CAPITULO 2

OBJETOS GEOMÉTRICOS Y TRANSFORMACIONES

2.1 REPRESENTACIÓN MATRICIAL - VENTANAS

NOTATION - Introduction

R is the set of real numbers; **C** is the set of complex numbers; R^+ (pronounced "R plus") is the set of positive real numbers; and R_0^+ (pronounced "R plus zero") is the set of non-negative reals.

The Cartesian product of the sets **B** and **C** is the set:

$$B \times C = \{(b,c) : b \in B, c \in C\}^1,$$

NOTATION - Introduction

The product $\mathbf{R} \times \mathbf{R}$ is denoted $\mathbf{R}^2$; higher order products are $\mathbf{R}^3$, $\mathbf{R}^4$, etc., with the n-fold product being $\mathbf{R}^n$.

The **closed interval** $[a, b]$ is the set of all real numbers between a and b, inclusive, that is,

$$[a, b] = \{x : a \leq x \leq b\}. \tag{7.2}$$

$$[a, b) = \{x : a \leq x < b\}, \qquad [a, \infty) = \{x : a \leq x\},$$

$$(a, b] = \{x : a < x \leq b\}. \qquad (-\infty, b] = \{x : x \leq b\}.$$

1. Functions

The notion of a *function* is already familiar to you from both mathematics and programming. We'll use a particular notation to express functions; an example is

$$f : \mathbf{R} \to \mathbf{R} : x \mapsto x^2. \tag{7.7}$$

The name of the function is f. Following the colon are two sets. The one to the left of the arrow is called the **domain;** the one to the right is called the **codomain**

This corresponds closely to the definition of a function in many programming languages, which tends to look like this:

```
double f(double x)
{
    return x * x;
}
```

1. Functions

In describing functions, we'll always describe the domain, the codomain, and the rule that associates elements in the first to elements in the second. Sometimes these rules may involve cases, as in

$$u : \mathbf{R} \to \mathbf{R} : x \mapsto \begin{cases} 1 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}, \tag{7.15}$$

$h : \mathbf{R}_0^+ \to \mathbf{R}_0^+ : x \mapsto x^2,$ **Inverse→** $h^{-1} : \mathbf{R}_0^+ \to \mathbf{R}_0^+ : x \mapsto \sqrt{x}.$

2. Coordinates

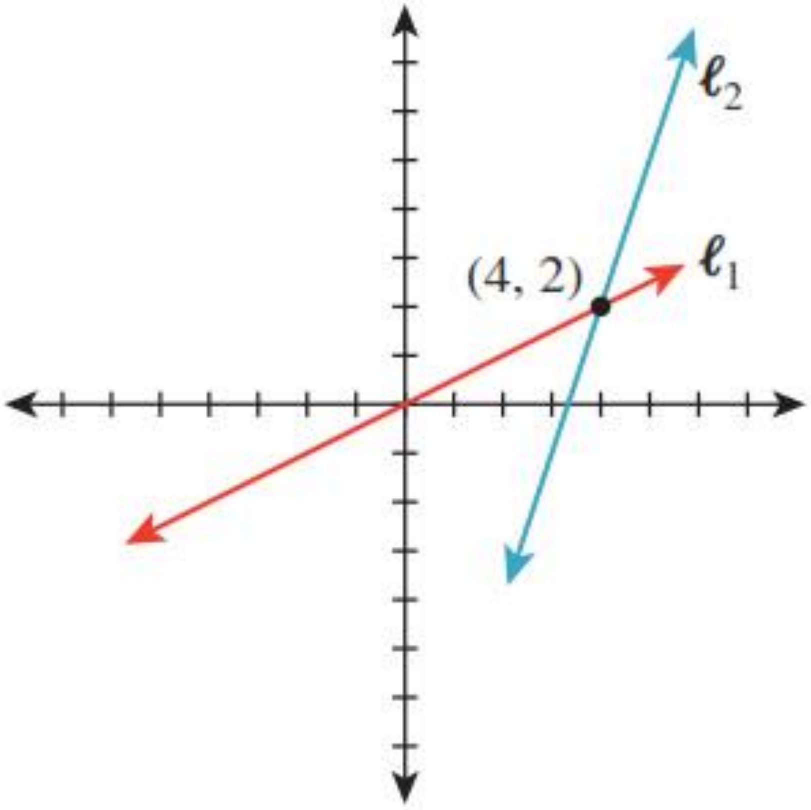


Figure 7.3: The Cartesian plane, in which points are specified by x- and y-coordinates.

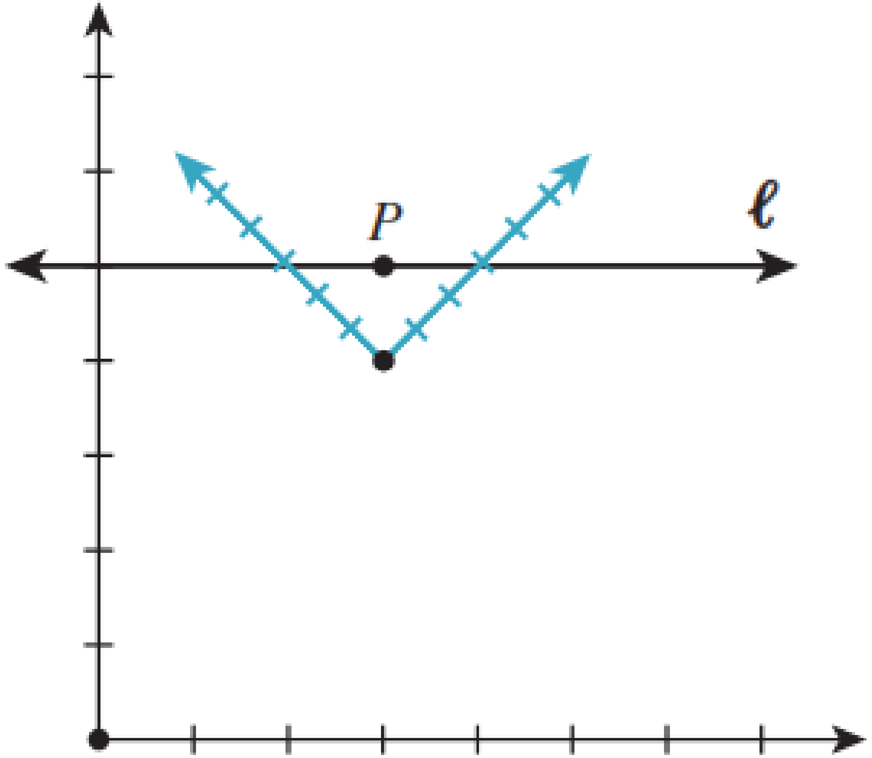


Figure 7.4: The Cartesian plane with multiple coordinate systems.

2. Coordinates

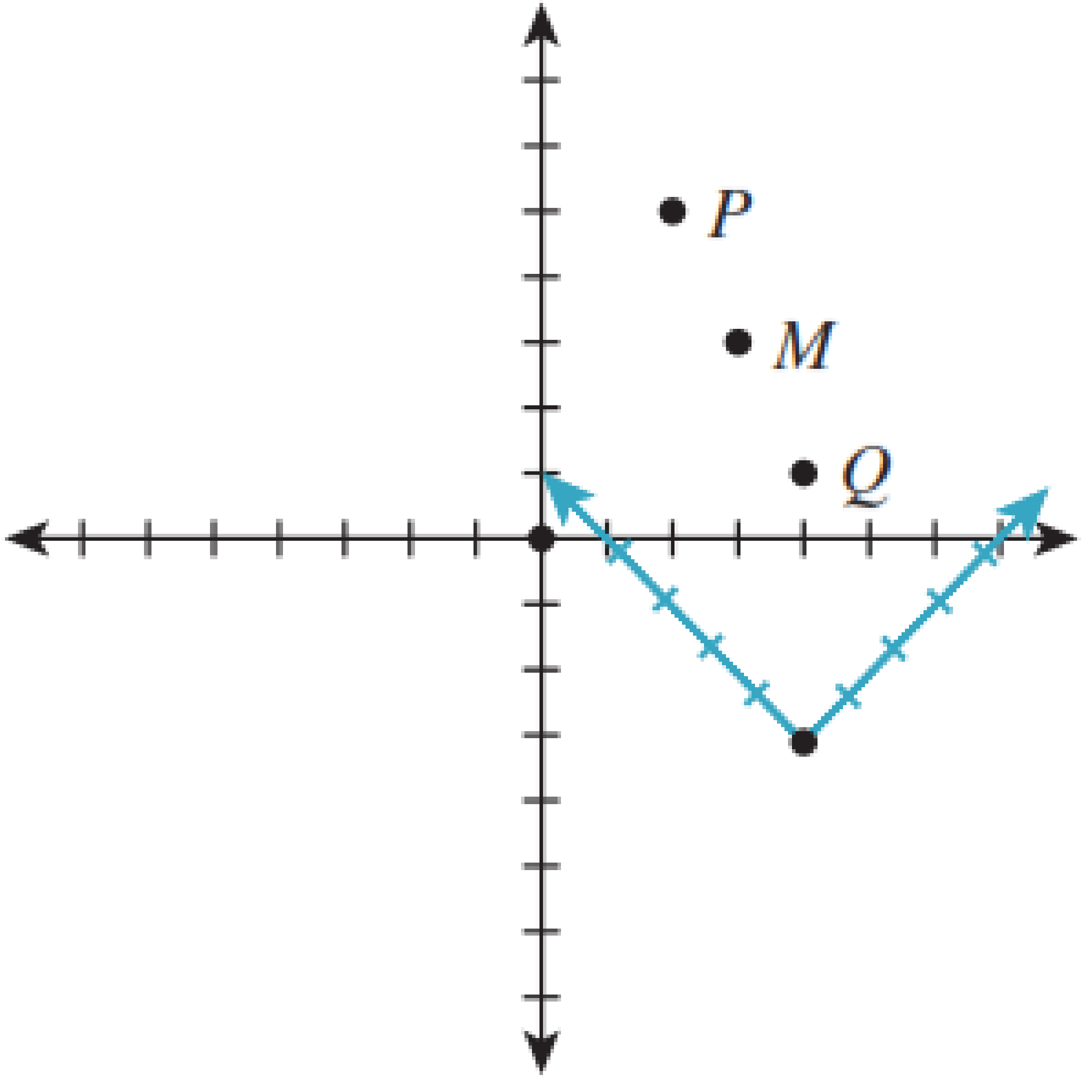
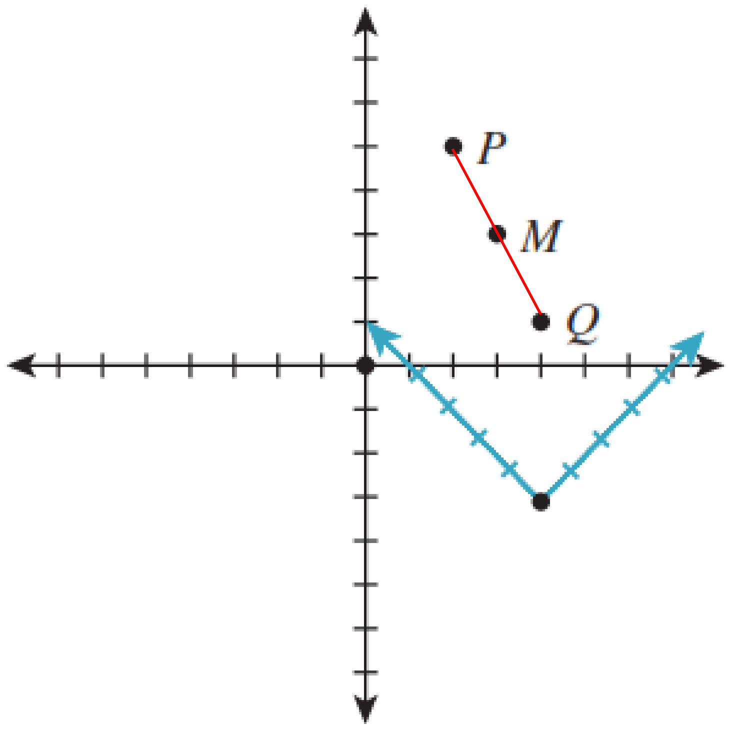


Figure 7.5: The coordinates of M in each coordinate system are the average of the coordinates of P and Q in that coordinate system; thus, the geometric *operation of finding the midpoint of a segment corresponds to the* algebraic *operation of averaging coordinates,* independent of what coordinate system we use.

2. Coordinates - parametric form of the line between P and Q

As α ranges over the real numbers, the points $(1-\alpha)P+\alpha Q$ range over the line containing P and Q, with $\alpha = 1$ corresponding to Q. and $\alpha = 0$ corresponding to P and values of α between 0 and 1 corresponding to points *between* P and Q.



$$\gamma : \mathbf{R} \to \mathbf{R}^2 : t \mapsto (1-t)P + tQ.$$

The image of this function is the line between P and Q; if we restrict the domain to the interval [0, 1], then the image is the line segment between P and Q.

t → parameter

3. Vectors

Vector de $\mathbb{R}^n$:

$$\mathbf{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}, \quad \mathbf{o} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

Los a_i se denominan *componentes* del vector.

Suma de vectores:

$$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} + \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} \quad \\ \vdots \\ \quad \end{pmatrix}$$

Producto de un vector por un escalar:

$$k \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} \quad \\ \vdots \\ \quad \end{pmatrix}$$

Vector traspuesto:

$$\mathbf{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}, \quad \mathbf{a}' = \ldots$$

3. Vectors

Combinación lineal (CL) de vectores: Sean $\mathbf{a}_1, \ldots, \mathbf{a}_m \in \mathbb{R}^n$ vectores y $k_1, \ldots, k_m \in \mathbb{R}$ escalares. El vector siguiente es una *combinación lineal* de $\mathbf{a}_1, \ldots, \mathbf{a}_m$:

$$\mathbf{a} = k_1\mathbf{a}_1 + \cdots + k_m\mathbf{a}_m$$

Vectores linealmente dependientes e independientes: Sea $m \in \mathbb{N}$. Se dice que un conjunto de vectores $\{\mathbf{a}_1, \ldots, \mathbf{a}_m\}$ de $\mathbb{R}^n$ es *linealmente dependiente*, si existen escalares $k_1, \ldots, k_m \in \mathbb{R}$ no todos nulos, tales que

$$k_1\mathbf{a}_1 + \cdots + k_m\mathbf{a}_m = \mathbf{o}\,, \tag{1.1}$$

Un conjunto de vectores se dice que es *linealmente independiente* si no es linealmente dependiente, es decir, si la única solución del sistema de ecuaciones (1.1) es

$$k_1 = \cdots = k_m = 0\,.$$

3. Vectors

Producto escalar de dos vectores: El producto escalar de $\mathbf{a} = (a_1, \ldots, a_n)'$ por $\mathbf{b} = (b_1, \ldots, b_n)'$ es el número real

$$\mathbf{a}'\mathbf{b} = \sum_{i=1}^{n} a_i b_i \, .$$

Módulo o norma de un vector: El módulo de un vector $\mathbf{a} = (a_1, \ldots, a_n)$ es el número real

$$|\mathbf{a}| = \sqrt{\mathbf{a}'\mathbf{a}} = \sqrt{\sum_{i=1}^{n} a_i^2} \, .$$

El módulo de un vector mide su longitud.

3. Vectors

Vectores unitarios: Un vector $\mathbf{a} \in \mathbb{R}^n$ con módulo $|\mathbf{a}| = 1$ se llama vector *unitario*.

Normalización de una vector: Si un vector $\mathbf{a} \neq \mathbf{o}$ no es unitario, se puede construir el vector

$$\mathbf{a}^\circ = \frac{1}{|\mathbf{a}|}\,\mathbf{a}\,.$$

¿Es $\mathbf{a}^\circ$ unitario? ...

Ángulo entre dos vectores: Sean $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n - \{\mathbf{o}\}$. El *ángulo entre a y b* es el número real contenido entre 0 y π definido por

$$\operatorname{ang}(\mathbf{a}, \mathbf{b}) = \arccos \frac{\mathbf{a}'\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

Vectores ortogonales: Dos vectores $\mathbf{a}$ y $\mathbf{b}$ son *ortogonales* si

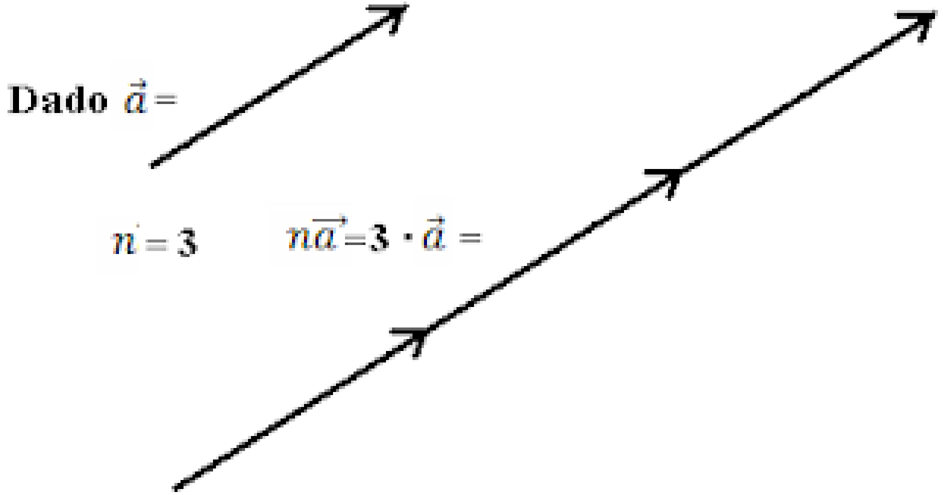
$$\mathbf{a}'\mathbf{b} = 0\,.$$

3. Vectors


Producto Vectorial
Escalar por un vector
$n\vec{a}$
Resulta otro *vector*
Producto Escalar de dos vectores
$\vec{a} \cdot \vec{b}$
Resulta un *escalar*
Producto Vectorial de dos Vectores
$\vec{a}\ x\ \vec{b}$
Resulta otro *vector*

3. Vectors - Producto de un vector por un escalar

sobre la misma línea de su dirección tomamos tantas veces el módulo de vector como marque el escalar, que de ser negativo cambia el sentido

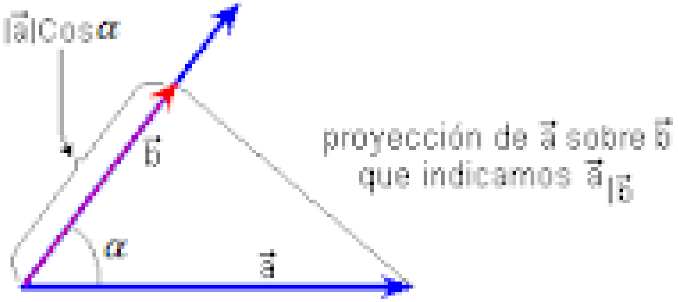


$$\boxed{n \cdot \vec{a} = (n \cdot a_x, n \cdot a_y, n \cdot a_z)}$$

$$\boxed{n \cdot \vec{a} = n \cdot a_x \hat{i} + n \cdot a_y \hat{j} + n \cdot a_z \hat{k}}$$

3. Vectors - Producto escalar de dos vectores (punto)

La multiplicación **da como resultado un número real** , no un vector, *por lo que esta operación se denomina* ***producto escalar***.



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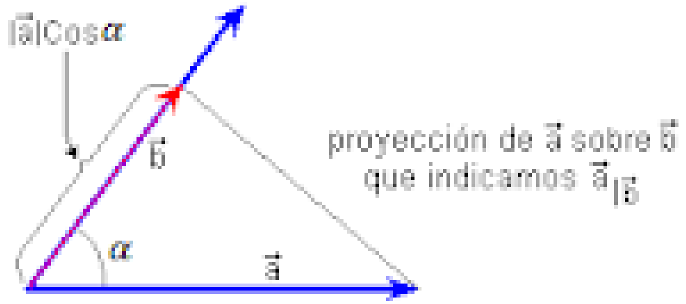


Conociendo el ángulo entre los vectores y el módulo de cada vector tenemos que el producto de escalar de los vectores analíticamente se calcula

$$\boxed{\vec{a}\,.\,\vec{b} = |\vec{a}|.|\vec{b}|\ \cos\alpha}$$

Conociendo las componentes de los vectores tenemos que el producto de ***escalar de los vectores*** en forma algebraica se calcula

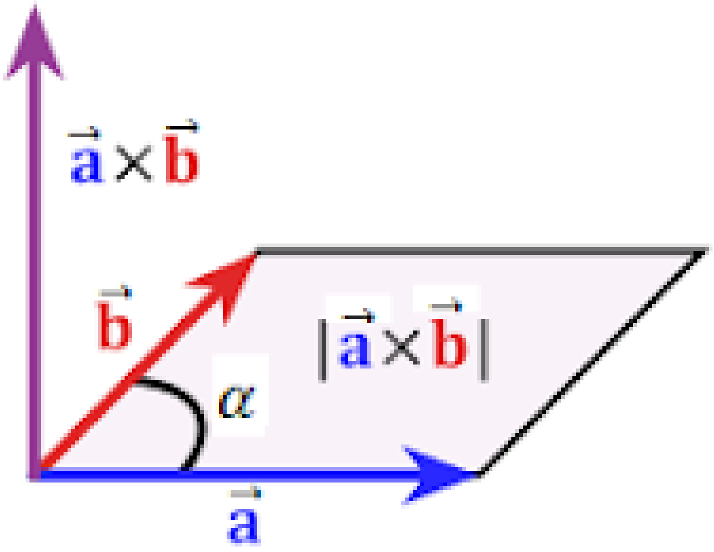
$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} \quad \text{y} \quad \vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\boxed{\vec{a}\,.\,\vec{b} = (a_x.b_x) + (a_y.b_y) + (a_z.b_z)}$$

$$\boxed{|\vec{a}|.|\vec{b}|\ \cos\alpha = (a_x.b_x) + (a_y.b_y) + (a_z.b_z)}$$

3. Vectors - Producto vectorial de dos vectores (cruz)

El producto vectorial de los vectores a y b, se define como un vector, donde su dirección es perpendicular al plano de a y b, en el sentido del movimiento de un tornillo que gira hacia la derecha por el camino más corto de a a b.



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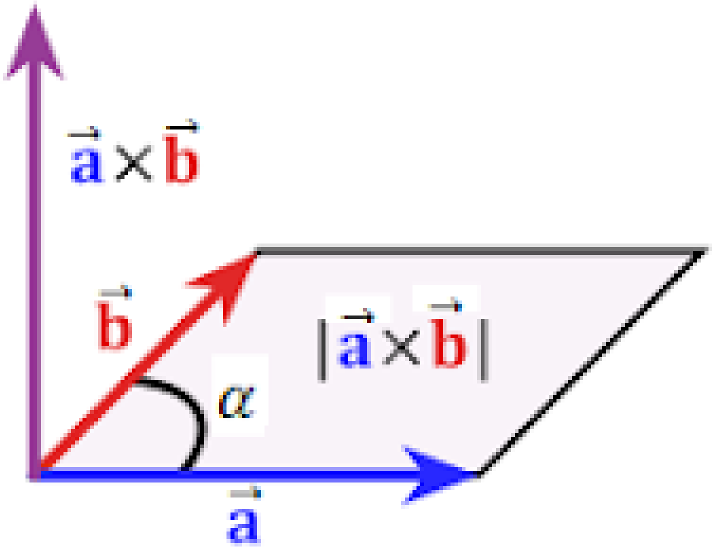

La expresión relaciona al producto vectorial con el área del paralelogramo que definen ambos vectores.

$$\boxed{|\vec{a} \times \vec{b}| = |\vec{a}|.|\vec{b}|\ sen\,\alpha}$$

Componentes del producto vectorial de dos vectores

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} \quad \text{y} \quad \vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$|\vec{a} \times \vec{b}| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \vec{c}$$

$$\boxed{|\vec{a} \times \vec{b}| = (a_y b_z - a_z b_y)\hat{\imath} - (a_x b_z - a_z b_x)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}}$$

$$\vec{c} = c_x\hat{\imath} + c_y\hat{\jmath} + c_z\hat{k}$$

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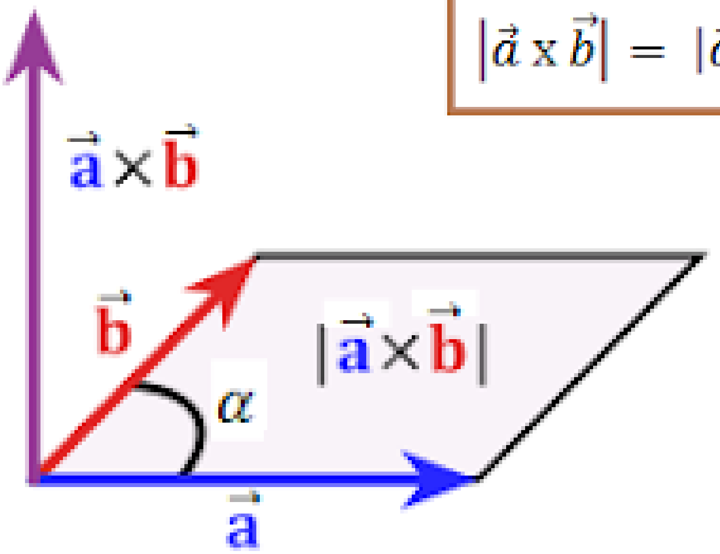

$$\times \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -v_y \\ v_x \end{bmatrix}. \tag{7.34}$$

This cross product has an important property: Going from $\mathbf{v}$ to $\times\mathbf{v}$ involves a rotation by 90° *in the same direction as the rotation that takes the positive x-axis to the positive y-axis*. Because of this, it's sometimes also denoted by $\mathbf{v}^{\perp}$.

3. Matrix

Matriz: Sean $n, p \in \mathbb{N}$. Una *matriz de orden* $n \times p$ *sobre* $\mathbb{R}$ es un conjunto rectangular de np elementos de $\mathbb{R}$, representados en n filas y p columnas

$$A = (a_{ij}) = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \ddots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{np} \end{pmatrix}$$

El conjunto de todas las matrices reales de orden $n \times p$ se designa por $\mathbb{R}^{(n,p)}$.

Vectores fila: Las filas de A se pueden considerar como matrices de orden $1 \times p$ o como vectores de tamaño p:

$$\mathbf{a}_i = (a_{i1}, a_{i2}, \ldots, a_{ip}),\ i = 1, \ldots, n\,.$$

3. Matrix

Vectores columna: Las columnas de A se pueden considerar como matrices de orden $n \times 1$ o como vectores de tamaño n:

$$\mathbf{a}^j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{nj} \end{pmatrix}, \quad j = 1, \ldots, p\,.$$

Operaciones con matrices:

Operación	Restricciones	Definición
Suma	A, B del mismo orden	$A + B = (a_{ij} + b_{ij})$
Producto escalar	$c \in \mathbb{R},\ A \in \mathbb{R}^{(n,p)}$	$cA = (ca_{ij})$
Multiplicación	$A \in \mathbb{R}^{(n,p)}, B \in \mathbb{R}^{(p,m)}$	$AB = (\mathbf{a}_i'\mathbf{b}^j)$
Traspuesta		$A' = (a_{ji})$
Traza	$n = p$	$\mathrm{tr}(A) = \sum_{i=1}^{n} a_{ii}$
Determinante	$n = p$	$\|A\|$
Inversa	$n = p$, $\|A\| \neq 0$	$AA^{-1} = A^{-1}A = \mathbf{I}$

3. Matrix

Matriz inversa: La *matriz inversa* de una matrix cuadrada A es la única matriz que verifica

$$AA^{-1} = A^{-1}A = \mathbf{I}\,.$$

Matrices ortogonales: Una matriz cuadrada A es *ortogonal* si cumple $AA' = \mathbf{I}$. Las propiedades más importantes son las siguientes:

(i) $A^{-1} = A'$

(ii) $A'A = \mathbf{I}$

(iii) Tanto las filas como las columnas de A son vectores ortogonales dos a dos.

(iv) Si A y B son ortogonales, entonces $C = AB$ es ortogonal.

3. Implicit Lines

Instead of writing a function t → γ(t) whose value at each real number t is a point of the line, we can write a different kind of function—one that tells, for each point (x, y) of R^2, whether the point (x, y) is on the line. Such a function is said to define the line implicitly rather than parametrically.

3.1 Level Set (Conjunto de nivel)

If $F : \mathbf{R}^2 \to \mathbf{R}$ is a function, then for each c, we can define the set

$$L_c = \{(x, y) : F(x, y) = c\},$$

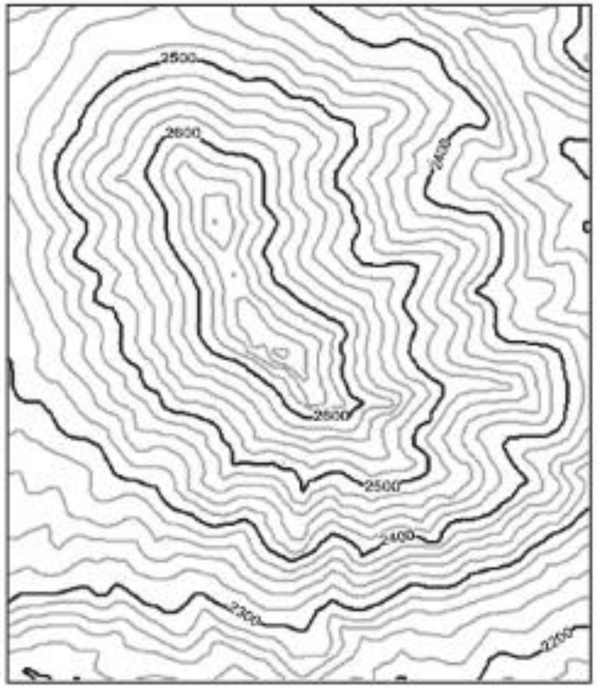

Figure 7.11: A contour map shows the height above sea level with contour lines.

3. Implicit Lines – explicit and implicit representation

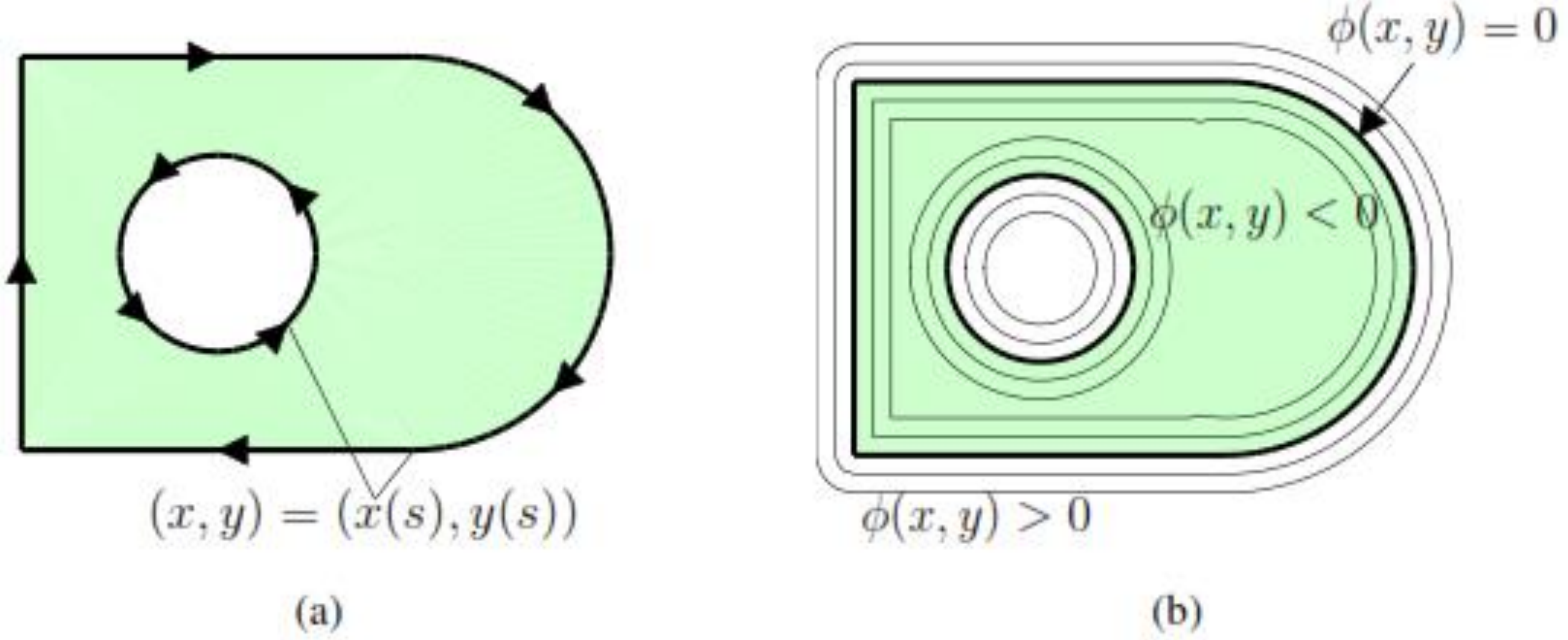

Figure 1: Geometric representation of a domain. (a) Explicit representation, with parametrized boundaries. (b) Implicit representation, with level sets of the distance function shown. The boundary corresponds to the zero level set. Figure from Persson (2004).

3. Implicit Lines – An implicit description of a Line in a Plane

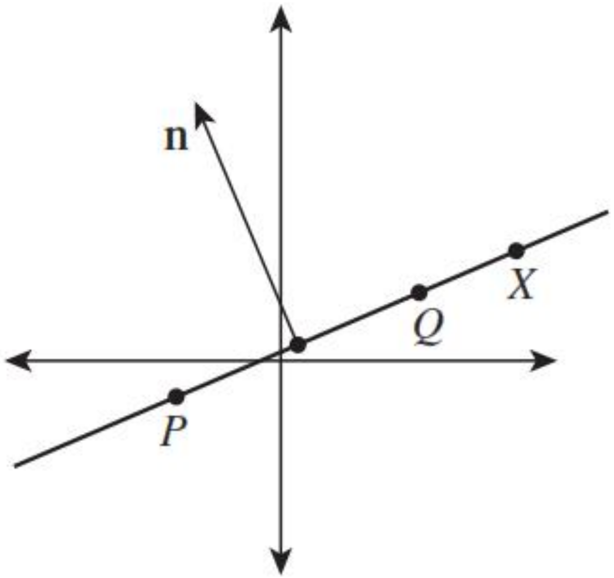

Figure 7.12: The vector $\mathbf{n} = (Q-P)^{\perp}$ *is perpendicular to the line through* P *and* Q*. A typical point* X *of this line has the property that* $(X - P)$ *is also perpendicular to* $\mathbf{n}$*. Indeed, a point* X *is on the line if and only if* $(X-P) \cdot \mathbf{n} = 0$*.*

First,[4] let $\mathbf{n} = \times(Q-P) = (Q-P)^{\perp}$; then the vector $\mathbf{n}$ is perpendicular to the line (see Figure 7.12). A nonzero vector with this property is said to be a **normal vector** or simply a **normal** to the line.

If X is a point of the line, then the vector $X - P$ points along the line, and so is also perpendicular to $\mathbf{n}$. If X is not on the line, then $X - P$ does *not* point along the line, and hence is not perpendicular to $\mathbf{n}$. Thus,

$$(X - P) \cdot \mathbf{n} = 0 \tag{7.67}$$

completely characterizes points X that lie on the line. We can therefore define

$$F(X) = (X - P) \cdot \mathbf{n}, \tag{7.68}$$

which serves as an implicit description of the line. We'll call this the **standard implicit form for a line.**

3. Implicit Lines – An implicit description of a Line in a Plane

To find the normal vector of the line we simply find the gradient of the implicit function $f(x, y)$:

$$\boldsymbol{n} = \nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right).$$

For our implicit form of the line $f(x, y) = ax+by+c = 0$ this is simply $\boldsymbol{n} = (a, b)$. The perpendicular vector from the line to a point is then some multiple of this normal vector, i.e. $k\boldsymbol{n}$, and the length of this vector is $d = k\|\boldsymbol{n}\| = k\sqrt{a^2 + b^2}$.

Taking a point $u = (x+ka, y+kb)$ a distance $k\sqrt{a^2 + b^2}$ from the point $p = (x, y)$ on the line (see figure 1),

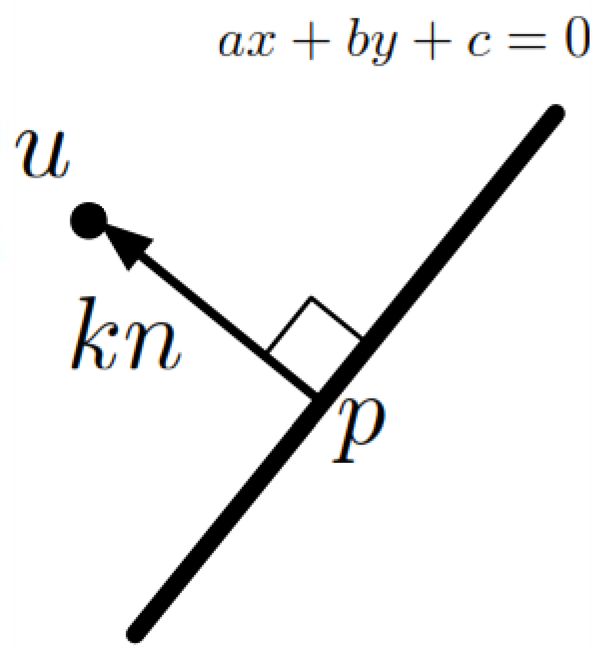

Figure 1: A point u a distance $k\|\boldsymbol{n}\|$ from the line. Here $\boldsymbol{n} = (a, b)$ since $f(x, y) = ax + by + c$.

3. Implicit Lines – An implicit description of a Line in a Plane - Example

Find the implicit form of the line for P=(1,0) and Q=(3,4)

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$$\mathbf{n} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}.$$

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Find the implicit form of the line for P=(1,0) and Q=(3,4)

$$Q - P = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\mathbf{n} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}.$$

Letting X have coordinates (x, y), we have

$$F(x, y) = \begin{bmatrix} x - 1 \\ y - 0 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 2 \end{bmatrix} = 0$$

Implicit form of the line

3. Implicit Lines – An implicit description of a Line in a Plane - Example

Find the implicit form of the line for P=(1,0) and Q=(3,4)

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Letting X have coordinates (x, y), we have

$$F(x, y) = \begin{bmatrix} x - 1 \\ y - 0 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 2 \end{bmatrix} = 0 \Rightarrow -4x+2y+4= 0 \Rightarrow$$

Implicit form of the line expressed in coordinates

4. Parametric-Parametric Line intersection

$$\gamma : \mathbf{R} \to \mathbf{R}^2 : t \mapsto tA + (1-t)B,$$
$$\eta : \mathbf{R} \to \mathbf{R}^2 : s \mapsto sC + (1-s)D,$$

the point P where these lines intersect.

$$t_0A + (1-t_0)B = s_0C + (1-s_0)D,$$

$$B - D = -t_0(A - B) + s_0(C - D),$$

$$\mathbf{u} = C - D,$$
$$\mathbf{v} = A - B$$

$$B - D = -t_0\mathbf{v} + s_0\mathbf{u}.$$

Taking the dot product of both sides with $\times\mathbf{v}$, we get

$$(B-D)\cdot(\times\mathbf{v}) = -t_0\mathbf{v}\cdot(\times\mathbf{v}) + s_0\mathbf{u}\cdot(\times\mathbf{v})$$
$$(B-D)\cdot(\times\mathbf{v}) = s_0\mathbf{u}\cdot(\times\mathbf{v})$$
$$\frac{(B-D)\cdot(\times\mathbf{v})}{\mathbf{u}\cdot(\times\mathbf{v})} = s_0$$

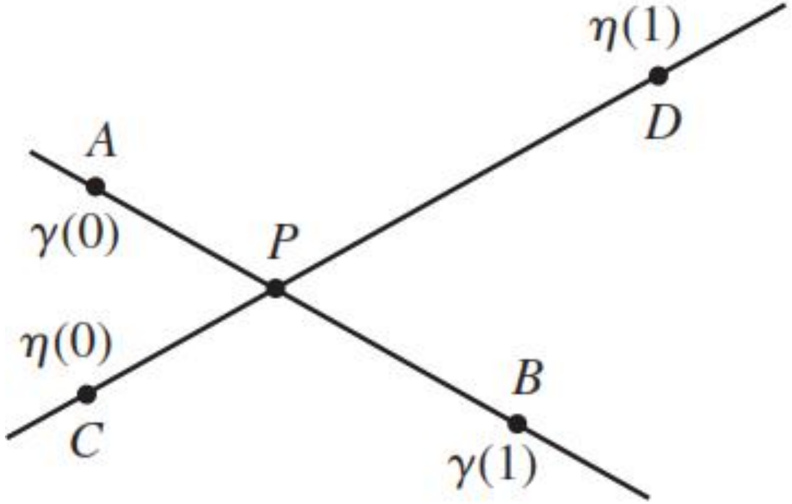

Figure 7.13: The lines AB and CD are the images of the parametric functions γ and η; they intersect at the unknown point P.

4. Parametric-Implicit Line intersection

$$\gamma(t_0) = (1 - t_0)P + t_0 Q$$

$$\ell = \{X : (X - S) \cdot \mathbf{n} = 0\}$$

the point t_0 where these lines intersect.

$$(\gamma(t_0) - S) \cdot \mathbf{n} = 0,$$

$$((1 - t_0)P + t_0 Q - S) \cdot \mathbf{n} = 0.$$

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$$(\gamma(t_0) - S) \cdot \mathbf{n} = 0,$$

$$((1 - t_0)P + t_0 Q - S) \cdot \mathbf{n} = 0.$$

Once again the vector form becomes useful as we simplify:

$$(P + t_0(Q - P) - S) \cdot \mathbf{n} = 0, \text{so}$$
$$(t_0(Q - P) + (P - S)) \cdot \mathbf{n} = 0.$$

Writing $\mathbf{u} = Q - P$ and $\mathbf{v} = P - S$, we have

$$\begin{aligned} t_0 \mathbf{u} \cdot \mathbf{n} + \mathbf{v} \cdot \mathbf{n} &= 0 \\ t_0 \mathbf{u} \cdot \mathbf{n} &= -\mathbf{v} \cdot \mathbf{n} \\ t_0 &= \frac{-\mathbf{v} \cdot \mathbf{n}}{\mathbf{u} \cdot \mathbf{n}}. \end{aligned}$$

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the point t_0 where these lines intersect.

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$$((1 - t_0)P + t_0 Q - S) \cdot \mathbf{n} = 0.$$

Once again the vector form becomes useful as we simplify:

$$(P + t_0(Q - P) - S) \cdot \mathbf{n} = 0, \text{so}$$
$$(t_0(Q - P) + (P - S)) \cdot \mathbf{n} = 0.$$

Writing $\mathbf{u} = Q - P$ and $\mathbf{v} = P - S$, we have

$$t_0 \mathbf{u} \cdot \mathbf{n} + \mathbf{v} \cdot \mathbf{n} = 0$$
$$t_0 \mathbf{u} \cdot \mathbf{n} = -\mathbf{v} \cdot \mathbf{n}$$
$$t_0 = \frac{-\mathbf{v} \cdot \mathbf{n}}{\mathbf{u} \cdot \mathbf{n}}.$$

Note that this computation gives us *two* things. It tells us where along the line determined by γ the intersection lies, by giving us t_0; if t_0 is between 0 and 1, for instance, we know that the intersection lies between P and Q. It also tells us the actual coordinates of the intersection point (x_0, y_0), that is, it provides an explicit point on the line that's determined implicitly by $Ax + By + C = 0$. If we only care about intersections between P and Q, but find t_0 is not between 0 and 1, we can avoid the second part of the computation.

4. Parametric-Implicit Line intersection - Example

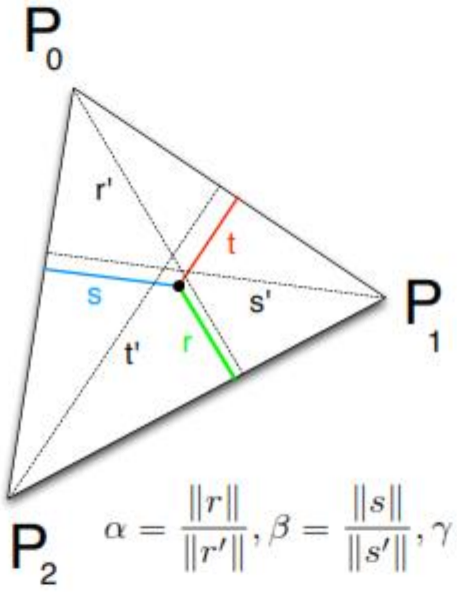
La línea L1 tiene como puntos A(2,6), B (7,2).

La línea L2 C(2,3), D (4,6).

a) Definir las líneas utilizando su expresión paramétrica e implícita.
b) Calcular la intersección de las líneas utilizando el método paramétrico – paramétrico.
c) Calcular la intersección de las líneas utilizando el método paramétrico – implícito.

4. Barycentric coordinates

The barycentric coordinates of a point in the 2D plane are defined as the ratios shown in figure 2. For β, the barycentric coordinate is the perpendicular distance of the point (x, y) from the line between P_0 and P_2, scaled so that at P_1, $\beta = 1$.



$$\alpha = \frac{\|r\|}{\|r'\|}, \beta = \frac{\|s\|}{\|s'\|}, \gamma = \frac{\|t\|}{\|t'\|}$$

Figure 2: Definition of barycentric coordinates of a point. The vectors r, r', s, s', t and t' are perpendicular to the corresponding edges of the triangle (*e.g.* r and r' are perpendicular to the line between P_1 and P_2).

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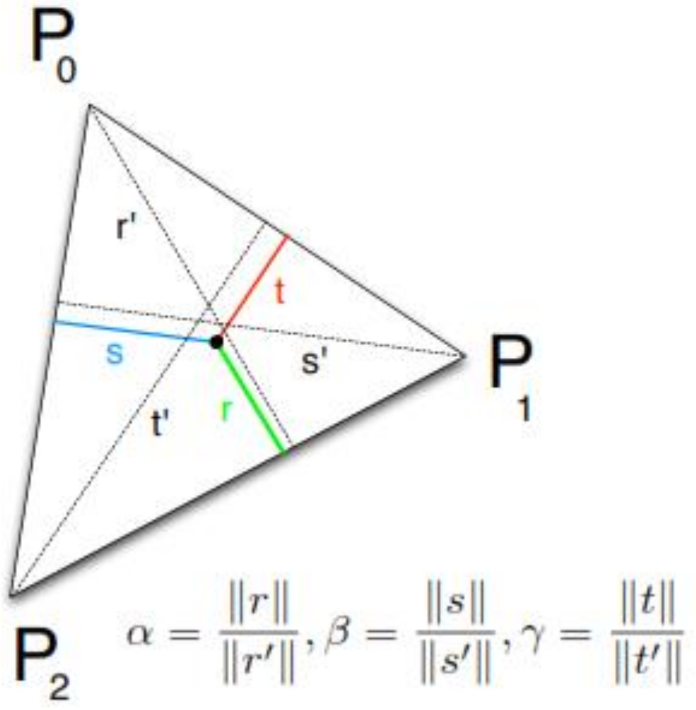
Then for a point (x, y) we can write

$$\beta = \frac{f_{P_2 P_0}(x, y)}{f_{P_2 P_0}(x_1, y_1)},$$

where $P_1 = (x_1, y_1)$.



$$\alpha = \frac{||r||}{||r'||}, \beta = \frac{||s||}{||s'||}, \gamma = \frac{||t||}{||t'||}$$

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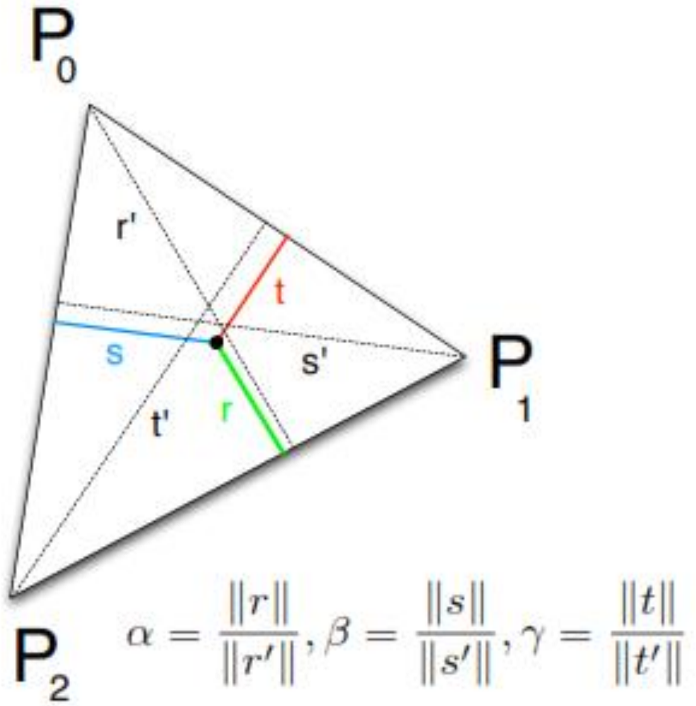
where $P_1 = (x_1, y_1)$. Similarly with $P_0 = (x_0, y_0)$ and $P_2 = (x_2, y_2)$ we can write

$$\gamma = \frac{f_{P_0P_1}(x, y)}{f_{P_0P_1}(x_2, y_2)},$$

and

$$\alpha = \frac{f_{P_1P_2}(x, y)}{f_{P_1P_2}(x_0, y_0)}.$$



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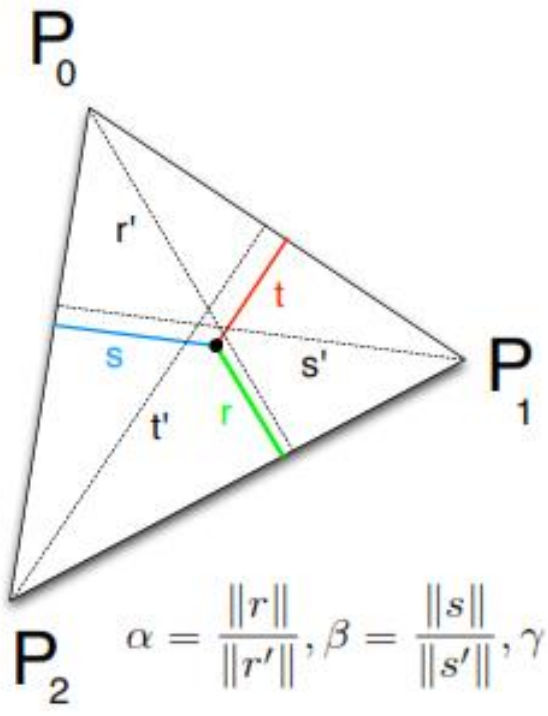
4. Barycentric coordinates

where $P_1 = (x_1, y_1)$. Similarly with $P_0 = (x_0, y_0)$ and $P_2 = (x_2, y_2)$ we can write

$$\alpha = \frac{f_{P_1P_2}(x,y)}{f_{P_1P_2}(x_0,y_0)} \qquad \text{and} \qquad \gamma = \frac{f_{P_0P_1}(x,y)}{f_{P_0P_1}(x_2,y_2)},$$

In words, γ is the perpendicular distance of the point (x, y) from the line between P_0 and P_1, scaled so that at P_2, $\gamma = 1$, and α is the perpendicular distance of the point (x, y) from the line between P_1 and P_2, scaled so that at P_0, $\alpha = 1$.

The division of *e.g.* $f_{P_2P_0}(x, y)$ by $f_{P_2P_0}(x_1, y_1)$ is simply to scale β so that at P_1, where $x = x_1$ and $y = y_1$, $\beta = \frac{f_{P_2P_0}(x_1,y_1)}{f_{P_2P_0}(x_1,y_1)} = 1$



$$\alpha = \frac{\|r\|}{\|r'\|}, \beta = \frac{\|s\|}{\|s'\|}, \gamma = \frac{\|t\|}{\|t'\|}$$

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