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Lista de Cálculo 1: Integral Definida

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Lista de exercícios

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Sumário

1 1^o Teorema Fundamental do Cálculo

$$\int_0^{\frac{\pi}{8}} \text{sen } 2x\, dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{8}} = -\frac{1}{2}\cos\frac{\pi}{4} + \frac{1}{2}$$

ou seja,

$$\int_0^{\frac{\pi}{8}} \text{sen } 2x\, dx = \frac{2-\sqrt{2}}{4}. \qquad \blacksquare$$

EXEMPLO 7. Calcule $\int_0^1 e^{-x}\, dx$.

Solução

$$\int_0^1 e^{-x}\, dx = [-e^{-x}]_0^1 = 1 - \frac{1}{e}. \qquad \blacksquare$$

Exercícios 11.5

Calcule.

1. $\int_0^1 (x+3)\, dx$
2. $\int_{-1}^1 (2x+1)\, dx$
3. $\int_0^4 \frac{1}{2}\, dx$
4. $\int_{-2}^1 (x^2-1)\, dx$
5. $\int_1^3 dx$
6. $\int_{-1}^2 4\, dx$
7. $\int_1^3 \frac{1}{x^3}$
8. $\int_{-1}^1 5\, dx$
9. $\int_0^2 (x^2+3x-3)\, dx$
10. $\int_0^1 \left(5x^3 - \frac{1}{2}\right) dx$
11. $\int_1^1 (2x+3)\, dx$
12. $\int_1^0 (2x+3)\, dx$

13. $\int_{-2}^{-1} \left(\frac{1}{x^2} + x\right) dx$

14. $\int_0^4 \sqrt{x}$

15. $\int_1^4 \frac{1}{\sqrt{x}}\, dx$

16. $\int_0^8 \sqrt[3]{x}\, dx$

17. $\int_{-1}^0 (x^3 - 2x + 3)\, dx$

18. $\int_0^1 \sqrt[8]{x}\, dx$

19. $\int_1^2 \left(x^3 + x + \frac{1}{x^3}\right) dx$

20. $\int_0^1 (x + \sqrt[4]{x})\, dx$

21. $\int_1^3 \left(5 + \frac{1}{x^2}\right) dx$

22. $\int_{-3}^3 x^3\, dx$

23. $\int_{-1}^1 (x^7 + x^3 + x)\, dx$

24. $\int_{\frac{1}{2}}^1 (x + 3)\, dx$

25. $\int_1^4 (5x + \sqrt{x})\, dx$

26. $\int_1^0 (x^7 - x + 3)\, dx$

27. $\int_1^2 \frac{1 + x}{x^3}\, dx$

28. $\int_0^1 (x + 1)^2\, dx$

29. $\int_1^4 \frac{1 + x}{\sqrt{x}}\, dx$

30. $\int_0^1 (x - 3)^2\, dx$

31. $\int_0^2 (t^2 + 3t - 1)\, dt$

32. $\int_1^2 \frac{1 + t^2}{t^4}\, dt$

33. $\int_{\frac{1}{2}}^1 (s + 2)\, ds$

34. $\int_0^3 (u^2 - 2u + 3)\, du$

35. $\int_1^2 (s^2 + 3s + 1)\, ds$

36. $\int_{-1}^1 \sqrt[3]{t}\, dt$

37. $\int_1^3 \left(1 + \frac{1}{x}\right) dx$

38. $\int_1^2 \frac{1 + 3x^2}{x}\, dx$

39. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \cos 2x\, dx$

40. $\int_{-\pi}^0 \operatorname{sen} 3x\, dx$

41. $\int_{-1}^1 e^{2x}\, dx$

42. $\int_0^1 \frac{1}{1 + t^2}\, dt$

43. $\int_0^{\frac{\pi}{4}} \operatorname{sen} x\, dx$

44. $\int_{-1}^0 e^{-2x}\, dx$

45. $\int_0^{\frac{\pi}{3}} (3 + \cos 3x)\, dx$

46. $\int_0^1 \operatorname{sen} 5x\, dx$

47. $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1 - x^2}}\, dx$

48. $\int_0^2 2^x\, dx$

49. $\int_0^1 2x e^{x^2}\, dx$

50. $\int_0^1 \frac{2x}{1 + x^2}\, dx$

51. $\int_0^1 \frac{1}{1 + x}\, dx$

52. $\int_{-1}^1 x^3 e^{x^4}\, dx$

53. $\int_0^{\frac{\pi}{3}} (\operatorname{sen} x + \operatorname{sen} 2x)\, dx$

54. $\int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x\right) dx$

55. $\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$ $\left(\textit{Sugestão}\text{: Verifique que } \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x.\right)$

56. $\int_0^{\frac{\pi}{2}} \operatorname{sen}^2 x\, dx$

57. $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$

58. $\int_0^1 3^x\, dx$

59. $\int_0^1 3^x e^x\, dx$

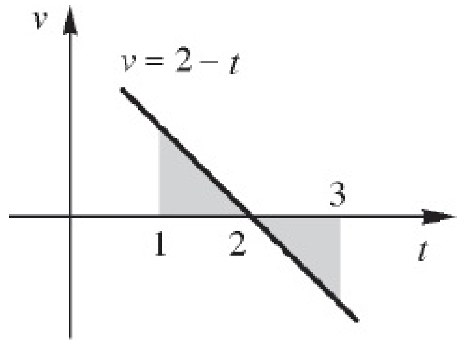
60. $\int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x\, dx$

11.6. CÁLCULO DE ÁREAS

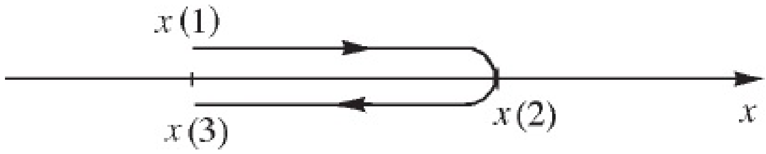
Seja f contínua em $[a, b]$, com $f(x) \geq 0$ em $[a, b]$. Estamos interessados em definir a *área* do conjunto A do plano limitado pelas retas $x = a$, $x = b$, $y = 0$ e pelo gráfico de $y = f(x)$.

2 Cálculo de áreas

a) $x(3) - x(1) = \int_1^3 (2-t)\,dt = \left[2t - \frac{t^2}{2}\right]_1^3 = 0.$



Em [1, 2 [, $v(t) > 0$, o que significa que no intervalo de tempo [1, 2] a partícula avança no sentido positivo; em]2, 3], $v(t) < 0$, o que significa que neste intervalo de tempo a partícula recua, de tal modo que no instante $t = 3$ ela volta a ocupar a mesma posição por ela ocupada no instante $t = 1$.



b) O espaço percorrido entre os instantes $t = 1$ e $t = 3$ é

$$\int_1^3 |2-t|\,dt = \int_1^2 (2-t)\,dt - \int_2^3 (2-t)\,dt = 1.$$

Observe que o espaço percorrido entre os instantes 1 e 2 é

$$\int_1^2 (2-t)\,dt = \frac{1}{2}$$

e que o espaço percorrido entre os instantes 2 e 3 é

$$\int_2^3 |2-t|\,dt = -\int_2^3 (2-t)\,dt = \frac{1}{2}.$$ ■

Exercícios 11.6

Nos Exercícios de 1 a 22, desenhe o conjunto A dado e calcule a área.

1. A é o conjunto do plano limitado pelas retas $x = 1$, $x = 3$, pelo eixo $0x$ e pelo gráfico de $y = x^3$.

2. A é o conjunto do plano limitado pelas retas $x = 1$, $x = 4$, $y = 0$ e pelo gráfico de $y = \sqrt{x}$.

3. A é o conjunto de todos (x, y) tais que $x^2 - 1 \leq y \leq 0$.

4. A é o conjunto de todos (x, y) tais que $0 \leq y \leq 4 - x^2$.

5. A é o conjunto de todos (x, y) tais que $0 \leq y \leq |\operatorname{sen} x|$, com $0 \leq x \leq 2\pi$.

6. A é a região do plano compreendida entre o eixo $0x$ e o gráfico de $y = x^2 - x$, com $0 \leq x \leq 2$.

7. A é o conjunto do plano limitado pela reta $y = 0$ e pelo gráfico de $y = 3 - 2x - x^2$, com $-1 \leq x \leq 2$.

8. A é o conjunto do plano limitado pelas retas $x = -1$, $x = 2$, $y = 0$ e pelo gráfico de $y = x^2 + 2x + 5$.

9. A é o conjunto do plano limitado pelo eixo $0x$, pelo gráfico de $y = x^3 - x$, $-1 \leq x \leq 1$.

10. A é o conjunto do plano limitado pela reta $y = 0$ e pelo gráfico de $y = x^3 - x$, com $0 \leq x \leq 2$.

11. A é o conjunto do plano limitado pelas retas $x = 0$, $x = \pi$, $y = 0$ e pelo gráfico de $y = \cos x$.

12. A é o conjunto de todos (x, y) tais que $x \geq 0$ e $x^3 \leq y \leq x$.

13. A é o conjunto do plano limitado pela reta $y = x$, pelo gráfico de $y = x^3$, com $-1 \leq x \leq 1$.

14. $A = \{(x, y) \in \mathbb{R}^2 \mid 0 \leqslant x \leqslant 1 \text{ e } \sqrt{x} \leqslant y \leqslant 3\}$.

15. A é o conjunto do plano limitado pelas retas $x = 0$, $x = \dfrac{\pi}{2}$ e pelos gráficos de $y = \operatorname{sen} x$ e $y = \cos x$.

16. A é o conjunto de todos os pontos (x, y) tais que $x^2 + 1 \leq y \leq x + 1$.

17. A é o conjunto de todos os pontos (x, y) tais que $x^2 - 1 \leq y \leq x + 1$.

18. A é o conjunto do plano limitado pelas retas $x = 0$, $x = \frac{\pi}{2}$ e pelos gráficos de $y = \cos x$ e $y = 1 - \cos x$.

19. $A = \{ (x, y) \in \mathbb{R}^2 \mid x \geq 0 \text{ e } x^3 - x \leq y \leq -x^2 + 5x \}$.

20. A é o conjunto do plano limitado pelos gráficos de $y = x^3 - x$, $y = \text{sen } \pi x$, com $-1 \leq x \leq 1$.

21. A é o conjunto de todos os pontos (x, y) tais que $x \geq 0$ e $-x \leq y \leq x \leq x^2$

22. A é o conjunto de todos (x, y) tais que $x > 0$ e $\frac{1}{x^2} \leqslant y \leqslant 5 - 4x^2$.

23. Uma partícula desloca-se sobre o eixo x com velocidade $v(t) = 2t - 3$, $t \geq 0$.

    a) Calcule o deslocamento entre os instantes $t = 1$ e $t = 3$.
    b) Qual o espaço percorrido entre os instantes $t = 1$ e $t = 3$?
    c) Descreva o movimento realizado pela partícula entre os instantes $t = 1$ e $t = 3$

24. Uma partícula desloca-se sobre o eixo $0x$ com velocidade $v(t) = \text{sen } 2t$, $t \geq 0$. Calcule o espaço percorrido entre os instantes $t = 0$ e $t = \pi$.
25. Uma partícula desloca-se sobre o eixo $0x$ com velocidade $v(t) = -t^2 + t$, $t \geq 0$. Calcule o espaço percorrido entre os instantes $t = 0$ e $t = 2$.
26. Uma partícula desloca-se sobre o eixo $0x$ com velocidade $v(t) = t^2 - 2t - 3$, $t \geq 0$. Calcule o espaço percorrido entre os instantes $t = 0$ e $t = 4$.

11.7. MUDANÇA DE VARIÁVEL NA INTEGRAL

Veremos, no Vol. 2, que toda *função contínua* num intervalo I admite, neste intervalo, uma primitiva. Por ora, vamos admitir tal resultado e usá-lo na demonstração do próximo teorema.

Teorema. Seja f contínua num intervalo I e sejam a e b dois reais quaisquer em I. Seja $g : [c, d] \to I$, com g' contínua em $[c, d]$, tal que $g(c) = a$ e $g(d) = b$. Nestas condições

$$\int_a^b f(x)\, dx = \int_c^d f(g(u))\, g'(u)\, du.$$

3 Mudança de variável na integral

$$\int_{-1}^{0} x^2 \sqrt{x+1}\, dx = \left[\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - 2\, \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 = \frac{16}{105}. \qquad \blacksquare$$

Exercícios 11.7

1. Calcule.

a) $\int_1^2 (x-2)^5\, dx$

b) $\int_0^1 (3x+1)^4\, dx$

c) $\int_0^1 \sqrt{3x+1}\, dx$

d) $\int_{-1}^0 (2x+5)^3\, dx$

e) $\int_{-3}^4 \sqrt[3]{5-x}\, dx$

f) $\int_1^2 \frac{2}{(3x-2)^3}\, dx$

g) $\int_0^1 \frac{1}{(x+1)^5}\, dx$

h) $\int_{-2}^1 \frac{3}{4+x}\, dx$

i) $\int_0^2 e^{2x}\, dx$

j) $\int_0^1 xe^{x^2}\, dx$

l) $\int_{-1}^0 x\sqrt{x+1}\, dx$

m) $\int_0^{\frac{\pi}{3}} \cos 2x\, dx$

n) $\int_0^1 \frac{x^2}{1+x^3}\, dx$

o) $\int_0^1 \frac{x^2}{(1+x^3)^2}\, dx$

p) $\int_{-1}^0 x^2\sqrt{1+x^3}\, dx$

q) $\int_1^3 \frac{2}{5+3x}\, dx$

r) $\int_{-1}^1 \sqrt[3]{x+1}\, dx$

s) $\int_0^1 \frac{x}{(x+1)^5}\, dx$

t) $\int_{-1}^0 x(x+1)^{100}\, dx$

u) $\int_1^2 x^2 (x-2)^{10}\, dx$

2. Suponha f contínua em $[-2, 0]$. Calcule $\int_0^2 f(x-2)\,dx$, sabendo que $\int_{-2}^0 f(x)\,dx = 3$.

3. Suponha f contínua em $[-1, 1]$. Calcule $\int_0^1 f(2x-1)\,dx$ sabendo que $\int_{-1}^1 f(u)\,du = 5$.

4. Suponha f contínua em $[0, 4]$. Calcule $\int_{-2}^2 x\,f(x^2)\,dx$.

5. Calcule $\int_{-\pi}^{\pi} \frac{\operatorname{sen} x}{x^4 + x^2 + 1}\,dx$.

6. Calcule a área do conjunto dado.

$a)$ $A = \left\{(x, y) \in \mathbb{R}^2 \mid 1 \leqslant x \leqslant 2 \text{ e } 0 \leqslant y \leqslant \sqrt{x-1}\right\}$

$b)$ $A = \left\{(x, y) \in \mathbb{R}^2 \mid 0 \leqslant x \leqslant 2 \text{ e } 0 \leqslant y \leqslant \frac{x}{1+x^2}\right\}$

$c)$ A é o conjunto do plano limitado pela reta $x = 1$ e pelos gráficos de $y = e^{-2x}$ e $y = e^{-x}$, com $x \geq 0$

7. Calcule.

$a)$ $\int_0^1 x\sqrt{x^2+3}\,dx$

$b)$ $\int_0^1 x\,(x^2+3)^5\,dx$

$c)$ $\int_1^2 x\,(x^2-1)^5\,dx$

$d)$ $\int_0^1 x\sqrt{1-x^2}\,dx$

$e)$ $\int_{-1}^0 x^2 e^{x^3}\,dx$

$f)$ $\int_0^1 x\sqrt{1+2x^2}\,dx$

$g)$ $\int_1^2 \frac{3s}{1+s^2}\,ds$

$h)$ $\int_0^1 \frac{1}{1+4s}\,ds$

i) $\displaystyle\int_0^3 \frac{x}{\sqrt{x+1}}\,dx$

j) $\displaystyle\int_0^1 \frac{s}{\sqrt{s^2+1}}\,ds$

l) $\displaystyle\int_0^3 \frac{x^2}{\sqrt{x+1}}\,dx$

m) $\displaystyle\int_0^1 \frac{x^2}{(x+1)^2}\,dx$

n) $\displaystyle\int_{-1}^1 x^3\,(x^2+3)^{10}\,dx$

o) $\displaystyle\int_0^{\sqrt{3}} x^3\,\sqrt{x^2+1}\,dx$

p) $\displaystyle\int_0^{\frac{\pi}{3}} \operatorname{sen} x \cos^2 x\,dx$

q) $\displaystyle\int_0^{\frac{\pi}{6}} \cos x \operatorname{sen}^5 x\,dx$

r) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{sen} x\,(1-\cos^2 x)\,dx$

s) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{sen} x \operatorname{sen}^2 x\,dx$

t) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{sen}^3 x\,dx$

u) $\displaystyle\int_0^{\frac{\pi}{6}} \cos^3 x\,dx$

8. Um aluno (precipitado), ao calcular a integral $\int_{-1}^1 \sqrt{1+x^2}\,dx$, raciocinou da seguinte forma: fazendo a mudança de variável $u = 1 + x^2$, os novos extremos de integração seriam iguais a 2 ($x = -1 \to u = 2$; $x = 1 \to u = 2$) e assim a integral obtida após a mudança de variável seria igual a zero e, portanto, $\int_{-1}^1 \sqrt{1+x^2}\,dx = 0$!! Onde está o erro?

9. Seja f uma função par e contínua em $[-r, r]$, $r > 0$. (Lembre-se: f par $\Leftrightarrow f(-x) = f(x)$.)

   a) Mostre que $\int_{-r}^0 f(x)\,dx = \int_0^r f(x)\,dx$

   b) Conclua de (*a*) que $\int_{-r}^r f(x)\,dx = 2\int_0^r f(x)\,dx$. Interprete graficamente

10. Suponha f contínua em $[a, b]$. Seja $g\colon [c, d] \to \mathbb{R}$ com g' contínua em $[c, d]$, $g(c) = a$ e $g(d) = b$. Suponha, ainda, que $g'(u) > 0$ em $]c, d[$. Seja $c = u_0 < u_1 < u_2 < \ldots < u_n = d$ uma partição de $[c, d]$ e seja $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ partição de $[a, b]$, em que $x_i = g(u_i)$, para i variando de 0 a n.

a) Mostre que, para todo i, $i = 1, 2, \ldots, n$, existe $\overline{u_i}$ em $[u_{i-1}, u_i]$ tal que

$$\Delta x_i = g'(\overline{u_i})\,\Delta u_i$$

b) Conclua de (*a*) que

$$\sum_{i=1}^{n} f(g(\overline{u_i}))\,g'(\overline{u_i})\,\Delta u_i = \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

em que $c_i = g(\overline{u_i})$.

c) Mostre que existe $M > 0$ tal que

$$\Delta x_i \leq M\,\Delta u_i$$

para i variando de 0 a n

d) Conclua que

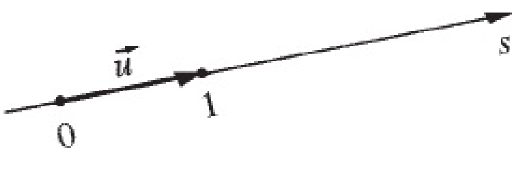
$$\lim_{\text{máx } \Delta u_i \to 0} \sum_{i=1}^{n} f(g(\overline{u_i}))\,g'(\overline{u_i})\,\Delta u_i = \lim_{\text{máx } \Delta x_i \to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

ou seja,

$$\int_c^d f(g(u))\,g'(u)\,du = \int_a^b f(x)\,dx$$

11.8. TRABALHO

Nesta seção, admitiremos que o leitor já saiba o que é um *vetor*. Consideremos, então, um eixo 0*s*



e indiquemos por $\vec{u}$ o vetor, de comprimento *unitário*, determinado pelo segmento orientado de *origem* 0 e *extremidade* 1.

Seja α um número real; $\vec{F} = \alpha\,\vec{u}$ é um vetor *paralelo* a $\vec{u}$. O número *a* é a

4 Primitivas imediatas

Solução

$$\cos 2x = \cos^2 x - \text{sen}^2\, x = 2\cos^2 x - 1$$

$$\boxed{\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x.}$$

Então:

$$\int \cos^2 x\, dx = \int \left[\frac{1}{2} + \frac{1}{2}\cos 2x\right] dx = \frac{1}{2}x + \frac{1}{4}\,\text{sen}\, 2x + k$$

ou seja,

$$\int \cos^2 x\, dx = \frac{1}{2}x + \frac{1}{4}\,\text{sen}\, 2x + k. \qquad \blacksquare$$

Exercícios 12.1

1. Calcule e verifique sua resposta por derivação.

a) $\int 3\, dx$

b) $\int x\, dx$

c) $\int x^5\, dx$

d) $\int \sqrt{x}\, dx$

e) $\int \sqrt[5]{x^2}\, dx$

f) $\int x^{-4}\, dx$

g) $\int \frac{1}{x^3}\, dx$

h) $\int \frac{x + x^2}{x^2}\, dx$

i) $\int \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$

j) $\int \left(x^2 + \frac{3}{x}\right) dx$

l) $\int \frac{x+1}{x}\, dx$

m) $\int (e^x + 4)\, dx$

n) $\int e^{5x}\, dx$

o) $\int e^{-2x}\, dx$

p) $\int (e^{2x} + e^{-x})\, dx$

q) $\int \left(\frac{1}{x} + \frac{1}{e^x}\right) dx$

r) $\int \left(e^{4x} + \frac{1}{x^2}\right) dx$

s) $\int \left(\frac{3}{x} + \frac{2}{x^3}\right) dx$

t) $\int \frac{x^5 + x + 1}{x^2}\, dx$

u) $\int e^{\sqrt{2}\,x}\, dx$

2. Calcule.

a) $\int_0^1 e^{2x}\, dx$

b) $\int_1^2 \left(x + \frac{1}{x}\right) dx$

c) $\int_{-1}^1 e^{-x}\, dx$

d) $\int_0^1 \frac{1}{1+x^2}\, dx$

e) $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\, dx$

f) $\int_1^2 \frac{x^3+1}{x}\, dx$

3. Calcule e verifique sua resposta por derivação.

a) $\int \operatorname{sen} x\, dx$

b) $\int \operatorname{sen} 2x\, dx$

c) $\int \cos 5x\, dx$

d) $\int \operatorname{sen} 4t\, dt$

e) $\int \cos 7t\, dt$

f) $\int \cos \sqrt{3}\, t\, dt$

g) $\int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

h) $\int \left(2 + \frac{1}{3}\operatorname{sen} 2x\right) dx$

i) $\int \left(x + \frac{1}{5}\cos 3x\right) dx$

j) $\int \left(\frac{1}{x} + 4\operatorname{sen} 3x\right) dx$

l) $\int \left(\frac{1}{3} + \frac{5}{2}\cos 7x\right) dx$

m) $\int \left(\cos 3x + \frac{1}{2}\operatorname{sen} 4x\right) dx$

n) $\int \left(\frac{1}{3}\operatorname{sen} 2x + \frac{1}{2}\cos 3x\right) dx$

o) $\int \frac{\operatorname{sen} 2x}{\cos x}\, dx$

p) $\int \left(\frac{1}{3}\cos 3x - \frac{1}{7}\operatorname{sen} 7x\right) dx$

q) $\int \left(\frac{1}{3}e^{3x} + \operatorname{sen} 3x\right) dx$

4. Calcule.

a) $\int_0^{\frac{\pi}{3}} \operatorname{sen} 2x \, dx$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \frac{x}{2} \, dx$

c) $\int_0^{\frac{\pi}{3}} (\operatorname{sen} 3x + \cos 3x) \, dx$

d) $\int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x\right) dx$

5. *a*) Verifique que $\operatorname{sen}^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$.

b) Calcule $\int \operatorname{sen}^2 x \, dx$.

6. Calcule.

a) $\int \cos^2 2x \, dx$

b) $\int \cos^2 5x \, dx$

c) $\int \operatorname{sen}^2 3x \, dx$

d) $\int \cos^2 \frac{x}{2} \, dx$

e) $\int \cos^4 x \, dx$

f) $\int \left(\frac{1}{2} + \frac{1}{2} \cos 2x\right)^2 dx$

g) $\int (\operatorname{sen} x + \cos x)^2 \, dx$

h) $\int (\operatorname{sen} x - \cos x)^2 \, dx$

i) $\int (5 + \operatorname{sen} 3x)^2 \, dx$

j) $\int (1 - \cos 2x)^2 \, dx$

7. Calcule.

a) $\int_0^{\frac{\pi}{8}} \cos^2 x \, dx$

b) $\int_0^{\frac{\pi}{4}} \operatorname{sen}^2 x \, dx$

c) $\int_0^{\frac{\pi}{2}} (\operatorname{sen} x + \cos x)^2 \, dx$

d) $\int_0^{\frac{\pi}{2}} \cos^4 x \, dx$

8. Calcule $\int_0^{2\pi} \sqrt{1 + \cos x} \, dx$.

9. *a*) Verifique que

$$\int \sec x \, dx = \ln(\sec x + \operatorname{tg} x) + k$$

com $x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$.

b) Mostre que

$$\int \sec x \, dx = \ln |\sec x + \operatorname{tg} x| + k.$$

10. Calcule.

a) $\int \operatorname{tg} x \, dx$

b) $\int \sec^2 x \, dx$

c) $\int \operatorname{tg}^2 x \, dx$

d) $\int \sec x \, dx$

e) $\int \operatorname{tg} 2x \, dx$

f) $\int \sec 3x \, dx$

g) $\int 3^x \, dx$

h) $\int \frac{5}{\sqrt{1 - x^2}} \, dx$

i) $\int (5^x + e^{-x}) \, dx$

j) $\int (x + \sec^2 3x) \, dx$

l) $\int (1 + \sec x)^2 \, dx$

m) $\int \frac{\cos x + \sec x}{\cos x} \, dx$

11. *a*) Determine α e β de modo que

$$\operatorname{sen} 6x \cos x = \frac{1}{2} (\operatorname{sen} \alpha x + \operatorname{sen} \beta x)$$

$$\left(\textit{Sugestão}: \operatorname{sen} a \cos b = \frac{1}{2} [\operatorname{sen} (a + b) + \operatorname{sen} (a - b)]. \right)$$

b) Calcule $\int \operatorname{sen} 6x \cos x \, dx.$

12. Calcule.

a) $\int \operatorname{sen} 5x \cos x \, dx$

b) $\int \operatorname{sen} 3x \cos 4x \, dx$

c) $\int \operatorname{sen} x \cos 3x \, dx$

d) $\int \operatorname{sen} 3x \cos 3x \, dx$

13. *a*) Determine α e β de modo que

$$\operatorname{sen} 3x \operatorname{sen} 2x = -\frac{1}{2} (\cos \alpha x - \cos \beta x)$$

$$\left(\textit{Sugestão}: \operatorname{sen} a \operatorname{sen} b = \frac{1}{2} [\cos (a - b) - \cos (a + b)]. \right)$$

b) Calcule $\int \operatorname{sen} 3x \operatorname{sen} 2x \, dx.$

14. Calcule $\int \cos 5x \cos 2x \, dx.$

$$\left(\textit{Sugestão}: \cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]. \right)$$

15. Calcule.

a) $\int \operatorname{sen} x \operatorname{sen} 3x \, dx$

b) $\int \operatorname{sen} 2x \operatorname{sen} 5x \, dx$

c) $\int \operatorname{sen} 3x \cos 2x \, dx$

d) $\int \cos 5x \cos x \, dx$

e) $\int \cos 7x \cos 3x \, dx$

16. Calcule.

a) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \operatorname{sen} 3x \, dx$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \operatorname{sen} 3x \operatorname{sen} 4x \, dx$

17. Sejam m e n naturais. Calcule.

a) $\int_{-\pi}^{\pi} \operatorname{sen} mx \operatorname{sen} nx \, dx$

b) $\int_{-\pi}^{\pi} \cos mx \operatorname{sen} nx \, dx$

12.2. TÉCNICA PARA CÁLCULO DE INTEGRAL INDEFINIDA DA FORMA $\int f(g(x))\, g'(x)\, dx$

Sejam f e g tais que Im $g \subset D_f$ com g derivável. Suponhamos que F seja uma primitiva de f, isto é, $F' = f$. Segue que $F(g(x))$ é uma primitiva de $f(g(x))\, g'(x)$, de fato,

$$(F(g(x)))' = F'(g(x))\, g'(x) = f(g(x))\, g'(x).$$

Deste modo, de

$$\int f(u)\, du = F(u) + k$$

segue

$$\int f(g(x))\, g'(x)\, dx = F(g(x)) + k.$$

5 Técnica para cálculo de integral indefinida da forma $\int f\left(g(x)\right) g'($

$$\int \frac{2}{3+2x^2}\,dx = \frac{\sqrt{6}}{3}\operatorname{arc\,tg}\frac{\sqrt{6}}{3}x + k. \qquad \blacksquare$$

EXEMPLO 14. Verifique que

$$\int \sec x\,dx = \ln|\sec x + \operatorname{tg} x| + k$$

Solução

$$\sec x = \frac{\sec x \operatorname{tg} x + \sec^2 x}{\sec x + \operatorname{tg} x}$$

$$u = \sec x + \operatorname{tg} x;\ du = (\sec x \operatorname{tg} x + \sec^2 x)\,dx.$$

Assim,

$$\int \sec x\,dx = \int \frac{\sec x \operatorname{tg} x + \sec^2 x}{\sec x + \operatorname{tg} x}\,dx = \int \frac{1}{u}\,du = \ln|u| + k$$

ou seja,

$$\int \sec x\,dx = \ln|\sec x + \operatorname{tg} x| + k. \qquad \blacksquare$$

Exercícios 12.2

1. Calcule.

a) $\int (3x-2)^3\,dx$

b) $\int \sqrt{3x-2}\,dx$

c) $\int \frac{1}{3x-2}\,dx$

d) $\int \frac{1}{(3x-2)^2}\,dx$

e) $\int x \operatorname{sen} x^2\,dx$

f) $\int x\,e^{x^2}\,dx$

g) $\int x^2\,e^{x^3}\,dx$

h) $\int \operatorname{sen} 5x\,dx$

i) $\int x^3 \cos x^4\,dx$

j) $\int \cos 6x\,dx$

l) $\int \cos^3 x \operatorname{sen} x\,dx$

m) $\int \operatorname{sen}^5 x \cos x\,dx$

n) $\int \frac{2}{x+3}\,dx$

o) $\int \frac{5}{4x+3}\,dx$

p) $\int \frac{x}{1+4x^2}\,dx$

q) $\int \frac{3x}{5+6x^2}\,dx$

r) $\int \frac{x}{(1+4x^2)^2}\,dx$

s) $\int x\sqrt{1+3x^2}\,dx$

t) $\int e^x\sqrt{1+e^x}\,dx$

u) $\int \frac{1}{(x-1)^3}\,dx$

v) $\int \frac{\operatorname{sen} x}{\cos^2 x}\,dx$

x) $\int x\,e^{-x^2}\,dx$

2. Calcule (veja a Seção 11.7).

a) $\int_0^1 x\,e^{-x^2}\,dx$

b) $\int_0^{\frac{\pi}{3}} \operatorname{sen}^4 x \cos x\,dx$

c) $\int_0^1 \frac{3}{2x+1}\,dx$

d) $\int_1^2 \frac{x}{1+3x^2}\,dx$

e) $\int_0^1 \frac{x}{\sqrt{1+x^2}}\,dx$

f) $\int_0^1 \frac{x^3}{\sqrt{1+x^2}}\,dx$

g) $\int_{-\frac{3}{2}}^{-1} (2x+3)^{100}\,dx$

h) $\int_0^{\sqrt{\pi}} x \operatorname{sen} 3x^2\,dx$

i) $\int_2^3 \frac{1}{(x-1)^3}\,dx$

j) $\int_0^{\frac{\pi}{3}} \frac{\operatorname{sen} x}{\cos^2 x}\,dx$

l) $\int_0^1 \frac{x}{1+x^4}\,dx$

m) $\int_0^{\frac{\pi}{4}} \cos^2 2x\,dx$

3. Calcule.

a) $\int \operatorname{sen}^2 x \cos x \, dx$

b) $\int \operatorname{sen}^2 x \cos^3 x \, dx$

c) $\int \cos^3 x \operatorname{sen}^3 x \, dx$

d) $\int \operatorname{sen} x \sqrt{\cos x} \, dx$

e) $\int \operatorname{sen} 2x \sqrt{1 + \cos^2 x} \, dx$

f) $\int \operatorname{sen} 2x \sqrt{5 + \operatorname{sen}^2 x} \, dx$

g) $\int \operatorname{sen}^3 x \, dx$

h) $\int \cos^5 x \, dx$

i) $\int \operatorname{tg}^3 x \sec^2 x \, dx$

j) $\int \operatorname{tg} x \sec^2 x \, dx$

l) $\int \operatorname{tg} x \sec^3 x \, dx$

m) $\int \operatorname{tg}^3 x \sec^4 x \, dx$

n) $\int \operatorname{sen} x \sqrt{3 + \cos x} \, dx$

o) $\int \operatorname{sen} x \sec^2 x \, dx$

p) $\int \operatorname{sen} x \sec^3 x \, dx$

q) $\int \operatorname{sen}^2 x \cos^2 x \, dx$

r) $\int \operatorname{tg}^3 x \cos x \, dx$

s) $\int \dfrac{\sec^2 x}{3 + 2 \operatorname{tg} x} \, dx$

4. Calcule.

a) $\int \dfrac{2}{x - 3} \, dx$

b) $\int \left(\dfrac{5}{x - 1} + \dfrac{2}{x} \right) dx$

c) $\int \dfrac{1}{2x + 3} \, dx$

d) $\int \left(x + \dfrac{3}{x - 2} \right) dx$

e) $\int \dfrac{x}{x + 1} \, dx$

f) $\int \dfrac{x + 2}{x - 1} \, dx$

g) $\int \dfrac{2x + 3}{x + 1} \, dx$

h) $\int \dfrac{x^2}{x + 1} \, dx$

5. Suponha α, β, m e n constantes, com $a \neq \beta$. Mostre que existem constantes A e B tais que

$$\frac{mx + n}{(x - \alpha)(x - \beta)} = \frac{A}{x - \alpha} + \frac{B}{x - \beta}$$

6. Utilizando o Exercício 5, calcule.

a) $\displaystyle\int \frac{1}{(x+1)(x-1)}\,dx$

b) $\displaystyle\int \frac{2x+3}{x(x-2)}\,dx$

c) $\displaystyle\int \frac{x}{x^2-4}\,dx$

d) $\displaystyle\int \frac{1}{x^2-4}\,dx$

e) $\displaystyle\int \frac{5x+3}{x^2-3x+2}\,dx$

f) $\displaystyle\int \frac{x+1}{x^2-x-2}\,dx$

g) $\displaystyle\int \frac{2}{x^2-5x+6}\,dx$

h) $\displaystyle\int \frac{x-3}{x^2+3x+2}\,dx$

7. Seja $a \neq 0$ uma constante. Verifique que

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\operatorname{arc\,tg}\frac{x}{a} + k.$$

8. Calcule.

a) $\displaystyle\int \frac{1}{5+x^2}\,dx$

b) $\displaystyle\int \frac{2}{4+x^2}\,dx$

c) $\displaystyle\int \frac{1}{2+5x^2}\,dx$

d) $\displaystyle\int \frac{3}{5+x^2}\,dx$

e) $\displaystyle\int \frac{x}{5+x^2}\,dx$

f) $\displaystyle\int \frac{3x+2}{1+x^2}\,dx$

g) $\displaystyle\int \frac{x-1}{4+x^2}\,dx$

h) $\displaystyle\int \frac{2x-3}{1+4x^2}\,dx$

i) $\displaystyle\int \frac{1}{1+(x+1)^2}\,dx$

j) $\displaystyle\int \frac{1}{x^2+2x+2}\,dx$

l) $\displaystyle\int \frac{2}{5+(x+2)^2}\,dx$

m) $\displaystyle\int \frac{1}{x^2+4x+8}\,dx$

n) $\displaystyle\int \frac{1}{x^2+x+1}\,dx$

o) $\displaystyle\int \frac{2}{x^2+2x+2}\,dx$

9. Sejam $\alpha \neq 0$ e β constantes. Verifique que

a) $\displaystyle\int \frac{1}{x^2-\alpha^2}\,dx = \frac{1}{2\alpha}\ln\left|\frac{x-\alpha}{x+\alpha}\right| + k.$

b) $\displaystyle\int \frac{1}{\alpha^2+(x+\beta)^2}\,dx = \frac{1}{\alpha}\operatorname{arc\,tg}\frac{x+\beta}{\alpha} + k.$

10. Calcule.

$a) \int \frac{x^3}{(16 + x^4)^3}\,dx$ $\quad$ $b) \int \frac{x^3}{16 + x^4}\,dx$

$c) \int \frac{x}{16 + x^4}\,dx$ $\quad$ $d) \int \operatorname{tg} 2x\,dx$

$e) \int \frac{1}{x \ln x}\,dx$ $\quad$ $f) \int \frac{1}{x(\ln x)^2}\,dx$

$g) \int \operatorname{tg}^2 x\,dx$ $\quad$ $h) \int \frac{1}{\sqrt{1 - x^2}}\,dx$

$i) \int \frac{5}{\sqrt{1 - 4x^2}}\,dx$ $\quad$ $j) \int \frac{x}{\sqrt{1 - 4x^2}}\,dx$

$l) \int \frac{1}{\sqrt{4 - x^2}}\,dx$ $\quad$ $m) \int \frac{2x + 3}{\sqrt{1 - 4x^2}}\,dx$

$n) \int \frac{2}{\sqrt{4 - 9x^2}}\,dx$ $\quad$ $o) \int \frac{x}{\sqrt{1 - x^4}}\,dx$

$p) \int \frac{e^x}{\sqrt{1 - e^{2x}}}\,dx$ $\quad$ $q) \int \frac{e^x}{\sqrt{1 - e^x}}\,dx$

$r) \int \frac{1}{x\sqrt{1 - (\ln x)^2}}\,dx$ $\quad$ $s) \int \frac{2}{\sqrt{1 - (x + 1)^2}}\,dx$

$t) \int \frac{e^x}{1 + e^{2x}}\,dx$ $\quad$ $u) \int \frac{e^x}{1 + 3e^x}\,dx$

$v) \int \frac{1}{x}\cos(\ln x)\,dx$ $\quad$ $x) \int \frac{x^3}{1 + x^8}\,dx$

12.3. INTEGRAÇÃO POR PARTES

Suponhamos f e g definidas e deriváveis num mesmo intervalo I. Temos:

$$[f(x)\,g(x)]' = f'(x)\,g(x) + f(x)\,g'(x)$$

ou

$$f(x)\,g'(x) = [f(x)\,g(x)]' - f'(x)\,g(x).$$

Supondo, então, que $f'(x)\,g(x)$ admita primitiva em I e observando que $f(x)\,g(x)$ é uma primitiva de $[f(x)\,g(x)]'$, então $f(x)\,g'(x)$ também admitirá primitiva em I e

6 Integração por partes

$$\int_0^{\frac{1}{2}} \text{arc sen } x \, dx = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1. \qquad \blacksquare$$

Exercícios 12.3

1. Calcule.

$a)$ $\int x e^x \, dx$

$b)$ $\int x \,\text{sen}\, x \, dx$

$c)$ $\int x^2 e^x \, dx$

$d)$ $\int x \ln x \, dx$

$e)$ $\int \ln x \, dx$

$f)$ $\int x^2 \ln x \, dx$

$g)$ $\int x \sec^2 x \, dx$

$h)$ $\int x (\ln x)^2 \, dx$

$i)$ $\int (\ln x)^2 \, dx$

$j)$ $\int x e^{2x} \, dx$

$l)$ $\int e^x \cos x \, dx$

$m)$ $\int e^{-2x} \,\text{sen}\, x \, dx$

$n)$ $\int x^3 e^{x^2} \, dx$

$o)$ $\int x^3 \cos x^2 \, dx$

$p)$ $\int e^{-x} \cos 2x \, dx$

$q)$ $\int x^2 \,\text{sen}\, x \, dx$

2. $a)$ Verifique que

$$\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \,\text{tg}\, x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

em que $n > 1$ é um natural.

$b)$ Calcule $\int \sec^5 x \, dx.$

3. Verifique que, para todo natural $n \neq 0$, tem-se

$a)$ $\int \text{sen}^n x \, dx = -\frac{1}{n} \text{sen}^{n-1} x \cos x + \frac{n-1}{n} \int \text{sen}^{n-2} x \, dx$

$b)$ $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \,\text{sen}\, x + \frac{n-1}{n} \int \cos^{n-2} x \, dx.$

4. Utilizando o item (a) do Exercício 3, calcule.

$a)$ $\int \text{sen}^3 x \, dx$

$b)$ $\int \text{sen}^4 x \, dx.$

5. Calcule $\int e^{-st} \operatorname{sen} t \, dt$; $s > 0$ constante.

6. Verifique que para todo natural $n \geq 1$ e todo real $s > 0$

$$\int t^n e^{-st} \, dt = -\frac{1}{s} t^n e^{-st} + \frac{n}{s} \int t^{n-1} e^{-st} \, dt.$$

7. Calcule.

a) $\int_0^1 x e^x \, dx$

b) $\int_1^2 \ln x \, dx$

c) $\int_0^{\frac{\pi}{2}} e^x \cos x \, dx$

d) $\int_0^x t^2 e^{-st} \, dt \; (s \neq 0)$

8. Sejam m e n dois naturais diferentes de zero. Verifique que

a) $\int_0^1 x^n (1 - x)^m \, dx = \frac{m}{n+1} \int_0^1 x^{n+1} (1-x)^{m-1} \, dx$

b) $\int_0^1 x^n (1 - x)^m \, dx = \frac{n! \, m!}{(m + n + 1)!}$

9. Verifique que, para todo natural $n \geq 2$,

$$\int_0^{\frac{\pi}{2}} \operatorname{sen}^n x \, dx = \frac{n-1}{n} \int_0^{\frac{\pi}{2}} \operatorname{sen}^{n-2} x \, dx$$

10. Verifique que, para todo natural $n \geq 1$, tem-se

a) $\int_0^1 (1 - x^2)^n \, dx = \frac{2n}{2n+1} \int_0^1 (1 - x^2)^{n-1} \, dx$

b) $\int_0^1 (1 - x^2)^n \, dx = \frac{2^{2n} (n!)^2}{(2n+1)!}$

11. Suponha que g tenha derivada contínua em $[0, +\infty[$ e que $g(0) = 0$. Verifique que

$$\int_0^x g'(t) \, e^{-st} \, dt = g(x) \, e^{-sx} + s \int_0^x g(t) \, e^{-st} \, dt.$$

12. Suponha f'' contínua em $[a, b]$. Verifique que

$$f(b) = f(a) + f'(a)(b - a) + \int_a^b (b - t)\, f''(t)\, dt.$$

13. Suponha f''' contínua em $[a, b]$. Conclua do Exercício 12 que

$$f(b) = f(a) + f'(a)(b - a) + \frac{f''(a)}{2}(b - a)^2 + \int_a^b \frac{(b - t)^2}{2} f'''(t)\, dt.$$

12.4. MUDANÇA DE VARIÁVEL

Seja f definida num intervalo I. Suponhamos que $x = \varphi(u)$ seja inversível, com inversa $u = \theta(x)$, $x \in I$, sendo φ e θ deriváveis.

① $$\int f(\varphi(u))\, \varphi'(u)\, du = F(u) + k \; (u \in D_\varphi)$$

então,

$$\int f(x)\, dx = F(\theta(x)) + k.$$

De fato, de ①

$$F'(u) = f(\varphi(u))\, \varphi'(u)$$

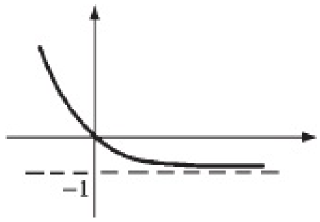
então,

$$\begin{aligned} (F(\theta(x)))' &= F'(\theta(x))\, \theta\ (x) \\ &= f(\varphi(\theta(x)))\, \varphi'(\theta(x))\, \theta'(x) \\ &= f(x) \end{aligned}$$

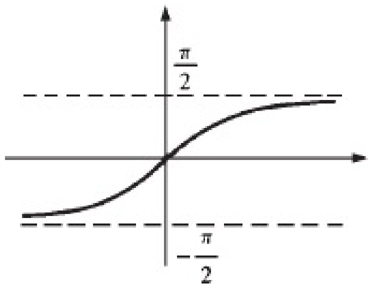
pois, $\varphi(\theta(x)) = x$ e $\varphi'(\theta(x))\, \theta'(x) = (\varphi(\theta(x)))' = 1$.

$$\int f(x)\, dx = ?$$

7 Respostas



***d*)** $y = \text{arc tg } x$



CAPÍTULO 11

11.5

1. 7/2

2. 2

3. 2

4. 0

5. 2

6. 12

7. 4/9

8. 10

9. 8/3

10. 3/4

11. 0

12. −4

13. −1

14. 16/3

15. 2

16. 12

17. 15/4

18. 8/9

19. 45/8

20. 13/10

21. 32/3

22. 0

23. 0

24. 15/8

25. 253/6

26. −21/8

27. 7/8

28. 7/3

29. 20/3

30. 19/3

31. 20/3

32. 19/24

33. 11/8

34. 9

35. 47/6

36. 0

37. $2 + \ln 3$

38. $\ln 2 + \dfrac{9}{2}$ **39.** $\dfrac{\sqrt{3}}{4}$ **40.** $-\dfrac{2}{3}$ **41.** $\dfrac{1}{2}(e^2 - e^{-2})$ **42.** $\dfrac{\pi}{4}$

43. $\dfrac{2-\sqrt{2}}{2}$ **44.** $\dfrac{1}{2}(e^2 - 1)$ **45.** π **46.** $\dfrac{1}{5}(1 - \cos 5)$

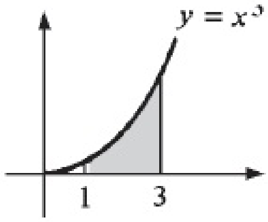
47. $\dfrac{\pi}{6}$ **48.** $\dfrac{3}{\ln 2}$ **49.** $e - 1$ **50.** $\ln 2$ **51.** $\ln 2$ **52.** 0

53. $\dfrac{5}{4}$ **54.** $\dfrac{\pi}{4}$ **55.** $\dfrac{\pi}{4}$ **56.** $\dfrac{\pi}{4}$ **57.** 1 **58.** $\dfrac{2}{\ln 3}$

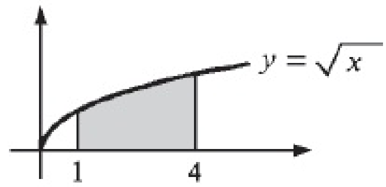
59. $\dfrac{3e-1}{1+\ln 3}$ **60.** $\dfrac{4-\pi}{4}$

11.6

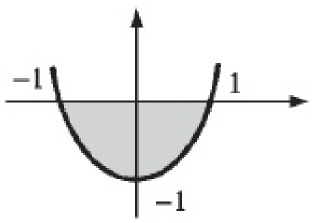
1. Área $= 20$



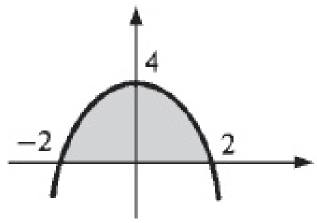
2. Área $= \dfrac{14}{3}$



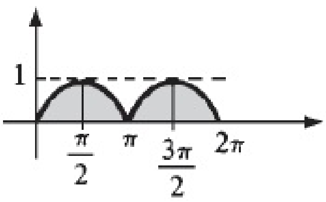
3. Área $= \dfrac{4}{3}$



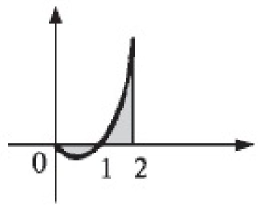
4. Área $= \dfrac{32}{3}$



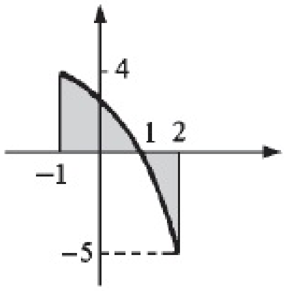
5. Área $= 4$



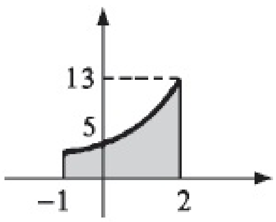
6. Área $= 1$



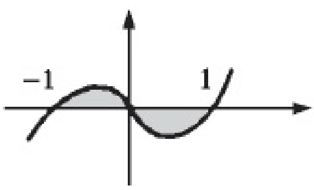
7. Área $= \dfrac{23}{3}$



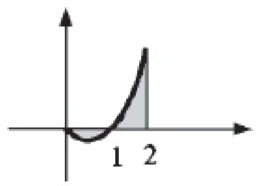
8. Área $= 21$



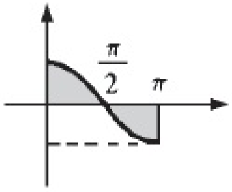
9. Área $= \dfrac{1}{2}$



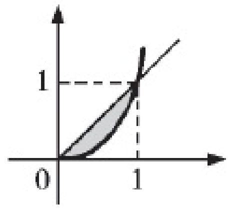
10. Área $= \dfrac{5}{2}$



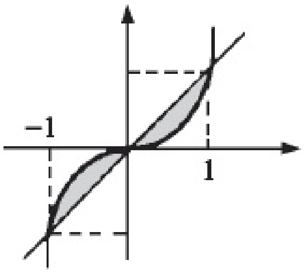
11. Área $= 2$



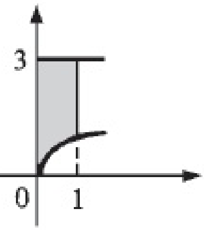
12. Área $= \frac{1}{4}$



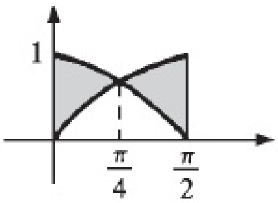
13. Área $= \frac{1}{2}$



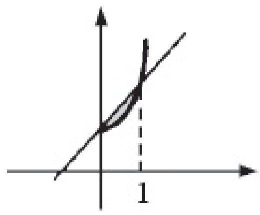
14. Área $= \frac{7}{3}$



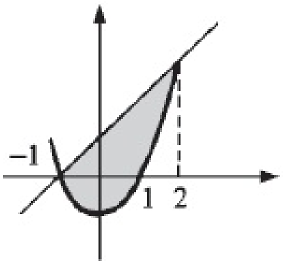
15. Área $= 2\,(\sqrt{2} - 1)$



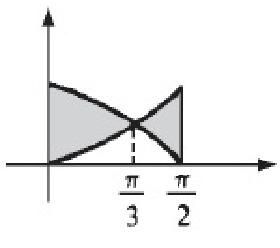
16. Área $= \frac{1}{6}$



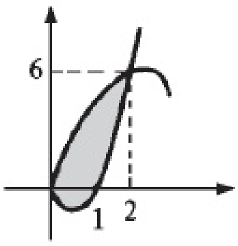
17. Área $= \frac{9}{2}$



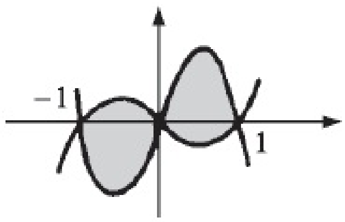
18. Área $= \frac{1}{6}\,(12\sqrt{3} - \pi - 12)$



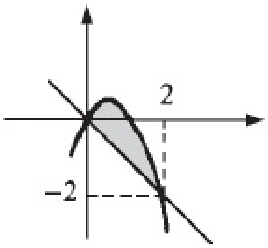
19. Área $= \dfrac{16}{3}$



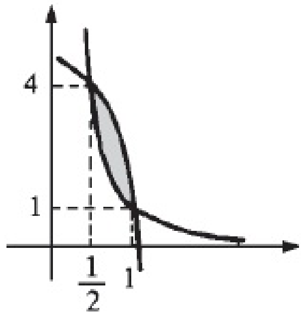
20. Área $= \dfrac{8 + \pi}{2\pi}$



21. Área $= \dfrac{4}{3}$



22. Área $= \dfrac{1}{3}$



23. ***a*)** 2

***b*)** $\dfrac{10}{4}$

24. 2

25. 1

26. $\dfrac{34}{3}$

11.7

1. *a)* $-\frac{1}{6}$ *b)* $\frac{1.023}{15}$ *c)* $\frac{14}{9}$ *d)* 68 *e)* $\frac{45}{4}$ *f)* $\frac{5}{16}$

g) $\frac{15}{64}$ *h)* $3 \ln \frac{5}{2}$ *i)* $\frac{1}{2}[e^4 - 1]$ *j)* $\frac{1}{2}[e - 1]$ *l)* $-\frac{4}{15}$

m) $\frac{\sqrt{3}}{4}$ *n)* $\frac{1}{3} \ln 2$ *o)* $\frac{1}{6}$ *p)* $\frac{2}{9}$ *q)* $\frac{2}{3} \ln \frac{7}{4}$ *r)* 0

s) $\frac{11}{192}$ *t)* $-\frac{1}{10.302}$ *u)* $\frac{46}{429}$

2. 3

3. $\frac{5}{2}$

4. 0

5. 0

6. *a)* $\frac{2}{3}$ *b)* $\frac{1}{2} \ln 5$ *c)* $\frac{e^{-2} - 2e^{-1} + 1}{2}$

7. *a)* $\frac{8 - \sqrt{27}}{3}$ *b)* $\frac{3.367}{12}$ *c)* $\frac{243}{4}$ *d)* $\frac{1}{3}$ *e)* $\frac{1}{3}(1 - e^{-1})$

f) $\frac{3\sqrt{3} - 1}{6}$ *g)* $\frac{3}{2} \ln \frac{5}{2}$ *h)* $\frac{1}{4} \ln 5$ *i)* $\frac{8}{3}$ *j)* $\sqrt{2} - 1$

l) $\frac{76}{15}$ *m)* $\frac{3 - 4\ln 2}{2}$ *n)* 0 *o)* $\frac{58}{15}$ *p)* $\frac{7}{24}$ *q)* $\frac{1}{384}$

r) $\frac{11}{24}$ *s)* $\frac{11}{24}$ *t)* $\frac{11}{24}$ *u)* $\frac{11}{24}$

11.8

1. ***a)*** $6\ J$

b) $4\ J$

c) $-1\ J$

***d*)** 0

2. $a)\ \int_1^x -3x\,dx = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$, logo, $\frac{3x^2}{2} + v^2 = \frac{3}{2}$

***b*)** $\sqrt{\frac{3}{2}}$

***c*)** 1 e −1

***d*)** $|x| = 1$

***e*)** Oscilatório

3. $b)\ \sqrt{5}$ e $-\sqrt{5}$ $\quad c)\ x = 0$ $\quad d)\ |x| = \sqrt{5}$

4. $a)\ |v| = \frac{\sqrt{10}}{5}\sqrt{40 - x^2}$ $\quad b)\ 4$ $\quad c)\ \sqrt{40}$ $\quad d)\ \sqrt{40}$ e $-\sqrt{40}$

5. $v = \sqrt{1 - \frac{1}{x}}$

6. $\int_{x_0}^{x} ma\,dx = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ ou $2a(x - x_0) = v^2 - v_0^2$.

7. $b)\ \frac{v_0^2}{2g}$

8. $a)\ v^2 - \frac{1}{x} = v_0^2 - 1$

***b*)** $v_0 = 1$

10. $\int_0^3 3x\cos 30^\circ\,dx = \frac{27\sqrt{3}}{4}\ J$

11. $a)\ \frac{3\sqrt{2}}{2}\ J$

***b*)** 0

12. $\int_{-2}^{-1} \frac{-x}{(4 + x^2)\sqrt{4 + x^2}}\,dx = \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{8}}.$

CAPÍTULO 12

12.1

1. *a)* $3x + k$ *b)* $\dfrac{x^2}{2} + k$ *c)* $\dfrac{x^6}{6} + k$ *d)* $\dfrac{2}{3}\sqrt{x^3} + k$

e) $\dfrac{5}{7}\sqrt[5]{x^7} + k$ *f)* $-\dfrac{1}{3x^3} + k$ *g)* $-\dfrac{1}{2x^2} + k$ *h)* $x + \ln|x| + k$

i) $\ln|x| - \dfrac{1}{x} + k$ *j)* $\dfrac{x^3}{3} + 3\ln|x| + k$ *l)* $x + \ln|x| + k$

m) $e^x + 4x + k$ *n)* $\dfrac{1}{5}e^{5x} + k$ *o)* $-\dfrac{1}{2}e^{-2x} + k$

p) $\dfrac{1}{2}e^{2x} - e^{-x} + k$ *q)* $\ln|x| - e^{-x} + k$ *r)* $\dfrac{1}{4}e^{4x} - \dfrac{1}{x} + k$

s) $3\ln|x| - \dfrac{1}{x^2} + k$ *t)* $\dfrac{x^4}{4} + \ln|x| - \dfrac{1}{x} + k$ *u)* $\dfrac{\sqrt{2}}{2}e^{\sqrt{2}x} + k$

2. *a)* $\dfrac{1}{2}(e^2 - 1)$ *b)* $\dfrac{3 + 2\ln 2}{2}$ *c)* $e - \dfrac{1}{e}$ *d)* $\dfrac{\pi}{4}$ *e)* $\dfrac{\pi}{6}$ *f)* $\dfrac{7 + 3\ln 2}{3}$

3. *a)* $-\cos x + k$ *b)* $-\dfrac{1}{2}\cos 2x + k$ *c)* $\dfrac{1}{5}\operatorname{sen} 5x + k$

d) $-\dfrac{1}{4}\cos 4t + k$ *e)* $\dfrac{1}{7}\operatorname{sen} 7t + k$ *f)* $\dfrac{1}{\sqrt{3}}\operatorname{sen}\sqrt{3}t + k$

g) $\dfrac{1}{2}x - \dfrac{1}{4}\operatorname{sen} 2x + k$ *h)* $2x - \dfrac{1}{6}\cos 2x + k$ *i)* $\dfrac{x^2}{2} + \dfrac{1}{15}\operatorname{sen} 3x + k$

j) $\ln|x| - \dfrac{4}{3}\cos 3x + k$ *l)* $\dfrac{1}{3}x + \dfrac{5}{14}\operatorname{sen} 7x + k$

m) $\dfrac{1}{3}\operatorname{sen} 3x - \dfrac{1}{8}\cos 4x + k$ *n)* $-\dfrac{1}{6}\cos 2x + \dfrac{1}{6}\operatorname{sen} 3x + k$

o) $-2\cos x + k$ *p)* $\dfrac{1}{9}\operatorname{sen} 3x + \dfrac{1}{49}\cos 7x + k$

q) $\dfrac{1}{9}e^{3x} - \dfrac{1}{3}\cos 3x + k$

4. *a)* $\dfrac{3}{4}$ *b)* $2\sqrt{2}$ *c)* $\dfrac{2}{3}$ *d)* $\dfrac{\pi}{4}$

5. *b)* $\dfrac{1}{2}x - \dfrac{1}{4}\operatorname{sen} 2x + k$

6. *a)* $\frac{1}{2}x + \frac{1}{8}\operatorname{sen} 4x + k$ *b)* $\frac{1}{2}x + \frac{1}{20}\operatorname{sen} 10x + k$

c) $\frac{1}{2}x - \frac{1}{12}\operatorname{sen} 6x + k$ *d)* $\frac{1}{2}x + \frac{1}{2}\operatorname{sen} x + k$

e) $\frac{3}{8}x + \frac{1}{4}\operatorname{sen} 2x + \frac{1}{32}\operatorname{sen} 4x + k$ *f)* $\frac{3}{8}x + \frac{1}{4}\operatorname{sen} 2x + \frac{1}{32}\operatorname{sen} 4x + k$

g) $x - \frac{1}{2}\cos 2x + k$ *h)* $x + \frac{1}{2}\cos 2x + k$

i) $\frac{51}{2}x - \frac{10}{3}\cos 3x - \frac{1}{12}\operatorname{sen} 6x + k$ *j)* $\frac{3}{2}x - \operatorname{sen} 2x + \frac{1}{8}\operatorname{sen} 4x + k$

7. *a)* $\frac{\pi}{16} + \frac{\sqrt{2}}{8}$ *b)* $\frac{\pi}{8}$ *c)* $\frac{\pi}{2} + 1$ *d)* $\frac{3\pi}{16}$

8. $4\sqrt{2}$

10. ***a)*** $-\ln|\cos x| + k$

b) $\operatorname{tg} x + k$

c) $\operatorname{tg} x - x + k$

d) $\ln|\sec x + \operatorname{tg} x| + k$

e) $-\frac{1}{2}\ln|\cos 2x| + k$

f) $\frac{1}{3}\ln|\sec 3x + \operatorname{tg} 3x| + k$ *g)* $\frac{1}{\ln 3}3^x + k$ *h)* $5 \operatorname{arc\,sen} x + k$

i) $\frac{5^x}{\ln 5} - e^{-x} + k$ *j)* $\frac{x^2}{2} + \frac{1}{3}\operatorname{tg} 3x + k$

l) $x + \operatorname{tg} x + 2\ln|\sec x + \operatorname{tg} x| + k$ *m)* $x + \operatorname{tg} x + k$

11. *a)* $\operatorname{sen} 6x \cos x = \frac{1}{2}[\operatorname{sen} 7x + \operatorname{sen} 5x]$ *b)* $-\frac{1}{14}\cos 7x - \frac{1}{10}\cos 5x + k$

12. *a)* $-\frac{1}{12}\cos 6x - \frac{1}{8}\cos 4x + k$ *b)* $-\frac{1}{14}\cos 7x + \frac{1}{2}\cos x + k$

c) $-\frac{1}{8}\cos 4x + \frac{1}{4}\cos 2x + k$ *d)* $-\frac{1}{12}\cos 6x + k$

13. *a)* $\operatorname{sen} 3x \operatorname{sen} 2x = -\frac{1}{2}(\cos 5x - \cos x)$ *b)* $-\frac{1}{10}\operatorname{sen} 5x + \frac{1}{2}\operatorname{sen} x + k$

14. $\frac{1}{14}\operatorname{sen} 7x + \frac{1}{6}\operatorname{sen} 3x + k$

15. *a)* $-\frac{1}{8}\operatorname{sen} 4x + \frac{1}{4}\operatorname{sen} 2x + k$ *b)* $-\frac{1}{14}\operatorname{sen} 7x + \frac{1}{6}\operatorname{sen} 3x + k$

c) $-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + k$ *d)* $\frac{1}{12}\operatorname{sen} 6x + \frac{1}{8}\operatorname{sen} 4x + k$

e) $\frac{1}{20}\operatorname{sen} 10x + \frac{1}{8}\operatorname{sen} 4x + k$

16. ***a)*** 0

b) $\frac{8}{7}$

17. ***a)*** 0 se $m \neq n$; π se $m = n$

b) 0

12.2

1. *a)* $\frac{(3x-2)^4}{12} + k$ *b)* $\frac{2}{9}\sqrt{(3x-2)^3} + k$ *c)* $\frac{1}{3}\ln|3x-2| + k$

d) $-\frac{1}{3(3x-2)} + k$ *e)* $-\frac{1}{2}\cos x^2 + k$ *f)* $\frac{1}{2}e^{x^2} + k$

g) $\frac{1}{3}e^{x^3} + k$ *h)* $-\frac{1}{5}\cos 5x + k$ *i)* $\frac{1}{4}\operatorname{sen} x^4 + k$ *j)* $\frac{1}{6}\operatorname{sen} 6x + k$

l) $-\frac{1}{4}\cos^4 x + k$ *m*) $\frac{1}{6}\,\mathrm{sen}^6 x + k$ *n*) $2\ln|x+3| + k$

o) $\frac{5}{4}\ln|4x+3| + k$ *p*) $\frac{1}{8}\ln(1+4x^2) + k$ *q*) $\frac{1}{4}\ln(5+6x^2) + k$

r) $-\frac{1}{8(1+4x^2)}$ *s*) $\frac{1}{9}\sqrt{(1+3x^2)^3} + k$ *t*) $\frac{2}{3}\sqrt{(1+e^x)^3}$

u) $-\frac{1}{2(x-1)^2} + k$ *v*) $\frac{1}{\cos x} + k$ *x*) $-\frac{1}{2}e^{-x^2} + k$

2. *a*) $\frac{1}{2}\left(1-\frac{1}{e}\right)$ *b*) $\frac{1}{5}\left(\frac{\sqrt{3}}{2}\right)^5$ *c*) $\frac{3}{2}\ln 3$ *d*) $\frac{1}{6}\ln\frac{13}{4}$

e) $\sqrt{2}-1$ *f*) $\frac{2-\sqrt{2}}{3}$ *g*) $\frac{1}{202}$ *h*) $\frac{1}{3}$ *i*) $\frac{3}{8}$ *j*) 1 *l*) $\frac{\pi}{8}$ *m*) $\frac{\pi}{8}$

3. *a*) $\frac{1}{3}\mathrm{sen}^3 x + k$ *b*) $\frac{\mathrm{sen}^3 x}{3} - \frac{\mathrm{sen}^5 x}{5} + k$ *c*) $\frac{\mathrm{sen}^4 x}{4} - \frac{\mathrm{sen}^6 x}{6} + k$

d) $-\frac{2}{3}\sqrt{\cos^3 x} + k$ *e*) $-\frac{2}{3}\sqrt{(1+\cos^2 x)^3} + k$

f) $\frac{2}{3}\sqrt{(5+\mathrm{sen}^2 x)^3} + k$ *g*) $-\cos x + \frac{1}{3}\cos^3 x + k$

h) $\mathrm{sen}\,x - \frac{2}{3}\mathrm{sen}^3 x + \frac{1}{5}\mathrm{sen}^5 x + k$ *i*) $\frac{1}{4}\mathrm{tg}^4 x + k$ *j*) $\frac{1}{2}\mathrm{tg}^2 x + k$

l) $\frac{1}{3}\sec^3 x + k$ *m*) $\frac{1}{6}\sec^6 x - \frac{1}{4}\sec^4 x + k$

n) $-\frac{2}{3}(3+\cos x)^{3/2} + k$ *o*) $\frac{1}{\cos x} + k$ *p*) $\frac{1}{2\cos^2 x} + k$

q) $\frac{1}{8}x - \frac{1}{32}\,\mathrm{sen}\,4x + k$ *r*) $\sec x + \cos x + k$ *s*) $\frac{1}{2}\ln|3+2\,\mathrm{tg}\,x| + k$

4. ***a)*** $2\ln|x-3| + k$

b) $5\ln|x-1| + 2\ln|x| + k$

c) $\frac{1}{2}\ln|2x+3| + k$

d) $\dfrac{x^2}{2} + 3 \ln|x-2| + k$

e) $x - \ln|x+1| + k$

f) $x + 3\ln|x-1| + k$

g) $2x + \ln|x+1| + k$

h) $\dfrac{(x+1)^2}{2} - 2(x+1) + \ln|x+1| + k$

6. *a)* $-\dfrac{1}{2}\ln|x+1| + \dfrac{1}{2}\ln|x-1| + k$ *b)* $-\dfrac{3}{2}\ln|x| + \dfrac{7}{2}\ln|x-2| + k$

c) $\dfrac{1}{2}\ln|x-2| + \dfrac{1}{2}\ln|x+2| + k$ *d)* $\dfrac{1}{4}\ln|x-2| - \dfrac{1}{4}\ln|x+2| + k$

e) $-8\ln|x-1| + 13\ln|x-2| + k$

f) $\ln|x-2| + k$

g) $-2\ln|x-2| + 2\ln|x-3| + k$

h) $-4\ln|x+1| + 5\ln|x+2| + k$

8. *a)* $\dfrac{1}{\sqrt{5}} \operatorname{arc\,tg} \dfrac{x}{\sqrt{5}} + k$ *b)* $\operatorname{arc\,tg} \dfrac{x}{2} + k$

c) $\dfrac{\sqrt{10}}{10} \operatorname{arc\,tg} \dfrac{\sqrt{10}\ x}{2} + k$ *d)* $\dfrac{3}{\sqrt{5}} \operatorname{arc\,tg} \dfrac{x}{\sqrt{5}} + k$

e) $\dfrac{1}{2}\ln(5+x^2) + k$ *f)* $\dfrac{1}{4}\ln(1+4x^2) - \dfrac{3}{2}\operatorname{arc\,tg} 2x + k$

g) $\dfrac{1}{2}\ln(4+x^2) - \dfrac{1}{2}\operatorname{arc\,tg}\dfrac{x}{2} + k$ *h)* $\dfrac{1}{4}\ln(1+4x^2) - \dfrac{3}{2}\operatorname{arc\,tg} 2x + k$

i) $\operatorname{arc\,tg}(x+1) + k$ *j)* $\operatorname{arc\,tg}(x+1) + k$ *l)* $\dfrac{2}{\sqrt{5}}\operatorname{arc\,tg}\dfrac{x+2}{\sqrt{5}} + k$

m) $\dfrac{1}{2}\operatorname{arc\,tg}\dfrac{x+2}{2} + k$ *n)* $\dfrac{2}{\sqrt{3}}\operatorname{arc\,tg}\dfrac{2x+1}{\sqrt{3}} + k$ *o)* $2\operatorname{arc\,tg}(x+1) + k$

10. *a*) $-\dfrac{1}{8(16+x^4)^2}+k$ *b*) $\dfrac{1}{4}\ln(16+x^4)+k$ *c*) $\dfrac{1}{8}\operatorname{arc\,tg}\dfrac{x^2}{4}+k$

d) $-\dfrac{1}{2}\ln|\cos 2x|+k$ *e*) $\ln|\ln x|+k$ *f*) $-\dfrac{1}{\ln x}+k$

g) $\operatorname{tg} x - x + k$ *h*) $\operatorname{arc\,sen} x + k$ *i*) $\dfrac{5}{2}\operatorname{arc\,sen} 2x + k$

j) $-\dfrac{1}{4}\sqrt{1-4x^2}+k$ *l*) $\operatorname{arc\,sen}\dfrac{x}{2}+k$

m) $-\dfrac{1}{2}\sqrt{1-4x^2}+\dfrac{3}{2}\operatorname{arc\,sen} 2x+k$ *n*) $\dfrac{2}{3}\operatorname{arc\,sen}\dfrac{3x}{2}+k$

o) $\dfrac{1}{2}\operatorname{arc\,sen} x^2+k$ *p*) $\operatorname{arc\,sen} e^x+k$ *q*) $-2\sqrt{1-e^x}+k$

r) $\operatorname{arc\,sen}(\ln x)+k$ *s*) $2\operatorname{arc\,sen}(x+1)+k$ *t*) $\operatorname{arc\,tg} e^x+k$

u) $\dfrac{1}{3}\ln(1+3e^x)+k$ *v*) $\operatorname{sen}(\ln x)+k$ *x*) $\dfrac{1}{4}\operatorname{arc\,tg} x^4+k$

12.3

1. ***a)*** $(x-1)\,e^x+k$

b) $-x\cos x+\operatorname{sen} x+k$

c) $e^x\,(x^2-2x+2)+k$

d) $\dfrac{x^2}{2}\left(\ln x-\dfrac{1}{2}\right)+k$

e) $x\,(\ln x-1)+k$

f) $\dfrac{1}{3}x^3\left(\ln x-\dfrac{1}{3}\right)+k$

g) $x \operatorname{tg} x + \ln|\cos x| + k$

h) $\frac{x^2}{2}\left[(\ln x)^2 - \ln x + \frac{1}{2}\right] + k$

i) $x(\ln x)^2 - 2x(\ln x - 1) + k$

j) $\frac{1}{2}e^{2x}\left(x - \frac{1}{2}\right) + k$

l) $\frac{1}{2}e^x(\operatorname{sen} x + \cos x) + k$

m) $-\frac{1}{5}e^{-2x}(\cos x + 2\operatorname{sen} x) + k$

n) $\frac{1}{2}(x^2 - 1)\,e^{x^2} + k$

o) $\frac{1}{2}(x^2 \operatorname{sen} x^2 + \cos x^2) + k$

p) $\frac{e^{-x}}{5}(2\operatorname{sen} 2x - \cos 2x) + k$

q) $-x^2\cos x + 2x\operatorname{sen} x + 2\cos x + k$

2. *b*) $\frac{1}{4}\sec^3 x \operatorname{tg} x + \frac{3}{8}\sec x \operatorname{tg} x + \frac{3}{8}\ln|\sec x + \operatorname{tg} x| + k$

4. *a*) $-\frac{1}{3}\operatorname{sen}^2 x\cos x - \frac{2}{3}\cos x$ *b*) $-\frac{1}{4}\operatorname{sen}^3 x\cos x - \frac{3}{8}\operatorname{sen} x\cos x + \frac{3}{8}x + k$

5. $-\frac{e^{-st}}{1+s^2}(\cos t + s\operatorname{sen} t) + k$

7. ***a*)** 1

***b*)** $2\ln 2 - 1$

***c*)** $\frac{1}{2}\left(e^{\frac{\pi}{2}} - 1\right)$

***d*)** $-\frac{1}{s}x^2e^{-sx} - \frac{2}{s^2}xe^{-sx} - \frac{2}{s^3}e^{-sx} + \frac{2}{s^3}$

12.4

1. *a)* $\frac{1}{4}[\text{arc sen } 2x + 2x\sqrt{1-4x^2}\,] + k$ *b)* $\text{arc sen } \frac{x}{2} + k$

c) $\ln(x + \sqrt{4+x^2}\,) + k$ *d)* $\frac{1}{2}\text{ arc tg } \frac{x}{2} + k$ *e)* $-\sqrt{1-x^2} + k$

f) $\frac{3}{4}\left[\text{arc sen } \frac{2x}{\sqrt{3}} + \frac{2x}{3}\sqrt{3-4x^2}\right] + k$

g) $\frac{1}{2}[\text{arc sen } x - x\sqrt{1-x^2}\,] + k$

h) $\frac{1}{8}[\text{arc sen } x - x\sqrt{1-x^2}\,(1-2x^2)] + k$

i) $\ln\left|\frac{x}{1+\sqrt{1+x^2}}\right| + k$

j) $\frac{9}{2}\text{ arc sen } \frac{x-1}{3} + \frac{(x-1)\sqrt{9-(x-1)^2}}{2} + k$

l) Faça $2x = 3 \text{ sen } t$ *m)* $-x^2 + 2x + 2 = 3 - (x-1)^2$; faça $x - 1 = \sqrt{3}\,t$

n) $2\text{ arc sen } \frac{x-1}{2} + \frac{x-1}{2}\sqrt{4-(x-1)^2} + k$ *o)* $-\frac{\sqrt{1+x^2}}{x} + k$

2. $\frac{\pi}{2}$

3. πab

4. *a)* $\dfrac{(x+1)^{13}}{13} - \dfrac{(x+1)^{12}}{6} + \dfrac{(x+1)^{11}}{11} + k$

b) $\dfrac{2}{7}(x-1)^{7/2} + \dfrac{4}{5}(x-1)^{5/2} + \dfrac{2}{3}(x-1)^{3/2} + k$

c) $2(\sqrt{x} - \ln(1+\sqrt{x})) + k$ *d)* $-\dfrac{4}{1+\sqrt{x}} + \dfrac{2}{(1+\sqrt{x})^2} + k$

e) $-\dfrac{1}{3(x+1)^3} - \dfrac{1}{4(x+1)^4} + k$ *f)* $\dfrac{1}{6}(2x+1)^{3/2} - \dfrac{3}{2}(2x+1)^{1/2} + k$

g) $2\sqrt{1-e^x} + \ln\dfrac{1-\sqrt{1-e^x}}{1+\sqrt{1-e^x}} + k$ *h)* $\dfrac{4}{5}(1+\sqrt{x})^{5/2} - \dfrac{4}{3}(1+\sqrt{x})^{3/2} + k$

i) $\dfrac{5}{2}\,\text{arc sen}\,(x-1) - \dfrac{1}{2}\sqrt{2x-x^2}\,(x+3) + k$

j) $\dfrac{1}{2}\,\text{arc tg}\,\dfrac{(x+1)}{2} + k$ *l)* $\left(\dfrac{x^2}{2} - \dfrac{1}{4}\right)\text{arc sen}\,x + \dfrac{x}{4}\sqrt{1-x^2} + k$

m) $\dfrac{1}{2}(\text{arc tg}\,x)^2(1+x^2) - x\,\text{arc tg}\,x + \dfrac{1}{2}\ln(1+x^2) + k$

n) $(x+1)\,\text{arc tg}\,\sqrt{x} - \sqrt{x} + k$ *o)* $-\dfrac{\text{arc tg}\,e^x}{e^x} + x - \dfrac{1}{2}\ln(1+e^{2x}) + k.$

6. *a)* $\dfrac{1}{2}\ln(4+x^2) + \dfrac{1}{2}\,\text{arc tg}\,\dfrac{x}{2} + k$ *b)* $\dfrac{1}{4}\ln(9+4x^2) - \dfrac{1}{6}\,\text{arc tg}\,\dfrac{2x}{3} + k$

c) $\dfrac{1}{2}\ln(x^2+2x+2) + 9\,\text{arc tg}\,(x+1) + k$

d) $\dfrac{3}{2}\ln(x^2+x+1) - \dfrac{7}{\sqrt{3}}\,\text{arc tg}\,\dfrac{2x+1}{\sqrt{3}} + k$

e) $\ln(x^2+4x+5) - 3\,\text{arc tg}\,(x+2) + k$

f) $\dfrac{1}{2}\ln(9+x^2) - \dfrac{1}{3}\,\text{arc tg}\,\dfrac{x}{3} + k$

7. $\dfrac{3\sqrt{2}}{2}\,\text{arc sen}\,\dfrac{1}{\sqrt{3}} + \dfrac{1}{3}$ **8.** $\dfrac{4-3\ln 3}{6}$

9. ***a*)** $x = 3 \operatorname{sen} t$

***b*)** $x = 3 \sec t$

***c*)** $x = 3 \operatorname{tg} t$

***d*)** $x = \operatorname{sen} t$

e) $2x = \sqrt{3} \operatorname{sen} t$ *f*) $2x = \sqrt{3} \sec t$ *g*) $2x = \sqrt{3} \operatorname{tg} t$

h) $\sqrt{3}\, x = \sqrt{2} \operatorname{sen} t$ *i*) $\sqrt{3}\, x = \sqrt{2} \operatorname{sen} t$ *j*) $\sqrt{3}\, x = \sqrt{2} \sec t$

***l*)** $x - 1 = u^2, u > 0$

***m*)** $1 + e^x = u^2, u > 0$

***n*)** $x + \frac{3}{2} = \frac{\sqrt{3}}{2} \operatorname{tg} t$

***o*)** $1 + \sqrt{x} = t^3$

12.5

1. $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + k$

2. $-2 \ln | x - 2 | + 3 \ln | x - 3 | + k$

3. $\frac{1}{2} \ln | x^2 - 4 | + k$

4. $\ln | x^2 - 1 | + \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + k$

5. $6 \ln | x - 1 | + 10 (x - 1) + \frac{5}{2} (x - 1)^2 + k$

6. $\ln | x - 1 | - \frac{4}{x-1} + k$

7. $x + \frac{1}{4} \ln | x + 1 | + \frac{19}{4} \ln | x - 3 | + k$

8. $\ln | x - 2 | - \frac{4}{x-2} - \frac{5}{2 (x-2)^2} + k$

9. $-3 \ln | x | + 4 \ln | x - 1 | + k$

10. $x - \ln | x | + 3 \ln | x - 1 | + k$

11. $\frac{x^2}{2} + 2x + 4 \ln | x - 1 | - \frac{3}{x-1} + k$

12. $\dfrac{x^2}{2} + 4x - \dfrac{3}{2}\ln|x-1| + \dfrac{31}{2}\ln|x-3| + k$

13. $\dfrac{1}{\sqrt{5}}\,\text{arc tg}\,\dfrac{x}{\sqrt{5}} + k$

14. $\dfrac{1}{2}\ln(x^2+9) + \dfrac{1}{3}\,\text{arc tg}\,\dfrac{x}{3} + k$

15. $x + 2\ln|x-3| - 2\ln|x+3| + k$

16. $-\dfrac{1}{3}\ln|x+1| + \dfrac{1}{3}\ln|x-2| + k$

12.6

1. *a*) $-\dfrac{2}{x-1} + \dfrac{1}{2(x-1)^2} + k$

b) $-\dfrac{1}{6}\ln|x| + \dfrac{3}{10}\ln|x-2| - \dfrac{2}{15}\ln|x+3| + k$

c) $\dfrac{x^2}{2} - \ln|x| + \dfrac{3}{2}\ln|x-1| + \dfrac{1}{2}\ln|x+1| + k$

d) $\dfrac{2}{9}\ln|x+2| - \dfrac{2}{9}\ln|x-1| - \dfrac{2}{3(x-1)} + k$

e) $-2\ln|x-1| + \dfrac{1}{3}\ln|x+1| + \dfrac{5}{3}\ln|x-2| + k$

f) $\dfrac{5}{4}\ln|x| - \dfrac{5}{4}\ln|x-2| - \dfrac{7}{2(x-2)} + k$

g) $\ln|x-2| - \dfrac{4}{x-2} - \dfrac{5}{2(x-2)^2} + k$

h) $\dfrac{x^3}{4} + 4x - \dfrac{3}{4}\ln|x| + \dfrac{35}{8}\ln|x-2| - \dfrac{29}{8}\ln|x+2| + k$

***i*)** *e*

***j*)** Verifique o resultado encontrado por derivação.

2. $b)$ $\frac{7}{27}\ln|x-1|+\frac{6}{27(x-1)}-\frac{7}{27}\ln|x+2|+\frac{15}{27(x+2)}+k$

3. $a)$ $-\frac{1}{2(x-1)^2}-\frac{2}{3(x-1)^3}+k$

$b)$ $-\frac{1}{2x^2}+\frac{1}{2x}+\frac{1}{4}\ln|x|-\frac{1}{4}\ln|x+2|+k$

$c)$ $\frac{1}{x}+3\ln|x|-3\ln|x+1|+\frac{2}{x+1}+k$

$d)$ $\frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|+\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+k$

12.7

1. $2\ln|x-1|+\ln(x^2+6x+10)+\text{arc tg}\,(x+3)+k$

2. $\frac{2}{5}\ln|x|-\frac{1}{5}\ln(x^2+2x+5)+\frac{3}{10}\text{arc tg}\,\frac{x+1}{2}+k$

3. $2\ln(x^2+6x+12)-\frac{11}{\sqrt{3}}\text{arc tg}\,\frac{x+3}{\sqrt{3}}+k$

4. $-\frac{7}{2}\ln|x+2|+\frac{15}{2}\ln|x+4|+k$

5. $2\ln|x-1|+\frac{1}{2}\ln(x^2+2x+3)+\frac{1}{\sqrt{2}}\text{arc tg}\,\frac{x+1}{\sqrt{2}}+k$

6. $\ln|x-2|+\frac{1}{2}\ln(x^2+2x+4)-\frac{1}{\sqrt{3}}\text{arc tg}\,\frac{x+1}{\sqrt{3}}+k$

7. e **8.** Verifique o resultado encontrado por derivação

12.8

1. *a)* $\dfrac{-\cos 9x}{18} - \dfrac{\cos 5x}{10} + k$ *b)* $\dfrac{\text{sen } 2x}{4} - \dfrac{\text{sen } 8x}{16} + k$

c) $\dfrac{\text{sen } 3x}{6} + \dfrac{\text{sen } x}{2} + k$ *d)* $\dfrac{-\cos 3x}{6} - \dfrac{\cos x}{2} + k$

e) $\dfrac{-\cos(n+m)x}{2(n+m)} - \dfrac{\cos(n-m)x}{2(n-m)} + k$ se $n \neq m$; $\dfrac{-\cos 2nx}{4n} + k$ se $n = m$

f) $\dfrac{-\cos 2x}{8} + \dfrac{\cos 6x}{24} - \dfrac{\cos 4x}{16} + k$

g) $\dfrac{\text{sen } 6x}{24} + \dfrac{\text{sen } 4x}{16} + \dfrac{\text{sen } 2x}{8} + \dfrac{x}{4} + k$

2. 0 (observe que o integrando é uma função ímpar)

3. 0 se $n \neq m$; π se $n = m$

12.9

1. *a)* $\dfrac{x}{2} + \dfrac{\text{sen } 10x}{20} + k$ *b)* $\dfrac{-\cos^3 x}{3} + k$

c) $\dfrac{\text{sen}^5 x}{5} + k$ *d)* $\dfrac{-\cos^3 2x}{6} + k$

e) $\dfrac{-\text{sen } x \cos^5 x}{6} + \dfrac{\cos^3 x \text{ sen } x}{24} + \dfrac{\cos x \text{ sen } x}{16} + \dfrac{x}{16} + k$

f) $\dfrac{x}{8} - \dfrac{\text{sen } 8x}{64} +$ (*Lembrete*: $\text{sen } 4x = 2 \text{ sen } 2x \cos 2x$)

g) $\dfrac{x}{4} + \dfrac{\text{sen } 6x}{24} - \dfrac{\text{sen } 4x}{16} - \dfrac{\text{sen } 10x}{80} - \dfrac{\text{sen } 2x}{16} + k$

h) $\dfrac{\text{sen } x}{2} + \dfrac{\text{sen } 9x}{36} + \dfrac{\text{sen } 7x}{28} + k$

3. *a)* $\dfrac{3}{4}\sqrt[3]{\text{sen}^4 x} + k$ *b)* $\text{sen } x + \dfrac{2}{3}\sqrt{\text{sen}^3 x} - \dfrac{\text{sen}^3 x}{3} - \dfrac{2}{7}\sqrt{\text{sen}^7 x} + k$

c) $\dfrac{1}{4\cos^4 x} + k$ *d)* $-\ln|\cos x| - \dfrac{\text{sen}^2 x}{2} + k$

e) $\dfrac{-1}{6 \text{ sen}^6 x} + \dfrac{1}{4 \text{ sen}^4 x} + k$ *f)* $\text{arctg}(\text{sen } x) + k$

12.10

1. *a)* $\dfrac{\operatorname{tg}^6 x}{6} + k$ *b)* $\dfrac{\sec^6 x}{6} - \dfrac{\sec^4 x}{4} + k$ *c)* $\dfrac{\sec^3 2x}{6} - \dfrac{\sec 2x}{2} + k$

d) $\dfrac{\sec^2 3x}{6} + \dfrac{1}{3}\ln|\cos 3x| + k$ *e)* $3\sqrt[3]{\sec x} + k$

f) $-\ln|\cos x| + \dfrac{1}{\sec^2 x} - \dfrac{1}{4\sec^4 x} + k$ *g)* $\operatorname{tg} x + \dfrac{\operatorname{tg}^3 x}{3} + k$

h) $\dfrac{\sec^5 3x}{15} + k$ *i)* $\dfrac{\operatorname{tg}^5 x}{5} - \dfrac{\operatorname{tg}^3 x}{3} + \operatorname{tg} x - x + k$

j) $\dfrac{\sec^3 x \operatorname{tg} x}{4} + \dfrac{3\sec x \operatorname{tg} x}{8} + \dfrac{3}{8}\ln|\sec x + \operatorname{tg} x| + k$

3. *a)* $\dfrac{-\operatorname{cosec} x \operatorname{cotg} x}{2} - \dfrac{1}{2}\ln|\operatorname{cosec} x + \operatorname{cotg} x| + k$

b) $\dfrac{-\operatorname{cosec} x \operatorname{cotg} x}{2} + \dfrac{1}{2}\ln|\operatorname{cosec} x + \operatorname{cotg} x| + k$

c) $\dfrac{-\operatorname{cotg}^3 x}{3} + \operatorname{cotg} x + x + k$

12.11

1. $\dfrac{1}{4}\ln\left(\dfrac{2 + \operatorname{sen} x}{2 - \operatorname{sen} x}\right) + k$ **2.** $\dfrac{\sqrt{2}}{2}\ln\left|\dfrac{\operatorname{tg}\dfrac{x}{2} - 1 + \sqrt{2}}{\operatorname{tg}\dfrac{x}{2} - 1 - \sqrt{2}}\right| + k$

3. $2\,[\,\ln(1 + \cos x) - \cos x\,] + k$

4. $\ln|\,2\sec x + 3\,| + k$

5. $\dfrac{1}{2}\ln\left|\sec\left(x + \dfrac{\pi}{6}\right) + \operatorname{tg}\left(x + \dfrac{\pi}{6}\right)\right| + k$

6. $\dfrac{2}{\sqrt{3}}\operatorname{arc\,tg}\dfrac{2\operatorname{tg}\dfrac{x}{2} + 1}{\sqrt{3}} + k$

CAPÍTULO 13

13.1