Compiler Construction

Lecture # 09

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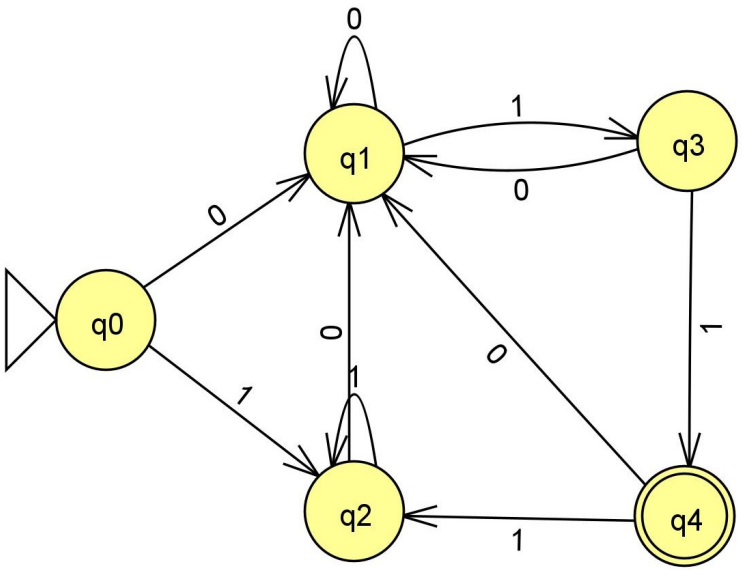
National University
of Computer & Emerging Sciences

DFA Minimization: Partitioning Method Example 1

DFA Minimization

For any given Deterministic Automaton with large number of states, we can construct its equivalent DFA with minimum number of states



	0	1
q0	q1	q2
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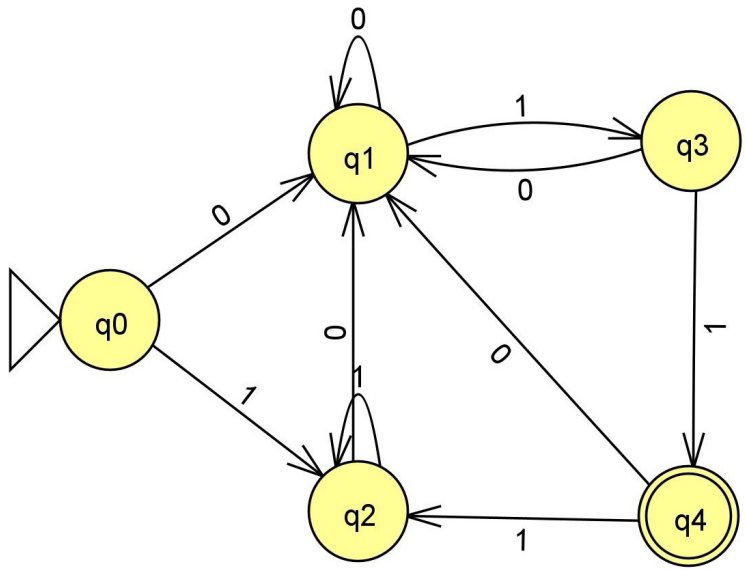

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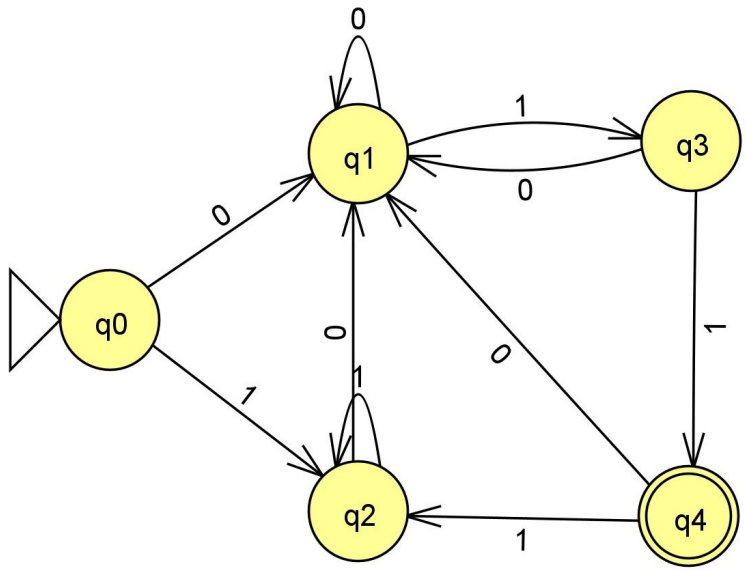

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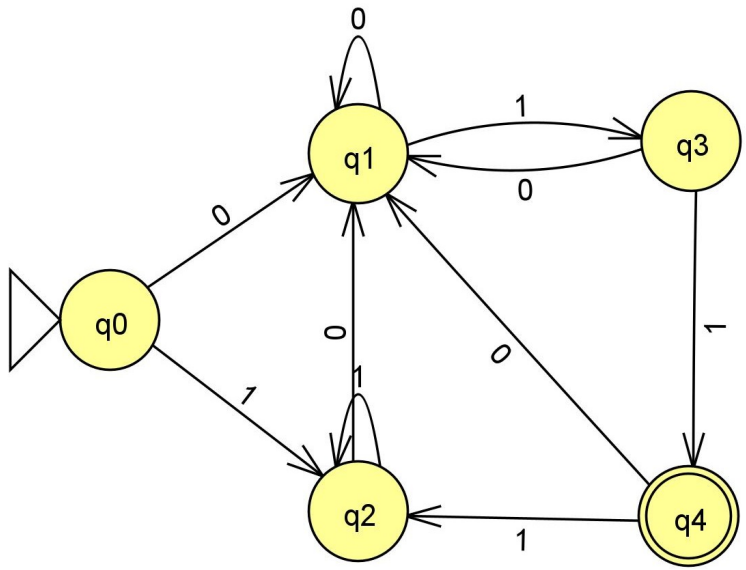

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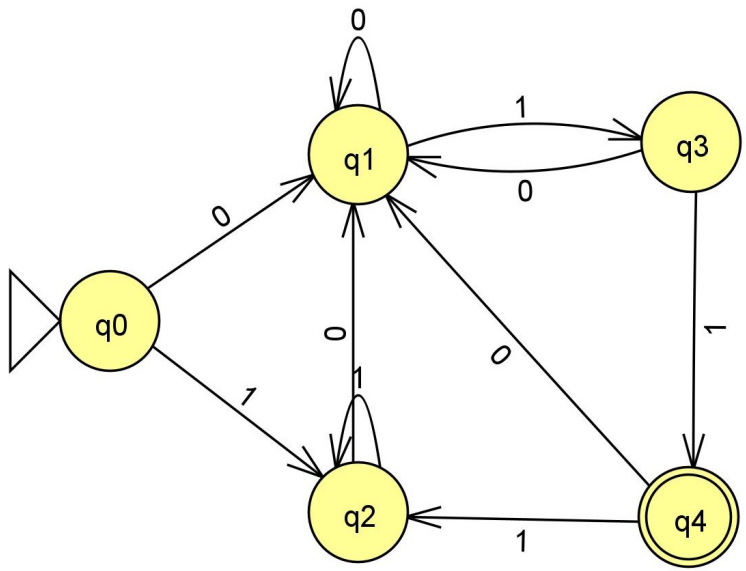

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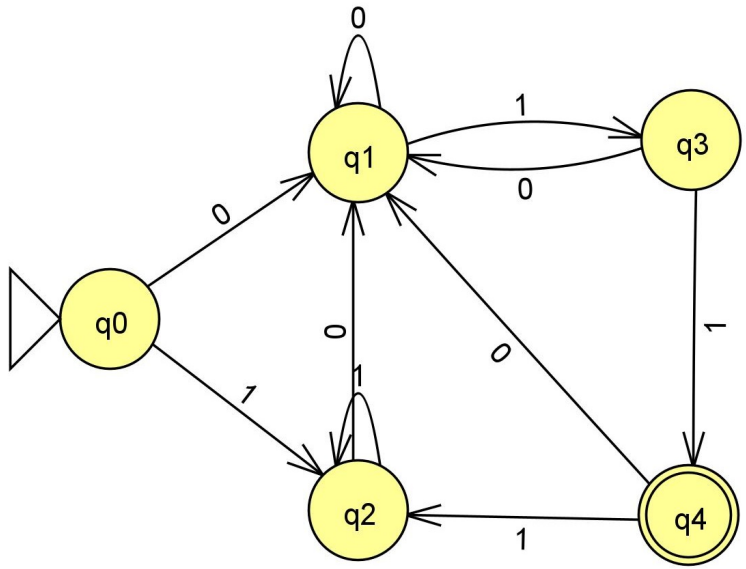

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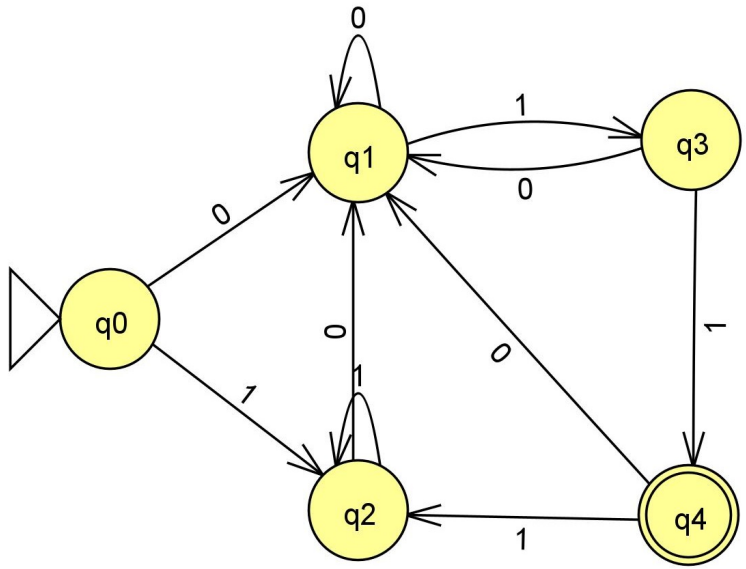

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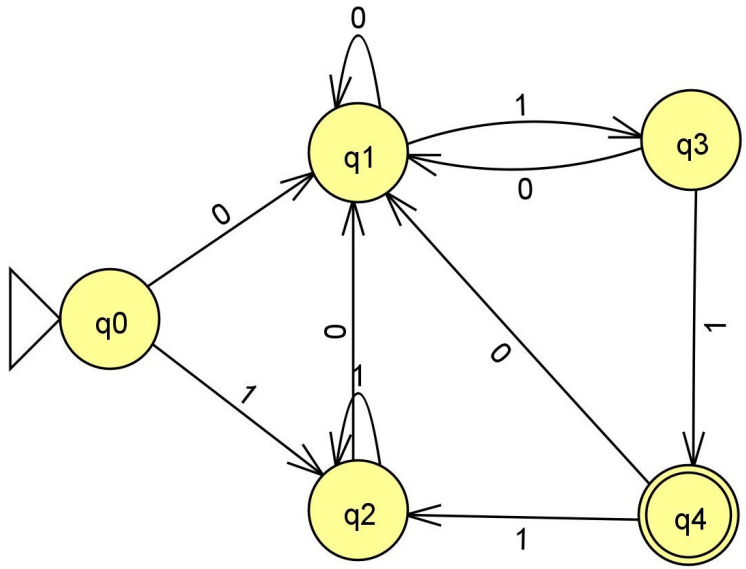

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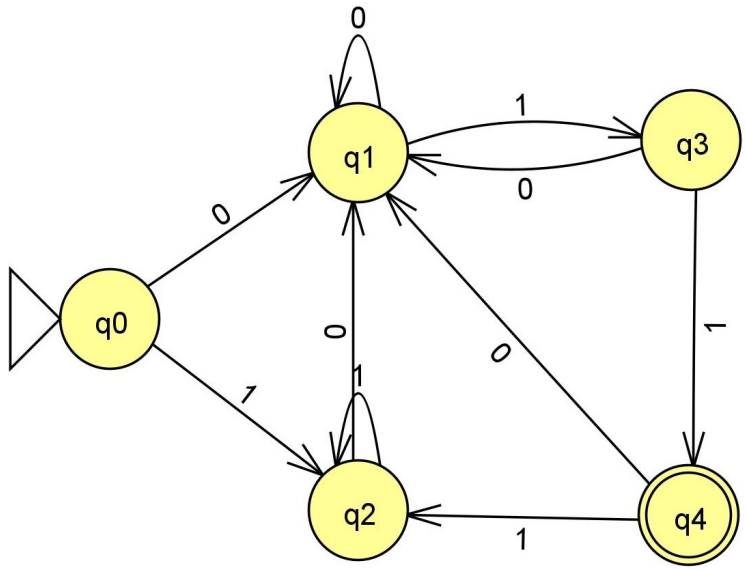

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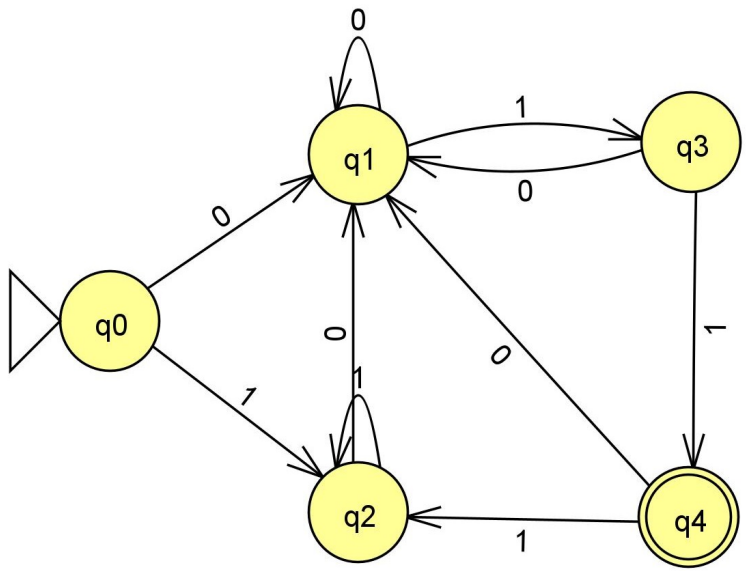

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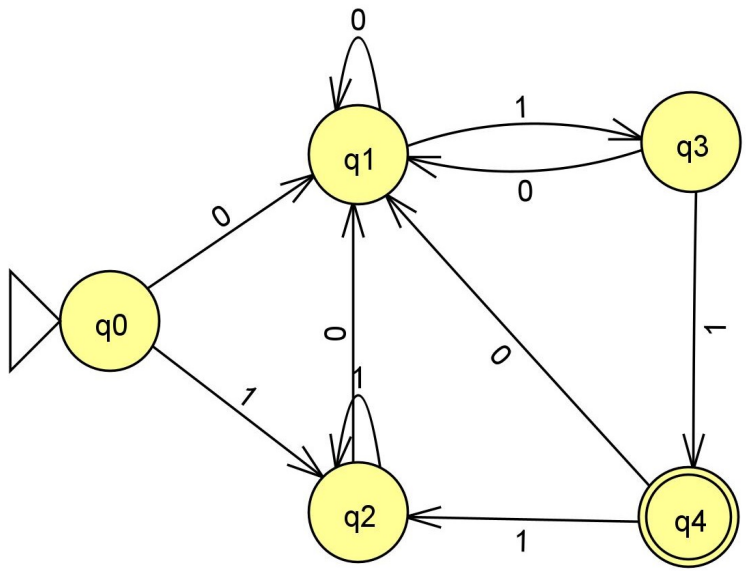

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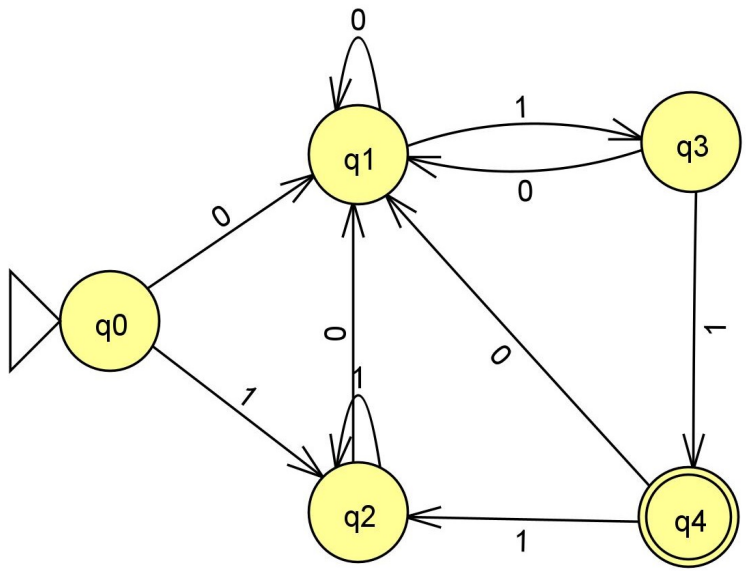

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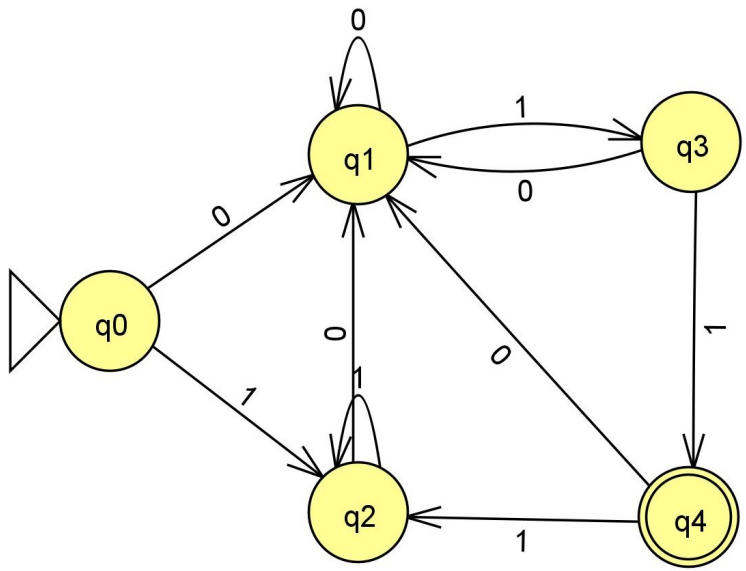

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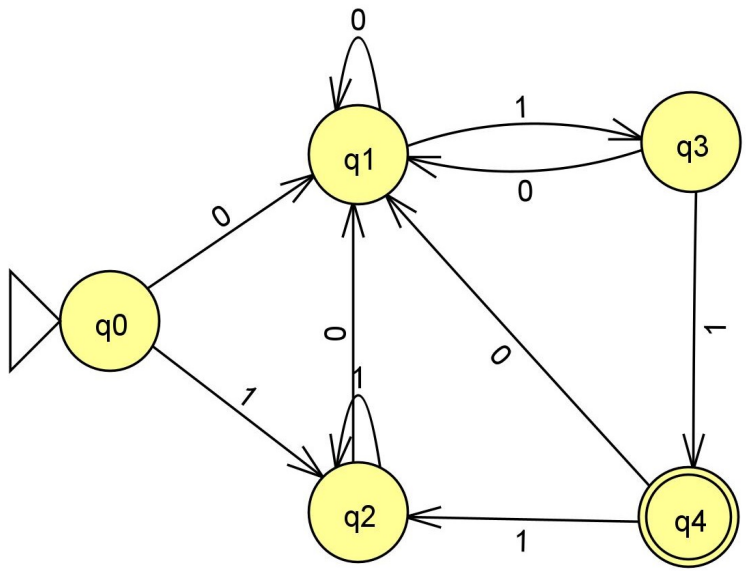

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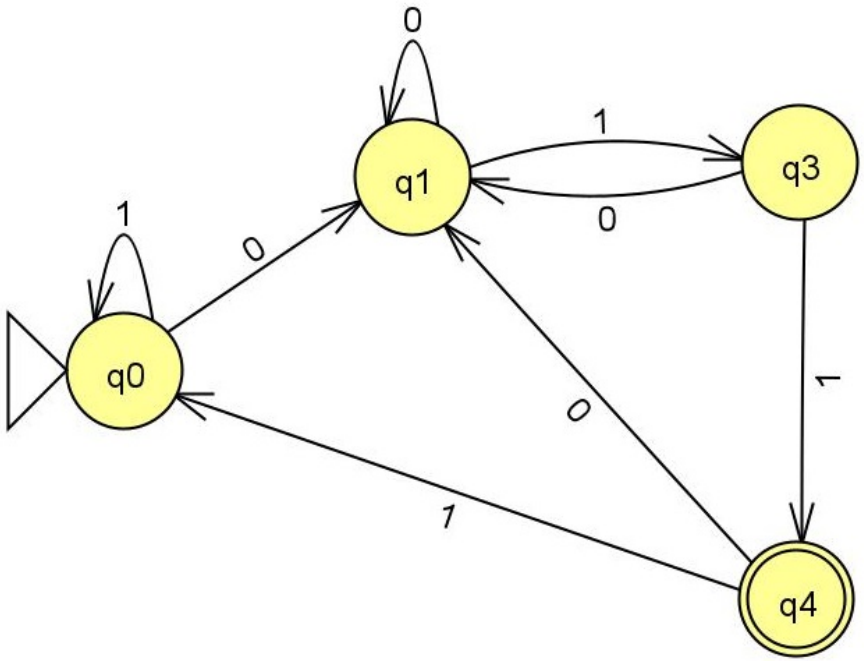
DFA Minimization: Partitioning Method Example 1 continued ...

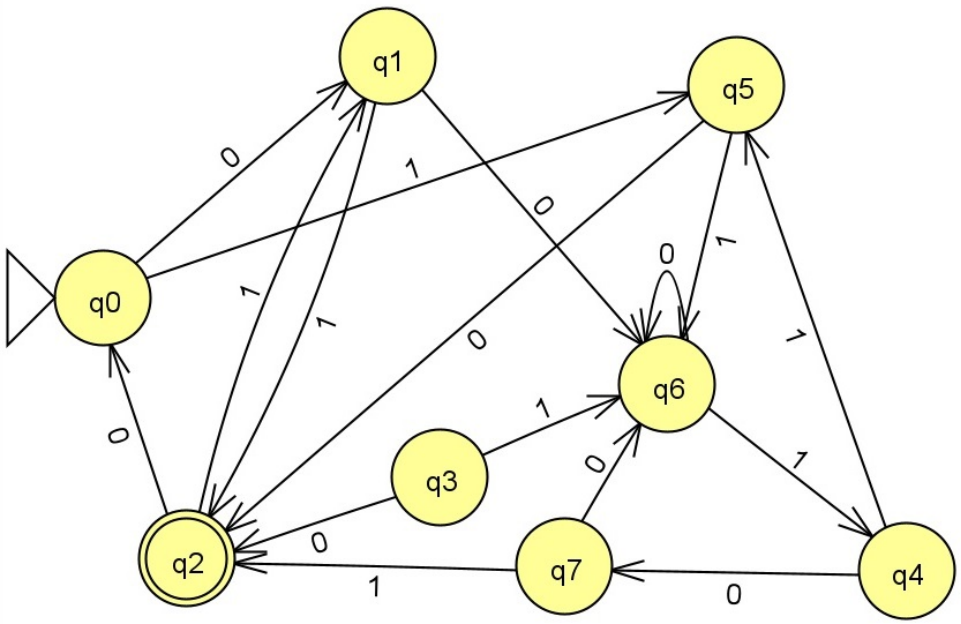
- The reduced but resultant equivalent DFA is,



	0	1
q0	q1	q1
q1	q1	q3
q3	q1	q4
q4	q1	q0

DFA Minimization: Partitioning Method Example 2

- Consider the following initial DFA,

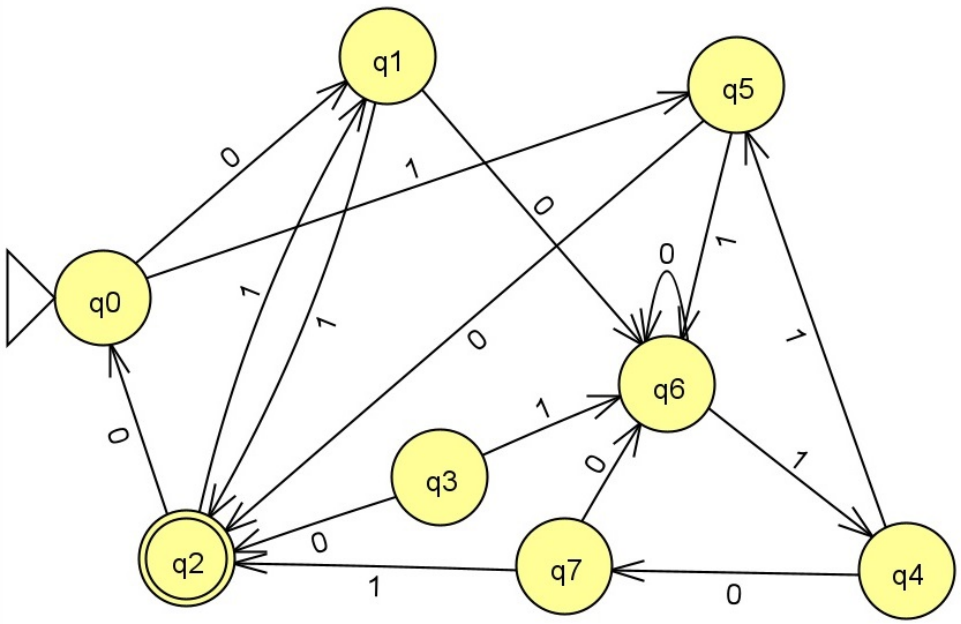


	0	1
q0	q1	q5
q1	q6	q2
q2	q0	q1
q3	q2	q6
q4	q7	q5
q5	q2	q6
q6	q6	q4
q7	q6	q2

- $\prod_0$ = { { q0, q1, q3, q4, q5, q6, q7}, {q2} }

DFA Minimization: Partitioning Method Example 2

- Consider the following initial DFA,



	0	1
q0	q1	q5
q1	q6	q2
q2	q0	q1
q3	q2	q6
q4	q7	q5
q5	q2	q6
q6	q6	q4
q7	q6	q2

- $\prod_0$ = { { q0, q1, q3, q4, q5, q6, q7}, {q2} }
- $\prod_1$ = { {q0, q4, q6}, {q1, q7}, {q3, q5},{q2} }

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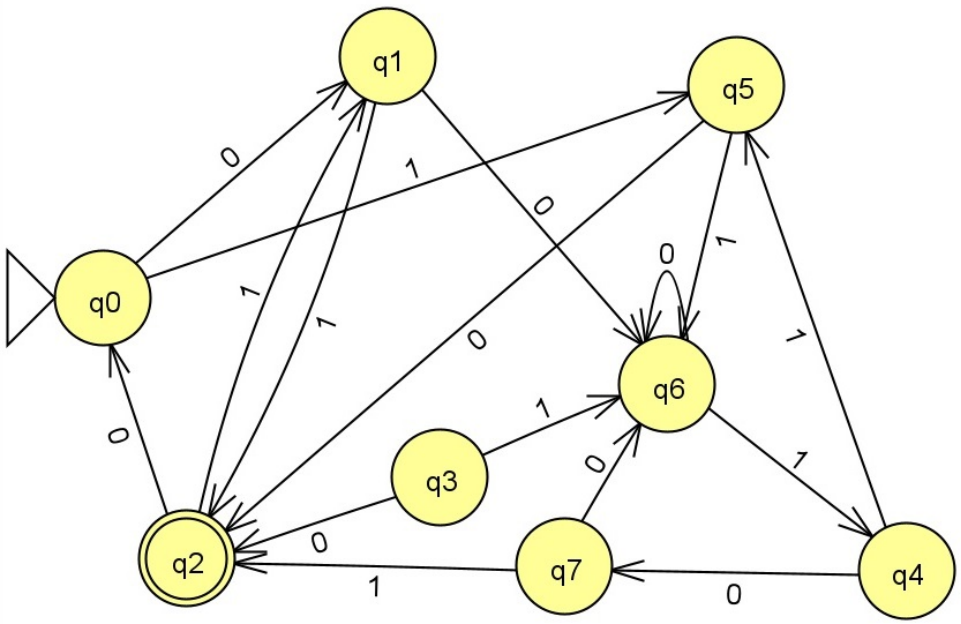

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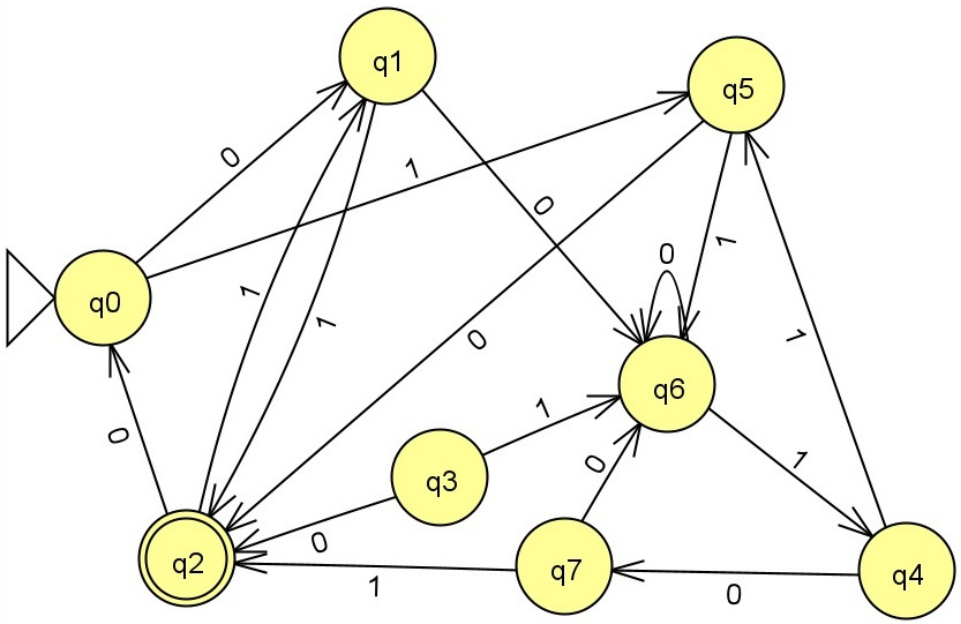

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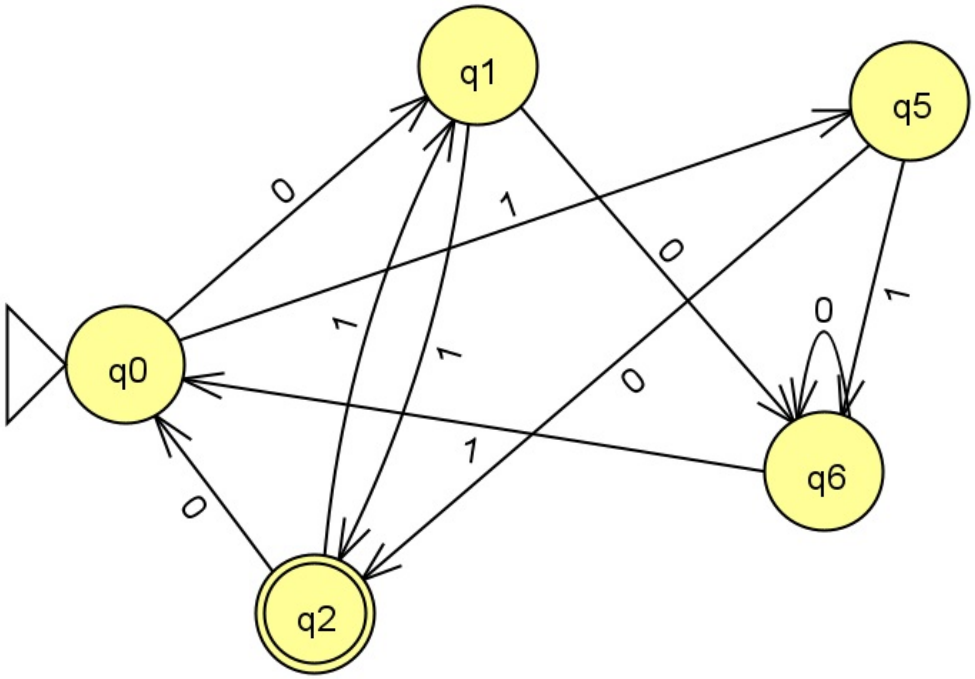
- $\prod_3$ = { {q0, q4}, {q6}, {q1, q7}, {q3, q5},{q2} }

DFA Minimization: Partitioning Method Example 2 continued ...

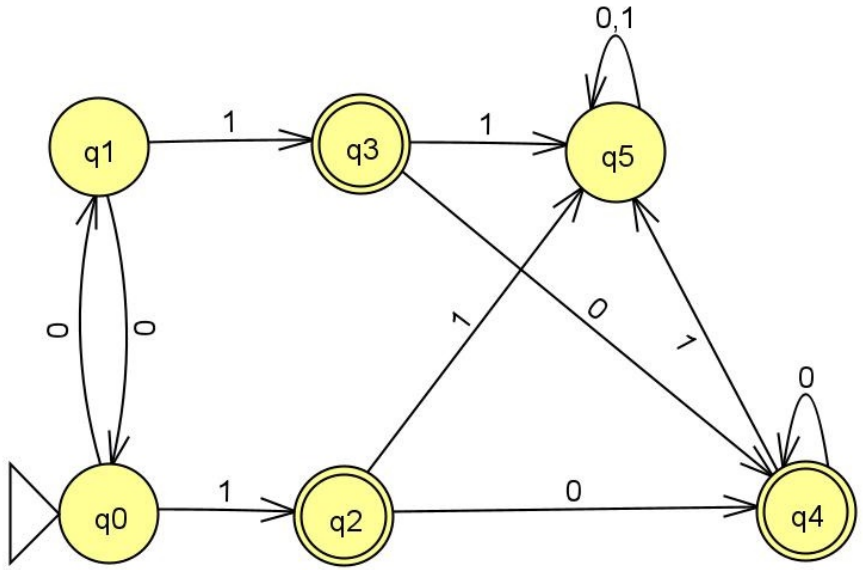
- The reduced but resultant equivalent DFA is,



	0	1
q0	q1	q5
q1	q6	q2
q2	q0	q1
q5	q2	q6
q6	q6	q0

DFA Minimization: Partitioning Method with Multiple Final States Example

- consider the following DFA,



	0	1
q0	q1	q2
q1	q0	q3
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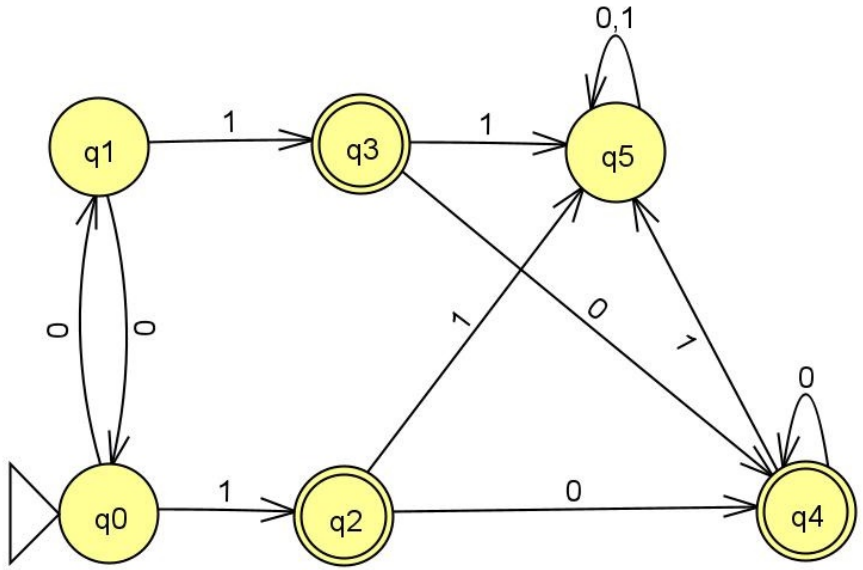

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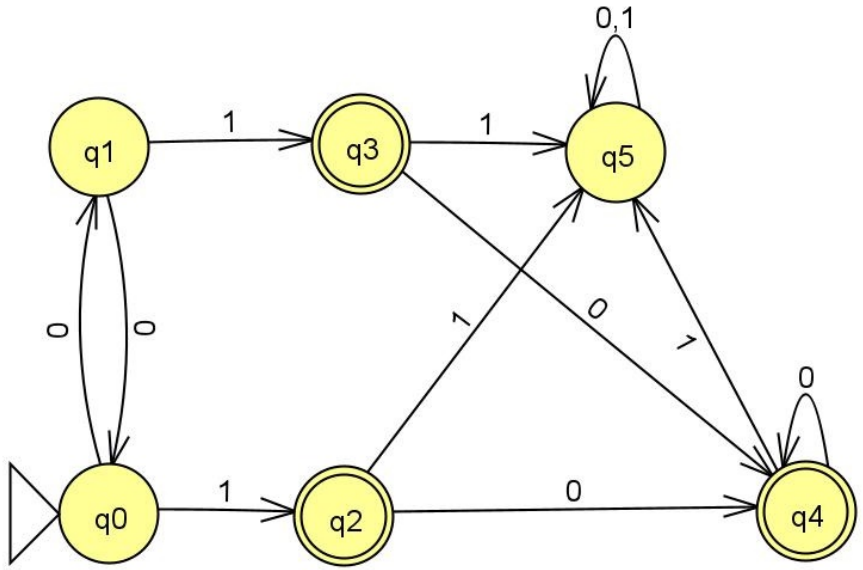

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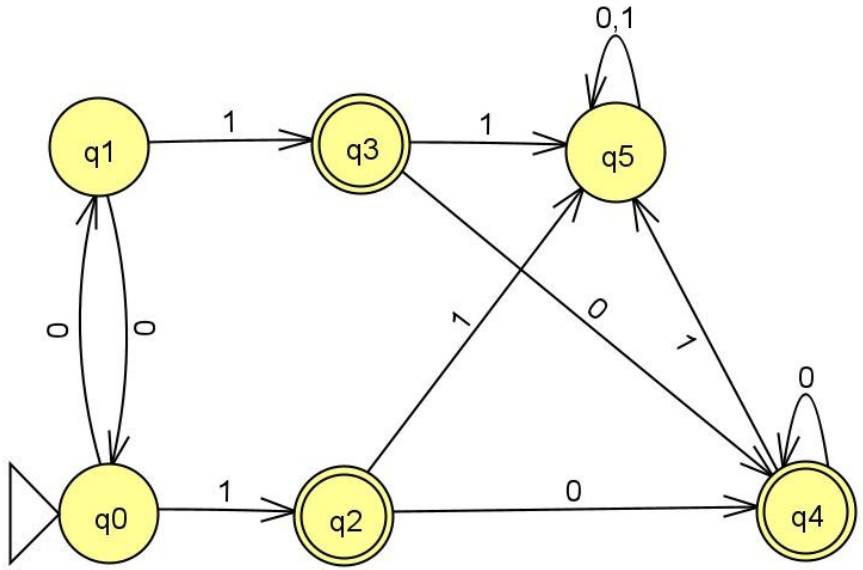

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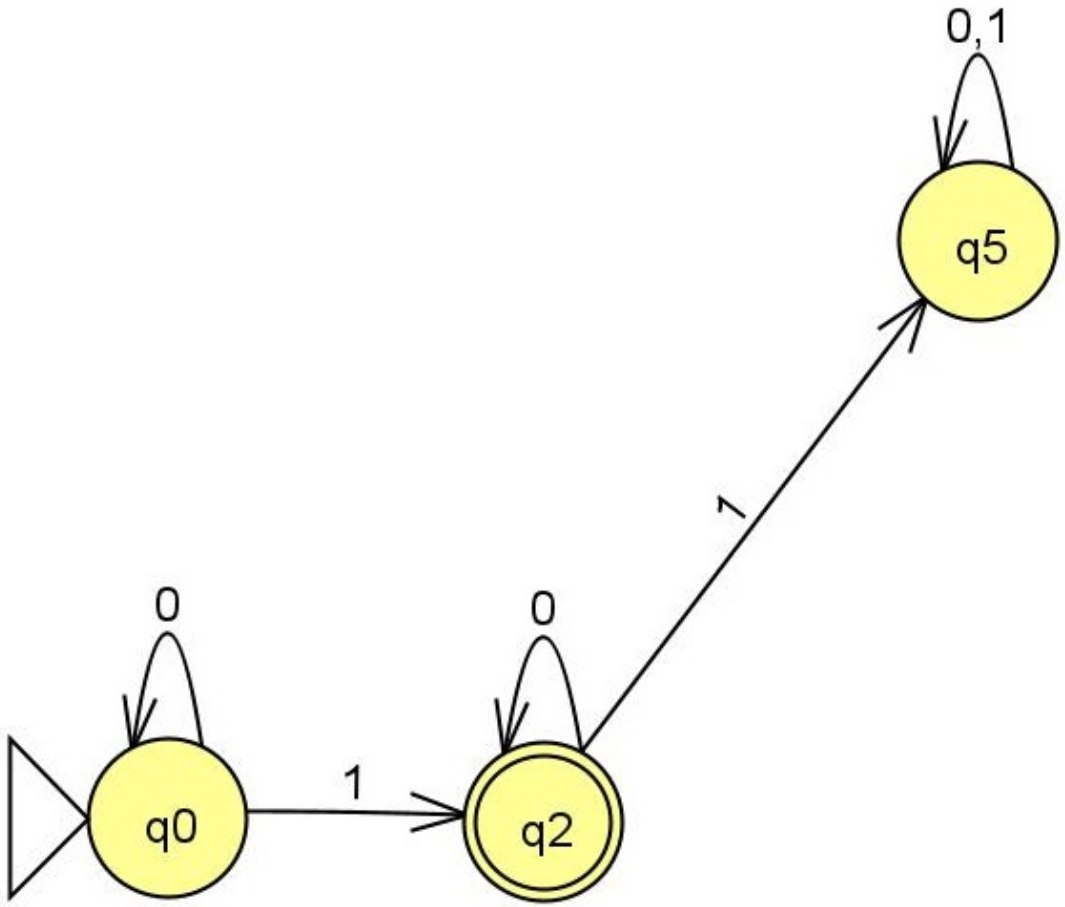
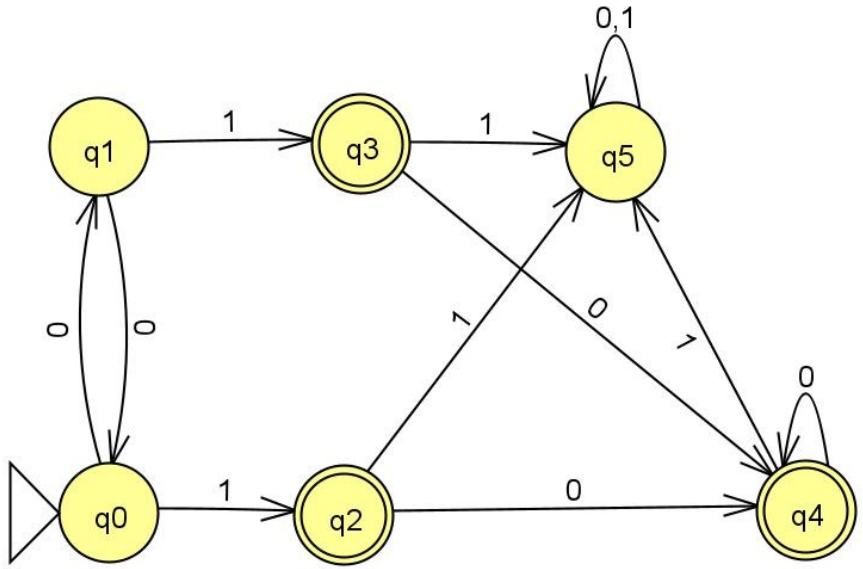
DFA Minimization: Partitioning Method with Multiple Final States Example continued …

- The minimized but equivalent DFA is,

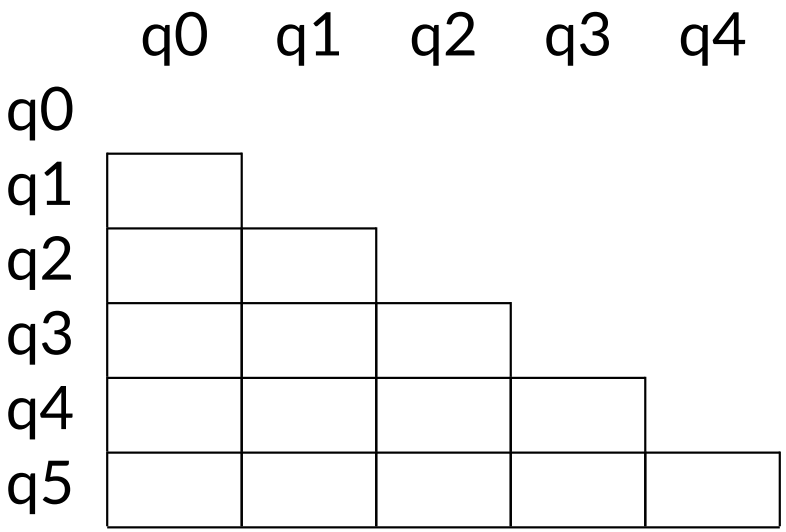


	0	1
q0	q0	q2
q2	q2	q5
q5	q5	q5

DFA Minimization: Table Filling Method Example



	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5



1. Draw a table for all pair of states, say (P,Q)

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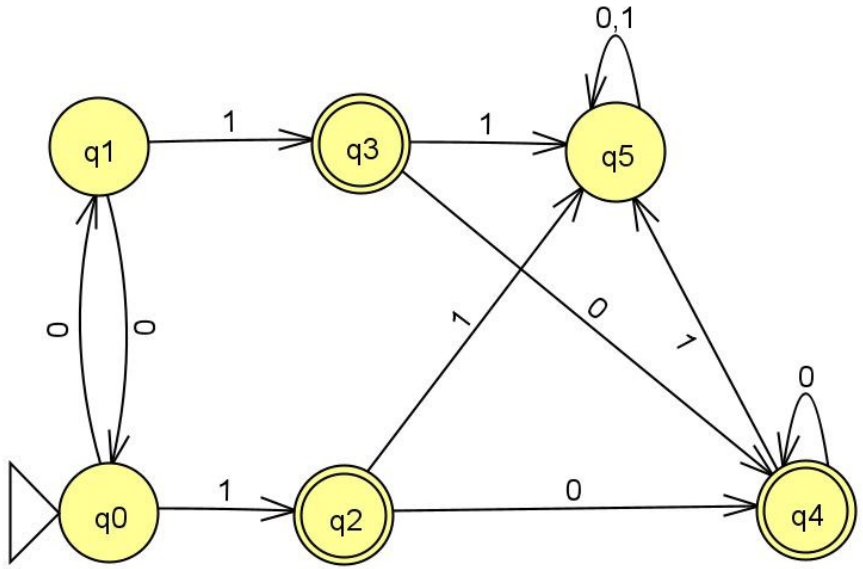

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q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2						
q3						
q4						
q5						

2. Mark all pairs where (P∈F and Q∈F)

   Ex: for (q1,q0) neither q1 nor q0 is final state. So we can not mark the cell (q1,q0)

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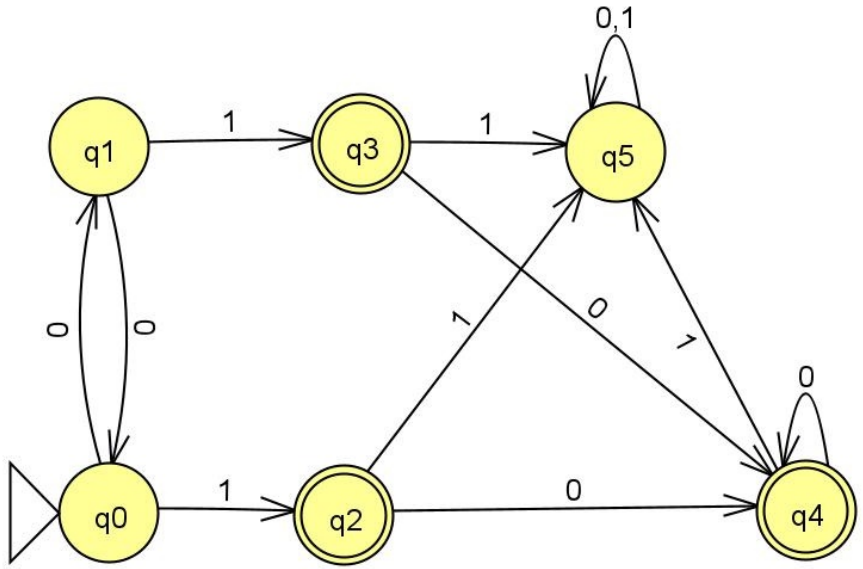

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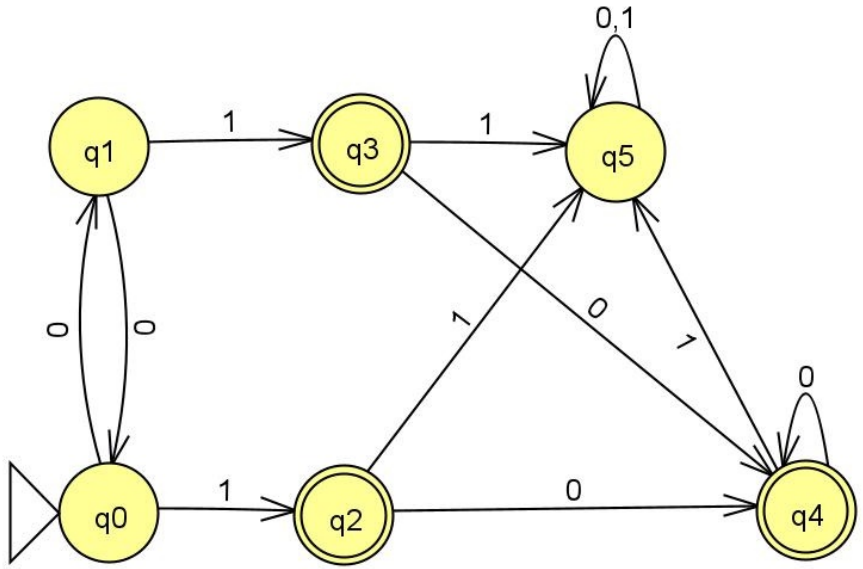
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓					
q3						
q4						
q5						

2. Mark all pairs where (P∈F and Q∈F)

   Ex: for (q2,q0) q2 is final and q0 is not. So mark the cell (q2,q0)

DFA Minimization: Table Filling Method Example



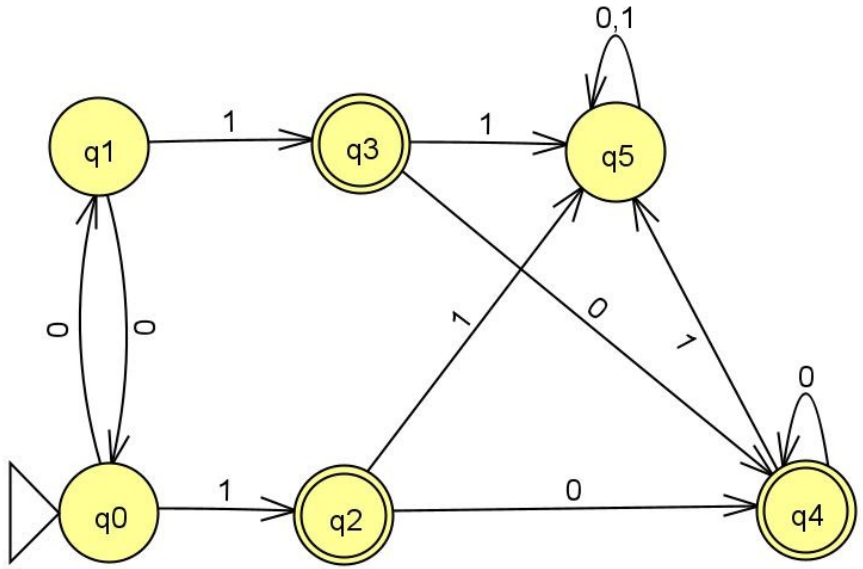
	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5			✓	✓	✓	

2. Mark all pairs where (P∈F and Q∈F)

   Similarly, fill the whole table

DFA Minimization: Table Filling Method Example

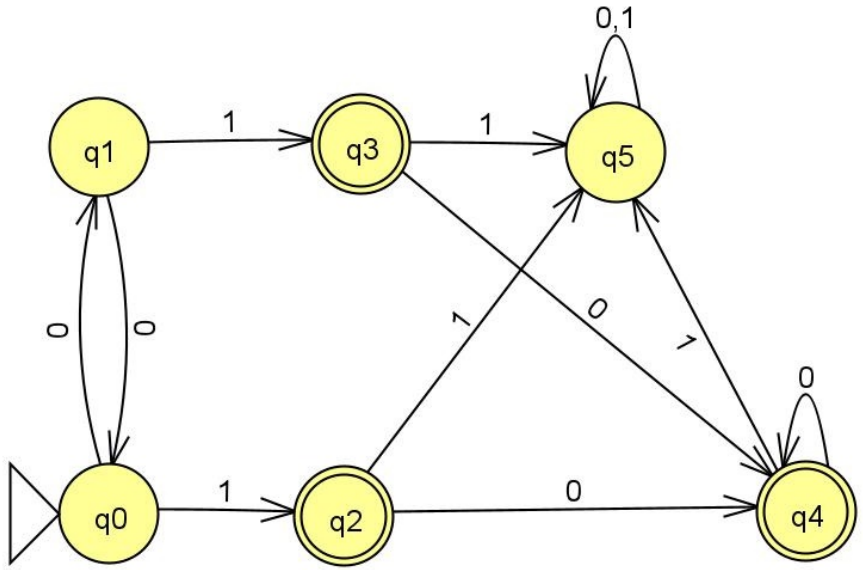


	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5			✓	✓	✓	

3. for any unmarked pair (P,Q) such that $[\delta(P, x)$ or $\delta(Q, x)]$ is marked, then mark(P,Q)

DFA Minimization: Table Filling Method Example



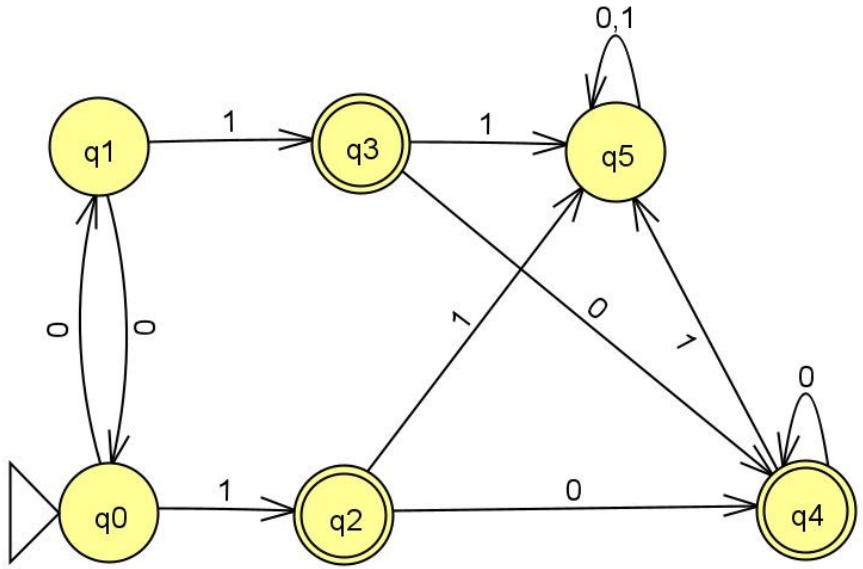
	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5			✓	✓	✓	

3. if $[\delta(P, x)$ or $\delta(Q, x)]$ is marked, then mark(P,Q)

Ex: for (q1,q0), (q1,q0) is unmarked , (q2,q3) is unmarked hence, leave (q1,q0) cell unmarked

DFA Minimization: Table Filling Method Example



	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5			✓	✓	✓	

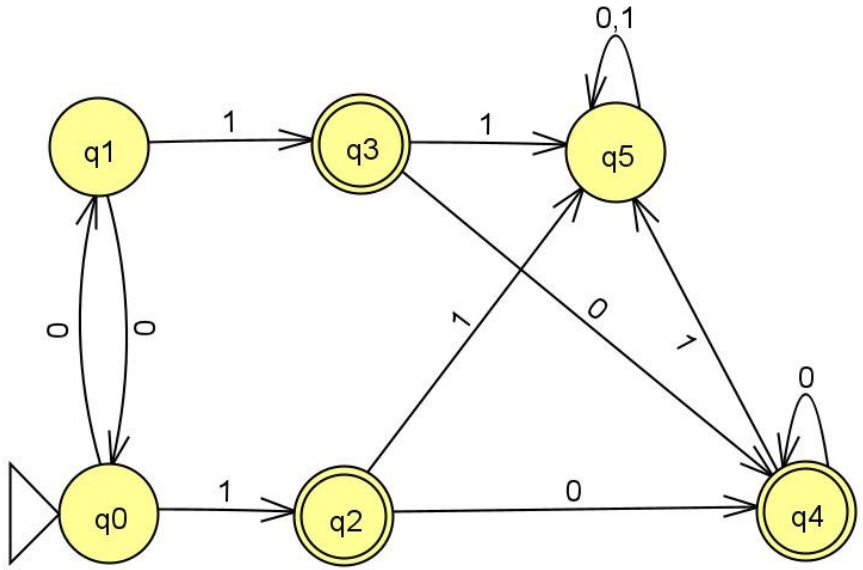
3. if $[\delta(P, x)$ or $\delta(Q, x)]$ is marked, then mark(P,Q)

   Ex: for (q5,q0),

   (q1,q5) is unmarked ,

   (q2,q5) is unmarked

DFA Minimization: Table Filling Method Example



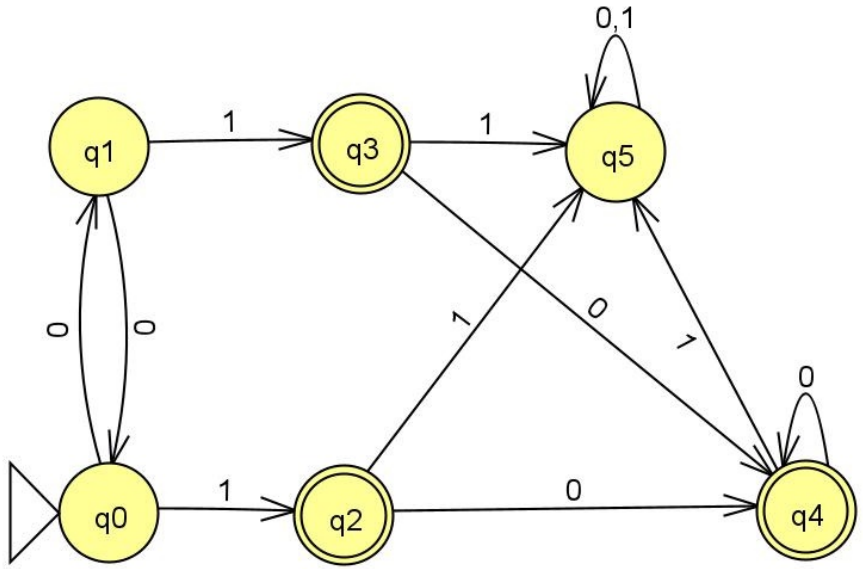
	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5	✓		✓	✓	✓	

3. if $[\delta(P, x)$ or $\delta(Q, x)]$ is marked, then mark(P,Q)

Ex: for (q5,q0),
(q1,q5) is unmarked ,
(q2,q5) is unmarked
hence, mark (q5,q0)

DFA Minimization: Table Filling Method Example



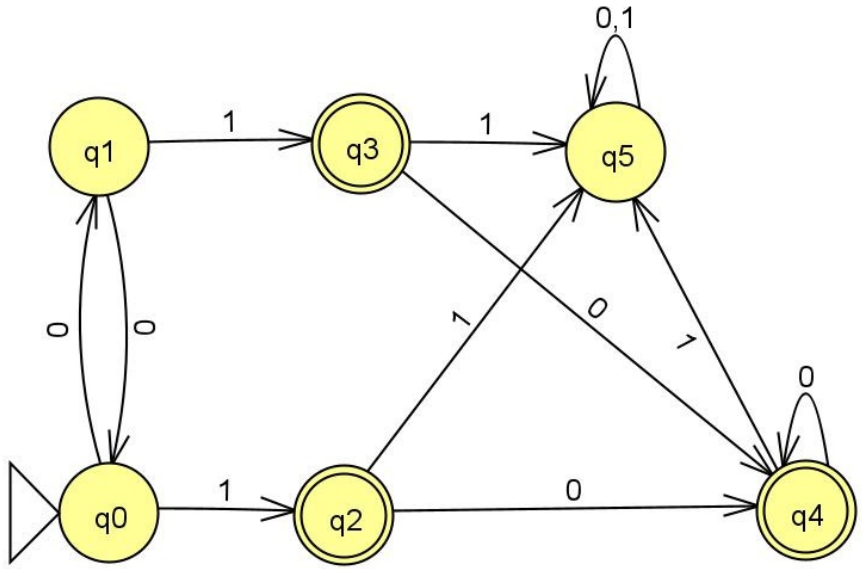
	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5	✓		✓	✓	✓	

3. if $[\delta(P,x)$ or $\delta(Q,x)]$ is marked, then mark(P,Q)

   Similarly, perform this process for the whole table, we get

DFA Minimization: Table Filling Method Example



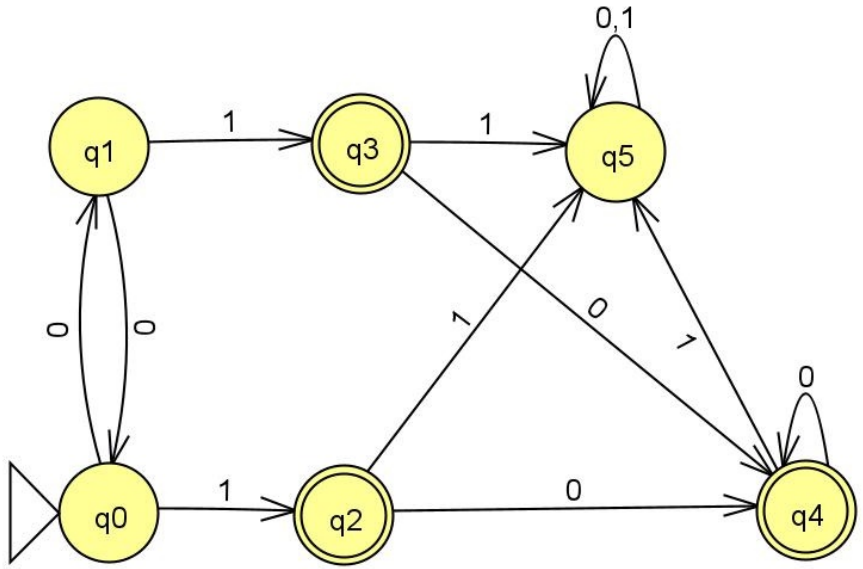
	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5	✓	✓	✓	✓	✓	

3. if $[\delta(P, x)$ or $\delta(Q, x)]$ is marked, then mark(P,Q)

   Similarly, perform this process for the whole table, we get

DFA Minimization: Table Filling Method Example

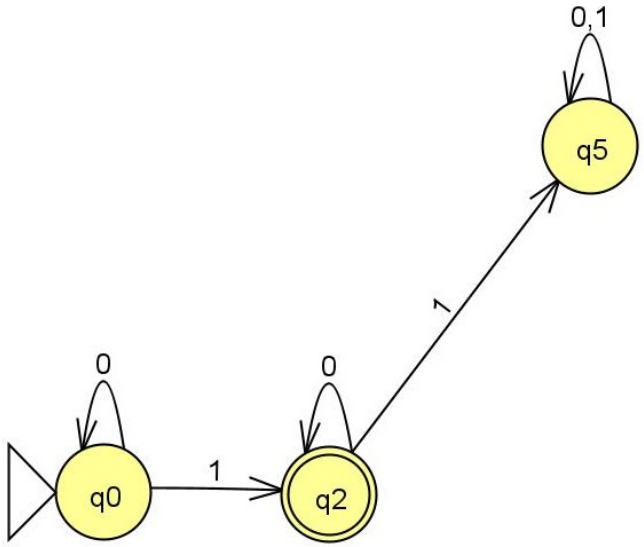


	0	1
q0	q1	q2
q1	q0	q3
q2	q4	q5
q3	q4	q5
q4	q4	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5	✓	✓	✓	✓	✓	

4. Finally, combine all the unmarked pairs and make them a single state in the minimized DFA

DFA Minimization: Table Filling Method Example



	0	1
q0	q0	q2
q2	q2	q5
q5	q5	q5

	q0	q1	q2	q3	q4	q5
q0						
q1						
q2	✓	✓				
q3	✓	✓				
q4	✓	✓				
q5	✓	✓	✓	✓	✓	

- The minimized DFA achieved is