2: Coordinate transformation

Considere cells $K\ \hat{K} \subset \mathbb{R}^n$ (as a finite element meshes)

$F: \hat{K} \to K$

$F(\hat{x}) = J_K \hat{x} + b_K$ bijective mapping (ie. with J_K invertible $F(\hat{K}) = K$)

$h_K = \mathrm{diam}(K)$

ρ_K diameter of the largest ball that can be inscribed in K

a) operator norm (induced by the usual Euclidean)
of the Jacobian $\|J_K\| \le \dfrac{h_K}{\rho_{\hat{K}}}$

sol
by definition

$$\|J_K\|_{\ell^2} = \sup_{\substack{x \in \hat{K} \\ x \neq 0}} \frac{\|J_K(x)\|}{\|x\|} =$$

$\exists\ \hat{x}$ such that $\|\hat{x}\| = \rho_{\hat{K}}$ and it correspond with this supremem, beacuse is the largest ball inscribed in $\hat{K}$

$$\Rightarrow \|J_K\|_{\ell^\infty} = \sup_{\|\hat{x}\| = \rho_{\hat{K}}} \frac{\|J_K x\|}{\rho_{\hat{K}}}$$

let be $\hat{x} \in \hat{K} \Rightarrow \exists\ \hat{y}, \hat{z} \in \partial J_{\hat{K}}$ s.t. $\hat{x} = \hat{y} - \hat{z}$

$\Rightarrow \cancel{F_K(\hat{x}) = J_K(\hat{y} - \hat{z}) = J_K(\hat{y}) - J_K(\hat{z}) = y}$

because F is linear

$F(\hat{y} - \hat{z}) = F(\hat{y}) - F(\hat{z}) = y - z$

linearity of F

$\|$

[redacted]$_K(\hat{x}) =$ [redacted].

$J_K(\hat{x}) = J_K(\hat{y} - \hat{z})$

$\Rightarrow \|J_K(\hat{x})\| = \|y - z\| \le h_K$

$\Rightarrow$
$\|J_K\|_{\ell^\infty} \le \dfrac{h_K}{\rho_{\hat{K}}}$

b) prove $\|J_K^{-1}\| \leq \frac{h_{\hat{K}}}{\rho_K}$

ρ_K - radius of largest ball in K

analogous

$$\|B_K^{-1}\|_{1,\infty} = \sup_{\substack{x \in K \\ \|x\| \neq 0}} \frac{\|J_K^{-1} x\|}{\|x\|} = \sup_{\|x\| = \rho_K} \frac{\|B_K^{-1} x\|}{\rho_K}$$

$\exists\ y, z \in \partial B_{\rho_K}$ s.t. $x = y - z$

$$J_K^{-1}(y-z) = F^{-1}(y) - F^{-1}(z) = \hat{y} - \hat{z} \in \hat{K}$$

$$\Rightarrow \quad \|B_K^{-1} x\| = \|\hat{y} - \hat{z}\| \leq h_{\hat{K}}$$

$$\Rightarrow \quad \|B_K^{-1}\| \leq \frac{h_{\hat{K}}}{\rho_K} \quad //$$