Why is correlation not allways a good indication of a pair being mean reverting?

Because correlation only looks at the distance between the return of both time series. It does not do any dependencie tests on the divergence of the series. Because of this we want to use co-integration.

Cointegration: When stationarity is found by combining several non-stationary price series together, these price series are called cointegrated.

Stationarity: Price series that are "mean reverting" are called stationary.

If we assume that y_t and x_t are non-stationary and cointegrated, then we can find a stationary linear combination. In other words: we are saying that the spread should be mean reverting. (assuming constant hedge ratio)

$$y_t - \beta x_t = u_t$$

Where,
y_t: Price series of stock Y
x_t: Price series of stock X
u_t : spread (price of the pair)
β : Hedge ratio