Example of the RAS technique

Véronique Robichaud, February 2000

RAS is a methodology widely used to evaluate, balance or update matrices. This technique starts from an initial share matrix from which a final matrix must be derived such as given rows and columns totals are respected. Here is a simple example in order to illustrate the methodology.

The first step consists in building a share matrix by dividing each element of the initial matrix by the total of the column in which it appears:

$$Initial\ \ matrix = \begin{vmatrix} 3 & 4 & 2 \\ 7 & 4 & 3 \end{vmatrix} \Rightarrow \begin{vmatrix} 3/10 & 4/8 & 2/5 \\ 7/10 & 4/8 & 3/5 \end{vmatrix} = \begin{vmatrix} 0.3 & 0.5 & 0.4 \\ 0.7 & 0.5 & 0.6 \end{vmatrix} = A$$

The total of every row and every column of the final matrix has to be know. Let's suppose that vector *ROW* and *COL* represent respectively the totals to be obtained in row and in column:

$$ROW = \begin{vmatrix} 10 \\ 12 \end{vmatrix}$$

Column totals to obtain

$$COL = \begin{vmatrix} 4 & 10 & 8 \end{vmatrix}$$

The first step in evaluating the matrix that will respect all these totals is to multiply the elements in the share matrix by the corresponding total to respect given in *COL*:

$$A' = \begin{vmatrix} 0.3\times4 & 0.5\times10 & 0.4\times8 \\ 0.7\times4 & 0.5\times10 & 0.6\times8 \end{vmatrix} = \begin{vmatrix} 1.2 & 5 & 3.2 \\ 2.8 & 5 & 4.8 \end{vmatrix}$$

Then we verify if the sum of each row of this new matrix, A', equal the ones in the *ROW* vector:

$$1.2 + 5 + 3.2 = 9.4 \neq 10$$
$$2.8 + 5 + 4.8 = 12.6 \neq 12$$

The following iteration consists in multiplying the elements of A' by the ratio of the total to obtain (in vector *ROW*) by the actual sum:

$$A'' = \begin{vmatrix} 1.2\times{}^{10}\!/_{9.4} & 5\times{}^{10}\!/_{9.4} & 3.2\times{}^{10}\!/_{9.4} \\ 2.8\times{}^{12}\!/_{12.6} & 5\times{}^{12}\!/_{12.6} & 4.8\times{}^{12}\!/_{12.6} \end{vmatrix} = \begin{vmatrix} 1.3 & 5.3 & 3.4 \\ 2.7 & 4.7 & 4.6 \end{vmatrix}$$

Symmetrically to the first iteration, we then verify if the total of each column gives the total to be obtained (*COL*):

$$1.3 + 2.7 = 4$$
$$5.3 + 4.7 = 10$$
$$3.4 + 4.6 = 8$$

Thus, the matrix A'' respects both total in column and in row, we stop the iterative process. If we did not obtain the right totals, we would have multiply each element by the ratio of the column total to be obtained on the actual column total and so on.