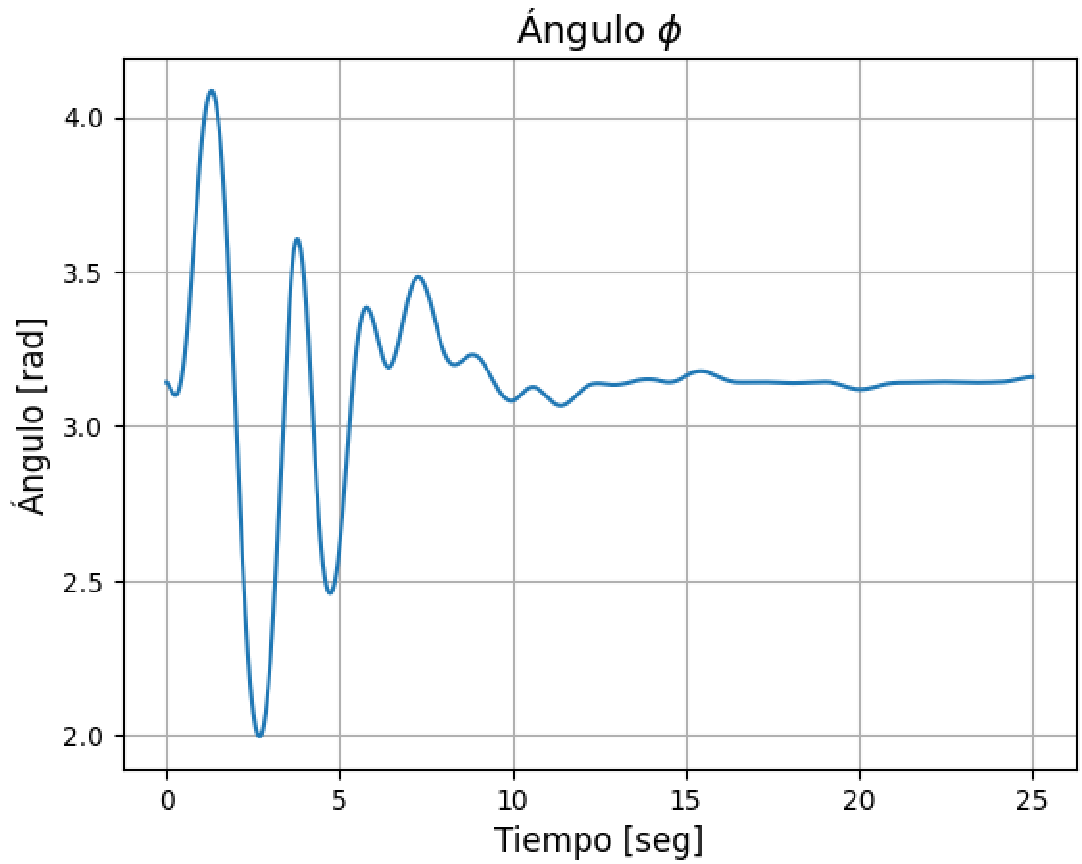
[190]:
```
plt.plot(t, delta[:len(t)], linewidth=1.5, label="Costo Jn")
plt.plot(t, reference[:len(t)], 'r--', label="Referencia")

plt.grid(True)
plt.title('Desplazamiento $\delta$', fontsize=14)
plt.xlabel('Tiempo [seg]', fontsize=12)
plt.ylabel('Jn', fontsize=12)

plt.legend()

plt.show()
```



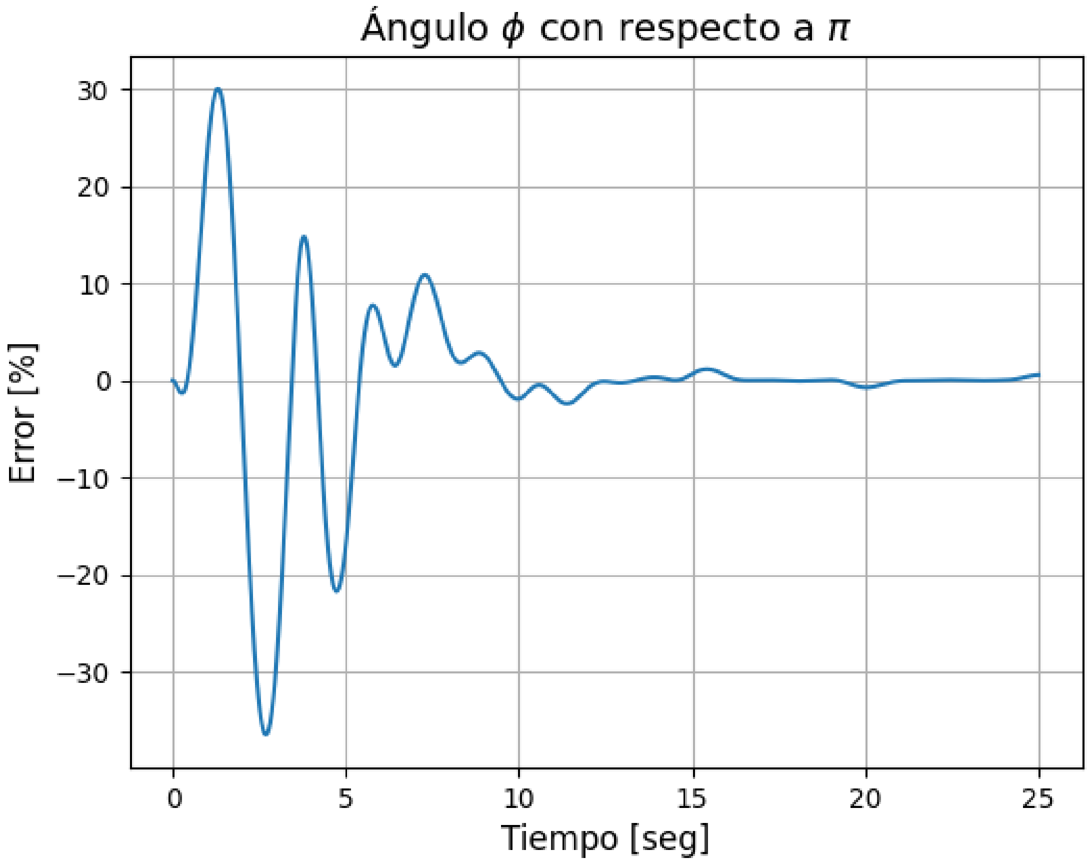
[183]:
```
import matplotlib.pyplot as plt

plt.plot(t, deltaP[:len(t)], linewidth=1.5, label="Velocidad de desplazamiento")

plt.grid(True)
plt.title('Velocidad de desplazamiento', fontsize=14)
plt.xlabel('Tiempo [seg]', fontsize=12)
plt.ylabel('Velocidad [m/s]', fontsize=12)

plt.show()
```



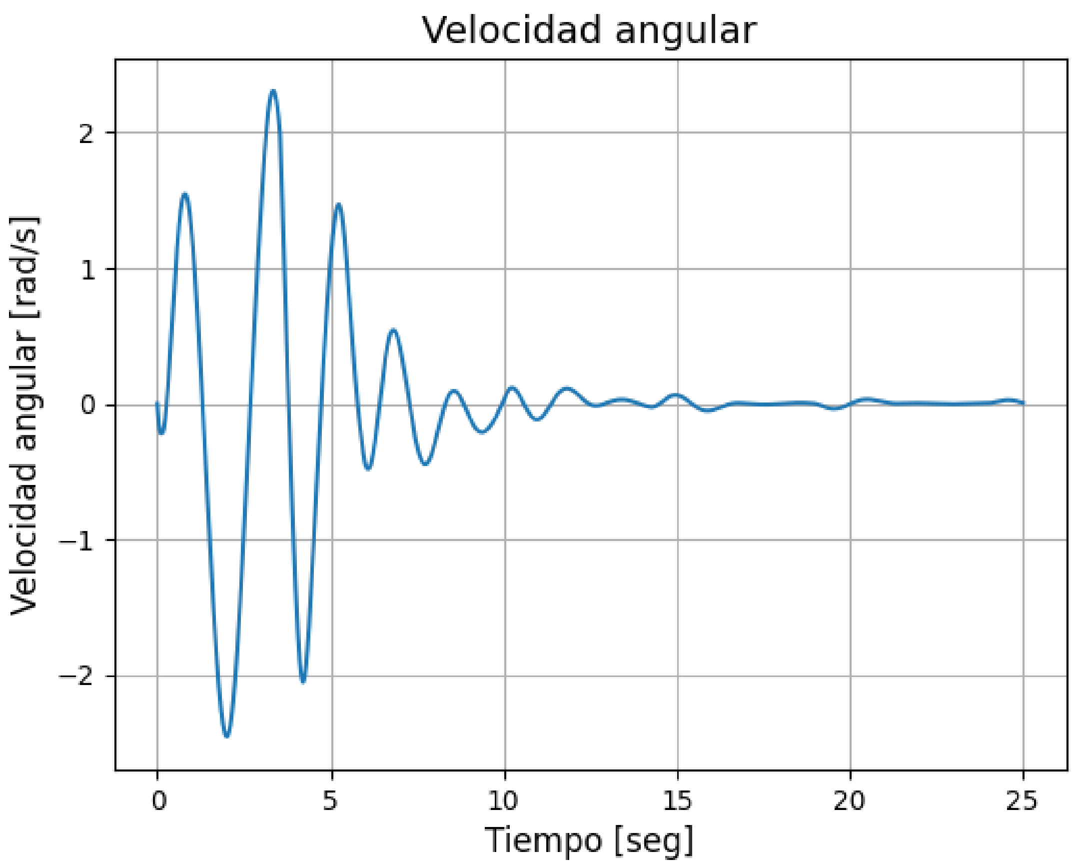
[184]:
```
import matplotlib.pyplot as plt

plt.plot(t, phi[:len(t)], linewidth=1.5, label="Ángulo $\phi$")

plt.grid(True)
plt.title('Ángulo $\phi$', fontsize=14)
plt.xlabel('Tiempo [seg]', fontsize=12)
plt.ylabel('Ángulo [rad]', fontsize=12)

plt.show()
```



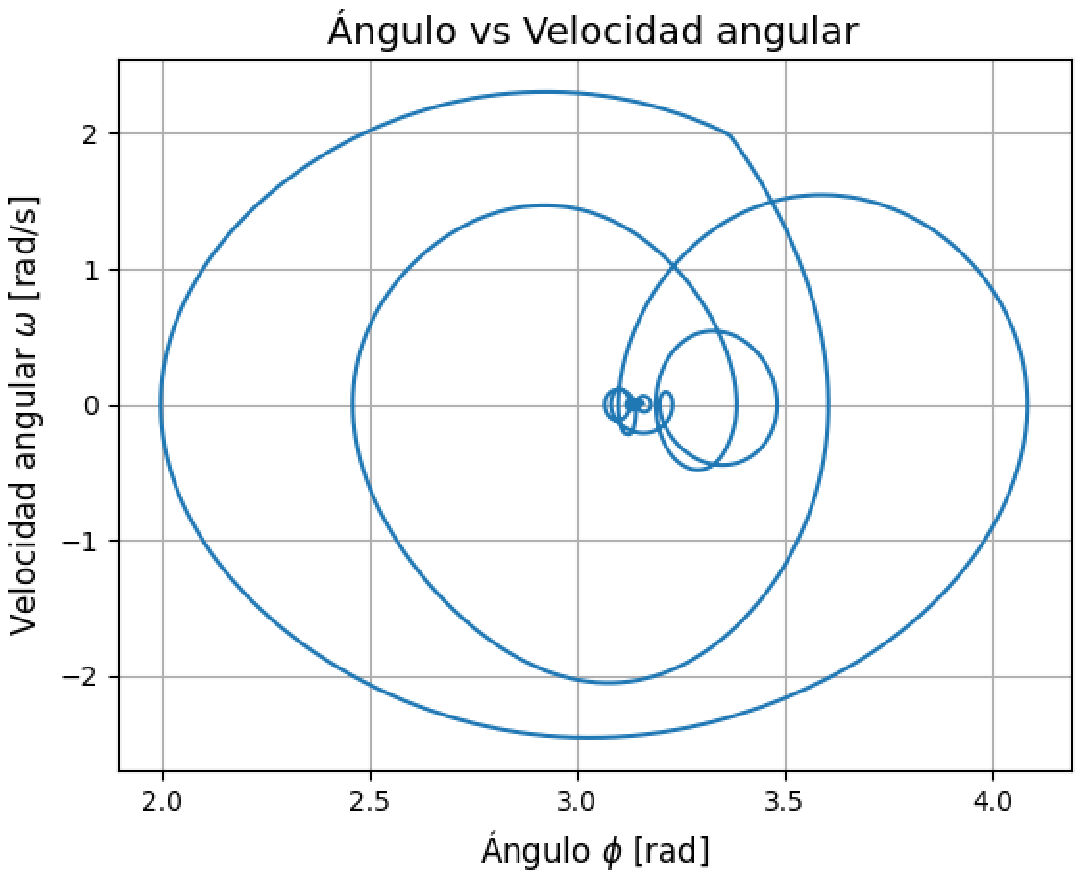
[185]:
```
plt.plot(t, (np.array(phi[:len(t)]) / np.pi) * 100 - 100, linewidth=1.5,␣
 ↪label="Error relativo respecto a $\pi$")

plt.grid(True)
plt.title('Ángulo $\phi$ con respecto a $\pi$', fontsize=14)
plt.xlabel('Tiempo [seg]', fontsize=12)
plt.ylabel('Error [%]', fontsize=12)

plt.show()
```



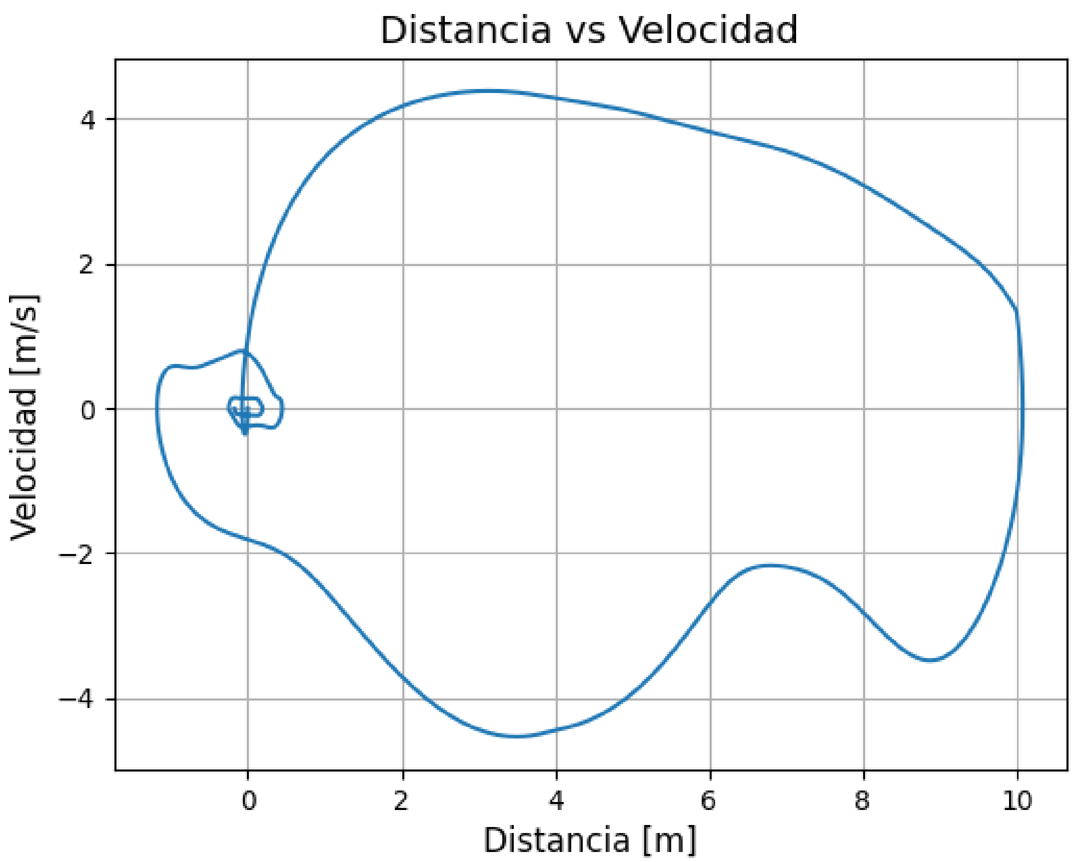
[186]:
```
plt.plot(t, omega[:len(t)], linewidth=1.5, label="Velocidad angular $\omega$")

plt.grid(True)
plt.title('Velocidad angular', fontsize=14)
plt.xlabel('Tiempo [seg]', fontsize=12)
plt.ylabel('Velocidad angular [rad/s]', fontsize=12)

plt.show()
```



[187]:
```
plt.plot(phi, omega, linewidth=1.5)

plt.title('Ángulo vs Velocidad angular', fontsize=14)
plt.xlabel('Ángulo $\phi$ [rad]', fontsize=12)
plt.ylabel('Velocidad angular $\omega$ [rad/s]', fontsize=12)
plt.grid(True)

plt.show()
```



[188]:
```python
import matplotlib.pyplot as plt

plt.plot(delta, deltaP, linewidth=1.5)

plt.title('Distancia vs Velocidad', fontsize=14)
plt.xlabel('Distancia [m]', fontsize=12)
plt.ylabel('Velocidad [m/s]', fontsize=12)
plt.grid(True)

plt.show()
```



3.0.2 Conculusiones

Conseguimos un actuador LQR en tiempo discreto que alcanza la referencia a los 25 segundos. Se aplico un actuador no lineal con un valor de 1 para la zona muerta.

[197]:

```

```