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Lista de Cálculo 1: Integral Definida

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Lista de exercícios

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Sumário

1 1^o Teorema Fundamental do Cálculo

$$\int_0^{\frac{\pi}{8}} \operatorname{sen} 2x\, dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{8}} = -\frac{1}{2}\cos\frac{\pi}{4} + \frac{1}{2}$$

ou seja,

$$\int_0^{\frac{\pi}{8}} \operatorname{sen} 2x\, dx = \frac{2-\sqrt{2}}{4}. \quad \blacksquare$$

EXEMPLO 7. Calcule $\int_0^1 e^{-x}\, dx$.

Solução

$$\int_0^1 e^{-x}\, dx = [-e^{-x}]_0^1 = 1 - \frac{1}{e}. \quad \blacksquare$$

Exercícios 11.5

Calcule.

1. $\int_0^1 (x+3)\, dx$
2. $\int_{-1}^1 (2x+1)\, dx$
3. $\int_0^4 \frac{1}{2}\, dx$
4. $\int_{-2}^1 (x^2-1)\, dx$
5. $\int_1^3 dx$
6. $\int_{-1}^2 4\, dx$
7. $\int_1^3 \frac{1}{x^3}$
8. $\int_{-1}^1 5\, dx$
9. $\int_0^2 (x^2+3x-3)\, dx$
10. $\int_0^1 \left(5x^3 - \frac{1}{2}\right) dx$
11. $\int_1^1 (2x+3)\, dx$
12. $\int_1^0 (2x+3)\, dx$

13. $\int_{-2}^{-1} \left(\frac{1}{x^2} + x \right) dx$

14. $\int_{0}^{4} \sqrt{x}$

15. $\int_{1}^{4} \frac{1}{\sqrt{x}}\, dx$

16. $\int_{0}^{8} \sqrt[3]{x}\, dx$

17. $\int_{-1}^{0} (x^3 - 2x + 3)\, dx$

18. $\int_{0}^{1} \sqrt[8]{x}\, dx$

19. $\int_{1}^{2} \left(x^3 + x + \frac{1}{x^3} \right) dx$

20. $\int_{0}^{1} (x + \sqrt[4]{x})\, dx$

21. $\int_{1}^{3} \left(5 + \frac{1}{x^2} \right) dx$

22. $\int_{-3}^{3} x^3\, dx$

23. $\int_{-1}^{1} (x^7 + x^3 + x)\, dx$

24. $\int_{\frac{1}{2}}^{1} (x + 3)\, dx$

25. $\int_{1}^{4} (5x + \sqrt{x})\, dx$

26. $\int_{1}^{0} (x^7 - x + 3)\, dx$

27. $\int_{1}^{2} \frac{1 + x}{x^3}\, dx$

28. $\int_{0}^{1} (x + 1)^2\, dx$

29. $\int_{1}^{4} \frac{1 + x}{\sqrt{x}}\, dx$

30. $\int_{0}^{1} (x - 3)^2\, dx$

31. $\int_{0}^{2} (t^2 + 3t - 1)\, dt$

32. $\int_{1}^{2} \frac{1 + t^2}{t^4}\, dt$

33. $\int_{\frac{1}{2}}^{1} (s + 2)\, ds$

34. $\int_{0}^{3} (u^2 - 2u + 3)\, du$

35. $\int_{1}^{2} (s^2 + 3s + 1)\, ds$

36. $\int_{-1}^{1} \sqrt[3]{t}\, dt$

37. $\int_{1}^{3} \left(1 + \frac{1}{x} \right) dx$

38. $\int_{1}^{2} \frac{1 + 3x^2}{x}\, dx$

39. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \cos 2x\, dx$

40. $\int_{-\pi}^{0} \operatorname{sen} 3x\, dx$

41. $\int_{-1}^{1} e^{2x}\, dx$

42. $\int_{0}^{1} \frac{1}{1 + t^2}\, dt$

43. $\int_{0}^{\frac{\pi}{4}} \operatorname{sen} x\, dx$

44. $\int_{-1}^{0} e^{-2x}\, dx$

45. $\int_0^{\frac{\pi}{3}} (3 + \cos 3x)\, dx$

46. $\int_0^1 \operatorname{sen} 5x\, dx$

47. $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1 - x^2}}\, dx$

48. $\int_0^2 2^x\, dx$

49. $\int_0^1 2x e^{x^2}\, dx$

50. $\int_0^1 \frac{2x}{1 + x^2}\, dx$

51. $\int_0^1 \frac{1}{1 + x}\, dx$

52. $\int_{-1}^1 x^3\, e^{x^4}\, dx$

53. $\int_0^{\frac{\pi}{3}} (\operatorname{sen} x + \operatorname{sen} 2x)\, dx$

54. $\int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 2x\right) dx$

55. $\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$ $\left(\textit{Sugestão}: \text{Verifique que } \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x.\right)$

56. $\int_0^{\frac{\pi}{2}} \operatorname{sen}^2 x\, dx$

57. $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$

58. $\int_0^1 3^x\, dx$

59. $\int_0^1 3^x\, e^x\, dx$

60. $\int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x\, dx$

11.6. CÁLCULO DE ÁREAS

Seja f contínua em $[a, b]$, com $f(x) \geq 0$ em $[a, b]$. Estamos interessados em definir a *área* do conjunto A do plano limitado pelas retas $x = a$, $x = b$, $y = 0$ e pelo gráfico de $y = f(x)$.