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Lista de Cálculo 1: Integral Definida

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Lista de exercícios

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1 1^o Teorema Fundamental do Cálculo

$$\int_0^{\frac{\pi}{8}} \operatorname{sen} 2x \, dx = \left[-\frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{8}} = -\frac{1}{2} \cos \frac{\pi}{4} + \frac{1}{2}$$

ou seja,

$$\int_0^{\frac{\pi}{8}} \operatorname{sen} 2x \, dx = \frac{2 - \sqrt{2}}{4}. \quad \blacksquare$$

EXEMPLO 7. Calcule $\int_0^1 e^{-x} \, dx$.

Solução

$$\int_0^1 e^{-x} \, dx = [-e^{-x}]_0^1 = 1 - \frac{1}{e}. \quad \blacksquare$$

Exercícios 11.5 =====

Calcule.

1. $\int_0^1 (x + 3) \, dx$

2. $\int_{-1}^1 (2x + 1) \, dx$

3. $\int_0^4 \frac{1}{2} \, dx$

4. $\int_{-2}^1 (x^2 - 1) \, dx$

5. $\int_1^3 dx$

6. $\int_{-1}^2 4 \, dx$

7. $\int_1^3 \frac{1}{x^3} \, dx$

8. $\int_{-1}^1 5 \, dx$

9. $\int_0^2 (x^2 + 3x - 3) \, dx$

10. $\int_0^1 \left(5x^3 - \frac{1}{2} \right) dx$

11. $\int_1^1 (2x + 3) \, dx$

12. $\int_1^0 (2x + 3) \, dx$

$$13. \int_{-2}^{-1} \left(\frac{1}{x^2} + x \right) dx$$

$$14. \int_0^4 \sqrt{x} \, dx$$

$$15. \int_1^4 \frac{1}{\sqrt{x}} \, dx$$

$$16. \int_0^8 \sqrt[3]{x} \, dx$$

$$17. \int_{-1}^0 (x^3 - 2x + 3) \, dx$$

$$18. \int_0^1 \sqrt[8]{x} \, dx$$

$$19. \int_1^2 \left(x^3 + x + \frac{1}{x^3} \right) dx$$

$$20. \int_0^1 (x + \sqrt[4]{x}) \, dx$$

$$21. \int_1^3 \left(5 + \frac{1}{x^2} \right) dx$$

$$22. \int_{-3}^3 x^3 \, dx$$

$$23. \int_{-1}^1 (x^7 + x^3 + x) \, dx$$

$$24. \int_{\frac{1}{2}}^1 (x + 3) \, dx$$

$$25. \int_1^4 (5x + \sqrt{x}) \, dx$$

$$26. \int_1^0 (x^7 - x + 3) \, dx$$

$$27. \int_1^2 \frac{1+x}{x^3} \, dx$$

$$28. \int_0^1 (x+1)^2 \, dx$$

$$29. \int_1^4 \frac{1+x}{\sqrt{x}} \, dx$$

$$30. \int_0^1 (x-3)^2 \, dx$$

$$31. \int_0^2 (t^2 + 3t - 1) \, dt$$

$$32. \int_1^2 \frac{1+t^2}{t^4} \, dt$$

$$33. \int_{\frac{1}{2}}^1 (s+2) \, ds$$

$$34. \int_0^3 (u^2 - 2u + 3) \, du$$

$$35. \int_1^2 (s^2 + 3s + 1) \, ds$$

$$36. \int_{-1}^1 \sqrt[3]{t} \, dt$$

$$37. \int_1^3 \left(1 + \frac{1}{x} \right) dx$$

$$38. \int_1^2 \frac{1+3x^2}{x} \, dx$$

$$39. \int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \cos 2x \, dx$$

$$40. \int_{-\pi}^0 \sin 3x \, dx$$

$$41. \int_{-1}^1 e^{2x} \, dx$$

$$42. \int_0^1 \frac{1}{1+t^2} \, dt$$

$$43. \int_0^{\frac{\pi}{4}} \sin x \, dx$$

$$44. \int_{-1}^0 e^{-2x} \, dx$$

$$45. \int_0^{\frac{\pi}{3}} (3 + \cos 3x) dx$$

$$46. \int_0^1 \sin 5x dx$$

$$47. \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$$

$$48. \int_0^2 2^x dx$$

$$49. \int_0^1 2x e^{x^2} dx$$

$$50. \int_0^1 \frac{2x}{1+x^2} dx$$

$$51. \int_0^1 \frac{1}{1+x} dx$$

$$52. \int_{-1}^1 x^3 e^{x^4} dx$$

$$53. \int_0^{\frac{\pi}{3}} (\sin x + \sin 2x) dx$$

$$54. \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right) dx$$

$$55. \int_0^{\frac{\pi}{2}} \cos^2 x dx \left(\text{Sugestão: Verifique que } \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x. \right)$$

$$56. \int_0^{\frac{\pi}{2}} \sin^2 x dx$$

$$57. \int_0^{\frac{\pi}{4}} \sec^2 x dx$$

$$58. \int_0^1 3^x dx$$

$$59. \int_0^1 3^x e^x dx$$

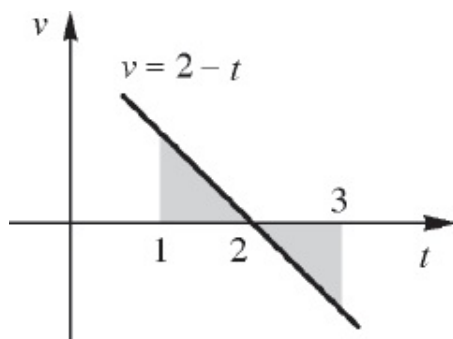
$$60. \int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x dx$$

11.6. CÁLCULO DE ÁREAS

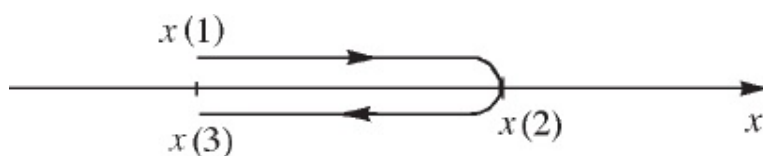
Seja f contínua em $[a, b]$, com $f(x) \geq 0$ em $[a, b]$. Estamos interessados em definir a *área* do conjunto A do plano limitado pelas retas $x = a$, $x = b$, $y = 0$ e pelo gráfico de $y = f(x)$.

2 Cálculo de áreas

$$a) x(3) - x(1) = \int_1^3 (2 - t) dt = \left[2t - \frac{t^2}{2} \right]_1^3 = 0.$$



Em $[1, 2[$, $v(t) > 0$, o que significa que no intervalo de tempo $[1, 2]$ a partícula avança no sentido positivo; em $]2, 3]$, $v(t) < 0$, o que significa que neste intervalo de tempo a partícula recua, de tal modo que no instante $t = 3$ ela volta a ocupar a mesma posição por ela ocupada no instante $t = 1$.



b) O espaço percorrido entre os instantes $t = 1$ e $t = 3$ é

$$\int_1^3 |2 - t| dt = \int_1^2 (2 - t) dt - \int_2^3 (2 - t) dt = 1.$$

Observe que o espaço percorrido entre os instantes 1 e 2 é

$$\int_1^2 (2 - t) dt = \frac{1}{2}$$

e que o espaço percorrido entre os instantes 2 e 3 é

$$\int_2^3 |2 - t| dt = -\int_2^3 (2 - t) dt = \frac{1}{2}.$$

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Exercícios 11.6

Nos Exercícios de 1 a 22, desenhe o conjunto A dado e calcule a área.

1. A é o conjunto do plano limitado pelas retas $x = 1$, $x = 3$, pelo eixo Ox e pelo gráfico de $y = x^3$.
2. A é o conjunto do plano limitado pelas retas $x = 1$, $x = 4$, $y = 0$ e pelo gráfico de $y = \sqrt{x}$.
3. A é o conjunto de todos (x, y) tais que $x^2 - 1 \leq y \leq 0$.
4. A é o conjunto de todos (x, y) tais que $0 \leq y \leq 4 - x^2$.
5. A é o conjunto de todos (x, y) tais que $0 \leq y \leq |\sin x|$, com $0 \leq x \leq 2\pi$.
6. A é a região do plano compreendida entre o eixo Ox e o gráfico de $y = x^2 - x$, com $0 \leq x \leq 2$.
7. A é o conjunto do plano limitado pela reta $y = 0$ e pelo gráfico de $y = 3 - 2x - x^2$, com $-1 \leq x \leq 2$.
8. A é o conjunto do plano limitado pelas retas $x = -1$, $x = 2$, $y = 0$ e pelo gráfico de $y = x^2 + 2x + 5$.
9. A é o conjunto do plano limitado pelo eixo Ox , pelo gráfico de $y = x^3 - x$, $-1 \leq x \leq 1$.
10. A é o conjunto do plano limitado pela reta $y = 0$ e pelo gráfico de $y = x^3 - x$, com $0 \leq x \leq 2$.
11. A é o conjunto do plano limitado pelas retas $x = 0$, $x = \pi$, $y = 0$ e pelo gráfico de $y = \cos x$.
12. A é o conjunto de todos (x, y) tais que $x \geq 0$ e $x^3 \leq y \leq x$.
13. A é o conjunto do plano limitado pela reta $y = x$, pelo gráfico de $y = x^3$, com $-1 \leq x \leq 1$.
14. $A = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \text{ e } \sqrt{x} \leq y \leq 3\}$.
15. A é o conjunto do plano limitado pelas retas $x = 0$, $x = \frac{\pi}{2}$ e pelos gráficos de $y = \sin x$ e $y = \cos x$.
16. A é o conjunto de todos os pontos (x, y) tais que $x^2 + 1 \leq y \leq x + 1$.
17. A é o conjunto de todos os pontos (x, y) tais que $x^2 - 1 \leq y \leq x + 1$.

18. A é o conjunto do plano limitado pelas retas $x = 0$, $x = \frac{\pi}{2}$ e pelos gráficos de $y = \cos x$ e $y = 1 - \cos x$.
19. $A = \{ (x, y) \in \mathbb{R}^2 \mid x \geq 0 \text{ e } x^3 - x \leq y \leq -x^2 + 5x \}$.
20. A é o conjunto do plano limitado pelos gráficos de $y = x^3 - x$, $y = \sin \pi x$, com $-1 \leq x \leq 1$.
21. A é o conjunto de todos os pontos (x, y) tais que $x \geq 0$ e $-x \leq y \leq x \leq x^2$
22. A é o conjunto de todos (x, y) tais que $x > 0$ e $\frac{1}{x^2} \leq y \leq 5 - 4x^2$.
23. Uma partícula desloca-se sobre o eixo x com velocidade $v(t) = 2t - 3$, $t \geq 0$.
- Calcule o deslocamento entre os instantes $t = 1$ e $t = 3$.
 - Qual o espaço percorrido entre os instantes $t = 1$ e $t = 3$?
 - Descreva o movimento realizado pela partícula entre os instantes $t = 1$ e $t = 3$
24. Uma partícula desloca-se sobre o eixo $0x$ com velocidade $v(t) = \sin 2t$, $t \geq 0$. Calcule o espaço percorrido entre os instantes $t = 0$ e $t = \pi$.
25. Uma partícula desloca-se sobre o eixo $0x$ com velocidade $v(t) = -t^2 + t$, $t \geq 0$. Calcule o espaço percorrido entre os instantes $t = 0$ e $t = 2$.
26. Uma partícula desloca-se sobre o eixo $0x$ com velocidade $v(t) = t^2 - 2t - 3$, $t \geq 0$. Calcule o espaço percorrido entre os instantes $t = 0$ e $t = 4$.

11.7. MUDANÇA DE VARIÁVEL NA INTEGRAL

Veremos, no Vol. 2, que toda *função contínua* num intervalo I admite, neste intervalo, uma primitiva. Por ora, vamos admitir tal resultado e usá-lo na demonstração do próximo teorema.

Teorema. Seja f contínua num intervalo I e sejam a e b dois reais quaisquer em I . Seja $g : [c, d] \rightarrow I$, com g' contínua em $[c, d]$, tal que $g(c) = a$ e $g(d) = b$. Nestas condições

$$\int_a^b f(x) dx = \int_c^d f(g(u)) g'(u) du.$$

3 Mudança de variável na integral

$$\int_{-1}^0 x^2 \sqrt{x+1} \, dx = \left[\frac{\frac{7}{2}}{\frac{7}{2}} - 2 \frac{\frac{5}{2}}{\frac{5}{2}} + \frac{\frac{3}{2}}{\frac{3}{2}} \right]_0^1 = \frac{16}{105}.$$

■

Exercícios 11.7

1. Calcule.

a) $\int_1^2 (x-2)^5 \, dx$

b) $\int_0^1 (3x+1)^4 \, dx$

c) $\int_0^1 \sqrt{3x+1} \, dx$

d) $\int_{-1}^0 (2x+5)^3 \, dx$

e) $\int_{-3}^4 \sqrt[3]{5-x} \, dx$

f) $\int_1^2 \frac{2}{(3x-2)^3} \, dx$

g) $\int_0^1 \frac{1}{(x+1)^5} \, dx$

h) $\int_{-2}^1 \frac{3}{4+x} \, dx$

i) $\int_0^2 e^{2x} \, dx$

j) $\int_0^1 x e^{x^2} \, dx$

l) $\int_{-1}^0 x \sqrt{x+1} \, dx$

m) $\int_0^{\frac{\pi}{3}} \cos 2x \, dx$

n) $\int_0^1 \frac{x^2}{1+x^3} \, dx$

o) $\int_0^1 \frac{x^2}{(1+x^3)^2} \, dx$

p) $\int_{-1}^0 x^2 \sqrt{1+x^3} \, dx$

q) $\int_1^3 \frac{2}{5+3x} \, dx$

r) $\int_{-1}^1 \sqrt[3]{x+1} \, dx$

s) $\int_0^1 \frac{x}{(x+1)^5} \, dx$

t) $\int_{-1}^0 x(x+1)^{100} \, dx$

u) $\int_1^2 x^2 (x-2)^{10} \, dx$

2. Suponha f contínua em $[-2, 0]$. Calcule $\int_0^2 f(x-2) dx$, sabendo que $\int_{-2}^0 f(x) dx = 3$.
3. Suponha f contínua em $[-1, 1]$. Calcule $\int_0^1 f(2x-1) dx$ sabendo que $\int_{-1}^1 f(u) du = 5$.
4. Suponha f contínua em $[0, 4]$. Calcule $\int_{-2}^2 x f(x^2) dx$.
5. Calcule $\int_{-\pi}^{\pi} \frac{\sin x}{x^4 + x^2 + 1} dx$.
6. Calcule a área do conjunto dado.
 - a) $A = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x \leq 2 \text{ e } 0 \leq y \leq \sqrt{x-1}\}$
 - b) $A = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2 \text{ e } 0 \leq y \leq \frac{x}{1+x^2}\}$
 - c) A é o conjunto do plano limitado pela reta $x = 1$ e pelos gráficos de $y = e^{-2x}$ e $y = e^{-x}$, com $x \geq 0$
7. Calcule.

a) $\int_0^1 x \sqrt{x^2 + 3} dx$

b) $\int_0^1 x (x^2 + 3)^5 dx$

c) $\int_1^2 x (x^2 - 1)^5 dx$

d) $\int_0^1 x \sqrt{1 - x^2} dx$

e) $\int_{-1}^0 x^2 e^{x^3} dx$

f) $\int_0^1 x \sqrt{1 + 2x^2} dx$

g) $\int_1^2 \frac{3s}{1+s^2} ds$

h) $\int_0^1 \frac{1}{1+4s} ds$

$$i) \int_0^3 \frac{x}{\sqrt{x+1}} dx$$

$$j) \int_0^1 \frac{s}{\sqrt{s^2+1}} ds$$

$$l) \int_0^3 \frac{x^2}{\sqrt{x+1}} dx$$

$$m) \int_0^1 \frac{x^2}{(x+1)^2} dx$$

$$n) \int_{-1}^1 x^3 (x^2+3)^{10} dx$$

$$o) \int_0^{\sqrt{3}} x^3 \sqrt{x^2+1} dx$$

$$p) \int_0^{\frac{\pi}{3}} \sin x \cos^2 x dx$$

$$q) \int_0^{\frac{\pi}{6}} \cos x \sin^5 x dx$$

$$r) \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x (1 - \cos^2 x) dx$$

$$s) \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x \sin^2 x dx$$

$$t) \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^3 x dx$$

$$u) \int_0^{\frac{\pi}{6}} \cos^3 x dx$$

8. Um aluno (precipitado), ao calcular a integral $\int_{-1}^1 \sqrt{1+x^2} dx$, raciocinou da seguinte forma: fazendo a mudança de variável $u = 1 + x^2$, os novos extremos de integração seriam iguais a 2 ($x = -1 \rightarrow u = 2$; $x = 1 \rightarrow u = 2$) e assim a integral obtida após a mudança de variável seria igual a zero e, portanto, $\int_{-1}^1 \sqrt{1+x^2} dx = 0!!$ Onde está o erro?

9. Seja f uma função par e contínua em $[-r, r]$, $r > 0$. (Lembre-se: f par $\Leftrightarrow f(-x) = f(x)$.)

a) Mostre que $\int_{-r}^0 f(x) dx = \int_0^r f(x) dx$

b) Conclua de (a) que $\int_{-r}^r f(x) dx = 2 \int_0^r f(x) dx$. Interprete graficamente

10. Suponha f contínua em $[a, b]$. Seja $g: [c, d] \rightarrow \mathbb{R}$ com g' contínua em $[c, d]$, $g(c) = a$ e $g(d) = b$. Suponha, ainda, que $g'(u) > 0$ em $]c, d[$. Seja $c = u_0 < u_1 < u_2 < \dots < u_n = d$ uma partição de $[c, d]$ e seja $a = x_0 < x_1 < x_2 < \dots < x_n = b$ partição de $[a, b]$, em que $x_i = g(u_i)$, para i variando de 0 a n .

a) Mostre que, para todo $i, i = 1, 2, \dots, n$, existe \bar{u}_i em $[u_{i-1}, u_i]$ tal que

$$\Delta x_i = g'(\bar{u}_i) \Delta u_i$$

b) Conclua de (a) que

$$\sum_{i=1}^n f(g(\bar{u}_i)) g'(\bar{u}_i) \Delta u_i = \sum_{i=1}^n f(c_i) \Delta x_i$$

em que $c_i = g(\bar{u}_i)$.

c) Mostre que existe $M > 0$ tal que

$$\Delta x_i \leq M \Delta u_i$$

para i variando de 0 a n

d) Conclua que

$$\lim_{\max \Delta u_i \rightarrow 0} \sum_{i=1}^n f(g(\bar{u}_i)) g'(\bar{u}_i) \Delta u_i = \lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i$$

ou seja,

$$\int_c^d f(g(u)) g'(u) du = \int_a^b f(x) dx$$

11.8. TRABALHO

Nesta seção, admitiremos que o leitor já saiba o que é um *vetor*. Consideremos, então, um eixo Os



e indiquemos por \vec{u} o vetor, de comprimento *unitário*, determinado pelo segmento orientado de *origem* 0 e *extremidade* 1.

Seja α um número real; $\vec{F} = \alpha \vec{u}$ é um vetor *paralelo* a \vec{u} . O número α é a

4 Primitivas inmediatas

Solução

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x.$$

Então:

$$\int \cos^2 x \, dx = \int \left[\frac{1}{2} + \frac{1}{2} \cos 2x \right] dx = \frac{1}{2} x + \frac{1}{4} \sin 2x + k$$

ou seja,

$$\int \cos^2 x \, dx = \frac{1}{2} x + \frac{1}{4} \sin 2x + k.$$

■

Exercícios 12.1

1. Calcule e verifique sua resposta por derivação.

a) $\int 3 \, dx$

b) $\int x \, dx$

c) $\int x^5 \, dx$

d) $\int \sqrt{x} \, dx$

e) $\int \sqrt[5]{x^2} \, dx$

f) $\int x^{-4} \, dx$

g) $\int \frac{1}{x^3} \, dx$

h) $\int \frac{x + x^2}{x^2} \, dx$

$$i) \int \left(\frac{1}{x} + \frac{1}{x^2} \right) dx$$

$$l) \int \frac{x+1}{x} dx$$

$$n) \int e^{5x} dx$$

$$p) \int (e^{2x} + e^{-x}) dx$$

$$r) \int \left(e^{4x} + \frac{1}{x^2} \right) dx$$

$$t) \int \frac{x^5 + x + 1}{x^2} dx$$

$$j) \int \left(x^2 + \frac{3}{x} \right) dx$$

$$m) \int (e^x + 4) dx$$

$$o) \int e^{-2x} dx$$

$$q) \int \left(\frac{1}{x} + \frac{1}{e^x} \right) dx$$

$$s) \int \left(\frac{3}{x} + \frac{2}{x^3} \right) dx$$

$$u) \int e^{\sqrt{2}x} dx$$

2. Calcule.

$$a) \int_0^1 e^{2x} dx$$

$$c) \int_{-1}^1 e^{-x} dx$$

$$e) \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$$

$$b) \int_1^2 \left(x + \frac{1}{x} \right) dx$$

$$d) \int_0^1 \frac{1}{1+x^2} dx$$

$$f) \int_1^2 \frac{x^3 + 1}{x} dx$$

3. Calcule e verifique sua resposta por derivação.

$$a) \int \sin x dx$$

$$c) \int \cos 5x dx$$

$$e) \int \cos 7t dt$$

$$g) \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) dx$$

$$i) \int \left(x + \frac{1}{5} \cos 3x \right) dx$$

$$l) \int \left(\frac{1}{3} + \frac{5}{2} \cos 7x \right) dx$$

$$n) \int \left(\frac{1}{3} \sin 2x + \frac{1}{2} \cos 3x \right) dx$$

$$p) \int \left(\frac{1}{3} \cos 3x - \frac{1}{7} \sin 7x \right) dx$$

$$b) \int \sin 2x dx$$

$$d) \int \sin 4t dt$$

$$f) \int \cos \sqrt{3} t dt$$

$$h) \int \left(2 + \frac{1}{3} \sin 2x \right) dx$$

$$j) \int \left(\frac{1}{x} + 4 \sin 3x \right) dx$$

$$m) \int \left(\cos 3x + \frac{1}{2} \sin 4x \right) dx$$

$$o) \int \frac{\sin 2x}{\cos x} dx$$

$$q) \int \left(\frac{1}{3} e^{3x} + \sin 3x \right) dx$$

4. Calcule.

a) $\int_0^{\frac{\pi}{3}} \sin 2x \, dx$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \frac{x}{2} \, dx$

c) $\int_0^{\frac{\pi}{3}} (\sin 3x + \cos 3x) \, dx$

d) $\int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right) dx$

5. a) Verifique que $\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$.

b) Calcule $\int \sin^2 x \, dx$.

6. Calcule.

a) $\int \cos^2 2x \, dx$

b) $\int \cos^2 5x \, dx$

c) $\int \sin^2 3x \, dx$

d) $\int \cos^2 \frac{x}{2} \, dx$

e) $\int \cos^4 x \, dx$

f) $\int \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right)^2 dx$

g) $\int (\sin x + \cos x)^2 dx$

h) $\int (\sin x - \cos x)^2 dx$

i) $\int (5 + \sin 3x)^2 dx$

j) $\int (1 - \cos 2x)^2 dx$

7. Calcule.

a) $\int_0^{\frac{\pi}{8}} \cos^2 x \, dx$

b) $\int_0^{\frac{\pi}{4}} \sin^2 x \, dx$

c) $\int_0^{\frac{\pi}{2}} (\sin x + \cos x)^2 dx$

d) $\int_0^{\frac{\pi}{2}} \cos^4 x \, dx$

8. Calcule $\int_0^{2\pi} \sqrt{1 + \cos x} \, dx$.

9. a) Verifique que

$$\int \sec x \, dx = \ln (\sec x + \tan x) + k$$

$$\text{com } x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[.$$

b) Mostre que

$$\int \sec x \, dx = \ln |\sec x + \operatorname{tg} x| + k.$$

10. Calcule.

a) $\int \operatorname{tg} x \, dx$

b) $\int \sec^2 x \, dx$

c) $\int \operatorname{tg}^2 x \, dx$

d) $\int \sec x \, dx$

e) $\int \operatorname{tg} 2x \, dx$

f) $\int \sec 3x \, dx$

g) $\int 3^x \, dx$

h) $\int \frac{5}{\sqrt{1-x^2}} \, dx$

i) $\int (5^x + e^{-x}) \, dx$

j) $\int (x + \sec^2 3x) \, dx$

l) $\int (1 + \sec x)^2 \, dx$

m) $\int \frac{\cos x + \sec x}{\cos x} \, dx$

11. a) Determine α e β de modo que

$$\operatorname{sen} 6x \cos x = \frac{1}{2} (\operatorname{sen} \alpha x + \operatorname{sen} \beta x)$$

$$\left(\text{Sugestão: } \operatorname{sen} a \cos b = \frac{1}{2} [\operatorname{sen} (a+b) + \operatorname{sen} (a-b)]. \right)$$

b) Calcule $\int \operatorname{sen} 6x \cos x \, dx$.

12. Calcule.

a) $\int \operatorname{sen} 5x \cos x \, dx$

b) $\int \operatorname{sen} 3x \cos 4x \, dx$

c) $\int \operatorname{sen} x \cos 3x \, dx$

d) $\int \operatorname{sen} 3x \cos 3x \, dx$

13. a) Determine α e β de modo que

$$\operatorname{sen} 3x \operatorname{sen} 2x = -\frac{1}{2} (\cos \alpha x - \cos \beta x)$$

$$\left(\text{Sugestão: } \operatorname{sen} a \operatorname{sen} b = \frac{1}{2} [\cos (a-b) - \cos (a+b)]. \right)$$

b) Calcule $\int \operatorname{sen} 3x \operatorname{sen} 2x \, dx$.

14. Calcule $\int \cos 5x \cos 2x \, dx$.

$$\left(\text{Sugestão: } \cos a \cos b = \frac{1}{2} [\cos (a + b) + \cos (a - b)]. \right)$$

15. Calcule.

$$a) \int \sin x \sin 3x \, dx$$

$$b) \int \sin 2x \sin 5x \, dx$$

$$c) \int \sin 3x \cos 2x \, dx$$

$$d) \int \cos 5x \cos x \, dx$$

$$e) \int \cos 7x \cos 3x \, dx$$

16. Calcule.

$$a) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \sin 3x \, dx$$

$$b) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 3x \sin 4x \, dx$$

17. Sejam m e n naturais. Calcule.

$$a) \int_{-\pi}^{\pi} \sin mx \sin nx \, dx$$

$$b) \int_{-\pi}^{\pi} \cos mx \sin nx \, dx$$

12.2. TÉCNICA PARA CÁLCULO DE INTEGRAL INDEFINIDA DA FORMA $\int f(g(x)) g'(x) \, dx$

Sejam f e g tais que $\text{Im } g \subset D_f$ com g derivável. Suponhamos que F seja uma primitiva de f , isto é, $F' = f$. Segue que $F(g(x))$ é uma primitiva de $f(g(x)) g'(x)$, de fato,

$$(F(g(x)))' = F'(g(x)) g'(x) = f(g(x)) g'(x).$$

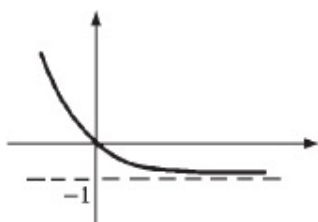
Deste modo, de

$$\int f(u) \, du = F(u) + k$$

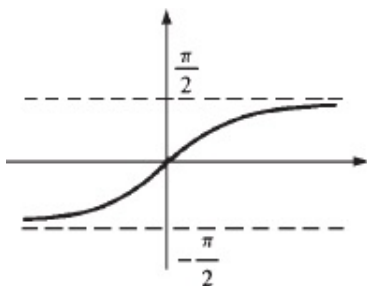
segue

$$\int f(g(x)) g'(x) \, dx = F(g(x)) + k.$$

5 Respostas



d) $y = \text{arc tg } x$



CAPÍTULO 11

11.5

1. $7/2$
2. 2
3. 2
4. 0
5. 2
6. 12
7. $4/9$
8. 10
9. $8/3$
10. $3/4$
11. 0
12. -4
13. -1

14. $16/3$

15. 2

16. 12

17. $15/4$

18. $8/9$

19. $45/8$

20. $13/10$

21. $32/3$

22. 0

23. 0

24. $15/8$

25. $253/6$

26. $-21/8$

27. $7/8$

28. $7/3$

29. $20/3$

30. $19/3$

31. $20/3$

32. $19/24$

33. $11/8$

34. 9

35. $47/6$

36. 0

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37. $2 + \ln 3$

38. $\ln 2 + \frac{9}{2}$

39. $\frac{\sqrt{3}}{4}$

40. $-\frac{2}{3}$

41. $\frac{1}{2}(e^2 - e^{-2})$

42. $\frac{\pi}{4}$

43. $\frac{2 - \sqrt{2}}{2}$

44. $\frac{1}{2}(e^2 - 1)$

45. π

46. $\frac{1}{5}(1 - \cos 5)$

47. $\frac{\pi}{6}$

48. $\frac{3}{\ln 2}$

49. $e - 1$

50. $\ln 2$

51. $\ln 2$

52. 0

53. $\frac{5}{4}$

54. $\frac{\pi}{4}$

55. $\frac{\pi}{4}$

56. $\frac{\pi}{4}$

57. 1

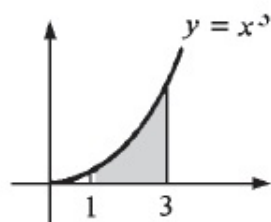
58. $\frac{2}{\ln 3}$

59. $\frac{3e - 1}{1 + \ln 3}$

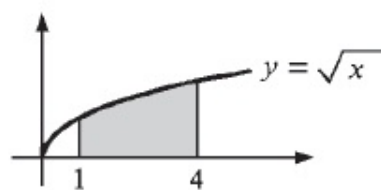
60. $\frac{4 - \pi}{4}$

11.6

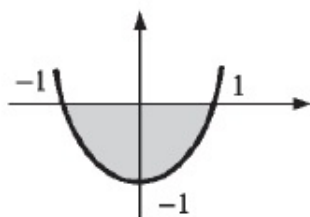
1. Área = 20



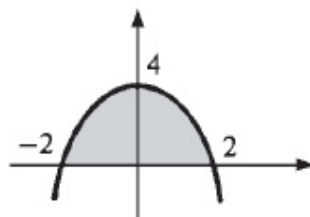
2. Área = $\frac{14}{3}$



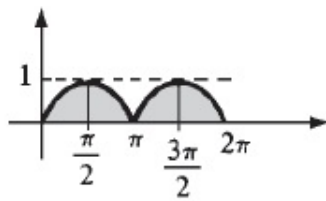
3. Área = $\frac{4}{3}$



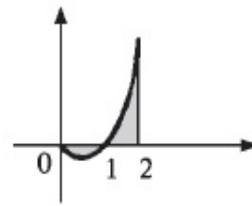
4. Área = $\frac{32}{3}$



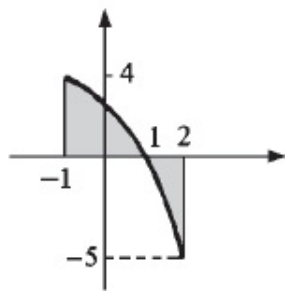
5. Área = 4



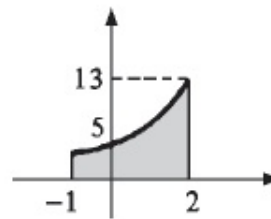
6. Área = 1



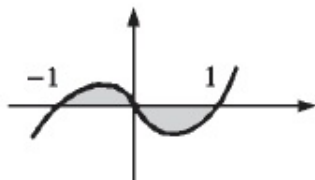
7. Área = $\frac{23}{3}$



8. Área = 21



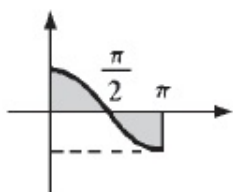
9. Área = $\frac{1}{2}$



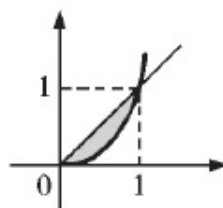
10. Área = $\frac{5}{2}$



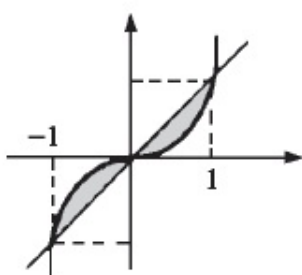
11. Área = 2



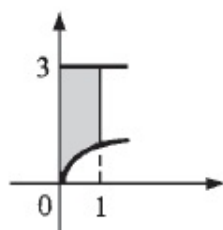
12. Área = $\frac{1}{4}$



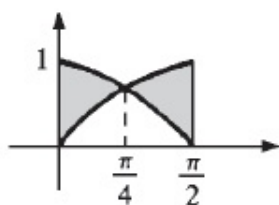
13. Área = $\frac{1}{2}$



14. Área = $\frac{7}{3}$



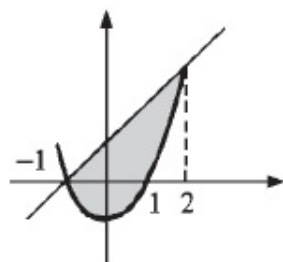
15. Área = $2(\sqrt{2} - 1)$



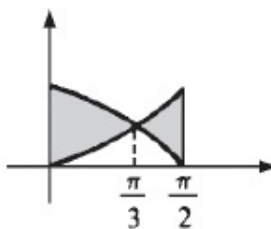
16. Área = $\frac{1}{6}$



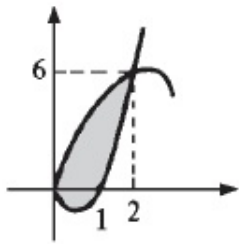
17. Área = $\frac{9}{2}$



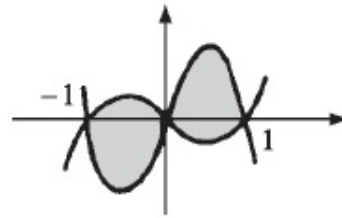
18. Área = $\frac{1}{6}(12\sqrt{3} - \pi - 12)$



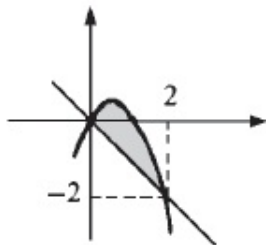
19. $\text{Área} = \frac{16}{3}$



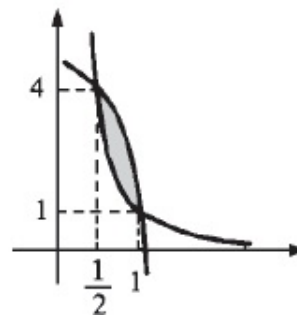
20. $\text{Área} = \frac{8 + \pi}{2\pi}$



21. $\text{Área} = \frac{4}{3}$



22. $\text{Área} = \frac{1}{3}$



23. a) 2

b) $\frac{10}{4}$

24. 2

25. 1

26. $\frac{34}{3}$

$$\begin{array}{llllll}
1. & a) -\frac{1}{6} & b) \frac{1.023}{15} & c) \frac{14}{9} & d) 68 & e) \frac{45}{4} & f) \frac{5}{16} \\
& g) \frac{15}{64} & h) 3 \ln \frac{5}{2} & i) \frac{1}{2}[e^4 - 1] & j) \frac{1}{2}[e - 1] & l) -\frac{4}{15} \\
& m) \frac{\sqrt{3}}{4} & n) \frac{1}{3} \ln 2 & o) \frac{1}{6} & p) \frac{2}{9} & q) \frac{2}{3} \ln \frac{7}{4} & r) 0 \\
& s) \frac{11}{192} & t) -\frac{1}{10.302} & u) \frac{46}{429}
\end{array}$$

2. 3

3. $\frac{5}{2}$

4. 0

5. 0

$$6. \quad a) \frac{2}{3} \quad b) \frac{1}{2} \ln 5 \quad c) \frac{e^{-2} - 2e^{-1} + 1}{2}$$

$$7. \quad a) \frac{8 - \sqrt{27}}{3} \quad b) \frac{3.367}{12} \quad c) \frac{243}{4} \quad d) \frac{1}{3} \quad e) \frac{1}{3}(1 - e^{-1})$$

$$f) \frac{3\sqrt{3} - 1}{6} \quad g) \frac{3}{2} \ln \frac{5}{2} \quad h) \frac{1}{4} \ln 5 \quad i) \frac{8}{3} \quad j) \sqrt{2} - 1$$

$$l) \frac{76}{15} \quad m) \frac{3 - 4 \ln 2}{2} \quad n) 0 \quad o) \frac{58}{15} \quad p) \frac{7}{24} \quad q) \frac{1}{384}$$

$$r) \frac{11}{24} \quad s) \frac{11}{24} \quad t) \frac{11}{24} \quad u) \frac{11}{24}$$

11.8

1. a) 6 J

b) 4 J

c) -1 J

d) 0

2. a) $\int_1^x -3x \, dx = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2$, logo, $\frac{3x^2}{2} + v^2 = \frac{3}{2}$

b) $\sqrt{\frac{3}{2}}$

c) 1 e -1

d) $|x| = 1$

e) Oscilatório

3. **b)** $\sqrt{5}$ e $-\sqrt{5}$ **c)** $x = 0$ **d)** $|x| = \sqrt{5}$

4. **a)** $|v| = \frac{\sqrt{10}}{5} \sqrt{40 - x^2}$ **b)** 4 **c)** $\sqrt{40}$ **d)** $\sqrt{40}$ e $-\sqrt{40}$

5. $v = \sqrt{1 - \frac{1}{x}}$

6. $\int_{x_0}^x ma \, dx = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2$ ou $2a(x - x_0) = v^2 - v_0^2$.

7. **b)** $\frac{v_0^2}{2g}$

8. **a)** $v^2 - \frac{1}{x} = v_0^2 - 1$

b) $v_0 = 1$

10. $\int_0^3 3x \cos 30^\circ \, dx = \frac{27\sqrt{3}}{4} \, J$

11. **a)** $\frac{3\sqrt{2}}{2} \, J$

b) 0

12. $\int_{-2}^{-1} \frac{-x}{(4+x^2)\sqrt{4+x^2}} \, dx = \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{8}}.$

CAPÍTULO 12

12.1

1. a) $3x + k$ b) $\frac{x^2}{2} + k$ c) $\frac{x^6}{6} + k$ d) $\frac{2}{3}\sqrt{x^3} + k$
e) $\frac{5}{7}\sqrt[5]{x^7} + k$ f) $-\frac{1}{3x^3} + k$ g) $-\frac{1}{2x^2} + k$ h) $x + \ln|x| + k$
i) $\ln|x| - \frac{1}{x} + k$ j) $\frac{x^3}{3} + 3\ln|x| + k$ l) $x + \ln|x| + k$
m) $e^x + 4x + k$ n) $\frac{1}{5}e^{5x} + k$ o) $-\frac{1}{2}e^{-2x} + k$
p) $\frac{1}{2}e^{2x} - e^{-x} + k$ q) $\ln|x| - e^{-x} + k$ r) $\frac{1}{4}e^{4x} - \frac{1}{x} + k$
s) $3\ln|x| - \frac{1}{x^2} + k$ t) $\frac{x^4}{4} + \ln|x| - \frac{1}{x} + k$ u) $\frac{\sqrt{2}}{2}e^{\sqrt{2}x} + k$
2. a) $\frac{1}{2}(e^2 - 1)$ b) $\frac{3 + 2\ln 2}{2}$ c) $e - \frac{1}{e}$ d) $\frac{\pi}{4}$ e) $\frac{\pi}{6}$ f) $\frac{7 + 3\ln 2}{3}$
3. a) $-\cos x + k$ b) $-\frac{1}{2}\cos 2x + k$ c) $\frac{1}{5}\sin 5x + k$
d) $-\frac{1}{4}\cos 4t + k$ e) $\frac{1}{7}\sin 7t + k$ f) $\frac{1}{\sqrt{3}}\sin \sqrt{3}t + k$
g) $\frac{1}{2}x - \frac{1}{4}\sin 2x + k$ h) $2x - \frac{1}{6}\cos 2x + k$ i) $\frac{x^2}{2} + \frac{1}{15}\sin 3x + k$
j) $\ln|x| - \frac{4}{3}\cos 3x + k$ l) $\frac{1}{3}x + \frac{5}{14}\sin 7x + k$
m) $\frac{1}{3}\sin 3x - \frac{1}{8}\cos 4x + k$ n) $-\frac{1}{6}\cos 2x + \frac{1}{6}\sin 3x + k$
o) $-2\cos x + k$ p) $\frac{1}{9}\sin 3x + \frac{1}{49}\cos 7x + k$
q) $\frac{1}{9}e^{3x} - \frac{1}{3}\cos 3x + k$
4. a) $\frac{3}{4}$ b) $2\sqrt{2}$ c) $\frac{2}{3}$ d) $\frac{\pi}{4}$
5. b) $\frac{1}{2}x - \frac{1}{4}\sin 2x + k$

6. a) $\frac{1}{2}x + \frac{1}{8} \sin 4x + k$ b) $\frac{1}{2}x + \frac{1}{20} \sin 10x + k$
 c) $\frac{1}{2}x - \frac{1}{12} \sin 6x + k$ d) $\frac{1}{2}x + \frac{1}{2} \sin x + k$
 e) $\frac{3}{8}x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + k$ f) $\frac{3}{8}x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + k$
 g) $x - \frac{1}{2} \cos 2x + k$ h) $x + \frac{1}{2} \cos 2x + k$
 i) $\frac{51}{2}x - \frac{10}{3} \cos 3x - \frac{1}{12} \sin 6x + k$ j) $\frac{3}{2}x - \sin 2x + \frac{1}{8} \sin 4x + k$

7. a) $\frac{\pi}{16} + \frac{\sqrt{2}}{8}$ b) $\frac{\pi}{8}$ c) $\frac{\pi}{2} + 1$ d) $\frac{3\pi}{16}$

8. $4\sqrt{2}$

10. a) $-\ln |\cos x| + k$

b) $\lg x + k$

c) $\lg x - x + k$

d) $\ln |\sec x + \tg x| + k$

e) $-\frac{1}{2} \ln |\cos 2x| + k$

f) $\frac{1}{3} \ln |\sec 3x + \tg 3x| + k$ g) $\frac{1}{\ln 3} 3^x + k$ h) $5 \arcsin x + k$

i) $\frac{5^x}{\ln 5} - e^{-x} + k$ j) $\frac{x^2}{2} + \frac{1}{3} \tg 3x + k$

l) $x + \tg x + 2 \ln |\sec x + \tg x| + k$ m) $x + \tg x + k$

11. a) $\sin 6x \cos x = \frac{1}{2} [\sin 7x + \sin 5x]$ b) $-\frac{1}{14} \cos 7x - \frac{1}{10} \cos 5x + k$

12. a) $-\frac{1}{12} \cos 6x - \frac{1}{8} \cos 4x + k$ b) $-\frac{1}{14} \cos 7x + \frac{1}{2} \cos x + k$
 c) $-\frac{1}{8} \cos 4x + \frac{1}{4} \cos 2x + k$ d) $-\frac{1}{12} \cos 6x + k$
13. a) $\sin 3x \sin 2x = -\frac{1}{2} (\cos 5x - \cos x)$ b) $-\frac{1}{10} \sin 5x + \frac{1}{2} \sin x + k$
14. $\frac{1}{14} \sin 7x + \frac{1}{6} \sin 3x + k$
15. a) $-\frac{1}{8} \sin 4x + \frac{1}{4} \sin 2x + k$ b) $-\frac{1}{14} \sin 7x + \frac{1}{6} \sin 3x + k$
 c) $-\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + k$ d) $\frac{1}{12} \sin 6x + \frac{1}{8} \sin 4x + k$
 e) $\frac{1}{20} \sin 10x + \frac{1}{8} \sin 4x + k$
16. a) 0
 b) $\frac{8}{7}$
17. a) 0 se $m \neq n$; π se $m = n$
 b) 0

12.2

1. a) $\frac{(3x-2)^4}{12} + k$ b) $\frac{2}{9} \sqrt{(3x-2)^3} + k$ c) $\frac{1}{3} \ln |3x-2| + k$
 d) $-\frac{1}{3(3x-2)} + k$ e) $-\frac{1}{2} \cos x^2 + k$ f) $\frac{1}{2} e^{x^2} + k$
 g) $\frac{1}{3} e^{x^3} + k$ h) $-\frac{1}{5} \cos 5x + k$ i) $\frac{1}{4} \sin x^4 + k$ j) $\frac{1}{6} \sin 6x + k$

$$l) -\frac{1}{4} \cos^4 x + k \quad m) \frac{1}{6} \sin^6 x + k \quad n) 2 \ln |x + 3| + k$$

$$o) \frac{5}{4} \ln |4x + 3| + k \quad p) \frac{1}{8} \ln (1 + 4x^2) + k \quad q) \frac{1}{4} \ln (5 + 6x^2) + k$$

$$r) -\frac{1}{8(1 + 4x^2)} \quad s) \frac{1}{9} \sqrt{(1 + 3x^2)^3} + k \quad t) \frac{2}{3} \sqrt{(1 + e^x)^3}$$

$$u) -\frac{1}{2(x - 1)^2} + k \quad v) \frac{1}{\cos x} + k \quad x) -\frac{1}{2} e^{-x^2} + k$$

$$2. \quad a) \frac{1}{2} \left(1 - \frac{1}{e} \right) \quad b) \frac{1}{5} \left(\frac{\sqrt{3}}{2} \right)^5 \quad c) \frac{3}{2} \ln 3 \quad d) \frac{1}{6} \ln \frac{13}{4}$$

$$e) \sqrt{2} - 1 \quad f) \frac{2 - \sqrt{2}}{3} \quad g) \frac{1}{202} \quad h) \frac{1}{3} \quad i) \frac{3}{8} \quad j) 1 \quad l) \frac{\pi}{8} \quad m) \frac{\pi}{8}$$

$$3. \quad a) \frac{1}{3} \sin^3 x + k \quad b) \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + k \quad c) \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + k$$

$$d) -\frac{2}{3} \sqrt{\cos^3 x} + k \quad e) -\frac{2}{3} \sqrt{(1 + \cos^2 x)^3} + k$$

$$f) \frac{2}{3} \sqrt{(5 + \sin^2 x)^3} + k \quad g) -\cos x + \frac{1}{3} \cos^3 x + k$$

$$h) \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + k \quad i) \frac{1}{4} \operatorname{tg}^4 x + k \quad j) \frac{1}{2} \operatorname{tg}^2 x + k$$

$$l) \frac{1}{3} \sec^3 x + k \quad m) \frac{1}{6} \sec^6 x - \frac{1}{4} \sec^4 x + k$$

$$n) -\frac{2}{3} (3 + \cos x)^{3/2} + k \quad o) \frac{1}{\cos x} + k \quad p) \frac{1}{2 \cos^2 x} + k$$

$$q) \frac{1}{8} x - \frac{1}{32} \sin 4x + k \quad r) \sec x + \cos x + k \quad s) \frac{1}{2} \ln |3 + 2 \operatorname{tg} x| + k$$

$$4. \quad a) 2 \ln |x - 3| + k$$

$$b) 5 \ln |x - 1| + 2 \ln |x| + k$$

$$c) \frac{1}{2} \ln |2x + 3| + k$$

$$d) \frac{x^2}{2} + 3 \ln |x - 2| + k$$

$$e) x - \ln |x + 1| + k$$

$$f) x + 3 \ln |x - 1| + k$$

$$g) 2x + \ln |x + 1| + k$$

$$h) \frac{(x + 1)^2}{2} - 2(x + 1) + \ln |x + 1| + k$$

$$6. \quad a) -\frac{1}{2} \ln |x + 1| + \frac{1}{2} \ln |x - 1| + k \quad b) -\frac{3}{2} \ln |x| + \frac{7}{2} \ln |x - 2| + k$$

$$c) \frac{1}{2} \ln |x - 2| + \frac{1}{2} \ln |x + 2| + k \quad d) \frac{1}{4} \ln |x - 2| - \frac{1}{4} \ln |x + 2| + k$$

$$e) -8 \ln |x - 1| + 13 \ln |x - 2| + k$$

$$f) \ln |x - 2| + k$$

$$g) -2 \ln |x - 2| + 2 \ln |x - 3| + k$$

$$h) -4 \ln |x + 1| + 5 \ln |x + 2| + k$$

$$8. \quad a) \frac{1}{\sqrt{5}} \operatorname{arc} \operatorname{tg} \frac{x}{\sqrt{5}} + k$$

$$b) \operatorname{arc} \operatorname{tg} \frac{x}{2} + k$$

$$c) \frac{\sqrt{10}}{10} \operatorname{arc} \operatorname{tg} \frac{\sqrt{10} x}{2} + k$$

$$d) \frac{3}{\sqrt{5}} \operatorname{arc} \operatorname{tg} \frac{x}{\sqrt{5}} + k$$

$$e) \frac{1}{2} \ln (5 + x^2) + k$$

$$f) \frac{1}{4} \ln (1 + 4x^2) - \frac{3}{2} \operatorname{arc} \operatorname{tg} 2x + k$$

$$g) \frac{1}{2} \ln (4 + x^2) - \frac{1}{2} \operatorname{arc} \operatorname{tg} \frac{x}{2} + k$$

$$h) \frac{1}{4} \ln (1 + 4x^2) - \frac{3}{2} \operatorname{arc} \operatorname{tg} 2x + k$$

$$i) \operatorname{arc} \operatorname{tg} (x + 1) + k \quad j) \operatorname{arc} \operatorname{tg} (x + 1) + k \quad l) \frac{2}{\sqrt{5}} \operatorname{arc} \operatorname{tg} \frac{x + 2}{\sqrt{5}} + k$$

$$m) \frac{1}{2} \operatorname{arc} \operatorname{tg} \frac{x + 2}{2} + k \quad n) \frac{2}{\sqrt{3}} \operatorname{arc} \operatorname{tg} \frac{2x + 1}{\sqrt{3}} + k \quad o) 2 \operatorname{arc} \operatorname{tg} (x + 1) + k$$

10. a) $-\frac{1}{8(16+x^4)^2} + k$ b) $\frac{1}{4} \ln(16+x^4) + k$ c) $\frac{1}{8} \operatorname{arc\,tg} \frac{x^2}{4} + k$
d) $-\frac{1}{2} \ln |\cos 2x| + k$ e) $\ln |\ln x| + k$ f) $-\frac{1}{\ln x} + k$
g) $\operatorname{tg} x - x + k$ h) $\operatorname{arc\,sen} x + k$ i) $\frac{5}{2} \operatorname{arc\,sen} 2x + k$
j) $-\frac{1}{4} \sqrt{1-4x^2} + k$ l) $\operatorname{arc\,sen} \frac{x}{2} + k$
m) $-\frac{1}{2} \sqrt{1-4x^2} + \frac{3}{2} \operatorname{arc\,sen} 2x + k$ n) $\frac{2}{3} \operatorname{arc\,sen} \frac{3x}{2} + k$
o) $\frac{1}{2} \operatorname{arc\,sen} x^2 + k$ p) $\operatorname{arc\,sen} e^x + k$ q) $-2 \sqrt{1-e^x} + k$
r) $\operatorname{arc\,sen} (\ln x) + k$ s) $2 \operatorname{arc\,sen} (x+1) + k$ t) $\operatorname{arc\,tg} e^x + k$
u) $\frac{1}{3} \ln(1+3e^x) + k$ v) $\operatorname{sen} (\ln x) + k$ x) $\frac{1}{4} \operatorname{arc\,tg} x^4 + k$

12.3

1. a) $(x-1)e^x + k$
b) $-x \cos x + \operatorname{sen} x + k$
c) $e^x(x^2 - 2x + 2) + k$
d) $\frac{x^2}{2} \left(\ln x - \frac{1}{2} \right) + k$
e) $x(\ln x - 1) + k$
f) $\frac{1}{3} x^3 \left(\ln x - \frac{1}{3} \right) + k$

$$g) x \operatorname{tg} x + \ln |\cos x| + k$$

$$h) \frac{x^2}{2} \left[(\ln x)^2 - \ln x + \frac{1}{2} \right] + k$$

$$i) x (\ln x)^2 - 2x (\ln x - 1) + k$$

$$j) \frac{1}{2} e^{2x} \left(x - \frac{1}{2} \right) + k$$

$$l) \frac{1}{2} e^x (\sin x + \cos x) + k$$

$$m) -\frac{1}{5} e^{-2x} (\cos x + 2 \sin x) + k$$

$$n) \frac{1}{2} (x^2 - 1) e^{x^2} + k$$

$$o) \frac{1}{2} (x^2 \sin x^2 + \cos x^2) + k$$

$$p) \frac{e^{-x}}{5} (2 \sin 2x - \cos 2x) + k$$

$$q) -x^2 \cos x + 2x \sin x + 2 \cos x + k$$

$$2. \quad b) \frac{1}{4} \sec^3 x \operatorname{tg} x + \frac{3}{8} \sec x \operatorname{tg} x + \frac{3}{8} \ln |\sec x + \operatorname{tg} x| + k$$

$$4. \quad a) -\frac{1}{3} \sin^2 x \cos x - \frac{2}{3} \cos x \quad b) -\frac{1}{4} \sin^3 x \cos x - \frac{3}{8} \sin x \cos x + \frac{3}{8} x + k$$

$$5. \quad -\frac{e^{-st}}{1+s^2} (\cos t + s \sin t) + k$$

$$7. \quad a) 1$$

$$b) 2 \ln 2 - 1$$

$$c) \frac{1}{2} \left(e^{\frac{\pi}{2}} - 1 \right)$$

$$d) -\frac{1}{s} x^2 e^{-sx} - \frac{2}{s^2} x e^{-sx} - \frac{2}{s^3} e^{-sx} + \frac{2}{s^3}$$

$$1. \quad a) \frac{1}{4} [\arcsin 2x + 2x \sqrt{1 - 4x^2}] + k \quad b) \arcsin \frac{x}{2} + k$$

$$c) \ln (x + \sqrt{4 + x^2}) + k \quad d) \frac{1}{2} \arctg \frac{x}{2} + k \quad e) -\sqrt{1 - x^2} + k$$

$$f) \frac{3}{4} \left[\arcsin \frac{2x}{\sqrt{3}} + \frac{2x}{3} \sqrt{3 - 4x^2} \right] + k$$

$$g) \frac{1}{2} [\arcsin x - x \sqrt{1 - x^2}] + k$$

$$h) \frac{1}{8} [\arcsin x - x \sqrt{1 - x^2} (1 - 2x^2)] + k$$

$$i) \ln \left| \frac{x}{1 + \sqrt{1 + x^2}} \right| + k$$

$$j) \frac{9}{2} \arcsin \frac{x-1}{3} + \frac{(x-1) \sqrt{9 - (x-1)^2}}{2} + k$$

$$l) \text{ Faça } 2x = 3 \operatorname{sen} t$$

$$m) -x^2 + 2x + 2 = 3 - (x-1)^2;$$

$$\text{faça } x-1 = \sqrt{3} t$$

$$n) 2 \arcsin \frac{x-1}{2} + \frac{x-1}{2} \sqrt{4 - (x-1)^2} + k \quad o) -\frac{\sqrt{1+x^2}}{x} + k$$

$$2. \frac{\pi}{2}$$

$$3. \pi ab$$

4. a) $\frac{(x+1)^{13}}{13} - \frac{(x+1)^{12}}{6} + \frac{(x+1)^{11}}{11} + k$
- b) $\frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + k$
- c) $2(\sqrt{x} - \ln(1 + \sqrt{x})) + k$ d) $-\frac{4}{1+\sqrt{x}} + \frac{2}{(1+\sqrt{x})^2} + k$
- e) $-\frac{1}{3(x+1)^3} - \frac{1}{4(x+1)^4} + k$ f) $\frac{1}{6}(2x+1)^{3/2} - \frac{3}{2}(2x+1)^{1/2} + k$
- g) $2\sqrt{1-e^x} + \ln \frac{1-\sqrt{1-e^x}}{1+\sqrt{1-e^x}} + k$ h) $\frac{4}{5}(1+\sqrt{x})^{5/2} - \frac{4}{3}(1+\sqrt{x})^{3/2} + k$
- i) $\frac{5}{2} \arcsin(x-1) - \frac{1}{2} \sqrt{2x-x^2} (x+3) + k$
- j) $\frac{1}{2} \arctg \frac{(x+1)}{2} + k$ l) $\left(\frac{x^2}{2} - \frac{1}{4} \right) \arcsin x + \frac{x}{4} \sqrt{1-x^2} + k$
- m) $\frac{1}{2} (\arctg x)^2 (1+x^2) - x \arctg x + \frac{1}{2} \ln(1+x^2) + k$
- n) $(x+1) \arctg \sqrt{x} - \sqrt{x} + k$ o) $-\frac{\arctg e^x}{e^x} + x - \frac{1}{2} \ln(1+e^{2x}) + k.$
6. a) $\frac{1}{2} \ln(4+x^2) + \frac{1}{2} \arctg \frac{x}{2} + k$ b) $\frac{1}{4} \ln(9+4x^2) - \frac{1}{6} \arctg \frac{2x}{3} + k$
- c) $\frac{1}{2} \ln(x^2+2x+2) + 9 \arctg(x+1) + k$
- d) $\frac{3}{2} \ln(x^2+x+1) - \frac{7}{\sqrt{3}} \arctg \frac{2x+1}{\sqrt{3}} + k$
- e) $\ln(x^2+4x+5) - 3 \arctg(x+2) + k$
- f) $\frac{1}{2} \ln(9+x^2) - \frac{1}{3} \arctg \frac{x}{3} + k$
7. $\frac{3\sqrt{2}}{2} \arcsin \frac{1}{\sqrt{3}} + \frac{1}{3}$ 8. $\frac{4-3 \ln 3}{6}$

9. a) $x = 3 \operatorname{sen} t$

b) $x = 3 \sec t$

c) $x = 3 \operatorname{tg} t$

d) $x = \operatorname{sen} t$

e) $2x = \sqrt{3} \operatorname{sen} t$ f) $2x = \sqrt{3} \sec t$ g) $2x = \sqrt{3} \operatorname{tg} t$

h) $\sqrt{3} x = \sqrt{2} \operatorname{sen} t$ i) $\sqrt{3} x = \sqrt{2} \sec t$ j) $\sqrt{3} x = \sqrt{2} \operatorname{tg} t$

l) $x - 1 = u^2, u > 0$

m) $1 + e^x = u^2, u > 0$

n) $x + \frac{3}{2} = \frac{\sqrt{3}}{2} \operatorname{tg} t$

o) $1 + \sqrt{x} = t^3$

12.5

1. $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + k$

2. $-2 \ln |x-2| + 3 \ln |x-3| + k$

3. $\frac{1}{2} \ln |x^2 - 4| + k$

4. $\ln |x^2 - 1| + \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + k$

5. $6 \ln |x-1| + 10(x-1) + \frac{5}{2}(x-1)^2 + k$

6. $\ln |x-1| - \frac{4}{x-1} + k$

7. $x + \frac{1}{4} \ln |x+1| + \frac{19}{4} \ln |x-3| + k$

8. $\ln |x-2| - \frac{4}{x-2} - \frac{5}{2(x-2)^2} + k$

9. $-3 \ln |x| + 4 \ln |x-1| + k$

10. $x - \ln |x| + 3 \ln |x-1| + k$

11. $\frac{x^2}{2} + 2x + 4 \ln |x-1| - \frac{3}{x-1} + k$

$$12. \quad \frac{x^2}{2} + 4x - \frac{3}{2} \ln |x - 1| + \frac{31}{2} \ln |x - 3| + k$$

$$13. \quad \frac{1}{\sqrt{5}} \operatorname{arc} \operatorname{tg} \frac{x}{\sqrt{5}} + k$$

$$14. \quad \frac{1}{2} \ln (x^2 + 9) + \frac{1}{3} \operatorname{arc} \operatorname{tg} \frac{x}{3} + k$$

$$15. \quad x + 2 \ln |x - 3| - 2 \ln |x + 3| + k$$

$$16. \quad -\frac{1}{3} \ln |x + 1| + \frac{1}{3} \ln |x - 2| + k$$

12.6

$$1. \quad a) -\frac{2}{x-1} + \frac{1}{2(x-1)^2} + k$$

$$b) -\frac{1}{6} \ln |x| + \frac{3}{10} \ln |x - 2| - \frac{2}{15} \ln |x + 3| + k$$

$$c) \frac{x^2}{2} - \ln |x| + \frac{3}{2} \ln |x - 1| + \frac{1}{2} \ln |x + 1| + k$$

$$d) \frac{2}{9} \ln |x + 2| - \frac{2}{9} \ln |x - 1| - \frac{2}{3(x-1)} + k$$

$$e) -2 \ln |x - 1| + \frac{1}{3} \ln |x + 1| + \frac{5}{3} \ln |x - 2| + k$$

$$f) \frac{5}{4} \ln |x| - \frac{5}{4} \ln |x - 2| - \frac{7}{2(x-2)} + k$$

$$g) \ln |x - 2| - \frac{4}{x-2} - \frac{5}{2(x-2)^2} + k$$

$$h) \frac{x^3}{4} + 4x - \frac{3}{4} \ln |x| + \frac{35}{8} \ln |x - 2| - \frac{29}{8} \ln |x + 2| + k$$

i) e

j) Verifique o resultado encontrado por derivação.

$$2. \quad b) \frac{7}{27} \ln |x-1| + \frac{6}{27(x-1)} - \frac{7}{27} \ln |x+2| + \frac{15}{27(x+2)} + k$$

$$3. \quad a) -\frac{1}{2(x-1)^2} - \frac{2}{3(x-1)^3} + k$$

$$b) -\frac{1}{2x^2} + \frac{1}{2x} + \frac{1}{4} \ln |x| - \frac{1}{4} \ln |x+2| + k$$

$$c) \frac{1}{x} + 3 \ln |x| - 3 \ln |x+1| + \frac{2}{x+1} + k$$

$$d) \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| + \frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + k$$

12.7

$$1. \quad 2 \ln |x-1| + \ln (x^2 + 6x + 10) + \arctg (x+3) + k$$

$$2. \quad \frac{2}{5} \ln |x| - \frac{1}{5} \ln (x^2 + 2x + 5) + \frac{3}{10} \arctg \frac{x+1}{2} + k$$

$$3. \quad 2 \ln (x^2 + 6x + 12) - \frac{11}{\sqrt{3}} \arctg \frac{x+3}{\sqrt{3}} + k$$

$$4. \quad -\frac{7}{2} \ln |x+2| + \frac{15}{2} \ln |x+4| + k$$

$$5. \quad 2 \ln |x-1| + \frac{1}{2} \ln (x^2 + 2x + 3) + \frac{1}{\sqrt{2}} \arctg \frac{x+1}{\sqrt{2}} + k$$

$$6. \quad \ln |x-2| + \frac{1}{2} \ln (x^2 + 2x + 4) - \frac{1}{\sqrt{3}} \arctg \frac{x+1}{\sqrt{3}} + k$$

7. e 8. Verifique o resultado encontrado por derivação

12.8

1. a) $\frac{-\cos 9x}{18} - \frac{\cos 5x}{10} + k$ b) $\frac{\sin 2x}{4} - \frac{\sin 8x}{16} + k$
 c) $\frac{\sin 3x}{6} + \frac{\sin x}{2} + k$ d) $\frac{-\cos 3x}{6} - \frac{\cos x}{2} + k$
 e) $\frac{-\cos (n+m)x}{2(n+m)} - \frac{\cos (n-m)x}{2(n-m)} + k$ se $n \neq m$; $\frac{-\cos 2nx}{4n} + k$ se $n = m$
 f) $\frac{-\cos 2x}{8} + \frac{\cos 6x}{24} - \frac{\cos 4x}{16} + k$
 g) $\frac{\sin 6x}{24} + \frac{\sin 4x}{16} + \frac{\sin 2x}{8} + \frac{x}{4} + k$
2. 0 (observe que o integrando é uma função ímpar)
3. 0 se $n \neq m$; π se $n = m$

12.9

1. a) $\frac{x}{2} + \frac{\sin 10x}{20} + k$ b) $\frac{-\cos^3 x}{3} + k$
 c) $\frac{\sin^5 x}{5} + k$ d) $\frac{-\cos^3 2x}{6} + k$
 e) $\frac{-\sin x \cos^5 x}{6} + \frac{\cos^3 x \sin x}{24} + \frac{\cos x \sin x}{16} + \frac{x}{16} + k$
 f) $\frac{x}{8} - \frac{\sin 8x}{64} + (\text{Lembrete: } \sin 4x = 2 \sin 2x \cos 2x)$
 g) $\frac{x}{4} + \frac{\sin 6x}{24} - \frac{\sin 4x}{16} - \frac{\sin 10x}{80} - \frac{\sin 2x}{16} + k$
 h) $\frac{\sin x}{2} + \frac{\sin 9x}{36} + \frac{\sin 7x}{28} + k$
3. a) $\frac{3}{4} \sqrt[3]{\sin^4 x} + k$ b) $\sin x + \frac{2}{3} \sqrt{\sin^3 x} - \frac{\sin^3 x}{3} - \frac{2}{7} \sqrt{\sin^7 x} + k$
 c) $\frac{1}{4 \cos^4 x} + k$ d) $-\ln |\cos x| - \frac{\sin^2 x}{2} + k$
 e) $\frac{-1}{6 \sin^6 x} + \frac{1}{4 \sin^4 x} + k$ f) $\arctg(\sin x) + k$

12.10

1. a) $\frac{\operatorname{tg}^6 x}{6} + k$ b) $\frac{\sec^6 x}{6} - \frac{\sec^4 x}{4} + k$ c) $\frac{\sec^3 2x}{6} - \frac{\sec 2x}{2} + k$
d) $\frac{\sec^2 3x}{6} + \frac{1}{3} \ln |\cos 3x| + k$ e) $\sqrt[3]{\sec x} + k$
f) $-\ln |\cos x| + \frac{1}{\sec^2 x} - \frac{1}{4 \sec^4 x} + k$ g) $\operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3} + k$
h) $\frac{\sec^5 3x}{15} + k$ i) $\frac{\operatorname{tg}^5 x}{5} - \frac{\operatorname{tg}^3 x}{3} + \operatorname{tg} x - x + k$
j) $\frac{\sec^3 x \operatorname{tg} x}{4} + \frac{3 \sec x \operatorname{tg} x}{8} + \frac{3}{8} \ln |\sec x + \operatorname{tg} x| + k$
3. a) $\frac{-\operatorname{cosec} x \cotg x}{2} - \frac{1}{2} \ln |\operatorname{cosec} x + \cotg x| + k$
b) $\frac{-\operatorname{cosec} x \cotg x}{2} + \frac{1}{2} \ln |\operatorname{cosec} x + \cotg x| + k$
c) $\frac{-\cotg^3 x}{3} + \cotg x + x + k$

12.11

1. $\frac{1}{4} \ln \left(\frac{2 + \sin x}{2 - \sin x} \right) + k$
2. $\frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tg} \frac{x}{2} - 1 - \sqrt{2}} \right| + k$
3. $2 [\ln (1 + \cos x) - \cos x] + k$
4. $\ln |2 \sec x + 3| + k$
5. $\frac{1}{2} \ln \left| \sec \left(x + \frac{\pi}{6} \right) + \operatorname{tg} \left(x + \frac{\pi}{6} \right) \right| + k$
6. $\frac{2}{\sqrt{3}} \operatorname{arc} \operatorname{tg} \frac{2 \operatorname{tg} \frac{x}{2} + 1}{\sqrt{3}} + k$

CAPÍTULO 13

13.1