Assignment # 01.

Note: ~~Date~~ Due date of submission is 5/04/2021.

Q1: State the order of the given ordinary differential equation. Also Check whether the equation is linear or non-linear.

(i) $$\frac{x\,d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0$$

(ii) $$\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r+u)$$

(iii) $$\frac{d^2y}{dx^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Q2: Verify that the indicated function $y = \phi(x)$ is the solution or not of the given ODE.

(i) $(y-x)y' = y-x+8$; $y = x+4\sqrt{x+2}$

(ii) $2y' = y^3\cos x$; $y = (1-\sin x)^{-1/2}$

(iii) $xy' - 2y = 0$; $y = \begin{cases} -x^2 & x<0 \\ x^2 & x \geq 0 \end{cases}$

(iv) $2x\frac{dy}{dx} - y = 2x\cos x$; $y = \sqrt{x}\int_4^x \frac{\cos t}{\sqrt{t}}\,dt$

Q3: Solve the given ODE by separation of variables

(i) $\dfrac{dy}{dx} = (x+1)^2$ (ii) $e^x y \dfrac{dy}{dx} = e^{-y} + e^{-2x-y}$

(iii) $\dfrac{dy}{dx} = \left(\dfrac{2y+3}{4x+5}\right)^2$ (iv) $x(1+y^2)^{1/2}\,dx = y(1+x^2)^{1/2}\,dy$

(v) $$\frac{dy}{dx} = \frac{xy+2y-x-2}{xy-3y+x-3}$$

Q4: Find the explicit solutions of the following ODE.

(i) $\dfrac{dy}{dx} = \dfrac{x\tan^{-1}x}{y}$ $y(0) = 3$.

(ii) $\dfrac{dy}{dx} = \dfrac{e^{\sqrt{x}}}{y}$ $y(1) = 4$.

(iii) $\dfrac{dy}{dx} = y + \dfrac{y}{x\ln x}$. $y(e) = 1$

Q:5:– Find the General solution of the following ODE.

(i) $\cos x \dfrac{dy}{dx} + (\sin x)y = 0$

(ii) $\dfrac{dr}{d\theta} + r\sec\theta = \cos\theta$.

(iii) $\dfrac{dP}{dt} + 2tP = P + 4t - 2$

(iv) $\frac{dT}{dt} = K(T - T_m)$ $T(0) = T_0$, where K, T_0 and T_m are constants.

(v) $L\frac{di}{dt} + Ri = E$ $i(0) = i_0$ L, R, E and i_0 are constant

(vi) Solve $\frac{dy}{dx} + p(x)y = 4x$ $y(0) = 3$ where

$$p(x) = \begin{cases} 2 & 0 \leq x \leq 1 \\ -\frac{2}{x} & x > 1 \end{cases}$$

(vii) $\frac{dy}{dx} + 2xy = f(x)$ $y(0) = 2$. where

$$f(x) = \begin{cases} x & 0 \leq x < 1 \\ 0 & x \geq 1 \end{cases}$$

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