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ON: 2A

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Integrating w.r.t (y)

$$\boxed{g(y) = \frac{3y^2}{2} + 7y}$$ ⇒ put in (A)

$$f(x,y) = x^2 + \frac{3y^2}{2} + 7y$$

Assignment.

1. $(2x-1)\,dx + (3y+7)\,dy = 0$

2. $M(x,y) = 2x-1, \quad N(x,y) = 3y+7$

$$\frac{\partial M}{\partial y} = 2x-1 \Rightarrow 0$$

$$\frac{\partial N}{\partial x} = 3y+7 \Rightarrow 0$$

Hence

$$\frac{\partial M}{\partial y} = 0 \text{ and } \frac{\partial N}{\partial x} = 0 \text{ Hence}$$

the equation is Exact.

$$\frac{\partial f}{\partial x} = 2x - 1 \rightarrow ① , \quad \frac{\partial f}{\partial y} = 3y+7 \rightarrow ②$$

Integrating eq ① w.r.t (x)

$$f(x,y) = 2\int x - \int 1\,dx$$

$$\cancel{f(x,y) = \frac{x^2}{2} - x + g(y)}$$

$$f(x,y) = x^2 - x + g(y) \rightarrow (A)$$

Integrating w.r.t (y) eq (A)

$$\boxed{\frac{\partial f}{\partial y} = g'(y)} \rightarrow$$ Putting value of $\frac{\partial f}{\partial y}$

$$3y + 7 = g'(y) \rightarrow$$ Integrating w.r.t (y)

$$\boxed{g(y) = 3\left(\frac{y^2}{2}\right) + 7y} \rightarrow$$ put in (A)

$$f(x,y) = x^2 - x + \frac{3y^2}{2} + 7y$$

So solution in implicity form $f(x,y) = c$

$$\boxed{c = x^2 - x + \frac{3y^2}{2} + 7y} \rightarrow$$ Implicit Solution.

$(2x+y)dx - (x+6y)dy = 0$

$M(x,y) = 2x+y$, $N(x,y) = -(x+6y)$

$= 1$, $\boxed{\frac{\partial N}{\partial x} = -1}$

$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ So Equation is not Exact.

$(5x+4y)dx + (4x-8y^3)dy = 0$

$M(x,y) = 5x+4y$, $N(x,y) = 4x-8y^3$

$= 4$, $\boxed{\frac{\partial N}{\partial x} = 4}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, Hence Equation is Exact.

Integrating Eq 1 w.r.t (x)

$$f(x,y) = 5\int x\,dx + 4y\int dx$$

$$f(x,y) = 5\frac{x^2}{2} + 4yx + g(y) \rightarrow (b)$$

Differenshiating ~~Integrating~~ (b) w.r.t (y)

$$\frac{\partial f}{\partial y} = 4x + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$.

$$\cancel{4x} - 8y^3 = \cancel{4x} + g'(y)$$

$$g'(y) = -8y^3 \rightarrow \text{Integrating w.r.t (y)}$$

$$g(y) = -8\int y^3\,dy$$

$$g(y) = -\cancel{8}^{2}\frac{y^4}{\cancel{4}}$$

$$\boxed{g(y) = -2y^4} \rightarrow \text{put in (b)}$$

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$$\boxed{F(x,y) = \frac{5}{2}x^2 + 4xy - 2y^4}$$

Implicit solution is $F(x,y) = c$

$$\boxed{c = \frac{5}{2}x^2 + 4xy - 2y^4}$$

4. $(\sin y$ [illegible] y [illegible]

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$f(x,y)$

4. $(\sin y - y\sin x)dx + (\cos x + x\cos y - y)dy = 0$

Sol

$M(x,y) = \sin y - y\sin x$

$N(x,y) = \cos x + x\cos y - y$

$$\boxed{\frac{\partial M}{\partial y} = \cos y - \sin x}$$

$$\boxed{\frac{\partial N}{\partial x} = -\sin x + \cos y}$$

Hence $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ so equation is exact.

$\frac{\partial f}{\partial x} = \sin y - y\sin x \rightarrow (1)$, $\frac{\partial f}{\partial y} = \cos x + x\cos y - y \rightarrow (2)$

Integrating eq (1) w.r.t (x)

$\int \frac{\partial f}{\partial x} dx = \int \sin y\, dx - \int y \sin x\, dx$

$\sin y\, x = y \int \sin x\, dx + g(y)$

$= x \sin y = y \cos x + g(y)$ (A)

shiating eq (A) w.r.t (y)

$x \cos y = \cos x + g'(y)$

Putting value of $\frac{\partial f}{\partial y}$

$+ x \cos y - y = x \cos x - \cos x + g'(y)$

$\cos x - y = g'(y)$

$-g'(y) + 2\cos x - y = 0$

$-g'(y) = -2\cos x + y$

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$$\boxed{g(y) = 2\cos xy - \frac{y^2}{2}} \rightarrow \text{put in (A)}$$

$$F(x,y) = x\sin y - y\cos x + 2\cos xy - \frac{y^2}{2}$$

$$f(x,y) = x\sin y + \cos xy - \frac{y^2}{2}$$

Implicit solution $F(x,y) = c$

$$\boxed{c = x\sin y + \cos xy - \frac{y^2}{2}} \rightarrow \text{Implicit Solution}$$

5. $(2xy^2 - 3)dx + (2x^2y + 4)dy = 0$

Sol

$$M(x,y) = 2xy^2 - 3$$

$$N(x,y) = 2x^2y + 4$$

$$\frac{\partial M}{\partial y} = \frac{d}{dy}\left[2xy^2 - 3\right]$$

$$\frac{\partial N}{\partial x} = \left[2x^2y + 4\right.$$

$$\Rightarrow 2y(2)x$$

$$\frac{\partial M}{\partial y} = 2x(2)y$$

$$\boxed{\frac{\partial M}{\partial y} = 4xy} \qquad \boxed{\frac{\partial N}{\partial x} = 4xy}$$

Hence Equation is Exact.

$$\frac{\partial f}{\partial x} = 2xy^2 - 3 \rightarrow (1), \qquad \frac{\partial f}{\partial y} = 2x^2y + 4 \rightarrow (2)$$

Integrating Eq (1) w.r.t (x)

$$\int \frac{\partial f}{\partial x} = \int 2xy^2 - 3$$

$$F(x,y) = 2y^2 \int x\,dx - 3\int dx$$

$$= \cancel{2}y^2 \frac{x^2}{\cancel{2}} - 3x + g(y)$$

$$F(x,y) = y^2x^2 - 3x + g(y) \rightarrow (A)$$

Differentiating w.r.t (y)

$$\frac{\partial f}{\partial y} = x^2 2y + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$2\cancel{x^2}y + 4 = 2\cancel{x^2}y + g'(y)$$

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$g'(y) = 4$

Integrating w.r.t (y)

$$\int g'(y)\,dy = 4\int dy$$

$$\boxed{g(y) = 4y} \rightarrow \text{put in (A)}$$

$$F(x,y) = y^2x^2 - 3x + 4y$$

$$\cancel{f(x,y) = y^2x^2 + x}$$

$$F(x,y) = x^2y^2 - 3x + 4y$$

~~Imp~~ Implicit solution will be

$$\boxed{c = x^2y^2 - 3x + 4y}$$

6. $\left(2y - \frac{1}{x} + \cos 3x\right)\frac{dy}{dx} + \frac{y}{x^2} - 4x^3 + 3y\sin 3x = 0$

~~Div~~ Multiplying with dx we get

$$\left(2y - \frac{1}{x} + \cos 3x\right)dy + \left[\frac{y}{x^2} - 4x^3 + 3y\sin 3x\right]dx$$

$M(x,y) = 2y - \frac{1}{x} + \cos 3x \rightarrow$ Differentiating w.r.t (y)

$N(x,y) = \frac{y}{x^2} - 4x^3 + 3y\sin 3x \rightarrow (2)$

$$\boxed{\frac{\partial M}{\partial y} = 2}$$

Differentiating eq (2) w.r.t (x)

$$\frac{\partial N}{\partial x} = y - 2(x)^{-3} - 4(3)(x)^{3-1} + 3y\cos 3x(3)$$

$$\boxed{\frac{\partial N}{\partial x} = y - 2x^{-3} - 12x^2 + 9y\cos 3x}$$

Hence Equation is not exact

Because $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

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7. $(x^2 - y^2)\,dx + (x^2 - 2xy)\,dy = 0$

$M(x,y) = x^2 - y^2 \rightarrow (1)$

$N(x,y) = x^2 - 2xy \rightarrow (2)$

Differentiating eq (1) w.r.t (y)

$$\boxed{\frac{\partial M}{\partial y} = -2y}$$

Differentiating eq (2) w.r.t (x)

$$\frac{\partial N}{\partial x} = 2x - 2y \quad (1)$$

$$\boxed{\frac{\partial N}{\partial x} = 2x - 2y}$$

As $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ so

equation is not exact.

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8. $\left(1+\ln x+\frac{y}{x}\right)dx=(1-\ln x)dy$

Sol

$$\left(1+\ln x+\frac{y}{x}\right)dx-(1-\ln x)dy=0$$

$$M(x,y)=1+\ln x+\frac{y}{x} \rightarrow (1)$$

$$N(x,y)=-(1-\ln x)\,dy$$

$$N(x,y)=(-1+\ln x) \rightarrow (2)$$

Differentiating eq (1) w.r.t (y)

$$\frac{\partial M}{\partial y}=\frac{1}{x}(1) \Rightarrow \frac{1}{x} \Rightarrow x^{-1}$$

Differentiating eq (2) w.r.t (x)

$$\frac{\partial N}{\partial y}=\frac{1}{x}=x^{-1}$$

As $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial x}$ so

Equation is Exact.

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$$\frac{\partial f}{\partial x} = 1 + \ln x + \frac{y}{x} \rightarrow (1) \quad , \quad \frac{\partial f}{\partial y} = -(1 - \ln y) \rightarrow (2)$$

Integrating eq (1) w.r.t (x)

$$f(x,y) = x + \frac{1}{x} + y \ln x + g(y) \rightarrow (3)$$

Differenshiating w.r.t (y)

$$\frac{\partial f}{\partial y} = 0 \ln x + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$-1 + \cancel{\ln x} = \cancel{\ln x} + g'(y)$$

$g'(y) = -1$ (Integrating w.r.t y)

$\int g'(y) = \int -1 \, dy$

$\boxed{g(y) = -y} \rightarrow$ put in (3)

$$f(x,y) = x + \frac{1}{x} + y \ln x + (-y)$$

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$f(x,y) = x + \frac{1}{x} + y\ln x - y$

$$\boxed{c = x + \frac{1}{x} + y\ln x - y} \rightarrow \text{Implicit solution.}$$

9. $(x - y^3 + y^2\sin x)dx = (3xy^2 + 2y\cos x)dy$

Sol

$(x - y^3 + y^2\sin x)dx - (3xy^2 + 2y\cos x)dy = 0$

$M(x,y) = x - y^3 + y^2\sin x \rightarrow ①$

$N(x,y) = -(3xy^2 + 2y\cos x) \rightarrow ②$

Differenshiating eq ① w.r.t (y)

$\frac{\partial M}{\partial y} = -3y^2 + \sin x\, 2y \Rightarrow 2y\sin x - 3y^2$

Differentiating eq ② w.r.t (x)

$\frac{\partial N}{\partial x} = -3xy^2 - 2y\cos x$

$= -3y^2 - 2y(-\sin x)$

$$\boxed{\frac{\partial N}{\partial x} = -3y^2 + 2y\sin x}$$

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Hence Equation is Exact

$$\frac{\partial f}{\partial x} = x - y^3 + y^2 \sin x \rightarrow (1) \quad , \quad \frac{\partial f}{\partial y} = -(3xy^2 + 2y\cos x) \rightarrow (2)$$

Integrating eq (1) w.r.t (x)

$$f(x,y) = \frac{x^2}{2} - y^3 x + y^2 \cos x + g(y) \rightarrow (A)$$

Differentiating w.r.t (y)

$$\frac{\partial f}{\partial y} = -x\,3y^2 + \cos x\, 2y + g'(y)$$

$$\frac{\partial f}{\partial y} = -3xy^2 + 2y\cos x + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$\cancel{-3xy^2} - 2y\cos x = \cancel{-3xy^2} + 2y\cos x + g'(y)$$

$$-4y\cos x = g'(y)$$

$$g'(y) = -4y\cos x$$

Integrating w.r.t (y)

$$g(y) = -4\int y\cos x\, dy$$

$$g(y) = -4\left[y\int \cos u\, dy = \right]\left[\right.$$

$$g(y) = -\cancel{4}^{2}\cos u \cdot \frac{y^2}{\cancel{2}}$$

$$\boxed{g(y) = -2y^2\cos u} \rightarrow \text{put in (A)}$$

$$f(u,y) = \frac{u^2}{2} - y^3u + y^2\cos u - 2y^2\cos u$$

$$f(u,y) = \frac{u^2}{2} - y^3u - y^2\cos u$$

Implicit Solution will be.

$$\boxed{c = \frac{u^2}{2} - y^3u - y^2\cos u}$$

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10. $(x^3+y^3)\,dx + 3xy^2\,dy = 0$

Sol:

$M(x,y) = x^3+y^3 \rightarrow (1)$

$N(x,y) = 3xy^2 \rightarrow (2)$

Differentiating eq (1) w.r.t (y)

$$\boxed{\frac{\partial M}{\partial y} = 3y^2}$$

Differentiating eq (2) w.r.t (x)

$$\boxed{\frac{\partial N}{\partial x} = 3y^2}$$

Hence Equation is Exact.

$\frac{\partial f}{\partial x} = x^3+y^3 \rightarrow (1)$, $\frac{\partial f}{\partial y} = 3xy^2 \rightarrow (2)$

Integrating eq (1) w.r.t (x)

$f(x,y) = \frac{x^4}{4} + y^3x + g(y)$

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10. $(x^3+y^3)\,dx + (3xy^2)\,dy = 0$

$$f(x,y) = \frac{1}{4}x^4 + y^3x + g(y)$$

Differenshiating w.r.t (y)

$$\cancel{\frac{\partial f}{\partial y} = \frac{1}{4}x^4 + y^3x + g'(y)}$$

$$\cancel{\frac{\partial f}{\partial y} = \frac{1}{4} \times x \frac{y^4}{4} + g}$$

$$\frac{\partial f}{\partial y} = 3y^2x + g(y)$$

$$\cancel{3xy^2} = \cancel{3xy^2} + g(y)$$

$$g(y) = 0$$

$$\boxed{f(x,y) = \frac{1}{4}x^4 + y^3x}$$

$$\boxed{C = \frac{1}{4}x^4 + y^3x}$$

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12. $(3x^2y + e^y)\,dx + (x^3 + xe^y - 2y)\,dy = 0$

$M(x,y) = 3x^2y + e^y$

$N(x,y) = x^3 + xe^y - 2y$

$\boxed{\frac{\partial M}{\partial y} = 3x^2 + e^y}$ $\boxed{\frac{\partial N}{\partial x} = 3x^2 + e^y}$

Hence Equations is Exact.

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$\frac{\partial f}{\partial x} = 3x^2y + e^y \rightarrow (1)$, $\frac{\partial f}{\partial y} = x^3 + xe^y - 2y \rightarrow (2)$

Integrating eq (1) w.r.t (x)

$f(x,y) = 3y\int x^2\,dx + e^y x + g(y)$

$f(x,y) = \cancel{3}y\frac{x^3}{\cancel{3}} + e^y x + g(y)$

$f(x,y) = x^3y + e^y x + g(y) \longrightarrow (A)$

Differentiating w.r.t (y)

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$$\frac{\partial f}{\partial y} = x^3 + xe^y + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$\cancel{x^3} + \cancel{xe^y} - 2y = \cancel{x^3} + \cancel{xe^y} + g'(y)$$

$$g'(y) = -2y$$

Integrating w.r.t (y)

$$\cancel{\boxed{g(y) = -2}} \rightarrow \text{put in (A)}$$

$$g(y) = -\cancel{2}\,\frac{y^2}{\cancel{2}}$$

$$f(x,y) = x^3y + e^y x - y^2$$

$$\boxed{C = x^3 y + e^y x - y^2}$$

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$$\frac{3}{\ } + x\frac{dy}{dx} + y = \frac{3}{x} - 1$$

$$\frac{dy}{dx} = \frac{3}{x} - 1 - y$$

$$dy = \left(\frac{3}{x} - 1 - y\right)dx$$

$$= -\left[\frac{3}{x} - 1 - y\right]$$

$$= 1 - \frac{3}{y} + x$$

$$-(-1) \Rightarrow 1 \qquad \boxed{\frac{\partial N}{\partial x} = 1}$$

Hence $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ so is exact.

$$-\left[\frac{3}{x} - 1 - y\right]dy$$

Integrating eq ① w.r.t x

$$f(x,y) = \int \left[-\frac{3}{x} + 1 + y \right] dx$$

$$f(x,y) = -3\ln x + x + yx + g(y) \quad (A)$$

Differenshiating w.r.t (y)

$$\frac{\partial f}{\partial y} = x + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$1 - \frac{3}{y} + \cancel{x} = \cancel{x} + g'(y)$$

$$g'(y) = 1 - \frac{3}{y}$$

Integrating w.r.t (y)

$$\boxed{g(y) = y - 3\ln|y|} \rightarrow \text{put in (A)}$$

$$f(x,y) = -3\ln x + x + yx + y - 3\ln|y|$$

$$f(x,y) = -3(\ln x + \ln y) + x + xy + y$$

$$\boxed{f(x,y) = -3\ln xy + x + xy + y}$$

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15. $\left(x^2y^3 - \frac{1}{1+9x^2}\right)\frac{dx}{dy} + x^3y^2 = 0$

$$\left(x^2y^3 - \frac{1}{1+9x^2}\right)dx + \left(x^3y^2\right)dy = 0$$

$$M(x,y) = x^2y^3 - \frac{1}{1+9x^2}$$

$$\frac{\partial M}{\partial y} = x^3(3)y^2 = 3x^3y^2$$

$$\boxed{\frac{\partial M}{\partial y} = 3x^3y^2}$$

$$N(x,y) = x^3y^2$$

$$\boxed{\frac{\partial M}{\partial y} = 3x^2y^2}$$

Hence the Given Equation is Exact

$$\frac{\partial f}{\partial x} = x^2y^3 - \frac{1}{1+9x^2} \longrightarrow (1)$$

$$\frac{\partial f}{\partial y} = x^3y^2 \longrightarrow (2)$$

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Integrating eq ② w.r.t (y)

$$f(x,y) = x^3 \frac{y^3}{3} + g(x)$$

Differentiating w.r.t (y)

$$\cancel{\frac{\partial f}{\partial y} = \frac{x^3}{\cancel{3}} \cancel{3}(y^2) + g'(y)}$$

$$\cancel{\frac{\partial f}{\partial y} = y^2 x^3 + g'(y)}$$

$$\frac{\partial f}{\partial x} = \frac{y^3 \cancel{3} x^2}{\cancel{3}} + g'(x)$$

$$\frac{\partial f}{\partial x} = y^3 x^2 + g'(x)$$

$$\cancel{x^2 y^3} - \frac{1}{1+9x^2} = \cancel{y^3 x^2} + g'(x)$$

$$g'(x) = -\frac{1}{1+9x^2}$$

Integrating w.r.t (x)

$$g(x) = -\frac{1}{18} \int \frac{18}{1+9x^2}$$

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16. $(5y-2x)\,y' - 2y = 0$

$$(5y-2x)\frac{dy}{dx} - 2y = 0$$

$$(5y-2x)\,dy - 2y\,dx = 0$$

$$M(x,y) = -2y$$

$$N(x,y) = 5y - 2x$$

$$\boxed{\frac{\partial M}{\partial y} = -2} \quad \boxed{\frac{\partial N}{\partial x} = -2}$$

Hence the given DE is Exact.

$$\frac{\partial f}{\partial x} = -2y \rightarrow (1) \quad , \quad \frac{\partial f}{\partial y} = 5y - 2x \rightarrow (2)$$

Integrating (1) w.r.t (x)

$$f(x,y) = -2yx + g(y) \rightarrow (A)$$

Differentiating w.r.t (y)

$$\frac{\partial f}{\partial y} = -2x + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$5y - \cancel{2x} = -\cancel{2x} + g'(y)$$

$$5y = g'(y)$$

Integrating w.r.t (y)

$$\boxed{g(y) = \frac{5}{2}y^2} \rightarrow \text{put in (A)}$$

$$\boxed{f(x,y) = -2xy + \frac{5}{2}y^2}$$

$$\boxed{C = -2xy + \frac{5}{2}y^2}$$

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17. $(\tan x - \sin x \sin y)\,dx + \cos x \cos y\,dy = 0$

Sol

$M(x,y) = \tan x - \sin x \sin y$

$N(x,y) = \cos x \cos y$

$$\boxed{\frac{\partial M}{\partial y} = -\sin x \cos y}$$

$$\boxed{\frac{\partial N}{\partial x} = \cos y - \sin x} \Rightarrow \boxed{\frac{\partial N}{\partial x} = -\sin x \cos y}$$

Hence the given DE is Exact.

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$\frac{\partial f}{\partial x} = \tan x - \sin x \sin y$$

$$\frac{\partial f}{\partial y} = \cos x \cos y \longrightarrow (2)$$

Integrating eq (2) w.r.t (y)

$$f(x,y) = \int \cos x \cos y\,dy + g(x)$$

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$$f(x,y) = \cos y \int \cos x \, dy + g(x)$$

$$f(x,y) = \cos y \sin x + g(x) \rightarrow (A)$$

Differenshiating w.r.t (x)

$$\frac{\partial f}{\partial x} = \cos y \sin x + g'(x)$$

Putting value of $\frac{\partial f}{\partial x}$

$$\tan x - \sin x \sin y = \cos y \sin x + g'(x)$$

$$g'(x) = -\cos y \sin x - \sin x \sin y + \tan x$$

Integrating

~~Differenshiating~~ w.r.t (x)

$$g(x) = -\cos y(-\cos x) - \sin y(-\cos x) + \sec^2 x$$

$$\boxed{g(x) = \cos x \cos y + \sin y \cos x + \sec^2 x}$$

~~F(x)~~

$$\boxed{f(x,y) = \cos y \sin x + \cos x \sin y + \cos x \cos y + \sec^2 x}$$

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13. $x\dfrac{dy}{dx} = 2xe^x - y + 6x^2$

Sol

$$x\,dy = (2xe^x - y + 6x^2)\,dx$$

$$(x)dy - (2xe^x - y + 6x^2)\,dx = 0$$

$$M(x,y) = -(2xe^x - y + 6x^2) \rightarrow (1)$$

$$N(x,y) = x \rightarrow (2)$$

$\dfrac{\partial M}{\partial y}$ Differentiating eq (1) w.r.t (y)

$$\frac{\partial M}{\partial y} = -2xe^x + y - 6x^2$$

$$\boxed{\frac{\partial M}{\partial y} = 1}$$

So given equation is Exact DE.

$$\boxed{\frac{\partial N}{\partial x} = 1}$$

$$\frac{\partial f}{\partial x} = -2xe^x + y - 6x^2 \rightarrow (3)$$

$$\frac{\partial f}{\partial y} = x \rightarrow (4)$$

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Integrating eq ① w.r.t (y)

$$f(x,y) = xy + g(x)$$

Differentiating w.r.t (x)

$$\frac{\partial f}{\partial x} = y + g'(x)$$

Putting value of $\frac{\partial f}{\partial x}$

$$-2xe^x + \cancel{y} - 6x^2 = \cancel{y} + g'(x)$$

$$g'(x) = -2xe^x - 6x^2$$

Integrating w.r.t (x)

$$g(x) = -2\int xe^x dx - \cancel{6}^2 \frac{x^3}{\cancel{3}}$$

$$g(x) = -2\int xe^x dx - 2x^3$$

$$= -2\left[x\int e^x dx - \int \frac{d}{dx}x \int e^x dx\right] dx - 2x^3$$

$$= -2\left(xe^x - \int e^x\right) - 2x^3$$

$$g(x) = -2xe^x + 2e^x - 2x^3$$

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$$\boxed{F(x,y) = xy - 2xe^{y} + 2e^{y} - 2x^{3}}$$

19. $(4t^3y - 15t^2 - y)\,dt + (t^4 + 3y^2 - t)\,dy = 0$

$M(t, y) = 4t^3y - 15t^2 - y$

Diff w.r.t (y)

$$\boxed{\frac{\partial M}{\partial y} = 4t^3 - 1}$$

$N(t, y) = t^4 + 3y^2 - t$

Diff w.r.t (t)

$$\boxed{\frac{\partial N}{\partial t} = 4t^3 - 1}$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}}$$

So given DE is Exact.

$\frac{\partial f}{\partial t} = 4t^3y - 15t^2 - y \rightarrow (1)$

$\frac{\partial f}{\partial y} = t^4 + 3y^2 - t \rightarrow (2)$

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Integrating eq ② w.r.t (y)

$$f(x,y) = t^4y + \frac{\cancel{3}y^3}{\cancel{3}} - ty + g(t) \rightarrow ①$$

$$f(x,y) = t^4y + y^3 - ty + g(t)$$

Diff w.r.t (t)

$$\frac{\partial f}{\partial t} = 4t^3y - y + g'(t)$$

putting $\frac{\partial f}{\partial t}$ value

$$\cancel{4t^3y} - 15t^2 - \cancel{y} = g'(t) + \cancel{4t^3y} - \cancel{y}$$

$$g'(t) = -15t^2$$

Integrating w.r.t (t)

$$g(t) = -\cancel{15}^{5}\frac{t^3}{\cancel{3}}$$

$$\boxed{g(t) = -5t^3} \rightarrow \text{put in ①}$$

$$\boxed{f(x,y) = t^4y + y^3 - ty - 5t^3}$$

$$\boxed{C = t^4y + y^3 - ty - 5t^3}$$

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$y(1) = 1$

21. $(x+y)^2 dx + (2xy + x^2 - 1) dy = 0$

$M(x,y) = (x+y)^2$

$M(x,y) = x^2 + y^2 + 2xy$

$$\boxed{\frac{\partial M}{\partial y} = 2y + 2x}$$

$N(x,y) = 2xy + x^2 - 1$

$$\boxed{\frac{\partial N}{\partial x} = 2y + 2x}$$

As $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ so given

DE is Exact.

$\frac{\partial f}{\partial x} = x^2 + 2xy + y^2 \rightarrow (1)$, $\frac{\partial f}{\partial y} = 2xy + x^2 - 1 \rightarrow (2)$

Integrating

~~Diff~~ eq (1) w.r.t (x)

$f(x,y) = \frac{x^3}{3} + 2y\frac{x^2}{2} + y^2 x + g(y)$

$f(x,y) = \frac{1}{3}x^3 + yx^2 + y^2 x + g(y) \rightarrow (1)$

Diff eq (A) w.r.t (y)

$$\frac{\partial f}{\partial y} = x^2 + 2yx + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$2\cancel{x}y + \cancel{x^2} - 1 = \cancel{x^2} + 2\cancel{x}y + g'(y)$$

$$g'(y) = -1$$

Integrating w.r.t (y)

$$\boxed{g(y) = -y} \rightarrow \text{put in A)}$$

$$f(x,y) = \frac{1}{3}x^3 + yx^2 + y^2x - y$$

put $x = 1, y = 1$

$$c = \frac{1}{3}(1)^3 + (1)(1)^2 + (1)^2(1) - \cancel{1}$$

$$\frac{1}{3} + 1$$

$$c = \frac{1+3}{3}$$

$$\boxed{c = \frac{4}{3}}$$

$$\boxed{\frac{1}{3}x^3 + yx^2 + y^2x - y = \frac{4}{3}}$$

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22. $(e^x + y)\,dx + (2 + x + ye^y)\,dy = 0$, $y(0) = 1$

Sol

$M(x,y) = e^x + y$

$$\boxed{\frac{\partial M}{\partial y} = 1}$$

$N(x,y) = (2 + x + ye^y)$

$$\boxed{\frac{\partial N}{\partial x} = 1}$$

Hence given DE is Exact.

$\frac{\partial f}{\partial x} = e^x + y \rightarrow (1)$, $\frac{\partial f}{\partial y} = 2 + x + ye^y \rightarrow (2)$

Integrating eq (1) w.r.t (x)

$f(x,y) = e^x + yx + g(y) \rightarrow (A)$

Diff w.r.t (y)

$$\frac{\partial f}{\partial y} = x + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$2 + \cancel{x} + ye^y = \cancel{x} + g'(y)$

$$\boxed{g'(y) = 2 + ye^y}$$

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Integrating w.r.t (y)

$$g(y) = 2y + [y]e^y + \int\left[\left[\frac{d}{dy}y\right]e^y\right]dy$$

$$g(y) = 2y + ye^y + e^y$$

$$\boxed{g(y) = 2y + ye^y + e^y} \rightarrow \text{put in (A)}$$

$$f(x,y) = e^x + yx + 2y + ye^y + e^y$$

$$\boxed{\text{put } x = 0,\ y = 1}$$

$$C = e^0 + (1)(0) + 2(1) + (1)(e)^1 + e^1$$

$$C = 1 + 2 + 2.71 + 2.71$$

$$C = 3 + 5.42$$

$$\boxed{C = 8.42}$$

$$\cancel{C = e^x}$$

$$\cancel{e^x + yx + gt}$$

$$\boxed{8 = e^x + yx + 2y + ye^y + e^y}$$

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23. $(4y+2t-5)\,dt + (6y+4t-1)\,dy = 0,\ y(-1)=2$

Sol:

$M(t,y) = 4y+2t-5$

$N(t,y) = 6y+4t-1$

$\boxed{\frac{\partial M}{\partial y} = 4}$ $\boxed{\frac{\partial N}{\partial t} = 4}$

Hence the Given DE is Exact.

$\frac{\partial f}{\partial t} = 4y+2t-5 \rightarrow (1)$, $\frac{\partial f}{\partial y} = 6y+4t-1 \rightarrow (2)$

Integratiy eq (1) w.r.t (t)

$f(t,y) = 4yt + \frac{2t^2}{2} - 5t + g(y)$

$f(t,y) = 4yt + t^2 - 5t + g(y) \rightarrow (A)$

Diff w.r.t (y)

$\frac{\partial f}{\partial y} = 4t + g'(y)$

Putting value of $\frac{\partial f}{\partial y}$

$6y + \cancel{4t} - 1 = \cancel{4t} + g'(y)$

$g'(y) = 6y - 1$

Integrating w.r.t (y)

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$$g(y) = \frac{\cancel{6}^{3}y^2}{\cancel{2}} - y$$

$$\boxed{g(y) = 3y^2 - y} \rightarrow \text{put in (A)}$$

$$f(x,y) = 4yt + t^2 - 5t + 3y^2 - y$$

$$\boxed{\text{put } t = -1,\ y = 2}$$

$$C = 4(2)(-1) + (-1)^2 - 5(-1) + 3(2)^2 - 2$$

$$= -8 + 1 + 5 + 12 - 2$$

$$= -7 + 5 + 12 - 2$$

$$= -7 + 5 + 10$$

$$= 15 - 7$$

$$\boxed{C = 8}$$

$$\boxed{4yt + t^2 - 5t + 3y^2 - y = 8}$$

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24. $\left(\frac{3y^2 - t^2}{y^5}\right)\frac{dy}{dt} + \frac{t}{2y^4} = 0$; $y(1) = 2$

Multiplying Both sides by dt.

$$\left(\frac{3y^2 - t^2}{y^5}\right)dy + \left(\frac{t}{2y^4}\right)dt = 0$$

$$M(t, y) = \frac{t}{2y^4}$$

$$\frac{\partial M}{\partial y} = \frac{t}{2}\frac{d}{dy}(y^{-4}) \Rightarrow \frac{t}{\cancel{2}}(\overset{-2}{\cancel{-4}})(y)^{-5}$$

$$= -2ty^{-5}$$

$$\boxed{\frac{\partial M}{\partial y} = -\frac{2t}{y^5}}$$

$$N(t, y) = \frac{3y^2 - t^2}{y^5}$$

$$\frac{\partial N}{\partial t} = \frac{3y^2}{y^5} - \frac{t^2}{y^5}$$

$$= \frac{3}{y^3} - \frac{t^2}{y^5}$$

$$\frac{\partial N}{\partial t} = \frac{3}{y^3} - \frac{1}{y^5}(t^2)$$

$$\boxed{\partial t = -\frac{2t}{y^5}}$$

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As $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$ so Equation is Exact

$$\frac{\partial f}{\partial t} = \frac{t}{2y^4} \rightarrow (1), \quad \frac{\partial f}{\partial y} = \frac{3y^2 - t^2}{y^5} \rightarrow (2)$$

Integrating eq (1) w.r.t (t)

$$f(t,y) = \frac{1}{2y^4} \cdot \frac{t^2}{2} + g(y)$$

$$f(t,y) = \frac{t^2}{4y^4} + g(y)$$

Differentiating w.r.t (y)

$$\frac{\partial f}{\partial y} = \frac{t^2}{\cancel{4}} \cdot \cancel{4}(y)^{-5} + g'(y)$$

$$\frac{\partial f}{\partial y} = -t^2 y^{-5} + g'(y)$$

$$\frac{3y^2 - t^2}{y^5} = -t^2 y^{-5} + g'(y)$$

$$\frac{3y^2}{y^5} - \cancel{\frac{t^2}{y^5}} = \cancel{\frac{-t^2}{y^5}} + g'(y)$$

$$g(y) = 3\int \frac{1}{y^3}\, dy$$

put in (A)

$$g(y) = 3 \cdot \frac{y^{-3+1}}{-3+1} \Rightarrow \boxed{\frac{3y^{-2}}{-2}}$$

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25. $(y^2\cos x - 3x$

Sol $M(x,y) = y^2\cos x - 3x^2y - 2x$

$$\frac{\partial M}{\partial y} = 2y\cos x - 3y(x^2)$$

$$\boxed{\frac{\partial M}{\partial y} = 2y\cos x - 3x^2}$$

$N(x,y) = 2y\sin x - x^3 + \ln y$

$$\boxed{\frac{\partial N}{\partial x} = 2y\cos x - 3x^2}$$

Hence the Given DE is Exact.

$$\frac{\partial f}{\partial x} = y^2\cos x - 3x^2y - 2x \longrightarrow (1)$$

$$\frac{\partial f}{\partial y} = 2y\sin x - x^3 + \ln y \longrightarrow (2).$$

Integrating eq (1) w.r.t (x)

$\cancel{f(x,y) = y^2(-\sin x) - 3y(2x)}$

$$f(x,y) = y^2\sin x - \frac{\cancel{3}x^3}{\cancel{3}}y - \cancel{2}\frac{x^2}{\cancel{2}}$$

$$\boxed{f(x,y) = y^2\sin x - x^3y - x^2 + g(y)} \rightarrow (A)$$

Diff w.r.t (y)

$$\frac{\partial f}{\partial y} = \sin x(2y) - x^3 + g'(y)$$

$$\frac{\partial f}{\partial y} = 2y\sin x - x^3 + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$\ln y + \cancel{2y\sin x} - \cancel{x^3} = \cancel{2y\sin x} - \cancel{x^3} + g'(y)$$

$$g'(y) = \ln y$$

Integrating w.r.t (y)

$$g(y) = \ln y \int 1\,dy + \int\left[\left(\frac{d}{dy}\ln y\right)\int 1\,dy\right]dy$$

$$= y\ln y + \int \frac{1}{y}\, y\, dy$$

$$\boxed{g(y) = y\ln y + y} \rightarrow \text{put in (A)}$$

$$\boxed{F(x,y) = y^2\sin x - x^3 y - x^2 + y\ln y + y}$$

put $x = 0$, $y = e$

$$c = e^2(0) - (0)(e) - 0 + e\ln e + e$$

$$c = e\ln e + e$$

$$\boxed{c = e(\ln e + 1)}$$

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$$\Rightarrow e(\ln e + 1) = y^2 \sin x - x^3 y - x^2 + y \ln y + y$$

26. $\left(\frac{1}{1+y^2} + \cos x - 2xy\right)\frac{dy}{dx} = y(y + \sin x), \; y(0) = 0$

$$\left(\frac{1}{1+y^2} + \cos x - 2xy\right) dy = (y^2 + y\sin x)\, dx$$

$$\left(\frac{1}{1+y^2} + \cos x - 2xy\right) dy - (y^2 + y\sin x)\, dx = 0$$

$$M(x,y) = -y^2 - y\sin x$$

$$\frac{\partial M}{\partial y} = -(2)(y) - \sin x$$

$$\boxed{\frac{\partial M}{\partial y} = -2y - \sin x}$$

$$\frac{\partial N}{\partial x} = \left(\frac{1}{1+y^2} + \cos x - 2xy\right)$$

$$\boxed{\frac{\partial N}{\partial x} = -\sin x - 2y}$$

Hence the given DE is exact.

$$\frac{\partial f}{\partial y} = \left(\frac{1}{1+y^2} + \cos x - 2xy\right)$$

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$$\frac{\partial f}{\partial x} = -y^2 - y\sin y \rightarrow (1) \quad , \quad \frac{\partial f}{\partial y} = \frac{1}{1+y^2} + \cos x - 2xy$$

~~Differen~~ Integratiy eq (1) w.r.t (x)

$$f(x,y) = -y^2 x + y\cos x + g(y) \rightarrow (A)$$

Differeshiating w.r.t (y)

$$\frac{\partial f}{\partial y} = -x\,2y + \cos x + g'(y)$$

Putting value of $\frac{\partial f}{\partial y}$

$$\frac{1}{1+y^2} + \cancel{\cos x} - \cancel{2xy} = -\cancel{2xy} + \cancel{\cos x} + g'(y)$$

$$g'(y) = \frac{1}{1+y^2}$$

Integratiy w.r.t (y)

$$g(y) = \int \frac{1}{1+y^2}\,dy$$

let $y = \tan u$
$dy = \sec^2 u\,du$

$$g(y) = \int \frac{\sec^2 u\,du}{1+\tan^2 u}$$

$$g(y) = \int 1\,du$$

$$g(y) = u$$

$\boxed{g(y) = \tan^{-1} y}$ put in (1)

$f(x,y) = -y^2 x + y\cos x + \tan^{-1} y$

Now put $\boxed{x = 0,\ y = 1}$

$C = -(1)^2(0) + (1)\cos(0) + \tan^{-1}(1)$

$= 1 + 1.619$

$\boxed{C = 2.619}$

$$\boxed{-xy^2 + y\cos x + \tan^{-1} y = 2.619}$$

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27. $(y^3 + kxy^4 - 2x)dx + (3xy^2 + 20x^2y^3)dy = 0$

$M(x,y) = y^3 + kxy^4 - 2x$

$N(x,y) = 3xy^2 + 20x^2y^3$

$\frac{\partial M}{\partial y} = 3y^2 + kx4y^3$

$$\boxed{\frac{\partial M}{\partial y} = 3y^2 + 4kxy^3}$$

$\frac{\partial N}{\partial x} = 3y^2 + 40xy^3$

Hence the given DE is Exact So,

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\cancel{3y^2} + kx4y^3 = \cancel{3y^2} + 40xy^3$

$4kxy^3 - 40xy^3 = 0$

$4xy^3(k - 10) = 0$

$k - 10 = 0$

$$\boxed{k = 10}$$

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$(-xy\sin x + 2y\cos x)dx + 2x\cos x\,dy = 0$

29. $\mu(x,y) = xy$

Multiplying with IF

$(-x^2y^2\sin x + 2xy^2\cos x)dx + (2x^2y\cos x)dy = 0$

$M(x,y) = -x^2y^2\sin x + 2xy^2\cos x$

$\frac{\partial M}{\partial y} = -x^2\,2y\sin x + 2x\cos x\,(2)\,y$

$$\boxed{\frac{\partial M}{\partial y} = -2x^2y\sin x + 4xy\cos x}$$

$N(x,y) = 2x^2y\cos x$

$N(x,y) = 2(x^2y\cos x)$

$\frac{\partial N}{\partial x} = 2y(x^2 - \sin x + \cos x\,2x)$

$$\boxed{\frac{\partial N}{\partial x} = -2yx^2\sin x + 4y\cos x}$$

As $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ so Equation is Exact

31. $(2y^2+3x)dx+2xy\,dy=0$

$M=2y^2+3x \quad , \quad N=2xy$

$\frac{\partial M}{\partial y}=4y \quad , \quad \frac{\partial N}{\partial x}=2y$

$\Rightarrow \frac{M_y-N_x}{N}=\frac{4y-2y}{2xy}=\frac{\cancel{2}y}{\cancel{2}xy}$

$=\frac{1}{x}$

$I.F=e^{\int\frac{1}{x}dx} \Rightarrow e^{\ln x}=x$

Multiplying (x).

$x(2y^2+3x)dx+x(2xy)dy=0$

$(2xy^2+3x^2)dx+(2x^2y)dy=0$

$M=2xy^2+3x^2, \quad N(x,y)=2x^2y$

$\frac{\partial M}{\partial y}=2x(2)y \Rightarrow 4xy$

$\frac{\partial N}{\partial x}=4xy$

Hence Equation is Exact

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$dx + (4y + 9x^2)dy = 0$

$6xy \quad , \quad N = 4y + 9x^2$

$6x \quad , \quad N_x = 18x$

$\frac{M_y - N_x}{N} \Rightarrow \frac{6x - 18x}{4y + 9x^2} = \frac{-12x}{4y + 9x^2}$

$\frac{N_x - M_y}{M} = \frac{18x - 6x}{6xy} = \frac{\cancel{12}^{2}x}{\cancel{6}xy} = \frac{2}{y}$

$e^{\int \frac{2}{y} dy} = e^{2\int \frac{1}{y} dy} = e^{2\ln|y|}$

$= \cancel{e}^{\ln|y|^2} = y^2$

$\boxed{I.F = y^2}$

$+ (4y^3 + 9x^2y^2)\, dy = 0$

$6xy^3$

$6x(3)y^2 \Rightarrow 18xy^2$

$4y^3 + 0$

34. $\cos x\, dx + \left(1 + \frac{2}{y}\right) \sin x\, dy = 0$

Sol $M = \cos x$

$\boxed{M_y = 0}$

$N = \left(1 + \frac{2}{y}\right) \sin x$

$N_x = \sin x + \frac{2 \sin x}{y}$

$= \cos x + \frac{1}{y} 2 \cos x$

$\boxed{N_x = \cos x + \frac{2 \cos x}{y}}$

$\boxed{N_x = \cos x \left(1 + \frac{2}{y}\right)}$

$\frac{N_x - M_y}{M} = \frac{\left(1 + \frac{2}{y}\right) \sin x - 0}{\cos x} = \frac{\left(1 + \frac{2}{y}\right) \sin x}{\cos x}$

$\frac{M_y - N_x}{N} = \frac{0 - \cos x \left(1 + \frac{2}{y}\right)}{\left(1 + \frac{2}{y}\right) \sin x}$

$= -\cot x$

$I.F = e^{\int -\cot x\, dx} = \sin x$

$\sin x\, dx + \sin x\, dy = 0$

$\boxed{M_y = 0, \quad N_x = 0}$

$$I \cdot F = e^{\int -\cot dx} \Rightarrow e^{-\int \cot x dx}$$

$$\boxed{I \cdot F \Rightarrow \csc x}$$

$$\cot x \, dx + \left(1 + \frac{2}{y}\right) dy = 0$$

$$M_y = 0 \quad , \quad M_x = 0$$

So Now Equation is Exact.

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36. $(y^2 + xy^3)\,dx + (5y^2 - xy + y^3 \sin y)\,dy = 0$

Sol

$\boxed{M_y = 2y + 3y^2 x}$, $\boxed{N_x = -y}$

$$\frac{N_x - M_y}{M} = \frac{-y - 2y - 3y^2 x}{y^2 + xy^3}$$

$$\Rightarrow \frac{-3y - 3y^2 x}{y^2 + xy^3} \Rightarrow \frac{-3y(1 + yx)}{y^2(1 + xy)}$$

$$= \frac{-3 - 3yx}{y + xy^2} \Rightarrow \frac{-3}{y}$$

$$\boxed{I.F = y^{-3}}$$

Xplyiy I.F with Given Equation,

$$\left(\frac{1}{y} + x\right) dx + \left(\frac{5}{y} - \frac{x}{y^2} + \sin y\right) dy = 0$$

$$M = \frac{1}{y} + x, \quad N = \frac{5}{y} - \frac{x}{y^2} + \sin y$$

$$\boxed{M_y = \frac{1}{y^2}, \quad N_x = \frac{1}{y}}$$

Hence Given Equation is Exact Now,

37. $x\,dx + (x^2y + 4y)\,dy = 0, \quad y(4) = 0$

$M = x \quad ; \quad N = x^2y + 4y$

$M_y = 0 \quad ; \quad N = 2xy$

Not Exact.

$\frac{N_x - M_y}{M} = 2y$

$\boxed{I.F = e^{y^2}}$

Xplying with I.F

$xe^{y^2}\,dx + (x^2ye^{y^2} + 4ye^{y^2})\,dy = 0$

$M = xe^{y^2} \quad ; \quad N = x^2ye^{y^2} + 4ye^{y^2}$

$M = 2xe^{y^2}y \quad ; \quad \boxed{N = 2xe^{y^2}y}$

$\int xe^{y^2}\,dx + \int 4ye^{y^2}\,dy = 0$

$\frac{1}{2}x^2e^{y^2} + 2e^{y^2} = c \rightarrow (1)$

$y(y) = 0$

$= \cancel{\frac{1}{2}(x)^2} = \frac{1}{2}(y)^2e^{(0)^2} + 2e^{(0)^2} = c$

$c = 8 + 2$

$\boxed{c = 10} \rightarrow$ put in (1)

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$$\boxed{\frac{1}{2}x^2 e^{y^2} + 2e^{y^2} = b}$$

39. $(4xy + 3x^2)dx + (2y + 2x^2)dy = 0$

(a)

$$\frac{d}{dx}(x^3 + 2x^2y + y^2) = \frac{d}{dx}(c)$$

$$3x^2 + 4xy + 2x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x^2 + 2y) = -(3x^2 + 4xy)$$

$$\frac{dy}{dx} = \frac{-3x^2 + 4xy}{2x^2 + 2y}$$

$$(2x^2 + 2y)\,dy = -(3x^2 + 4xy)\,dx$$

so $(3x^2 + 4xy)\,dx + (2x^2 + 2y)\,dy = 0$

has not solution $\boxed{x^2 + 2x^2y + y^2 = c}$

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(b) put $x=0$ $\quad y=-2$ in $x^3+2x^2y+y^2=c$

$$c=(0)^3+2(0)^2(-2)+(-2)^2$$

$$\boxed{c=4}$$

$x=1;\ y=1$ in $x^3+2x^2y+y^2=c$

$$c=(1)^3+2(1)^2(1)+(1)^2$$

$$\boxed{c=4}$$

$\boxed{x^3+2y^2y+y^2=4}$ required.

(c) $x^3+2x^2y+y^2=4$

$$x^3+2x^2y+y^2-4=c$$

$$y_{1,2}=\frac{-2x^2\pm\sqrt{(2x^2)^2-4(1)(x^3-4)}}{2(1)}$$

$$=-2x^2\pm\sqrt{4x^4-4x^3+16}\,/2$$

$$=-x^2+\sqrt{x^4-x^3+4}$$

$y_1(x)=-x^2+\sqrt{x^4-x^3+x}$ is satisfied by $y_1(0)=-2$

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Excercise No 2.5

1. $(x-y)dx + x\,dy = 0$

Sol:

let $y = ux$

$dy = u\,dx + x\,du$

$(x - ux)\,dx + x[u\,dx + x\,du] = 0$

$x\,dx - ux\,dx + x\,u\,dx + x^2 du = 0$

$dx(x - \cancel{ux} + \cancel{ux}) + x^2 du = 0$

$x\,dx + x^2 du = 0$

Dividing by (x)

$dx + x\,du = 0$

$dx + x\,du = 0$

$du = -\frac{1}{x}dx$

$\int du = \int -\frac{1}{x}dx$

$\boxed{u = -\ln|x| + c}$ $\boxed{u = \frac{y}{x}}$

$\frac{y}{x} = -\ln|x| + c$

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$(y$

2. $(x+y)dx + x\,dy = 0$

let $y = vx$

$dy = v\,dx + x\,dv$

$(x+vx)\,dx + x\,[v\,dx + x\,dv] = 0$

$x\,dx + vx\,dx + xv\,dx + x^2 dv = 0$

$dx(x + 2vx) + x^2 dv = 0$

$\frac{dx\,\cancel{x}(1+2v)}{x^{\cancel{2}}(1+2v)} + \frac{\cancel{x^2}\,dv = 0}{\cancel{x^2}(1+2v)}$

$\frac{dv}{1+2v} + \frac{1}{x}dx = 0$

$\frac{1}{1+2v}dv = -\frac{1}{x}dx$

sol

Integrate

$\frac{1}{2}\int \frac{2}{2v+1} = -\ln|x| + c$

$\frac{1}{2}\ln|2v+1| = -\ln|x| + c$

Taking e on B/s

$2v+1 = -2\ln|x| - 2c$

$$2v + 1 = cx^{-2}$$

Back substution $v = \frac{y}{x}$

$$2\left(\frac{y}{x}\right) + 1 = cx^{-2}$$

Multiply B/s by (x)

$$2y + x = cx^{-2}$$

$$2y = cx^{-1} - x$$

$$\boxed{y = \frac{c}{2x} - \frac{x}{2}}$$

3. $x\,dx + (y - 2x)\,dy = 0$

put $y = vx$

$$dy = v\,dx + x\,dv$$

$$vx\,dx + (vx - 2x)(v\,dx + x\,dv)$$

$$vx\,dx + v^2x\,dx + vx^2\,dv - 2xv\,dx - 2x^2\,dv = 0$$

$$vx\,dx + v^2x\,dx - 2xv\,dx + vx^2\,dv - 2x^2\,dv = 0$$

$$dx(vx + v^2x - 2xv) + (vx^2 - 2x^2)\,dv = 0$$

$$dx(v^2x - 2xv + vx) + (vx^2 - 2x^2)\,dv = 0$$

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$$u\left[1+v(v-2)\right]du + (v-2)\,u^2\,dv = 0$$

$$u(v-1)^2\,du + (v-2)\,u^2\,dv = 0$$

$$\frac{v-2}{(v-1)^2}\,dv = \frac{-1}{u}\,du$$

Integrating B/S

$$\int \frac{v-2}{(v-1)^2}\,dv = -\int \frac{1}{u}\,du$$

$$\int \frac{v-1-1}{(v-1)^2}\,dv = -\int \frac{1}{u}\,du$$

$$\ln(v-1) + \frac{1}{v-1} = -\ln u + c$$

$$\boxed{\text{Put } v = \frac{y}{u}}$$

so

$$\ln\left(\frac{y}{u}-1\right) + \frac{1}{\frac{y}{u}-1} = -\ln|u| + c$$

$$\ln\left[u\left(\frac{y}{u}-1\right)\right] + \frac{u}{y-u} = c$$

Ta

$$\ln(y-x) + \frac{x}{y-x} = c$$

$$c = \ln(y-x) + \frac{x}{y-x}$$

$$\boxed{(y-x)\ln(y-x) + x = c(y-x)}$$

4. $y\,dx = 2(x+y)\,dy$

$$y\,dx - 2(x+y)\,dy = 0$$

put $y = vx$

$$dy = v\,dx + x\,dv$$

$$vx\,dx - 2(x+vx)[v\,dx + x\,dv]$$

$$vx\,dx - 2[xv\,dx + x^2\,dv + v^2x\,dx + vx^2\,dv] = 0$$

$$vx\,dx - 2vx\,dx - 2x^2\,dv - 2v^2x\,dx - 2vx^2\,dv = 0$$

$$vx\,dx - 2vx\,dx - 2v^2x\,dx - 2x^2\,dv - 2vx^2\,dv = 0$$

$$dx(vx - 2vx - 2v^2x) - 2x^2\,dv(1+v) = 0$$

$$(-v - 2v^2)x\,dx = 2x^2(1+v)\,dv$$

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Separating variable

$$\int \frac{1}{2} u^{-1}\, du = \int \frac{v+1}{v-2v^2}\, dv$$

$$\frac{1}{2} \ln|u| = \int \left(\frac{A}{v} + \frac{B}{2v+1}\right) dv$$

$$\Rightarrow A(2v+1) + B(v) = v+1$$

$$2Av + A + Bv = v+1$$

$$2A + B = 1$$

$$\boxed{\begin{matrix} A = 1 \\ B = -1 \end{matrix}}$$

$$\frac{1}{2} \ln|u| = -\int \frac{1}{v} + \frac{-1}{2v+1}\, dv$$

$$\frac{1}{2} \ln|u| = \int \frac{1}{2v+1}\, dv - \int \frac{1}{y}\, dv$$

$$y = 2v+1$$

$$dy = 2dv$$

$$\boxed{\frac{1}{2} dy = dv}$$

$$\frac{1}{2} \ln|u| = \frac{1}{2} \ln(2v+1) - \ln|v| + c$$

$$y = vx$$

$$\boxed{v = \frac{y}{x}}$$

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$$\frac{1}{2}\ln(u) = \frac{1}{2}\ln\left(2(y/u)+1\right) - \ln\left(\frac{y}{u}\right) + c$$

$$\ln(u) = \ln\left(2(y/u)+1\right) - 2\ln\left(\frac{y}{u}\right) + c$$

$$\boxed{\because \; y = vu}$$

$$y\,du - 2(u+y)\,dy = 0$$

Back Substitution

$$u = vy \qquad \boxed{du = v\,dy + y\,dv}$$

$$y(v\,dy + y\,dv) - 2(vy + y)\,dy = 0$$

$$vy\,dy + y\,dv - 2vy - 2y\,dy = 0$$

$$y^2\,dv = (v+2)y\,dy$$

$$\frac{1}{v+2}\,dv = \frac{1}{y}\,dy$$

$$\ln(v+2) + c = \ln(y)$$

Applying e on B/S

$$y = (v+2) + e^c$$

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$x = vy$

$e^c = c_2$

$x = vy$

$$\boxed{y = c_2\left(\frac{x}{y} + 2\right)}$$

Q#5 $(y^2 + yx)\,dx - x^2\,dy = 0$

$\boxed{\text{Put } y = vx}$

$\boxed{dy = dvx + dxv}$

$((vx)^2 + (vx)(x))\,dx + x^2(x\,dv + v\,dx) = 0$

$(v^2x^2 + vx^2)\,dx + x^3\,dv + x^2 v\,dx = 0$

$v^2x^2\,dx + vx^2\,dx + x^3\,dv + x^2 v\,dx = 0$

$v^2x^2\,dx + vx^2\,dx + x^3\,dv + x^2 v\,dx = 0$

so

~~dx(v² + x~~

x

$dx(v^2x^2 + vx^2 + x^2 v) + x^3\,dv = 0$

$dx(v^2x^2 + 2vx^2) + x^3\,dv = 0$

$\cancel{x^2(v^2 + 2v)}\,dx + \cancel{x^3}\,dv = 0$

$\frac{x \cdot \cancel{x^2}(v^2 + 2v)}{} \qquad \frac{\cancel{x^3}(v^2 + 2v)}{}$

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$$\frac{dy}{x} + \frac{dv}{v^2+2v} = 0$$

$$\frac{dv}{v^2+2v} = \frac{dy}{x}$$

$$\frac{dv}{v(v+2)} = \frac{dx}{x}$$

$$\int \frac{dv}{v(v+2)} = \frac{dx}{x}$$

$$\int \frac{A}{v} + \int \frac{B}{v+2} = \ln|x| + c$$

$$\frac{1}{v(v+2)} = \frac{A}{v} + \frac{B}{v+2}$$

$$1 = A(v+2) + B(v)$$

Put $\boxed{v=-2}$

$$1 = B(-2)$$

$$\boxed{B = -\frac{1}{2}}$$

Put $v = 0$

$$2A = 1$$

$$\boxed{A = \frac{1}{2}}$$

$$\int \frac{1}{2(v)} dv + \int \frac{-1}{2(v+2)} = \ln|x| + c$$

$$\frac{1}{2}\ln|v| - \frac{1}{2}\ln|v+2| = \ln|x| + c$$

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$$\frac{1}{2}\ln|v| - \frac{1}{2}\ln|v+2| = \ln|x| + c$$

$$-\frac{1}{2}\left(\ln|v+2| - \ln|v|\right) = \ln|x| + c$$

$$-(\ln|v+2| - \ln|v|) = 2\ln|x| + 2c$$

$$\ln|v| + \ln|v+2| = 2\ln|x| + 2c$$

Taking e on B/S

$$\cancel{e}^{\ln|v|} + \cancel{e}^{\ln|v+2|} = e^{2\ln|x|} + e^{2c}$$

$$v + v+2 = (x)^2 + e^{2c}$$

$$2v+2 = x^2 + c_1$$

$$2v+2 - x^2 = c_1$$

put $\boxed{v = \frac{y}{x}}$

$$2\left(\frac{y}{x}\right) + 2 - x^2 = c_1$$

$$\frac{2y}{x} + 2 - x^2 = c_1$$

$$\boxed{\frac{2y}{x} + 2 - x^2 - c_1 = 0}$$

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Q#6 $(y^2+yx)\,dx + x^2 dy = 0$

Put $y = vx$

$$\boxed{dy = v\,dx + x\,dv}$$

$$= \left((vx)^2 + (vx)(x)\right)dx + x^2\left[v\,dx + x\,dv\right] = 0$$

$$= (v^2x^2 + x^2v)\,dx + x^2v\,dx + x^3\,dv = 0$$

$$\Rightarrow v^2x^2\,dx + x^2v\,dx + x^2v\,dx + x^3\,dv = 0$$

$$dx\,(v^2x^2 + x^2v + x^2v) + x^3\,dv = 0$$

$$dx\,x^2(v^2 + v + v) + x^3\,dv = 0$$

$$dx\,x^2(v^2 + 2v) + x^3\,dv = 0$$

$$\frac{dx\,\cancel{x^2}\cancel{(v^2+2v)}}{x^3\cancel{(v^2+2v)}} + \frac{\cancel{x^3}\,dv}{x^3(v^2+2v)}$$

$$\frac{dx}{x} + \frac{dv}{v^2+2v} = 0$$

$$\frac{dv}{v^2+2v} = -\frac{dx}{x}$$

$$\frac{dv}{v(v+2)} = -\frac{dx}{x}$$

$$\ln|v| + \ln|v+2| = 2\ln|x| + 2C$$

$$v + v + 2 = x^2 + C_1$$

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$$2v + 2 = x^2 + c_1$$

$$2\left(\frac{y}{x}\right) + 2 = x^2 + c_1$$

$$\frac{2y}{x} + 2 = x^2 + c_1$$

$$\boxed{2\frac{y}{x} + 2 - x^2 - c_1 = 0}$$

7. $\frac{dy}{dx} = \frac{y-x}{y+x}$

put $y = vx$

$$\boxed{dy = dv\,x + dx\,v}$$

$$\frac{x\,dv + dx\,v}{dx} = \cancel{y-x}\ \frac{vx - x}{vx + x}$$

$$x\,dv + dx\,v(vx + x) = dx(vx - x)$$

$$x^2 v\,dv + x^2 dv + dx\,v^2 x + dx\,v\,x = dx\,v\,x - dx\,x$$

$$x^2 v\,dv + x^2 dv = v\,x\,dx - x\,dx - dx\,v^2 x + dx\,v\,x$$

$$dv\,x^2(1+v) = dx(vx - x - v^2 x + vx)$$

$$\cancel{dx(v^2(x}$$

7. $\frac{dy}{dx} = \frac{y-x}{y+x}$

put $y = vx$

$$\left[\frac{dy}{dx} = v + x\frac{dv}{dx}\right]$$

$$\left[v + x\frac{dv}{dx}\right] = \frac{vx - x}{vx + x}$$

$$v + x\frac{dv}{dx} = \frac{vx - x}{vx + x}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$x\frac{dv}{dx} = \frac{vx - x}{vx + x} - v$$

$$x\frac{dv}{dx} = \frac{vx - x}{vx + x} - \frac{v(vx + x)}{vx + x}$$

$$x\frac{dv}{dx} = \frac{-v^2x - x}{vx + x}$$

$$x\frac{dv}{dx} = \frac{-x(v^2 + 1)}{x(v + 1)}$$

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$$x\frac{dv}{dx} = -\frac{(v^2+1)}{v+1}$$

$$\int \frac{v+1}{v^2+1}\,dx = -\int \frac{1}{x}\,dx$$

$$\int \frac{v}{v^2+1} + \frac{1}{v^2+1}\,dv = -\int \frac{1}{x}\,dx$$

$$\frac{1}{2}\ln|v^2+1| + \tan^{-1} v = -\ln|x| + c$$

$$\frac{1}{2}\ln\left(\left(\frac{y}{x}\right)^2+1\right) + \tan^{-1}\frac{y}{x} = -\ln x + c$$

$$\ln\left(\left(\frac{y}{x}\right)^2+1\right) + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\ln x + c$$

$$\boxed{\ln(x^2+y^2) + 2\tan^{-1}\left(\frac{y}{x}\right) = c}$$

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Q# 8: $\frac{dy}{dx} = \frac{x+3y}{3x+y}$

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\left(v + x\frac{dv}{dx}\right) = \frac{x+3(vx)}{3x+(vx)}$$

$$v + x\frac{dv}{dx} = \frac{x+3vx}{3x+vx}$$

$$x\frac{dv}{dx} = \frac{x(1+3v)}{x(3+v)} - v$$

$$= \frac{1+3v}{3+v} - 3\frac{(3+v)}{3+v}$$

$$= \frac{1+3v-3v-v^2}{3+v}$$

$$x\frac{dv}{dx} = \frac{1-v^2}{3+v}$$

$$\int \frac{3+v}{1-v}\,dv = \int \frac{1}{x}\,dx$$

$$\int \left(\frac{2}{1-v} + \frac{1}{1+v}\right) dv = \int \frac{1}{x}\,dx$$

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$$-2\ln(1-v) + \ln(1+v) = \ln x + \ln c$$

$$\ln\left(\frac{1+v}{1-v^2}\right) = \ln cx$$

Applying e on B/s

$$\frac{1+v}{1-v^2} = cx$$

$$\frac{1+y/x}{(1-y/x)^2} = cx$$

$$\frac{x(x+y)}{(x-y)^2} = cx$$

$$\boxed{(x-y)^2 = c(x+y)}$$

Q# 9 $\quad -y\,dx + (x+\sqrt{xy}\,dy) = 0$

put $x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$-y\,dx + (x+\sqrt{xy})\,dy = 0$$

$$-y(v\,dy + y\,dv) + (vy + \sqrt{(vy)y}\,dy) = 0$$

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$$-vy\,dy - y^2 dv + vy\,dy + y\sqrt{v}\,dy = 0$$

$$-vy\,dy - y^2 dv + vy\,dy + y\sqrt{v}\,dy = 0$$

$$\Rightarrow -y^2 dv + y\sqrt{v}\,dy = 0$$

$$-y\,dv + \sqrt{v}\,dy = 0$$

$$\int \frac{1}{\sqrt{v}}\,dv = \int \frac{1}{y}\,dv$$

$$2\sqrt{v} = \ln y + c$$

$$2\sqrt{\frac{u}{y}} = \ln y + c$$

$$\boxed{4\left(\frac{u}{y}\right) = (\ln y + c)^2}$$

Q# 10 $\quad u\dfrac{dy}{du} = y + \sqrt{u^2 - y^2}\ , \ u > 0$

Sol: put $y = vu$

$dy = v\,du + u\,dv$

$$u\,dy = \left(y + \sqrt{u^2 + y^2}\right)du$$

$$u\,(v\,du + u\,dv) = \left(vu + \sqrt{u^2 + v^2u^2}\right)du$$

$$uv\,du + u^2 dv = vu\,du + \sqrt{u^2 + v^2u^2}\,du$$

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$$x^2 dv + v x dx = v x dx + x\sqrt{1-v^2}\, dx$$

$$x^2 dv = v x dx - v x dx + x\sqrt{1-v^2}\, dx$$

$$\frac{1}{\sqrt{1-v^2}}\, dv = \frac{1}{x}\, dx$$

$$\int \frac{1}{\sqrt{1-v^2}}\, dv = \int \frac{1}{x}\, dx$$

$$\sin^{-1}(v) = \ln|x| + c$$

Back substitution

$$\sin^{-1}\left(\frac{y}{x}\right) = \ln|x| + c$$

$$\sin\left(\sin^{-1}\left(\frac{y}{x}\right)\right) = \sin(\ln|x| + c)$$

$$\frac{y}{x} = \sin(\ln|x| + c)$$

$$\boxed{y = x \sin(\ln|x| + c)}$$

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Initial value Problem.

$\boxed{11-14}$

11. $x^4 \frac{dy}{dx} = y^3 - x^3$, $y(1) = 2$

Multiplying B/s by dx and by

putting $y = vx$ & $\frac{dy}{dx} = x\frac{dv}{dx} + v\frac{dx}{dx}$

$\Rightarrow dy = x\,dv + v\,dx$

$x(xv)^2 \, [x\,dv + v\,dx] = [(vx)^3 - x^3]\, dx$

$v^2 x^4 dv + v^3 x^3 dx = v^3 x^3 - x^3 dx$

$v^2 x^4 dx = -x^3 dx$

$v^2 dv = \frac{-1}{x} dx$

$\div$ B/s by x^4

$\int v^2 dv = -\int \frac{1}{x} dx$

$\frac{1}{3} v^3 = -\ln|x| + c$ (Back Substitution

$\frac{y^3}{3x^3} = -\ln|x| + c$

$y = vx \Rightarrow v = \frac{y}{x}$

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$y^3 = -3x^3 \ln(x) + 3x^3 c$

Multiplying B/s by $3x^3$ we have

$8 = -3(1)^3 \ln(1) + 3(1)^3 c$

Since $y(1) = 2$ then $y^3(1) = 2^3$

Also $\ln(1) = 0$

$8 = 3c \Rightarrow \boxed{c = \frac{8}{3}}$

Put value of c in (A)

$$\boxed{y^3 + 3x^3 \ln(x) = 8x^3}$$

Q#12 $(x^2 + 2y^2)\frac{dx}{dy} = xy \; ; \; y(-1) = 1$

Sol: Put $y = vx$ & $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$(x^2 + 2y^2)\frac{dx}{dy} = xy$

$xy\frac{dy}{dx} = x^2 + 2y^2$

$x(vx)\left[v + x\frac{dv}{dx}\right] = x^2 + 2(vx)^2$

$$v^2u^2 + vu^3\frac{dv}{du} = u^2 + 2v^2u^2$$

$$vu^3\frac{dv}{du} = u^2 + v^2u^2$$

$$vu^3\frac{dv}{du} = u^2 + v^2u^2$$

$$vu^2\frac{dv}{du} = u^2(1+v^2)$$

$$\int\left(\frac{v}{v^2+1}\right)dv = \int\frac{1}{u}\,du$$

$$\frac{1}{2}\ln|v^2+1| = \ln|u| + c$$

$$\ln|v^2+1| = 2\ln u + c$$

$$v^2+1 = u^2c$$

$$v^2 = u^2c - 1 \qquad \left(\because v = \frac{y}{u}\right)$$

$$\left(\frac{y}{u}\right)^2 = cu^2 - 1 \qquad \boxed{\because y(-1) = 1}$$

$$1 = c - 1$$

$$\boxed{c = 2}$$

$$y^2 = cu^4 - u^2$$

$$\boxed{y^2 = 2u^4 - u^2}$$

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Q#14: $y\,dx + x(\ln x - \ln y - 1)\,dy = 0$; $y(1) = e$

Solution: Put $x = vy$ & $dv = v\,dy + y\,dv$

$$vy\,dy + y^2\,dv + vy\ln vy\,dy - vy\ln y\,dy - vy\,dy = 0$$

$$y^2\,dv + vy(\ln vy - \ln y)\,dy = 0$$

$$vy\ln v\,dy = -y^2\,dv$$

$$\frac{-1}{y^2}\,dy = \frac{1}{vy\ln v}\,dv$$

$$\int \frac{1}{v\ln v}\,dv = \int -\frac{1}{y}\,dy$$

$$\ln v = c/y$$

$$y(1) = e$$

$$\ln\left(\frac{1}{e}\right) = \frac{c}{e}$$

$$\boxed{c = -e}$$

$$\boxed{\ln\frac{x}{y} = \frac{e}{y}}$$

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15 - 22 (Bernoulli's eq)

Q 105: $x\frac{dy}{dx} + y = \frac{1}{y^2}$

÷ing B/s ~~by~~ ~~wey~~ by x we get

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{y^2 x}$$

Dividing B/s by y^2 we get

$$y^2\frac{dy}{dx} + \frac{y^3}{x} = \frac{1}{x}$$

put $u = y^3$

$$\frac{du}{dy} = 3y^2$$

$$\frac{dy}{du} = \frac{1}{3y^2}$$

$$\frac{dy}{du} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx} \Rightarrow \text{chain Rule}$$

$$\frac{dy}{dx} = \frac{1}{3y^2}\frac{du}{dx}$$

$$y^2\left[\frac{1}{3y^2}\frac{du}{dx}\right] + \frac{u}{x} = \frac{1}{x}$$

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$$\frac{1}{3}\frac{du}{dx} = \frac{1-u}{x}$$

$$-\int \frac{du}{1-u} = \int \frac{3}{x}\,dx$$

$$-\ln|1-u| = 3\ln x + c$$

$$\frac{1}{1-u} = cx^3$$

$$u = 1 - \frac{1}{cx^3}$$

$$\boxed{u = y^3}$$

$$\boxed{y^3 = 1 + cx^{-3}}$$

Q#16: $\frac{dy}{dx} - y = e^x y^2$

Xplying by y^2 we get

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y} = e^x$$

Put $u = \frac{1}{y}$

$$\frac{du}{dx} = \frac{-1}{y^2}$$

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$$\frac{dy}{du} = \frac{dy}{dv} \times \frac{dv}{dy}$$

$$= -y^2 \frac{dv}{dy}$$

$$\Rightarrow \frac{1}{y^2}\left[-y^2 \frac{dv}{dy}\right] - v = e^y$$

$$\Rightarrow -\frac{dv}{dy} - v = e^y$$

$$= \frac{dv}{dy} + v = -e^y$$

$$e^{\int dy} = e^y$$

$$e^y\left[\frac{dv}{dy} + v\right] = e^y(-e^y)$$

$$\frac{d}{dy}\left[e^y v\right] = e^{-2y}$$

$$\int d(e^y v) = \int e^{-2y}\, dy$$

$$e^y v = -\frac{1}{2} e^{-2y} + c$$

$$\boxed{v = -\frac{1}{2} e^y + ce^{-y}}$$

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Now Back substitution

$$\frac{1}{y} = \frac{-1}{2} e^{x} + ce^{-x}$$

$$y = \frac{1}{\frac{-1}{2} e^{x} + ce^{-x}}$$

$$\boxed{y = \frac{2}{-e^{x} - 2e^{-x}}}$$

Qu# 17. $\frac{dy}{dx} = y(xy^3 - 1)$

Sol Dividing B/s by y^x we have

$$\frac{1}{y^x}\frac{dy}{dx} + \frac{1}{y^3} = x$$

put $\boxed{v = \frac{1}{y^3}}$

$$\frac{dv}{dy} = \frac{-3}{y^4}$$

$$\frac{dy}{dx} = -\frac{3}{y^4}\frac{dv}{dx}$$

$$\frac{1}{y^4}\left[-\frac{y^4}{3}\frac{dv}{dx}\right] + v = x$$

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$$-\frac{1}{3}\frac{du}{dx} + v = x$$

$$\frac{dv}{dy} - 3v = -3y$$

$$e^{\int -3\,dy} = e^{-3x}$$

$$e^{-3y}\left[\frac{dv}{dy} - 3v\right] = e^{-3y}(-3y)$$

$$v = x + \frac{1}{3} + ce^{3y}$$

$$\frac{1}{y^3} = x + \frac{1}{3} + ce^{3y}$$

$$\boxed{y^{-3} = x + \frac{1}{3} + ce^{3y}}$$

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Q#18:

$$\frac{u\,dy}{du} - (1+u) = uy^2$$

$\div$ ing by 'u' we get

$$\frac{dy}{du} - \left(1 + \frac{1}{u}\right) = y^2$$

$\div$ by y^2

$$\frac{1}{y^2}\frac{dy}{du} - \left(1 + \frac{1}{u}\right)\frac{1}{y} = 1$$

Put $\frac{1}{y} = v$

$$\frac{dv}{dy} = -\frac{1}{y^2}$$

$$\frac{dy}{dv} = -y^2$$

Using Chain Rule

$$\frac{dy}{du} = \frac{dy}{dv} \times \frac{dv}{du}$$

$$\frac{dv}{du} = -y^2\frac{dv}{du}$$

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$$-\frac{du}{dx} - \left(1+\frac{1}{x}\right)u = 1$$

$$+\frac{du}{dx} + \left(1+\frac{1}{x}\right)u = -1$$

$$e^{\int\left(1+\frac{1}{x}\right)dx} = e^{x+\ln x} = xe^{x}$$

Integrating Factor

Multiplying B/s by I.F we have

$$xe^{x}\left[\frac{du}{dx} + \left(1+\frac{1}{x}\right)u\right] = xe^{x}(-1)$$

Integrating B/s

$$\cancel{\int \frac{d}{dx}}\left(xe^{x}u\right)dx = -\int x e^{x}dx$$

$$xe^{x}u = -xe^{x} + e^{x} + c$$

$$u = -1 + \frac{1}{x} + cx^{-1}e^{-x}$$

$$\frac{1}{y} = -1 + \frac{1}{x} + cx^{-1}e^{-x}$$

$$\boxed{y^{-1} = -1 + \frac{1}{x} + cx^{-1}e^{-x}}$$

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Q#9 $t^2 \frac{dy}{dt} + y^2 = ty$

÷ ling by t^2 & y^2 we get

$$\frac{1}{y^2}\frac{dy}{dt} - \frac{1}{t}\frac{1}{y} = \frac{-1}{t^2}$$

Put $v = \frac{1}{y}$

$$\frac{dv}{dy} = \frac{-1}{y^2}$$

$$\frac{dv}{dy} = -y^{-2}$$

$$\frac{dy}{dt} = \frac{dy}{dt}\frac{dv}{dt}$$

$$\frac{dy}{dt} = -y^2\frac{dv}{dt}$$

$$\frac{1}{y^2}\left[-y^2\frac{dv}{dt}\right] - \frac{1}{t}(v) = \frac{-1}{t^2}$$

$$-\frac{dv}{dt} - \frac{1}{t}v = \frac{-1}{t^2}$$

$$\frac{dv}{dt} + \frac{1}{t}v = \frac{1}{t^2}$$

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$$e^{\int \frac{1}{t} dt} = e^{\ln t} = t$$

$$I.F = t$$

$$t\left[\frac{du}{dt} + \frac{1}{t}u\right] = 1$$

$$\int d(tu) = \int \frac{1}{t} dt$$

$$tu = \ln t + c$$

Applying e on B/s

$$e^{tu} = ct$$

$$e^{t(1/y)} = ct$$

$$\boxed{e^{t/y} = ct}$$

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21-22

$$x^2 \frac{dy}{dx} - 2xy = 3y^4 \; ; \; y(1) = \frac{1}{2}$$

Dividing B.S by x^2 then y^4

$$\frac{1}{y^4} \frac{dy}{dx} - \frac{2}{x} \frac{1}{y^3} = \frac{3}{x^2}$$

$$v = \frac{1}{y^3}$$

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{dx}$$

$$= -\frac{y^4}{3} \frac{dv}{dx}$$

$$\frac{1}{y^4}\left[-\frac{y^4}{3} \frac{dv}{dx}\right] - \frac{2}{x}(v) = \frac{3}{x^2}$$

$$-\frac{1}{3} \frac{dv}{dx} - \frac{2}{x} v = \frac{3}{x^2}$$

$$\frac{dv}{dx} + \frac{6}{x} v = -\frac{9}{x^2}$$

$$e^{\int 6/x \, dx} \Rightarrow \cancel{e^{6\ln x}} \Rightarrow x^6$$

Multiplying with I.F $= x^6$

$$x^6\left[\frac{dv}{dx} + \frac{6}{x} x\right] = x^6\left[-\frac{9}{x^2}\right]$$

$$\cancel{\int \frac{d}{dx}(x^6 u)\,dx} = \int -9x^4$$

$$x^6 u = \frac{-9}{5} x^5 + c$$

$$\frac{1}{y^3} = -\frac{9}{5} x^{-1} + cx^{-6}$$

$$\frac{1}{y^3} = \frac{-9}{5x} + cx^{-6}$$

$$y^{-3} = \frac{-9}{5x} + cx^{-6}$$

$$y(1) = \frac{1}{2}$$

$$y^{-3} = \frac{9}{5x} + cx^{-6}$$

$$\left(\frac{1}{2}\right)^{-3} = \frac{9}{5x} + cx^{-6}$$

$$\boxed{c = \frac{49}{5}}$$

$$\boxed{y^{-3} = \frac{9}{5x} + \frac{49}{5} x^{-6}}$$

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