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ROLL NO : 20P-0165

SECTION : 3A

Assignment No : 1

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Q:-1 There are three kinds of people: knights who always tell the lie, knaves who speak the truth, and spies who either Lie or Tell the truth. You encounter three people A, B, C. You know two of them are knights and one is knave.... ?

a) A says "C is the knave"
B says "A is the knight"
C says "I am the spy"

Solution:

Let p1 = A is a knight.
p2 = A is a knave.
p3 = A is a spy.

q1 = B is a knight.
q2 = B is a knave.
q3 = B is a spy.

Let $r1 = C$ is a knight.

$r2 = C$ is a knave.

$r3 = C$ is a spy.

Equations for A, B & C:

$A = r2$

$B = p1$

$C = r3$

Formula for No. of combinations

$$3^3 = 27 \text{ combinations}$$

Cases	A	B	C
1.	knight	knight	knight
2.	knight	knight	knave ✓
3.	knight	knave	knight ✓
4.	knight	knave	knave
5.	knave	knave	knave
6.	knave	knave	knight
7.	knave	knight	knave
8.	knave	knight	knight ✓
9.	spy	spy	spy
10.	spy	spy	knight
11.	spy	knight	spy
12.	spy	knight	knight
13.	spy	spy	knave
14.	spy	knave	spy
15.	spy	knave	knave
16.	knight	knight	spy
17.	knight	spy	knight
18.	knight	spy	spy
19.	knave	knave	spy
20.	knave	spy	knave

Cases	A	B	C
21.	knave	spy	spy
22.	knave	knight	spy
23.	knight	knave	spy
24.	spy	knight	knave
25.	spy	knave	knight
26.	knave	spy	knight
27.	knight	spy	knave

"You know two of them are knights and one is knave."

Cases To check

	A	B	C	
Case 1:	knight	knight	knave	✓
Case 2:	knight	knave	knight	✓
Case 3:	knave	knight	knight	✓

Case 1: knight, knight, knave

(1) $r2 = F$

(2) $p1 = F$

(3) $r3 = T$

Put values

from (1) $T = T$

(2) gives $T \neq F$

$p1 = T$, $q1 = T$
$p2 = F$, $q2 = F$
$p3 = F$, $q3 = F$

$r1 = F$
$r2 = T$
$r3 = F$

Case 1 Does Not Holds Due to (2).

Case 2: (A) knight, (B) knave, (C) knight

(1) $r2 = F$

(2) $p1 = T$

(3) $r3 = F$

Putting values

(1) → $F = F$

(2) → $T = T$

(3) → $F = F$

$p1 = T$, $q1 = F$
$p2 = F$, $q2 = T$
$p3 = F$, $q3 = F$

$r1 = T$
$r2 = F$
$r3 = F$

Case 2 Holds

Case 3: knave, knight, knight

(1) $r2 = T$

(2) $p1 = F$

(3) $r3 = F$

$p1 = F$, $q1 = T$
$p2 = T$, $q2 = F$
$p3 = F$, $q3 = F$

$r1 = T$
$r2 = F$
$r3 = F$

Put Values

① → $F \neq T$

Case 3 Not Holds

According to Case 2

A = Knight

B = Knave

C = Knight

(b) A says "I am the knight", B says "I am the knave", and C says "B is the knight".

Solution:

Let

p1 = A is a knight. q1 = B is a knight.

p2 = A is a knave. q2 = B is a knave.

p3 = A is a spy. q3 = B is a spy.

r1 = C is a knight.

r2 = C is a knave.

r3 = C is a spy.

Equations for A, B, & C

$A = p1$

$B = q2$

$C = q1$

Cases

	A	B	C
Case1:	knight	knight	knave
Case2:	knight	knave	knight
Case3.	knave	knight	knight

Case 1: Knight, knight, knave

(1) $p1 = F$

(2) $q2 = F$

(3) $q1 = T$

$p_1 = T$	$, q1 = T$	$, r1 = F$
$p2 = F$	$, q2 = F$	$, r2 = T$
$p3 = F$	$, q3 = F$	$, r3 = F$

(1) → $T = F$

Case 1 Not Holds

Case 2: Knight, Knave, knight

(1) $p1 = F$

(2) $q2 = T$

(3) $q1 = F$

$p1 = T$	$q1 = F$	$r1 = T$
$p2 = F$	$q2 = T$	$r2 = F$
$p3 = F$	$q3 = F$	$r3 = F$

(1) → $T \neq F$

Case 2 No Holds

Case 3: Knave, Knight, Knight

(1) $p1 = T$

(2) $q2 = F$

(3) $q1 = F$

$p1 = F$	,	$q1 = T$, $r1 = T$
$p2 = T$	,	$q2 = F$, $r2 = F$
$p3 = F$	,	$q3 = F$, $r3 = F$

① → F = T

Case 3 Not Holds

Result: All cases Not Hold so we can't determine who is knight or knave.

(C) A says "I am the knave."

B says "I am the knave."

C says "I am the knave."

Sol:

$p1 = A$ is a knight.
$p2 = A$ is a knave.
$p3 = A$ is a spy.
$q1 = B$ is a knight.
$q2 = B$ is a knave.
$q3 = B$ is a spy.

$r1 = C$ is a knight.
$r2 = C$ is a knave.
$r3 = C$ is a spy.

Equation for A, B & C

$A = p2$

$B = q2$

$C = r2$

Cases

	A	B	C
Case1:	Knight	Knight	Knave
Case2:	Knight	Knave	Knight
Case3:	Knave	Knight	Knight

Case 1: Knight, Knight, Knave

(1) $p2 = F$

(2) $q2 = F$

(3) $r2 = T$

$p1 = T \quad , q1 = T \quad , r1 = F$

$p2 = F \quad , q2 = F \quad , r2 = T$

$p3 = F \quad , q3 = F \quad , r3 =$

① → $F = F$

② → $F = F$

③ → $T = T$

Case 1 Holds

Case 2: Knight, Knave, Knight

1) $p2 = F$

2) $q2 = T$

3) $r2 = F$

$p1 = T$, $q1 = F$, $r1 = T$

$p2 = F$, $q2 = T$, $r2 = F$

$p3 = F$, $q3 = F$, $r3 = F$

① → $F = F$

② → $T = T$

③ → $F = F$

Case 2 Holds

Case 3: Knave, Knight, knight

(1) $p2 = T$

(2) $q2 = F$

(3) $r2 = F$

$p1 = F$, $q1 = T$, $r1 = T$

$p2 = T$, $q2 = F$, $r2 = F$

$p3 = F$, $q3 = F$, $r3 = F$

① → $T = T$

② → $F = F$

③ → $F = F$

Case 3 Holds

Result

All cases Hold so there is a contraction so we cannot say who is knave or knight.

(d) A says "I am the knight".
B says "A is telling the truth".
C says "I am the spy."

Sol:

Let $p1$ = A is a knight. $q1$ = B is a knight.
$p2$ = A is a knave. $q2$ = B is a knave.
$p3$ = A is a spy. $q3$ = B is a spy.

$r1$ = C is a knight.
$r2$ = C is a knave.
$r3$ = C is a spy.

Equations for A, B & C

$A = p1$

$B = p1$

$C = r3$

CASES

	A	B	C
1⇒	Knight	Knight	Knave
2⇒	Knight	Knave	Knight
3⇒	Knave	Knight	Knight

Case 1: Knight, Knight, Knave

(1) $p1 = F$

(2) $p1 = F$

(3) $r3 = T$

$p1 = T, q1 = T, r1 = F$

$p2 = F, q2 = F, r2 = T$

$p3 = F, q3 = F, r3 = F$

(1) → $T = F$

Case 1 Not Holds

Case 2: Knight, Knave, Knight

(1) $p1 = F$

(2) $p1 = T$

(3) $r3 = F$

$p1 = T \quad q1 = F \quad r1 = T$

$p2 = F \quad q2 = T \quad r2 = F$

$p3 = F \quad q3 = F \quad r3 = F$

(1) → $T \neq F$

Case 2 Not Holds

Case 3: Knave, knight, knight

(1) $p1 = T$

(2) $p1 = F$

(3) $r3 = F$

$p1 = F$	$q1 = T$	$r1 = T$
$p2 = T$	$q2 = F$	$r2 = F$
$p3 = F$	$q3 = F$	$r3 = F$

(1) → F ≠ T

Case3 No Holds

Result: All the cases Not Holds so there is a contraction so we cannot say who is knave or who is knight.

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