NAME : JAWAD AHMED

ROLL NO : 20P - 0165

ASSIGNMENT : 01

Section : 2A

Dr. IKRAM - UL - HAQ

Q # 1

Taimur
20P-0165

1

(i) $x \frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0$

Solution:

* It is a non linear Differential equation and order of this ordinary differential is 3.

* The Non-Linearity is due to $\left(\frac{dy}{dx}\right)^4$

(ii) $\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r+u)$

Solution:

- Order of given differential equation is 2.
- Non Linear ordinary Differential Equation.

(iii) $\dfrac{d^2y}{dx^2} = \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$

2017-2016 Jawad

Solution

- Order of the given Differential equation is 2.

$$\frac{d^2y}{dx^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Taking square on B.S

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^2$$

$$\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right)^2 = 1$$

* So Non-Linear Ordinary differential equation.

Sol: $(y-x)$

Scanned with CamScanner

3

1) $(y-x)y' = y-x+8 \quad ; \quad y = x+4\sqrt{x+2}$

Sol:

$$y = x + 4\sqrt{x+2}$$

$$\frac{dy}{dx} = 1 + \cancel{4}^{2}\left(\frac{1}{\cancel{2}\sqrt{x+2}}\right)$$

$$\boxed{y' = 1 + \frac{2}{\sqrt{x+2}}}$$

20P-0165

Jawad

So,

$$= (\cancel{x} + 4\sqrt{x+2} - \cancel{x})\left(1 + \frac{2}{\sqrt{x+2}}\right) = \cancel{x} + 4\sqrt{x+2} - \cancel{x} + 8$$

$$(4\cancel{\sqrt{x+2}})\left(\frac{\sqrt{x+2}+2}{\cancel{\sqrt{x+2}}}\right) = 4\sqrt{x+2} + 8$$

$$4(\sqrt{x+2}+2) = 4(\sqrt{x+2}+2)$$

$$L.H.S = R.H.S$$

Hence Proved.

The $y = x+4\sqrt{x+2}$ is the solution of $(y-x)y' = y-x+8$.

Scanned with CamScanner

$y^3 \cos x \rightarrow$ 1 ; $y = (1-\sin x)^{-1/2}$

$(1-\sin x)^{-1/2}$ $\left(\because \frac{d}{dx}\sin x = \cos x\right)$ Solution //

$= \frac{-1}{2}(1-\sin x)^{-\frac{1}{2}-1}(-\cos x)$ $\cancel{\Rightarrow \frac{-1}{2}(-\cos x)}$

$\frac{-1}{2}(1-\sin x)^{-\frac{1}{2}-1}(-\cos x)$

$\boxed{\frac{1}{2}(1-\sin x)^{-3/2}(\cos x)}$ put in 1

$(1-\sin x)^{-3/2}\Big)\cos x = \left((1-\sin x)^{-1/2}\right)^3 \cos x$

$(1-\sin x)^{-3/2}\Big]\cos x = (1-\sin x)^{-3/2}\cos x$

HENCE L.H.S = R.H.S

Proved.

$xy' - 2y = 0 \rightarrow (1)$, $y = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$ (5)

Jawad
20P-0165

Solution

if $(x < 0)$

$\boxed{y = -x^2}$

$\boxed{y' = -2x} \Rightarrow$ put in (1)

$x(-2x) - 2(-x^2) = 0$

$-2x^2 + 2x^2 = 0$

$\boxed{0 = 0}$

if $(x \geq 0)$

$y = x^2 \Rightarrow y' = 2x$

$x(2x) - 2(x^2) = 0$

$\boxed{0 = 0}$

Hence Proved.

The given sol is the solution of Give ODE.

$xy' - 2y = 0 \rightarrow (1)$; $y = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$ (5)

(4)

Jawad
20P-0165

<u>Solution</u>

if $(x < 0)$

$\boxed{y = -x^2}$

$\boxed{y' = -2x} \rightarrow$ put in (1)

$x(-2x) - 2(-x^2) = 0$

$-2x^2 + 2x^2 = 0$

$\boxed{0 = 0}$

if $(x \geq 0)$

$y = x^2 \Rightarrow y' = 2x$

$x(2x) - 2(x^2) = 0$

$\boxed{0 = 0}$

<u>Hence Proved</u>

The given sol is the solution of Give ODE.

4. $2x\frac{dy}{dx} - y = 2x\cos x \quad , \quad y = \sqrt{x}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt$

* First derivative of y:

$$y' = \frac{1}{2}x^{-1/2}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt + \cancel{\sqrt{x}}\frac{\cos x}{\cancel{\sqrt{x}}}$$

$$y' = \frac{1}{2\sqrt{x}}\left(\int_4^x \frac{\cos t}{\sqrt{t}}\,dt\right) + \cos x$$

$$y' = \frac{1}{2x}\left(\sqrt{x}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt\right) + \cos x$$

$$\boxed{y' = \frac{y}{2x} + \cos x}$$

By Substituiting.

L.H.S of ODE

$$2x\frac{dy}{dx} = y - 2x\left(\frac{y}{2x} + \cos x\right) - y$$

$$2x\frac{dy}{dx} - y = \cancel{y} + 2x\cos x - \cancel{y}$$

Scanned with CamScanner

⑦

$$\boxed{2x \frac{dy}{dx} - y = 2x\cos x}$$

Since Sides are equal so solution is verified.

Jawad
20P-0165

Question #4

(i) $\frac{dy}{dx} = (x+1)^2$

$\int dy = \int dx(x+1)^2 \Rightarrow y = \frac{(x+1)^{2+1}}{2+1} + c$

By Integrating B.S

$$\boxed{y = \frac{1}{3}(x+1)^3 + c}$$

(ii) $e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$

$= e^x y \frac{dy}{dx} = e^{-y} + e^{-2x} \cdot e^{-y}$

$= e^x y \frac{dy}{dx} = e^{-y}(1 - e^{-2x})$

$$y\frac{dy}{e^y} = \frac{(1+e^{-2x})}{e^x}dx$$

$$\int y e^y \, dy = \int e^{-x} dx + \int e^{-3x} dx$$

$$y e^y - e^y = -e^{-x} - \frac{1}{3}e^{-3x} + c$$

Taking common e^y

$$\boxed{e^y(y-1) = -e^{-x} - \frac{1}{3}e^{-3x} + c}$$

→ General solution.

$$y - 1 = \frac{-e^{-x} - \frac{1}{3}e^{-3x} + c}{e^y}$$

$$\boxed{y = \frac{-e^{-x} - \frac{1}{3}e^{-3x} + c}{e^y} + 1}$$

(iii) $$\frac{dy}{dx} = \left(\frac{2y+3}{4x+5}\right)^2$$

$$\frac{dy}{dx} = \frac{(2y+3)^2}{(4x+5)^2}$$

$$\frac{dy}{(2y+3)^2} = \frac{dx}{(4x+5)^2}$$

Jawad
20P-0165

$$\frac{dy}{dx} = \frac{(2y+3)^2}{(4x+5)^2}$$

By Cross Multiplication

$$\frac{dy}{(2y+3)^2} = \frac{1}{(4x+5)^2} dx$$

$$(2y+3)^{-2} dy = (4x+5)^{-2} dx$$

Integrating B S

$$\int (2y+3)^{-2} dy = \int (4x+5)^{-2} dx$$

$$\frac{-1}{2(2y+3)} = -\frac{1}{4(4x+5)} + c$$

$\boxed{\text{Multiplying by } -4}$

$$-\cancel{4}^{-2} \times \frac{-1}{\cancel{2}(2y+3)} = -\cancel{4} \times \frac{-1}{\cancel{4}(4x+5)} + c$$

$$\frac{+2}{2y+3} = \frac{1}{4x+5} + c$$

Scanned with CamScanner

$$\boxed{\frac{2}{2y+3} = \frac{1}{4x+5} + c}$$

(iv) $x(1+y^2)^{1/2}\,dx = y(1+x^2)^{1/2}\,dy$

$$\frac{y}{(1+y^2)^{1/2}}\,dy = \frac{x}{(1+x^2)^{1/2}}\,dx$$

Integrating B.S

$$\int \frac{y}{(1+y^2)^{1/2}}\,dy = \int \frac{x}{(1+x^2)^{1/2}}\,dx$$

$$(1+y^2)^{1/2} = (1+x^2)^{1/2} + c$$

(v) $$\frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y+x-3}$$

Sol:

$$\frac{dy}{dx} = \frac{y(x+2)-1(x+2)}{y(x-3)+1(x-3)}$$

$$\frac{dy}{dx} = \frac{(y-1)(x+2)}{(y+1)(x-3)}$$

$$\frac{y+1}{y-1}\frac{dy}{dx} = \frac{x+2}{x+3} \Rightarrow$$

Jawad

20P-0165

$$\frac{y+1}{y-1}\,dy = \frac{x+2}{x-3}\,dx$$

Taking Integral on B.S

$$\int \frac{(y+1)\cancel{+0}}{y-1}\,dy = \int \frac{x+2}{x-3}\,dx$$

$$\int \frac{(y-1)+2}{y-1}\,dy = \int \frac{(x-3)+5}{x-3}\,dx$$

$$\int \left(1+\frac{2}{y-1}\right)dy = \int \left(1+\frac{5}{x-3}\right)dx$$

$$y + 2\ln(y-1) = x + 5\ln(x-3) + c$$

$$e^{y+2\ln(y-1)} = e^{x+5\ln(x-3)+c}$$

$$\boxed{e^{y}(y-1)^2 = c\,e^{x}(x-3)^5}$$

Q#4 (i) $\frac{dy}{dx} = \frac{x \tan^{-1} x}{y}$, $y(0) = 3$

$y\,dy = x \tan^{-1} x$

Integrating B.S

$$\int y\,dy = \int x \tan^{-1} x$$

$$\frac{y^2}{2} = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c$$

(multiplying by 2)

$$\boxed{y^2 = x^2 \tan^{-1} x + \tan^{-1} x - x + c} \rightarrow \text{General Solution}$$

Using Initial condition

$y(0) = 3$, $y = 3, x = 0$

$$(3)^2 = (0)^2 \tan^{-1} 0 + (0) - (0) + c$$

$$\boxed{c = 9}$$

$$y^2 = x^2 \tan^{-1} x + \tan^{-1} x - x + c$$

$$y = \pm\sqrt{x^2 \tan^{-1} x + \tan^{-1} x - x + c}$$

(1)

$$\boxed{y = \sqrt{x^2 \tan^{-1}x + \tan^{-1}x - x + 9}}$$

↳ Particular Solution

20P-0165
Jawad

(ii) $\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{y}$, $y(1) = 4$

$y\,dy = e^{\sqrt{x}}\,dx$

Taking Integral on B.S

$\int y\,dy = \int e^{\sqrt{x}}\,dx$

$\frac{y^2}{2} = 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

$$\boxed{y^2 = 4e^{\sqrt{x}}(\sqrt{x} - 1) + c_1} \rightarrow (1)$$

This is the General Solution.

Using Intial condition.

$y(1) = 4 \quad \because y = 4,\ x = 1$

So,

$(4)^2 = 4e^{\sqrt{1}}(\sqrt{1} - 1) + c_1$

$\boxed{c_1 = 16} \rightarrow$ put in (1)

$$\boxed{y^2 = 4e^{\sqrt{x}}(\sqrt{x}-1)+16}$$ → particular Solution

$$\boxed{y = \pm\sqrt{4e^{\sqrt{x}}(\sqrt{x}-1)+16}}$$

③ $\frac{dy}{dx} = y + \frac{y}{x\ln x}$, $y(e) = 1$

Sol: Separating the variables.

$$\frac{dy}{y} = \left(1 + \frac{1}{x\ln x}\right)dx$$

Integrating B S

$$\int \frac{dy}{y} = \int \left(1 + \frac{1}{x\ln x}\right)dx$$

$$\ln|y| = x + \ln|\ln x| + c$$

Now using the Initial condition to find the particular solution.

$y(e) = 1$, $x = e,\ y = 1$

$$\ln|1| = e + \ln|\ln e| + c \rightarrow ①$$

$\boxed{\because \ln e = 1}$

$\boxed{\because \ln 1 = 0}$

$$0 = e + \ln|1| + c$$

$$c + e = 0$$

Scanned with CamScanner

Jawad
20P-0165

$\boxed{c = -e}$

Now substituting $c = -e$ in eq ①

$$\ln|y| = x + \ln|\ln x| - e$$

$$\boxed{y = e^{x} + \ln x - e^{e}}$$

This is the particular solution.

Question # 5

Find the General sol of ODE

(i) $\cos x \frac{dy}{dx} + \sin x (y) = 1$

Dividing by $\cos x$

$$\frac{dy}{dx} + \frac{\sin x}{\cos x} y = \frac{1}{\cos x}$$

$$\frac{dy}{dx} + (\tan x) y = \sec x$$

Comparing with standarm. form

$p(x) = \tan x, \ f(x) = \sec x$

Now Finding Integrating Factor.

$$e^{\int p(x)} \Rightarrow e^{\int \tan x dx}$$

$= e^{\ln(\sec x)} \Rightarrow \sec x$

$\boxed{I.F = \sec x}$

Multiplying with the I.F

$$\sec x\left(\frac{dy}{dx} + (\tan x)\,y\right) = \sec x(\sec x)$$

$$\frac{d}{dx}\left((\sec x)(y)\right) = \sec^2 x$$

Integrating B.S

$$\int \cancel{d}\,(\sec x)y = \int \sec^2 x\,dx$$

$$(\sec x)y = \tan x + c$$

$$\boxed{y = \sin x + c\cos x}$$

This is the General Solution.

$\frac{dr}{d\theta} + r\sec\theta = \cos\theta$ (17)

Sol:

Comparing with standard form.

Jawad
20P-9b5

$p(x) = \sec\theta$

Find Integrating factor

$= e^{\int p(x)\,dx} \Rightarrow e^{\int \sec\theta\, d\theta}$

$= e^{\ln(\sec\theta + \tan\theta)\,d\theta}$

$$\boxed{I.F = \sec\theta + \tan\theta}$$

Multiplying with the I.F

$(\sec\theta + \tan\theta)\left(\frac{dr}{d\theta} + r\sec\theta\right) = (\sec\theta + \tan\theta)(\cos\theta)$ $\because \cancel{\tan\theta}$

$\frac{d}{d\theta}((\sec\theta + \tan\theta)r) = 1 + \sin\theta$

$\int d(\sec\theta + \tan\theta)r = \theta \int 1 + \sin\theta$ $\quad \because \int \sin\theta = -\cos$

$(\sec\theta + \tan\theta)r = \theta - \cos\theta + c$

$$\boxed{r = \frac{\theta - \cos\theta + c}{\sec\theta + \tan\theta}}$$

Unilpeul

(iii) $\frac{dp}{dt} + 2tp = p + 4t - 2$

Sol

$$\frac{dP}{dt} + (2t-1)p = 4t-2$$

$$p(x) = 2t-1,$$

Find Integrating Factor

$$= e^{\int p(x)} \Rightarrow e^{\int (2t-1)dt}$$

$$\boxed{I.F = e^{t^2-t}}$$ (Multiplying with IF)

$$= e^{t^2-t}\left(\frac{dP}{dt} + (2t-1)p\right)$$

$$= e^{t^2-t}(4t-2)$$

$$\boxed{\frac{d}{dt}(e^{t^2}-tp) = e^{t^2-t}(4t-2)}$$

$$\cancel{\int d(e^{t^2}-tp) = e^{t^2-t}(4t-2)}$$

$$\int d(e^{t^2}+p) = \int (4t-2)e^{t^2-t}\,dt$$

$$e^{t^2-t}p = 2e^{t^2-t} + c$$

$$\boxed{p = 2 + ce^{-(t^2-t)}}$$

$\frac{dT}{dt} = k(T - Tm)$, $T(0) = T_0$

Jawad
20P-0165

Sol

$$\frac{dT}{dt} - kT = -kTm$$

$$I.F = e^{\int -k\,dt}$$

$$\boxed{I.F = e^{-kt}}$$

$$e^{-kt}\left(\frac{dT}{dt} - kT\right) = e^{-kt}(-kTM)$$

$$\frac{d}{dt}\left(e^{-kt}\,T\right) = -kTme^{-kt}$$

$$\int d\left(e^{-kt}\,T\right) = \int -kTme^{-kt}\,dt$$

$$e^{-kt}T = -kTm - \frac{1}{k}e^{-kt} + c$$

$$T = Tm + Ce^{kt}$$

Use the Intial Condition

$$T(0) = T_0$$

$$T_0 = Tm + c$$

$$\boxed{c = T_0 - Tm}$$

$$\boxed{T = T_m + (\ldots}$$

(v) $L\dfrac{di}{dt} + Ri = E$ $\quad i(0) = i_0,$

L, R, E and i_0 are constant.

Dividing Both sides by L.

$$\frac{di}{dt} + \frac{Ri}{L} = \frac{E}{L}$$

$$I.F = e^{\int \frac{R}{L} t}$$

$$I.F = e^{R/L\, t}$$

$$e^{\frac{R}{L}t}\left(\frac{di}{dt} + \frac{R}{L}i\right) = e^{\frac{R}{L}t}\left(\frac{E}{L}\right)$$

$$\boxed{\frac{d}{dt}\left(e^{R/L\, t}\, i\right) = \frac{E}{L}e^{R/L\, t}}$$

$$\int d\left(e^{R/L\, t} i\right) = \int \frac{E}{L} e^{R/L\, t}\, dt$$

$$e^{R/L\, t}\, i = \frac{E}{L} \cdot \frac{L}{R} e^{R/L\, t} + c$$

Scanned with CamScanner

Multiplying the I F we get

$$\frac{dy}{dx} e^{x^2} + 2e^{x^2} x \cdot y = xe^{x^2}$$

Javad
20P-0165

$$\frac{d}{dx}(e^{x^2} y) = xe^{x^2}$$

$$\int \left[\frac{d}{dx} [e^{x^2} \cdot y) \right] dx = \int xe^{x^2} dx$$

$\boxed{\text{Let } x^2 = u}$ u is a dummy variable.

$$e^{x^2} y = \frac{1}{2} \int e^{u} du$$

$$e^{x^2} y = \frac{1}{2} e^{u} + c$$

$$\boxed{e^{x^2} y = \frac{1}{2} e^{x^2} + c}$$

This is a general solution.

$$\boxed{i = \frac{E}{R} + ce^{-\frac{R}{L}t}}$$ Solution

Now For Particular Solution.

$$i(0) = i_0$$

$$i_0 = \frac{E}{R} + c$$

$$c = i_0 - \frac{E}{R}$$

$$\boxed{i = \frac{E}{R} + \left(i_0 - \frac{E}{R}\right) e^{-R/L\, t}}$$

(vii) $\frac{dy}{dx} + 2xy = f(x)$, $y(0) = 2$

where $f(x) = \begin{cases} x & 0 \le x < 1 \\ 0 & x \ge 1 \end{cases}$

$$\frac{dy}{dx} + 2xy = x$$

Comparing with the standard form

$$p(x) = 2x$$

$$I.F = e^{\int p(x)} = e^{\int 2x}$$

$$\Rightarrow e^{\cancel{2}\frac{x^2}{\cancel{2}}} = e^{x^2}$$

$$\boxed{I.F = e^{x^2}}$$

For Particular Solution.

Using Intial Condition

$y(0) = 2$

$$\boxed{C = \frac{3}{2}}$$

Jawad
20P-0165

$$y(1) = \frac{1}{2} + \frac{3}{2} e^{(-1)^2}$$

$$\frac{dy}{du} e^{u^2} + 2ue^{u^2} \cdot y = 0 \cdot e^{u^2}$$

$$\int \left(\frac{d}{du} \left(e^{u^2} y \right) \right] du = \int 0 \, du$$

$$e^{u^2} \cdot y = c$$

$$\boxed{y = c e^{-u^2}}$$ → general form.