Matrix balancing under conflicting information

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RAS matrix balancing under conflicting information

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Abstract – Using the notation and concepts from constrained optimisation, we have developed a RAS variant (CRAS) that is able to balance and reconcile input-output tables and SAMs under conflicting external information and inconsistent constraints. In addition, CRAS fulfils all requirements of earlier RAS variants such as GRAS, and of constrained optimisation techniques:

a) constraints on arbitrarily sized and shaped subsets of matrix elements;
b) consideration of the reliability of the initial estimate and the external constraints;
c) ability to handle negative values and to preserve the sign of matrix elements.

We compare the CRAS variant with an equivalent technique for matrix balancing using constrained optimisation. In our case, we employ a conjugate gradient algorithm. Applying these two methods to the 1993-94 Australian National Accounts, we find that

- as constrained optimisation, CRAS is able to find a compromise solution between inconsistent constraints; this feature does not exist in conventional RAS variants such as GRAS;
- CRAS is significantly faster than the conjugate gradient algorithm;
- the conjugate gradient algorithm produces slightly more accurate results.

CRAS can constitute a major advance for the practice of balancing input-output tables and Social Accounting Matrices, in that it removes the necessity of manually tracing inconsistencies in external information. In contrast to constrained optimisation, this quality does not come at the expense of complicated programming requirements, and long run times.

Keywords: RAS matrix balancing, conflicting information

1 Introduction

A common problem in compiling and updating Social Accounting Matrices (SAM) or input-output tables is that of incomplete data. Missing matrix elements may be due to a variety of reasons such as costly and therefore incomplete industry surveys, or the suppression of confidential information. External data points can be used to formulate a system of equations that constrain the unknown matrix elements. However, unknowns usually outnumber external constraints, resulting in the system being underdetermined, that is exhibiting too many degrees of freedom to be solved analytically. The two most prominent numerical approaches for reconciling such an underdetermined system are probably the RAS method, and constrained optimisation.

During the past 40 years, both approaches have successfully tackled a number of challenges, leading to a number of useful features[1]: Ideally, the technique should

a) incorporate constraints on arbitrarily sized and shaped subsets of matrix elements, instead of only fixing row and column sums;
b) allow considering the reliability of the initial estimate;
c) allow considering the reliability of external constraints;
d) be able to handle negative values and to preserve the sign of matrix elements if required;
e) be able to handle conflicting external data.

While all criteria have been addressed by constrained optimisation methods, there is currently no RAS-type technique that satisfies criterion e). In particular the inability of RAS to deal with conflicting external data represents a considerable drawback for practice, because for most statistical agencies such data are often rather the norm than the exception.

The most simple case of conflicting data is probably a situation in which two data sources are located that prescribe two different values for the same matrix entry, resulting in inconsistent constraints. When faced with such constraints, existing RAS variants adjust the respective matrix element in turn to both directly conflicting values, and thus enter into oscillations without ever converging to a satisfactory solution.

More generally, sets of external data can be conflicting indirectly amongst each other. Cole, 1992 mentions convergence problems, and gives a simple example as a matrix $\begin{pmatrix} a & b \\ c & 1 \end{pmatrix}$ with a, b, c ≥ 0, and with inconsistent row and column totals $\{1, 3\}^t$ and $\{1, 3\}$. In practice, indirect conflict might present itself for example when on one hand, data on final demand and gross output of wheat suggest a certain intermediate demand of wheat, however on the other hand this intermediate demand is too large to be absorbed by the flour milling sector. Further examples involving conflicting external information are GDP measures[2], and multi-national and regional input-output systems. In practice, such inconsistencies are often traced and adjusted manually by statisticians.[3]

In this work we present a new RAS variant that is able to handle conflicting external data and inconsistent constraints. We achieve this capability by introducing standard error estimates for external data. We build on previous RAS variants that satisfy the remaining criteria, and thus arrive at a RAS-type method that matches the capabilities of constrained optimisation. We will refer to this method as CRAS (Conflicting RAS).

2 Literature review

The RAS method – in its basic form – bi-proportionally scales a matrix $\mathbf{A}_0$ of unbalanced preliminary estimates of an unknown real matrix **A**, using **A**'s known row and column sums. The balancing process is usually aborted when the discrepancy between the row and column sums of $\mathbf{A}_0$ and **A** is less than a previously fixed threshold. Bacharach, 1970 has analysed the bi-proportional constrained matrix problem in great detail, in particular in regard to the economic meaning of bi-proportional change[4], the existence and uniqueness of the iterative RAS solution, its properties of minimisation of distance metric[5], and uncertainty associated with errors in row and column sum data and with the assumption of bi-proportionality. The origins of the method go back several decades (Deming & Stephan, 1940). Stone & Brown, 1962, Bacharach, 1970, and Polenske, 1997 provide a historical background.

2.1 Constraints on arbitrarily sized and shaped subsets of matrix elements

A special situation arises when some of the matrix elements of **A** are known in addition to its row and column sums, for example from an industry survey. The 'modified RAS' (MRAS) approach (Paelinck & Walbroeck, 1963; Allen, 1974; Lecomber, 1975a) deals with this partial information as follows: the preliminary estimate $\mathbf{A}_0$ has to be "netted", that is the known elements are subtracted, and $\mathbf{A}_0$ contains 0 at the corresponding positions. The net $\mathbf{A}_0$ is then subjected to the standard RAS procedure, and the known elements are added back on after balancing.

In practice, situations can arise where, in addition to certain elements of **A**, some aggregates of elements of **A** are known. For example, a published table $\mathbf{A}^G$ of national aggregates may constitute partial information when constructing a multi-regional input-output system, or a more disaggregated national table. Accordingly, Oosterhaven *et al.*, 1986 add a "national cell constraint" to the standard row and column sum constraints. Similarly, Jackson & Comer, 1993 use partition coefficients for groups of cells of a disaggregated base year matrix to disaggregate cells in an updated but aggregated matrix. Batten & Martellato, 1985 (p. 52-55) discuss further constraints structures, involving intermediate and final demand data. Gilchrist & St Louis, 1999; 2004 propose a three-stage "TRAS" for the case when aggregation rules exist under which the partial aggregated information $\mathbf{A}^G$ can be constructed from its disaggregated form **A**. Subjecting an input-output matrix to random censoring, these authors demonstrate that the inclusion of partial aggregated information into the TRAS procedure leads to superior outcomes than applying the standard RAS method. Cole, 1992 describes the general TRAS type that accepts constrained subsets of any size or shape. However, no TRAS variant deals with uncertainties, or handles negative matrix elements and conflicting external information.

2.2 Reliability of the initial estimate and external information

Another variant of the MRAS method takes into account the uncertainty of the preliminary estimates, and contains the occurrence of perfectly known elements as a special case (Lecomber, 1975a; b, with case studies in Allen, 1974 and Allen & Lecomber, 1975). This is accomplished by introducing a matrix **E** containing "reliability information" about the elements in $\mathbf{A}_0$. $\mathbf{E}$ instead of $\mathbf{A}_0$ is then balanced in order to take up the difference between the preliminary and true totals:

$$\mathbf{A}^* = \left(\mathbf{A}_0 - \mathbf{E}\right) + \hat{\mathbf{r}}\mathbf{E}\hat{\mathbf{s}}\,. \qquad (1)$$

$\mathbf{A}^*$ is the balanced estimate, and $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ are diagonal scaling matrices, as in the conventional RAS algorithm. Where $E_{ij} = 0$, A_{ij} remains unchanged during balancing. Lecomber, 1975a; b also investigates the influence of errors in the "true" totals.

A shortcoming of Lecomber's approach is that the elements of $\mathbf{E}$ cannot be interpreted as standard deviations. If we follow Lecomber in maintaining $0 \le E_{ij} \le A_{0ij}$, and consider that RAS preserves the positive signs in $\mathbf{E}$, then $A^*_{ij} \ge A_{oij} - E_{ij}\,\forall i, j$. In other words if E_{ij} were the standard deviations of A_{0ij}, then the balanced estimate $\mathbf{A}^*$ could never go more than one standard deviation below the initial estimate $\mathbf{A}_0$. An upper limit for $\mathbf{A}^*$ does not exist however. Thus, as Lecomber points out, the elements of $\mathbf{E}$ must be sufficiently large to ensure the controlling vectors are non-negative – but there is no method to ensure this, whilst still interpreting the elements of $\mathbf{E}$ as standard errors. Thus considering that conflicting external information may well diverge by more than one standard deviation, it follows that MRAS will not reach a solution under sufficiently inconsistent constraints, unless more (unspecified) information on errors is obtained.

Lahr, 2001 takes into account the uncertainties of external constraints in treating the tolerances of the RAS termination criteria as functions of the varying reliabilities of row and column sums. Dalgaard & Gysting, 2004 incorporate information about the reliability of external constraints (again row and column totals) into the balancing process as "confidence factors" λ, and successively adjust the target totals $\mathbf{u}_n$ of the nth iteration as a weighted sum $u_{n,j} = \lambda_j^{n-1} u_{0,j} + \left(1 - \lambda_j^{n-1}\right) u_{n-1,j}$ of the initial unbalanced totals $u_{0,j}$ and the totals $u_{n-1,j}$ of the previous iteration. With subsequent iterations, the confidence factors $0 \le \lambda_j^n \le 1$ become smaller and smaller, thus gradually converging away from the unbalanced initial totals $\mathbf{u}_0$, towards the balanced totals $\mathbf{u}_\infty$. The innovation is that totals with high confidence ($\lambda_j \le 1$) get adjusted away from the initial totals much slower than those totals with low confidence ($\lambda_j \ge 0$).

While both approaches consider the varying reliability of totals, they cannot deal with inconsistent totals. In applying conventional RAS scaling factors, Lahr's algorithm would always end up balancing matrix elements to satisfy only one of a number of conflicting external constraints. Similarly, for large enough n, Dalgaard and Gysting's algorithm would oscillate around those inconsistent totals $u_{n-1,j}$ with non-zero confidence.[1]

2.3 Negative elements

[1] Dalgaard & Gysting, 2004 do describe balancing matrices with "unequal net row and column sum" and "macro differences between supply and use". However, rather than inconsistencies in external information, this means correct differences in the sum over supply by *industry* and use by *product*, which naturally occur in asymmetric commodity-by-industry supply and use tables.

Junius & Oosterhaven, 2003 derive a generalised RAS ("GRAS") algorithm that can balance negative elements, by splitting the matrix **A** into positive and negative parts **P** and **N**, and balancing **A** = **P** – **N** according to

$$\begin{aligned}&\left(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}\right)\mathbf{i} = \mathbf{u}^* \\ &\mathbf{i}\left(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}\right) = \mathbf{v}^*\end{aligned} , \quad (2)$$

where **i** is the summation vector. Note that in order to minimise information gain, the balanced matrix $\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}$ conform to totals $\mathbf{u}^* = e\,\mathbf{u}$, $\mathbf{v}^* = e\,\mathbf{v}$, and $\mathbf{i}\,\mathbf{u}^* = \mathbf{i}\,\mathbf{v}^*$, where e = 2.718… is the base of the exponential function, and **u** and **v** are the prescribed row and column sum vectors, respectively, of **A** (Oosterhaven, 2005). The results $\{A_{ij}\}$ of GRAS have to be scaled down by e in order to satisfy the initially prescribed totals **u** and **v**.

In its basic formulation by Junius & Oosterhaven, 2003, GRAS neither incorporates constraints on subsets, nor does it deal with uncertainty and data conflict.

2.4 Constrained optimisation

Already Bacharach, 1970 has shown that the conventional RAS technique is equivalent to the constrained minimisation of an information gain function $f = \Sigma_{ij} A_{ij} \ln(A_{ij}/A_{0ij})$. Naturally, this circumstance lead to the parallel developments of both RAS and constrained optimisation techniques for the purpose of balancing input-output tables or SAMs. It is interesting to see that researchers working on either technique have faced almost the same challenges.

The basic structure of a constrained optimisation problem applied to SAMs is

$$\text{Minimise } f(\mathbf{A}, \mathbf{A}_0), \text{ subject to } \Sigma_i A_{ij} = x_j \text{ and } \Sigma_j A_{ij} = x_i , \quad (3)$$

where f is the objective function, and x_i and x_j are row and column totals. Morrison & Thumann, 1980 minimise a weighted sum of squares of deviations $f = \Sigma_{ij} (A_{ij} - A_{0ij})^2 / w_{ij}$, where the w_{ij} are the weights. They also explicitly describe the incorporation of external information referring to general subsets of matrix elements, into a Lagrange multiplier approach. Using a vectorised representation of $\mathbf{A} = \{a_i\}_{i=1,N\times N}$, a system of N_C constraints of any shape and size on $N\times N$ variables (including row and column totals) can be conveniently described in matrix notation:

$$\mathbf{G}\,\mathbf{a} = \mathbf{c} , \quad (4)$$

Where the "aggregator matrix" **G** ($N_C \times N$) holds the coefficients linking the N variables a_i with the external data c_i on the N_C constraints.

Byron, 1978 incorporates variances $\mathbf{\Sigma}$ for the initial estimate $\mathbf{a}_0$ into a quadratic Lagrange function $f = (\mathbf{a} - \mathbf{a}_0)' \,\Sigma^{-1} (\mathbf{a} - \mathbf{a}_0) + \lambda'(\mathbf{Ga} - \mathbf{c})$, and uses the first-order conditions to solve for the Lagrange multipliers and the balanced SAM:

$$\lambda = (\mathbf{G\Sigma G'})^{-1}(\mathbf{Ga}_0 - \mathbf{c}) , \quad (5a)$$
$$\mathbf{a} = \mathbf{a}_0 - \mathbf{\Sigma G}\lambda . \quad (5b)$$

van der Ploeg, 1982; 1984; 1988 elegantly extends Byron's formulation by a) adding disturbances ε to the external constraint information **c**, so that **G a** = **c** + ε, and b) extending the unknown vector **a** with the unknown disturbances ε, to a compound vector **p**, distributed as

$$\mathbf{p} = \begin{pmatrix} \mathbf{a} \\ \boldsymbol{\varepsilon} \end{pmatrix} \sim \mathrm{D}\left[\begin{pmatrix} \mathbf{a}_0 \\ 0 \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_\mathbf{a} & \\ & \boldsymbol{\Sigma}_\mathbf{c} \end{pmatrix}\right] = \mathrm{D}[\mathbf{p}_0, \boldsymbol{\Sigma}] \tag{6}$$

with means $\mathbf{a}_0$ and 0, and variances Σ_a and Σ_c. Exactly known constraints are a special case with the corresponding element of Σ_c being zero. Extending **C** = (**G**, –**I**), where **I** is the unity matrix, the generalised problem becomes

$$\text{Minimise } f = (\mathbf{p} - \mathbf{p}_0)' \, \Sigma^{-1} \, (\mathbf{p} - \mathbf{p}_0), \text{ subject to } \mathbf{C}\,\mathbf{p} = \mathbf{c}, \tag{7}$$

with solutions analog to Eqs. 5a and 5b. Since the solution for the Lagrange multipliers involves the inversion of **CΣC**', computing times are strongly influenced by the sizes N and N_C of the SAM and constraint system. Both Byron and van der Ploeg go to great lengths in exploiting the sparse structure of the coefficients matrix, and in devising efficient algorithms in order to be able to solve large SAMs. In effect, it is the introduction of ε and Σ_c that enables handling conflicting external data (van der Ploeg calls it "constraint violation"), because the disturbances ε in Eqs. 6 and 7 allows the adjusted constraint value **G a** to deviate from its prescribed value **c**.

Lecomber, 1975a, Morrison & Thumann, 1980, and Harrigan & Buchanan, 1984 explicitly note that the conventional Langrange multiplier procedure (Eqs. 3a and 3b) does not guarantee non-negative solutions. This is undesirable because negative matrix entries can present problems in input-output analysis (ten Raa & van der Ploeg, 1989).

With the requirement of non-negativity, the constrained optimisation problem essentially becomes a *bounded* constrained optimisation. In general, one asks that the unknown SAM elements are within lower and upper bounds $l_i \leq a_i \leq u_i$. The mixing of equality and inequality conditions requires quadratic programming methods, which renders the solution of the optimisation problem considerably more complicated, as the expositions of Harrigan & Buchanan, 1984, Zenios *et al.*, 1989, and Nagurney & Robinson, 1992 may testify.

Tarancon & Del Rio, 2005 present an interesting variant of the bounded optimisation problem, by deriving lower and upper bounds from criteria for the stable structural evolution of input-output coefficients, and introducing supplementary variables to take up the slack between the bounds and the matrix entries. If the model turns out to be inconsistent because some constraints cannot be met within those bounds, then the analyst manually chosen certain constraints to be relaxed, until no variable exceeds the bounds.

Criterion	RAS-type technique	Constrained optimisation
a)	Gilchrist & St Louis, 1999	Morrison & Thumann, 1980
b)	Lecomber, 1975a; b	Stone *et al.*, 1942; Byron, 1978

c)	Lecomber, 1975a; b; Lahr, 2001; Dalgaard & Gysting, 2004	van der Ploeg, 1982
d)	Junius & Oosterhaven, 2003	Harrigan & Buchanan, 1984
e)	This work	van der Ploeg, 1982

Tab. 1: Recent extensions to RAS and optimisation techniques for balancing SAMs and input-output tables.

3 CRAS

Tarancon & Del Rio, 2005 explicitly state that (p. 2) "… the RAS process cannot be developed with interval estimates of the margins. Hence, point estimates are used, which may carry an implicit error." On the other hand, compared to constrained optimisation techniques, RAS has enjoyed higher popularity, which is probably due to ease of programming. Considering that the use of RAS in statistical agencies requires the manual and therefore often tedious removal of inconsistencies in the constraint system, it would be desirable to have a RAS technique that deals with such common occurrences in a systematic and automated way. The description of such a RAS variant is the topic of this Section. We will base our derivation strongly on the GRAS notation of Junius & Oosterhaven, 2003.

In the standard GRAS method, the preliminary estimate $\mathbf{A}_0 = \mathbf{P}_0 - \mathbf{N}_0$ is alternately row- and column-scaled using diagonal matrices $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$, so that after the n-th round of balancing, $\mathbf{A}_n = \hat{\mathbf{r}}_{n-1}\mathbf{P}_{n-1}\hat{\mathbf{s}}_{n-1} - \hat{\mathbf{r}}_{n-1}^{-1}\mathbf{N}_{n-1}\hat{\mathbf{s}}_{n-1}^{-1}$. $\mathbf{A}_n$ is then subjected to the next scaling operation. GRAS uses scalers

$$r_{n,i} = \frac{u_i^* + \sqrt{u_i^{*2} + 4\sum_j P_{n,ij} \sum_j N_{n,ij}}}{2\sum_j P_{n,ij}}, \text{ with}$$

$$P_{n,ij} = P_{n-1,ij} s_{n-1,j}, N_{n,ij} = N_{n-1,ij} s_{n-1,j}^{-1}, \text{ and} \tag{8}$$

$$s_{n-1,j} = \frac{v_j^* + \sqrt{v_j^{*2} + 4\sum_i P_{n-1,ij} \sum_i N_{n-1,ij}}}{2\sum_i P_{n-1,ij}}.$$

The algorithm converges if

$$\begin{aligned} \left\| \left(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}\right)\mathbf{i} - \mathbf{u}^* \right\| &< \delta \left\| \mathbf{u}^* \right\| \\ \left\| \mathbf{i}\left(\hat{\mathbf{r}}\mathbf{P}\hat{\mathbf{s}} - \hat{\mathbf{r}}^{-1}\mathbf{N}\hat{\mathbf{s}}^{-1}\right) - \mathbf{v}^* \right\| &< \delta \left\| \mathbf{v}^* \right\| \end{aligned}, \tag{9}$$

for a sufficiently small δ.

3.1 Incorporating constraints on arbitrary subsets of matrix elements

Consider now a generalised formulation of constraints as in **G a** = **c** (Eq. 4). Such a formulation includes constrained row and column sums, constraint single elements, constrained subsets, and negative elements as special cases. Constraints can include any number of elements, which may be fully, partly or non-adjacent.[6] Constraints may also exclude some of the row and column totals (compare Thissen & Löfgren, 1998, p. 1994). Let $\mathbf{G} = \mathbf{G}^{+} - \mathbf{G}^{-}$ be a decomposition of the constraint coefficients matrix, analog to the decomposition $\mathbf{A} = \mathbf{P} - \mathbf{N}$ of **A**. Let there be N_C constraints, and let $\mathbf{c}^* = e\,\mathbf{c}$. Eq. 8 can then be generalised to

$$r_n = \frac{c_i^* + \sqrt{c_i^{*2} + 4\sum_j G_{ij}^{+} a_{n-1,j} \sum_j G_{ij}^{-} a_{n-1,j}}}{2\sum_j G_{ij}^{+} a_{n-1,j}} \quad \text{and}$$

$$a_{n,j} = a_{n-1,j}\, r_n^{\mathrm{Sgn}(G_{ij})}, \qquad \text{with } i = n - \left\lfloor \frac{n}{N_C} \right\rfloor N_C . \tag{10}$$

In Eq. 10, the negative elements in Eq. 8 have been replaced with negative coefficients on positive elements, but otherwise the formulation is exactly the same. There is only one scaler r_i for each constraint i, and these scalers are applied consecutively for all $i = 1,\ldots,N_C$.[2] The r_i and a_j are calculated alternately. The GRAS feature of scaling negative elements by the inverse of the positive scaler is evident in the exponent $\mathrm{Sgn}(G_{ij})$ in Eq. 10. The algorithm converges if

$$\left\| \mathbf{Ga} - \mathbf{c}^* \right\| < \delta \left\| \mathbf{c}^* \right\| , \tag{11}$$

for a sufficiently small δ.

3.2 Incorporating reliability and conflict of external data

In cases of inconsistent constraints brought about by conflicting external data, the termination condition (11) may never be met, and GRAS has to be terminated if the distance function between the constraints **c** and their realisations **G a** does not improve anymore, that is if for two subsequent iterations $n - 1$ and n

$$\left\| \mathbf{Ga} - \mathbf{c}^* \right\|_n - \left\| \mathbf{Ga} - \mathbf{c}^* \right\|_{n-1} < \delta , \tag{12}$$

[2] The symbol $\lfloor\ \rfloor$ in equation 10 is the floor function and refers to the largest integer smaller than the number inside.

for a sufficiently small δ. Following this termination, we propose a GRAS-type algorithm that modifies the constraints **c*** as well:

$$r_n = \frac{c^*_{n,i} + \sqrt{c^{*2}_{n,i} + 4\sum_j G^+_{ij} a_{n-1,j} \sum_j G^-_{ij} a_{n-1,j}}}{2\sum_j G^+_{ij} a_{n-1,j}},$$

$$c^*_{n,i} = c^*_{n-1,i} - \mathrm{Sgn}\left(c^*_{n-1,i} - \sum_j G_{ij} a_{n-1,j} \right) \times \mathrm{Min}\left(\left| c^*_{n-1,i} - \sum_j G_{ij} a_{n-1,j} \right|, \alpha\sigma_i \right) \text{ with } c^*_{0,i} = c^*_i \text{, (13)}$$

$$a_{n,j} = a_{n-1,j} r_n^{\mathrm{Sgn}(G_{ij})}, \qquad \text{and } i = n\text{-}\left\lfloor \frac{n}{N_C} \right\rfloor N_C,$$

where $0 \le \alpha \le 1$ and σ_i is the standard error of c_i. We refer to this algorithm as CRAS ('Conflicting RAS'). The essence of this idea is that once GRAS terminates in oscillations without reaching convergence, the original external constraints c_i can clearly not all be satisfied simultaneously, and either some of them or all of them must be erroneous. In order to achieve convergence, the c_i must be modified "towards" their realisations $\{\mathbf{Ga}\}_i$. Since each constraint is known to a higher or lower degree of accuracy. Therefore, an amount $\alpha\sigma_i$ is added or subtracted from each $c^*_{n-1,i}$, depending on the sign Sgn($c^*_{n-1,i} - \sum_j G_{ij} a_{n-1,j}$). The constant α can be chosen freely: The higher its value, the more rapid the adjustment process, but also the more inaccurate the adjustment. Note that in order to prevent overshoot in situations where the realisation $\{\mathbf{Ga}\}_i$ is closer to the c_i than σ_i, the maximum adjustment allowed is $|c^*_{n-1,i} - \sum_j G_{ij} a_{n-1,j}|$. With constraint values mdified as in Eq. 13, the termination criterion of CRAS is equal to that in Eq. 11.

4 Applications

In the following, we will examine the performance of the CRAS method for the example provided by Cole, 1992, shown in Tab. 2, and for the 1993-94 Australian Make-Use framework, using a variety of distance measures. The results of the CRAS balancing process are compared with the GRAS balanced estimates (i.e. distance measure of CRAS from the initial estimate GRAS, as no 'correct' solution is known). A number of authors[7] examine concepts of relative distance between two matrices in order to characterise the comparative performance of matrix balancing methods. According to Butterfield & Mules, 1980, p. 293), "there exists no single statistical test for assessing the accuracy with which one matrix corresponds to another. Analysts working in this area have tended to use a number of [complementary] tests." Accordingly, we chose eight measures (compare Harrigan *et al.*, 1980, Günlük-Senesen & Bates, 1988, and Lahr, 2001).

– the relative arithmetic mean of absolute differences $AMAD = \dfrac{\sum_i \frac{|c_i - G_i a_n|}{\sigma_i}}{\sum_i \frac{c_i}{\sigma_i}}$;

– the standardised weighted absolute difference $SWAD = \frac{\sum_i \frac{c_i}{\sigma_i} \frac{|c_{ij} - G_i a_n|}{\sigma_i}}{\sum_i \frac{c_i}{\sigma_i}^2}$

– the relative geometric mean of absolute differences[8] $GMAD = \frac{\sqrt{\sum_i \left(\frac{|c_i - G_i a_n|}{\sigma_i}\right)^2}}{\sqrt{\sum_i \frac{c_i}{\sigma_i}^2}}$;

– the Isard/Romanoff Similarity Index[9] $SIM = 1 - \frac{\sum_i \frac{|c_i - G_i a_n|}{(c_i + G_i a_n)}}{N^2}$;

– the χ^2 distribution of absolute differences $CHI = \sum_i \frac{\left(\frac{(c_i - G_i a_n)}{\sigma_i}\right)^2}{\frac{c_i}{\sigma_i}}$;

– the arithmetic mean of relative differences $AMRD = \frac{\sum_i \frac{|c_i - G_i a_n|}{c_i}}{N^2}$;

– the information index $INFO = \sum_i \frac{G_i a}{\sigma_i}_n \log \frac{G_i a_n}{c_i}$; and

– the correlation coefficient $CORR = \frac{\mathrm{Cov}\left(\frac{c_i}{\sigma_i}, \frac{G_i a_n}{\sigma_i}\right)}{\sqrt{\mathrm{Var}(\frac{\mathrm{c}_i}{\sigma_i})}\sqrt{\mathrm{Var}(\frac{G_i\,\mathrm{a}_\mathrm{n}}{\sigma_i})}}$,

where $\mathbf{a}_n$ is the n^{th} iteration of **a**, and N the dimension of **a**. As we are interested in the change relative to sigma, the distance measures were calculated with respect to sigma, as shown.

$$\begin{pmatrix} a & b \\ c & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 \end{pmatrix}$$

Tab 2. Inconsistent constraint example of Cole, 1992

The results are presented in Tab 3. All algorithms were programmed and run in Visual Basic. The constrained optimisation approach was based on the Numerical Recipes package using conjugate gradients (Press, 1997). Results are presented for constrained optimisation, and for CRAS with different selections of alpha.

		AMAD	SWAD	GMAD	SIM	CHI	AMRD	INFO	CORR	Time (sec)
Constrained Optimisation		0.2414	0.2679	0.2873	0.9156	0.2574	0.0720	0.3459	0.353754	0.51
CRAS	α = 1	0.2511	0.2552	0.2557	0.9593	0.2341	0.0824	0.3967	0.363158	0.22
	α = 0.1	0.2527	0.2570	0.2575	0.9590	0.2371	0.0829	0.3988	0.362421	0.30
	α = 0.01	0.2529	0.2572	0.2577	0.9590	0.2375	0.0829	0.3991	0.362344	0.58
	α = 0.001	0.2529	0.2572	0.2577	0.9590	0.2375	0.0829	0.3991	0.362337	4.53

Tab 3. Comparison of optimisation and CRAS techniques for different values of alpha

Results show a wide disparity over the different distance measures. However, the constrained optimisation does appear to provide the most precise measure, both in terms of information gain, and across four of the other seven distance measures. The results show that the choice of alpha for CRAS is less important than what might at first be thought. For this example, the choice of a large alpha has less information gain, and outperforms smaller alphas for all but the measure of the similarity index, *SIM*, and the correlation co-efficient. The run-time of CRAS shows a significant improvement over constrained optimisation. Whilst the times shown in this example are fairly insubstantial, the speed of the convergence can be quite important when applied to big systems.

The empirical application chosen was the Australian Make-Use tables for 1993-94. Only since 1994-95 has the Australian Bureau of Statistics (ABS) forced the balance of the totals between the Make-Use framework. For 1993-94, there is a discrepancy of row totals of up to 11% for 56% of the rows. The 1993-94 data contains 107 products and 107 industries, with 7 categories of final demand, and 6 categories of value added. The supply and use tables were balanced according to the ABS published data, as well as for row totals, column totals, final and intermediate demand totals, value added totals and inter-table row and column totals. Error estimates for the published data was obtained from Australian Bureau of Statistics, 2005, and set to zero for the balancing constraints. Again, runs were obtained for a constrained optimisation, and for CRAS with varying alpha. On this occasion the programming was done in FORTRAN90 and run in a LINUX environment in order to improve efficiency of the large scale problem. The results of the runs are presented in Tab. 4.

		AMAD	SWAD	GMAD	SIM	AMRD	CHI	INFO	CORR	Time (sec)
Constrained Optimisation		no optima reached after several hours computing								
CRAS	α = 1	0.0014	7.64E-04	0.00118	1.000	6.78E-07	3.05E+05	-8.44E+05	1.000	120.76
	α = 0.1	0.0014	7.65E-04	0.00118	1.000	6.79E-07	3.06E+05	-8.15E+05	1.000	120.45
	α = 0.01	0.0014	7.74E-04	0.00119	1.000	7.42E-07	3.10E+05	-4.95E+05	1.000	120.69
	α = 0.001	0.0014	7.84E-04	0.00120	1.000	8.35E-07	3.14E+05	-2.62E+05	1.000	124.83
	α = 0.0001	0.0014	7.74E-04	0.00119	1.000	8.33E-07	3.16E+05	-5.13E+05	1.000	181.51

Tab. 4. Comparison of distance measure for the 1993-94 Australian Make-Use tables.

The fact that the constrained optimisation did not reach an optimum after several hours computing shows how problematic this approach can be for large systems.

For the CRAS runs, we find first that it appears to be significantly faster than the conjugate gradient algorithm. Second, in general, increasing α decreases the runtime, up to a point of saturation around $\alpha = 0.01$.

Of note are the variations in the information functional. Since we have not formally derived the CRAS algorithm from an optimisation principle, we cannot state that CRAS attempts to minimise information gain. This aspect is subject to further research.

5 Conclusions

Using the notation and concepts from constrained optimisation, we have developed a RAS variant (CRAS) that is able to balance and reconcile input-output tables and SAMs under conflicting external information and inconsistent constraints. In addition, CRAS fulfils all requirements of earlier RAS variants such as GRAS, and of constrained optimisation techniques:

d) constraints on arbitrarily sized and shaped subsets of matrix elements;
e) consideration of the reliability of the initial estimate and the external constraints;
f) ability to handle negative values and to preserve the sign of matrix elements.

We compare the CRAS variant with an equivalent technique for matrix balancing using constrained optimisation. In our case, we employ a conjugate gradient algorithm. Applying these two methods to the 1993-94 Australian National Accounts, we find that

- as constrained optimisation, CRAS is able to find a compromise solution between inconsistent constraints; this feature does not exist in conventional RAS variants such as GRAS;
- CRAS is significantly faster than the conjugate gradient algorithm;
- the conjugate gradient algorithm produces slightly more accurate results.

CRAS can constitute a major advance for the practice of balancing input-output tables and Social Accounting Matrices, in that it removes the necessity of manually tracing inconsistencies in external information. In contrast to constrained optimisation, this quality does not come at the expense of complicated programming requirements, and long run times.

While CRAS appears to be able to solve these problems, it should not be forgotten that collecting data requires skilled manual input. As Barker *et al.*, 1984 (p. 475) emphasise, automated balancing of national accounts “is not a replacement for knowledge of the data and its sources but an enhancement of it, allowing us to produce fully balanced accounts with the adjustments reflecting the quality of the data.”

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Tables

Endnotes

[1] Lahr & de Mesnard, 2004 provide a recent overview of extensions to the classic RAS technique.

[2] Barker *et al.*, 1984 (p. 475) write: "… we observed that the income, expenditure, production and financial estimates of data are typically inconsistent. The presence of such accounting inconsistencies emphasises the unreliable nature of economic data." See also Smith *et al.*, 1998.

[3] Barker *et al.*, 1984 (p. 475) remark that "…trading off the relative degrees of uncertainty of the various data items in the system in order to adjust the prior data to fit the accounting identities […] is essentially what national income accountants do during the last stages of compiling the accounts when faced mith major discrepancies between data from different sources". Dalgaard & Gysting, 2004 (p. 170) from Statistik Denmark report that many analysts responsible for compiling input-output tables favour manual adjustment, because "based on the experience that many errors in primary statistics are spotted in the course of a balancing process that is predominantly manual, compilers are typically convinced that a (mainly) manual balancing process yields results of higher quality than those emanating from a purely automatic balancing of the accounts. From that point of view, the resources involved in manual balancing are justified as a very efficient consistency check on the accounts."

[4] When applied to the forecasting of monetary input-output matrices, bi-proportional changes have been interpreted as productivity, substitution or fabrication effects (Leontief, 1941; Stone & Brown, 1962) affecting industries over time. Miernyk, 1976 view however is that the RAS method "substitutes computational tractability for economic logic", and that the production interpretation loses its meaning when the entire input-output table is balanced, and not only inter-industry transactions (see also Giarratani, 1975).

[5] The RAS, Linear Programming and minimum information gain algorithms yield a balanced matrix estimate that is – in terms of some measure of multidimensional 'distance' – closest to the unbalanced preliminary estimate. When applied to temporal forecasting, this property is explained as a conservative hypothesis of attributing inertia to inter-industrial relations (Bacharach, 1970, p. 26). While the classic RAS method is aimed at maintaining the value structure of the balanced matrix, the closely related Cross-Entropy methods (Robinson *et al.*, 2001) are aimed at maintaining the coefficient structure.

[6] Single-element constraints need not be part of the scaling procedure, but could be "netted out" using the "modified RAS" method.

[7] Czamanski & Malizia, 1969; Harrigan *et al.*, 1980; Jackson & Comer, 1993; Lecomber, 1975a; McMenamin & Haring, 1974; Morrison & Smith, 1974

[8] The square of this measure has also been referred to as 'Theil's inequality index' (Lahiri, 1984).

[9] 1 – *SIM* has been used as a "Dissimilarity Index" by Thissen & Löfgren, 1998).