8. Use De Morgan's laws to find the negation of each of the following statements.
   a) Kwame will take a job in industry or go to graduate school.
   b) Yoshiko knows Java and calculus.
   c) James is young and strong.
   d) Rita will move to Oregon or Washington.
9. Show that each of these conditional statements is a tautology by using truth tables.
   a) $(p \wedge q) \to p$ b) $p \to (p \vee q)$
   c) $\neg p \to (p \to q)$ d) $(p \wedge q) \to (p \to q)$
   e) $\neg(p \to q) \to p$ f) $\neg(p \to q) \to \neg q$
10. Show that each of these conditional statements is a tautology by using truth tables.
    a) $[\neg p \wedge (p \vee q)] \to q$
    b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$
    c) $[p \wedge (p \to q)] \to q$
    d) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
11. Show that each conditional statement in Exercise 9 is a tautology without using truth tables.
12. Show that each conditional statement in Exercise 10 is a tautology without using truth tables.
13. Use truth tables to verify the absorption laws.
    a) $p \vee (p \wedge q) \equiv p$ b) $p \wedge (p \vee q) \equiv p$
14. Determine whether $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology.
15. Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.

Each of Exercises 16–28 asks you to show that two compound propositions are logically equivalent. To do this, either show that both sides are true, or that both sides are false, for exactly the same combinations of truth values of the propositional variables in these expressions (whichever is easier).

16. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent.
17. Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.
18. Show that $p \to q$ and $\neg q \to \neg p$ are logically equivalent.
19. Show that $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.
20. Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.
21. Show that $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$ are logically equivalent.
22. Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.
23. Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.
24. Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.
25. Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.
26. Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.
27. Show that $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are logically equivalent.
28. Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent.
29. Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.
30. Show that $(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology.
31. Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.
32. Show that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent.
33. Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

The **dual** of a compound proposition that contains only the logical operators $\vee$, $\wedge$, and $\neg$ is the compound proposition obtained by replacing each $\vee$ by $\wedge$, each $\wedge$ by $\vee$, each **T** by **F**, and each **F** by **T**. The dual of s is denoted by s^*.

34. Find the dual of each of these compound propositions.
    a) $p \vee \neg q$ b) $p \wedge (q \vee (r \wedge \mathbf{T}))$
    c) $(p \wedge \neg q) \vee (q \wedge \mathbf{F})$
35. Find the dual of each of these compound propositions.
    a) $p \wedge \neg q \wedge \neg r$ b) $(p \wedge q \wedge r) \vee s$
    c) $(p \vee \mathbf{F}) \wedge (q \vee \mathbf{T})$
36. When does $s^* = s$, where s is a compound proposition?
37. Show that $(s^*)^* = s$ when s is a compound proposition.
38. Show that the logical equivalences in Table 6, except for the double negation law, come in pairs, where each pair contains compound propositions that are duals of each other.

**39. Why are the duals of two equivalent compound propositions also equivalent, where these compound propositions contain only the operators $\wedge$, $\vee$, and $\neg$?
40. Find a compound proposition involving the propositional variables p, q, and r that is true when p and q are true and r is false, but is false otherwise. [*Hint:* Use a conjunction of each propositional variable or its negation.]
41. Find a compound proposition involving the propositional variables p, q, and r that is true when exactly two of p, q, and r are true and is false otherwise. [*Hint:* Form a disjunction of conjunctions. Include a conjunction for each combination of values for which the compound proposition is true. Each conjunction should include each of the three propositional variables or its negations.]
42. Suppose that a truth table in n propositional variables is specified. Show that a compound proposition with this truth table can be formed by taking the disjunction of conjunctions of the variables or their negations, with one conjunction included for each combination values for which the compound proposition is true. The resulting compound proposition is said to be in **disjunctive normal form**.

A collection of logical operators is called **functionally complete** if every compound proposition is logically equivalent to a compound proposition involving only these logical operators.

43. Show that $\neg$, $\wedge$, and $\vee$ form a functionally complete collection of logical operators. [*Hint:* Use the fact that every compound proposition is logically equivalent to one in disjunctive normal form, as shown in Exercise 42.]

*44. Show that $\neg$ and $\wedge$ form a functionally complete collection of logical operators. [*Hint:* First use a De Morgan law to show that $p \vee q$ is logically equivalent to $\neg(\neg p \wedge \neg q)$.]

*45. Show that $\neg$ and $\vee$ form a functionally complete collection of logical operators.

The following exercises involve the logical operators *NAND* and *NOR*. The proposition p *NAND* q is true when either p or q, or both, are false; and it is false when both p and q are true. The proposition p *NOR* q is true when both p and q are false, and it is false otherwise. The propositions p *NAND* q and p *NOR* q are denoted by $p \mid q$ and $p \downarrow q$, respectively. (The operators $\mid$ and $\downarrow$ are called the **Sheffer stroke** and the **Peirce arrow** after H. M. Sheffer and C. S. Peirce, respectively.)

46. Construct a truth table for the logical operator *NAND*.
47. Show that $p \mid q$ is logically equivalent to $\neg(p \wedge q)$.
48. Construct a truth table for the logical operator *NOR*.
49. Show that $p \downarrow q$ is logically equivalent to $\neg(p \vee q)$.
50. In this exercise we will show that $\{\downarrow\}$ is a functionally complete collection of logical operators.
    a) Show that $p \downarrow p$ is logically equivalent to $\neg p$.
    b) Show that $(p \downarrow q) \downarrow (p \downarrow q)$ is logically equivalent to $p \vee q$.
    c) Conclude from parts (a) and (b), and Exercise 49, that $\{\downarrow\}$ is a functionally complete collection of logical operators.

*51. Find a compound proposition logically equivalent to $p \rightarrow q$ using only the logical operator $\downarrow$.

52. Show that $\{\mid\}$ is a functionally complete collection of logical operators.
53. Show that $p \mid q$ and $q \mid p$ are equivalent.
54. Show that $p \mid (q \mid r)$ and $(p \mid q) \mid r$ are not equivalent, so that the logical operator $\mid$ is not associative.

*55. How many different truth tables of compound propositions are there that involve the propositional variables p and q?

56. Show that if p, q, and r are compound propositions such that p and q are logically equivalent and q and r are logically equivalent, then p and r are logically equivalent.
57. The following sentence is taken from the specification of a telephone system: "If the directory database is opened, then the monitor is put in a closed state, if the system is not in its initial state." This specification is hard to understand because it involves two conditional statements. Find an equivalent, easier-to-understand specification that involves disjunctions and negations but not conditional statements.
58. How many of the disjunctions $p \vee \neg q$, $\neg p \vee q$, $q \vee r$, $q \vee \neg r$, and $\neg q \vee \neg r$ can be made simultaneously true by an assignment of truth values to p, q, and r?
59. How many of the disjunctions $p \vee \neg q \vee s$, $\neg p \vee \neg r \vee s$, $\neg p \vee \neg r \vee \neg s$, $\neg p \vee q \vee \neg s$, $q \vee r \vee \neg s$, $q \vee \neg r \vee \neg s$, $\neg p \vee \neg q \vee \neg s$, $p \vee r \vee s$, and $p \vee r \vee \neg s$ can be made simultaneously true by an assignment of truth values to p, q, r, and s?

A compound proposition is **satisfiable** if there is an assignment of truth values to the variables in the compound proposition that makes the compound proposition true.

60. Which of these compound propositions are satisfiable?
    a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$
    b) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$
    c) $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$
61. Explain how an algorithm for determining whether a compound proposition is satisfiable can be used to determine whether a compound proposition is a tautology. [*Hint:* Look at $\neg p$, where p is the compound proposition that is being examined.]

1.3 PREDICATES AND QUANTIFIERS

Introduction Propositional logic, studied in Sections 1.1 and 1.2, cannot adequately express the meaning of statements in mathematics and in natural language. For example, suppose that we know that

"Every computer connected to the university network is functioning properly."

No rules of propositional logic allow us to conclude the truth of the statement

"MATH3 is functioning properly,"

where MATH3 is one of the computers connected to the university network. Likewise, we cannot use the rules of propositional logic to conclude from the statement

"CS2 is under attack by an intruder,"

where CS2 is a computer on the university network, to conclude the truth of

Solution The bitwise *OR*, bitwise *AND*, and bitwise *XOR* of these strings are obtained by taking the *OR*, *AND*, and *XOR* of the corresponding bits, respectively. This gives us

01 1011 0110	
11 0001 1101	
11 1011 1111	bitwise *OR*
01 0001 0100	bitwise *AND*
10 1010 1011	bitwise *XOR*

Exercises

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
   a) Boston is the capital of Massachusetts.
   b) Miami is the capital of Florida.
   c) $2 + 3 = 5$. d) $5 + 7 = 10$.
   e) $x + 2 = 11$. f) Answer this question.
2. Which of these are propositions? What are the truth values of those that are propositions?
   a) Do not pass go.
   b) What time is it?
   c) There are no black flies in Maine.
   d) $4 + x = 5$.
   e) The moon is made of green cheese.
   f) $2^n \geq 100$.
3. What is the negation of each of these propositions?
   a) Today is Thursday.
   b) There is no pollution in New Jersey.
   c) $2 + 1 = 3$.
   d) The summer in Maine is hot and sunny.
4. Let p and q be the propositions
   p : I bought a lottery ticket this week.
   q : I won the million dollar jackpot on Friday.
   Express each of these propositions as an English sentence.
   a) $\neg p$ b) $p \vee q$
   c) $p \rightarrow q$ d) $p \wedge q$
   e) $p \leftrightarrow q$ f) $\neg p \rightarrow \neg q$
   g) $\neg p \wedge \neg q$ h) $\neg p \vee (p \wedge q)$
5. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.
   a) $\neg q$ b) $p \wedge q$
   c) $\neg p \vee q$ d) $p \rightarrow \neg q$
   e) $\neg q \rightarrow p$ f) $\neg p \rightarrow \neg q$
   g) $p \leftrightarrow \neg q$ h) $\neg p \wedge (p \vee \neg q)$
6. Let p and q be the propositions "The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence.
   a) $\neg p$ b) $p \vee q$
   c) $\neg p \wedge q$ d) $q \rightarrow p$
   e) $\neg q \rightarrow \neg p$ f) $\neg p \rightarrow \neg q$
   g) $p \leftrightarrow q$ h) $\neg q \vee (\neg p \wedge q)$
7. Let p and q be the propositions
   p : It is below freezing.
   q : It is snowing.
   Write these propositions using p and q and logical connectives.
   a) It is below freezing and snowing.
   b) It is below freezing but not snowing.
   c) It is not below freezing and it is not snowing.
   d) It is either snowing or below freezing (or both).
   e) If it is below freezing, it is also snowing.
   f) It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
   g) That it is below freezing is necessary and sufficient for it to be snowing.
8. Let p, q, and r be the propositions
   p : You have the flu.
   q : You miss the final examination.
   r : You pass the course.
   Express each of these propositions as an English sentence.
   a) $p \rightarrow q$ b) $\neg q \leftrightarrow r$
   c) $q \rightarrow \neg r$ d) $p \vee q \vee r$
   e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
   f) $(p \wedge q) \vee (\neg q \wedge r)$
9. Let p and q be the propositions
   p : You drive over 65 miles per hour.
   q : You get a speeding ticket.
   Write these propositions using p and q and logical connectives.
   a) You do not drive over 65 miles per hour.
   b) You drive over 65 miles per hour, but you do not get a speeding ticket.
   c) You will get a speeding ticket if you drive over 65 miles per hour.
   d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
   e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

f) You get a speeding ticket, but you do not drive over 65 miles per hour.
g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

10. Let p, q, and r be the propositions
p : You get an A on the final exam.
q : You do every exercise in this book.
r : You get an A in this class.
Write these propositions using p, q, and r and logical connectives.
a) You get an A in this class, but you do not do every exercise in this book.
b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
c) To get an A in this class, it is necessary for you to get an A on the final.
d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

11. Let p, q, and r be the propositions
p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.
Write these propositions using p, q, and r and logical connectives.
a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

12. Determine whether these biconditionals are true or false.
a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.
b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.
c) $1 + 1 = 3$ if and only if monkeys can fly.
d) $0 > 1$ if and only if $2 > 1$.

13. Determine whether each of these conditional statements is true or false.
a) If $1 + 1 = 2$, then $2 + 2 = 5$.
b) If $1 + 1 = 3$, then $2 + 2 = 4$.
c) If $1 + 1 = 3$, then $2 + 2 = 5$.
d) If monkeys can fly, then $1 + 1 = 3$.

14. Determine whether each of these conditional statements is true or false.
a) If $1 + 1 = 3$, then unicorns exist.
b) If $1 + 1 = 3$, then dogs can fly.
c) If $1 + 1 = 2$, then dogs can fly.
d) If $2 + 2 = 4$, then $1 + 2 = 3$.

15. For each of these sentences, determine whether an inclusive or or an exclusive or is intended. Explain your answer.
a) Coffee or tea comes with dinner.
b) A password must have at least three digits or be at least eight characters long.
c) The pre-requisite for the course is a course in number theory or a course in cryptography.
d) You can pay using U.S. dollars or euros.

16. For each of these sentences, determine whether an inclusive or an exclusive or is intended. Explain your answer.
a) Experience with C++ or Java is required.
b) Lunch includes soup or salad.
c) To enter the country you need a passport or a voter registration card.
d) Publish or perish.

17. For each of these sentences, state what the sentence means if the or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?
a) To take discrete mathematics, you must have taken calculus or a course in computer science.
b) When you buy a new car from Acme Motor Company, you get \$2000 back in cash or a 2% car loan.
c) Dinner for two includes two items from column A or three items from column B.
d) School is closed if more than 2 feet of snow falls or if the wind chill is below -100.

18. Write each of these statements in the form "if p, then q" in English. [*Hint:* Refer to the list of common ways to express conditional statements provided in this section.]
a) It is necessary to wash the boss's car to get promoted.
b) Winds from the south imply a spring thaw.
c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
d) Willy gets caught whenever he cheats.
e) You can access the website only if you pay a subscription fee.
f) Getting elected follows from knowing the right people.
g) Carol gets sea-sick whenever she is on a boat.

19. Write each of these statements in the form "if p, then q" in English. [*Hint:* Refer to the list of common ways to express conditional statements.]
a) It snows whenever the wind blows from the northeast.
b) The apple trees will bloom if it stays warm for a week.

c) That the Pistons win the championship implies that they beat the Lakers.
d) It is necessary to walk 8 miles to get to the top of Long's Peak.
e) To get tenure as a professor, it is sufficient to be world-famous.
f) If you drive more than 400 miles, you will need to buy gasoline.
g) Your guarantee is good only if you bought your CD player less than 90 days ago.
h) Jan will go swimming unless the water is too cold.

20. Write each of these statements in the form "if p, then q" in English. [*Hint:* Refer to the list of common ways to express conditional statements provided in this section.]
a) I will remember to send you the address only if you send me an e-mail message.
b) To be a citizen of this country, it is sufficient that you were born in the United States.
c) If you keep your textbook, it will be a useful reference in your future courses.
d) The Red Wings will win the Stanley Cup if their goalie plays well.
e) That you get the job implies that you had the best credentials.
f) The beach erodes whenever there is a storm.
g) It is necessary to have a valid password to log on to the server.
h) You will reach the summit unless you begin your climb too late.

21. Write each of these propositions in the form "p if and only if q" in English.
a) If it is hot outside you buy an ice cream cone, and if you buy an ice cream cone it is hot outside.
b) For you to win the contest it is necessary and sufficient that you have the only winning ticket.
c) You get promoted only if you have connections, and you have connections only if you get promoted.
d) If you watch television your mind will decay, and conversely.
e) The trains run late on exactly those days when I take it.

22. Write each of these propositions in the form "p if and only if q" in English.
a) For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.
b) If you read the newspaper every day, you will be informed, and conversely.
c) It rains if it is a weekend day, and it is a weekend day if it rains.
d) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him.

23. State the converse, contrapositive, and inverse of each of these conditional statements.
a) If it snows today, I will ski tomorrow.
b) I come to class whenever there is going to be a quiz.
c) A positive integer is a prime only if it has no divisors other than 1 and itself.

24. State the converse, contrapositive, and inverse of each of these conditional statements.
a) If it snows tonight, then I will stay at home.
b) I go to the beach whenever it is a sunny summer day.
c) When I stay up late, it is necessary that I sleep until noon.

25. How many rows appear in a truth table for each of these compound propositions?
a) $p \to \neg p$
b) $(p \vee \neg r) \wedge (q \vee \neg s)$
c) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$
d) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

26. How many rows appear in a truth table for each of these compound propositions?
a) $(q \to \neg p) \vee (\neg p \to \neg q)$
b) $(p \vee \neg t) \wedge (p \vee \neg s)$
c) $(p \to r) \vee (\neg s \to \neg t) \vee (\neg u \to v)$
d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

27. Construct a truth table for each of these compound propositions.
a) $p \wedge \neg p$
b) $p \vee \neg p$
c) $(p \vee \neg q) \to q$
d) $(p \vee q) \to (p \wedge q)$
e) $(p \to q) \leftrightarrow (\neg q \to \neg p)$
f) $(p \to q) \to (q \to p)$

28. Construct a truth table for each of these compound propositions.
a) $p \to \neg p$
b) $p \leftrightarrow \neg p$
c) $p \oplus (p \vee q)$
d) $(p \wedge q) \to (p \vee q)$
e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

29. Construct a truth table for each of these compound propositions.
a) $(p \vee q) \to (p \oplus q)$
b) $(p \oplus q) \to (p \wedge q)$
c) $(p \vee q) \oplus (p \wedge q)$
d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
f) $(p \oplus q) \to (p \oplus \neg q)$

30. Construct a truth table for each of these compound proposition
a) $p \oplus p$
b) $p \oplus \neg p$
c) $p \oplus \neg q$
d) $\neg p \oplus \neg q$
e) $(p \oplus q) \vee (p \oplus \neg q)$
f) $(p \oplus q) \wedge (p \oplus \neg q)$

31. Construct a truth table for each of these compound propositions.
a) $p \to \neg q$
b) $\neg p \leftrightarrow q$
c) $(p \to q) \vee (\neg p \to q)$
d) $(p \to q) \wedge (\neg p \to q)$
e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

32. Construct a truth table for each of these compound propositions.
a) $(p \vee q) \vee r$
b) $(p \vee q) \wedge r$

c) $(p \wedge q) \vee r$
d) $(p \wedge q) \wedge r$
e) $(p \vee q) \wedge \neg r$
f) $(p \wedge q) \vee \neg r$

33. Construct a truth table for each of these compound propositions.
a) $p \rightarrow (\neg q \vee r)$
b) $\neg p \rightarrow (q \rightarrow r)$
c) $(p \rightarrow q) \vee (\neg p \rightarrow r)$
d) $(p \rightarrow q) \wedge (\neg p \rightarrow r)$
e) $(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

34. Construct a truth table for $((p \rightarrow q) \rightarrow r) \rightarrow s$.

35. Construct a truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.

36. What is the value of x after each of these statements is encountered in a computer program, if $x = 1$ before the statement is reached?
a) **if** $1 + 2 = 3$ **then** $x := x + 1$
b) **if** $(1 + 1 = 3)$ *OR* $(2 + 2 = 3)$ **then** $x := x + 1$
c) **if** $(2 + 3 = 5)$ *AND* $(3 + 4 = 7)$ **then** $x := x + 1$
d) **if** $(1 + 1 = 2)$ *XOR* $(1 + 2 = 3)$ **then** $x := x + 1$
e) **if** $x < 2$ **then** $x := x + 1$

37. Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of each of these pairs of bit strings.
a) 101 1110, 010 0001
b) 1111 0000, 1010 1010
c) 00 0111 0001, 10 0100 1000
d) 11 1111 1111, 00 0000 0000

38. Evaluate each of these expressions.
a) $1\ 1000 \wedge (0\ 1011 \vee 1\ 1011)$
b) $(0\ 1111 \wedge 1\ 0101) \vee 0\ 1000$
c) $(0\ 1010 \oplus 1\ 1011) \oplus 0\ 1000$
d) $(1\ 1011 \vee 0\ 1010) \wedge (1\ 0001 \vee 1\ 1011)$

Fuzzy logic is used in artificial intelligence. In fuzzy logic, a proposition has a truth value that is a number between 0 and 1, inclusive. A proposition with a truth value of 0 is false and one with a truth value of 1 is true. Truth values that are between 0 and 1 indicate varying degrees of truth. For instance, the truth value 0.8 can be assigned to the statement "Fred is happy," because Fred is happy most of the time, and the truth value 0.4 can be assigned to the statement "John is happy," because John is happy slightly less than half the time.

39. The truth value of the negation of a proposition in fuzzy logic is 1 minus the truth value of the proposition. What are the truth values of the statements "Fred is not happy" and "John is not happy"?

40. The truth value of the conjunction of two propositions in fuzzy logic is the minimum of the truth values of the two propositions. What are the truth values of the statements "Fred and John are happy" and "Neither Fred nor John is happy"?

41. The truth value of the disjunction of two propositions in fuzzy logic is the maximum of the truth values of the two propositions. What are the truth values of the statements "Fred is happy, or John is happy" and "Fred is not happy, or John is not happy"?

***42.** Is the assertion "This statement is false" a proposition?

***43.** The nth statement in a list of 100 statements is "Exactly n of the statements in this list are false."
a) What conclusions can you draw from these statements?
b) Answer part (a) if the nth statement is "At least n of the statements in this list are false."
c) Answer part (b) assuming that the list contains 99 statements.

44. An ancient Sicilian legend says that the barber in a remote town who can be reached only by traveling a dangerous mountain road shaves those people, and only those people, who do not shave themselves. Can there be such a barber?

45. Each inhabitant of a remote village always tells the truth or always lies. A villager will only give a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit; the other branch leads deep into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?

46. An explorer is captured by a group of cannibals. There are two types of cannibals—those who always tell the truth and those who always lie. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always lies or always tells the truth. He is allowed to ask the cannibal exactly one question.
a) Explain why the question "Are you a liar?" does not work.
b) Find a question that the explorer can use to determine whether the cannibal always lies or always tells the truth.

47. Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives.
a) "The message is scanned for viruses whenever the message was sent from an unknown system."
b) "The message was sent from an unknown system but it was not scanned for viruses."
c) "It is necessary to scan the message for viruses whenever it was sent from an unknown system."
d) "When a message is not sent from an unknown system it is not scanned for viruses."

48. Express these system specifications using the propositions p "The user enters a valid password," q "Access is granted," and r "The user has paid the subscription fee" and logical connectives.
a) "The user has paid the subscription fee, but does not enter a valid password."
b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
c) "Access is denied if the user has not paid the subscription fee."
d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

49. Are these system specifications consistent? "The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."

50. Are these system specifications consistent? "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."

51. Are these system specifications consistent? "The router can send packets to the edge system only if it supports the new address space. For the router to support the new address space it is necessary that the latest software release be installed. The router can send packets to the edge system if the latest software release is installed, The router does not support the new address space."

52. Are these system specifications consistent? "If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer."

53. What Boolean search would you use to look for Web pages about beaches in New Jersey? What if you wanted to find Web pages about beaches on the isle of Jersey (in the English Channel)?

54. What Boolean search would you use to look for Web pages about hiking in West Virginia? What if you wanted to find Web pages about hiking in Virginia, but not in West Virginia?

Exercises 55–59 relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?

55. A says "At least one of us is a knave" and B says nothing.

56. A says "The two of us are both knights" and B says "A is a knave."

57. A says "I am a knave or B is a knight" and B says nothing.

58. Both A and B say "I am a knight."

59. A says "We are both knaves" and B says nothing.

Exercises 60–65 are puzzles that can be solved by translating statements into logical expressions and reasoning from these expressions using truth tables.

60. The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith, Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was if
   a) one of the three men is guilty, the two innocent men are telling the truth, but the statements of the guilty man may or may not be true?
   b) innocent men do not lie?

61. Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

62. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.

63. A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.

64. Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said "Carlos did it." John said "I did not do it." Carlos said "Diana did it." Diana said "Carlos lied when he said that I did it."
   a) If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.
   b) If the authorities also know that exactly one is lying, who did it? Explain your reasoning.

*65. Solve this famous logic puzzle, attributed to Albert Einstein, and known as the **zebra puzzle.** Five men with different nationalities and with different jobs live in consecutive houses on a street. These houses are painted different colors. The men have different pets and have different favorite drinks. Determine who owns a zebra and whose favorite drink is mineral water (which is one of the favorite drinks) given these clues: The Englishman lives in the red house. The Spaniard owns a dog. The Japanese man is a painter. The Italian drinks tea. The Norwegian lives in the first house on the left. The green house is immediately to the right of the white one. The photographer breeds snails. The diplomat lives in the yellow house. Milk is drunk in the middle house. The owner of the green house drinks coffee. The Norwegian's house is next to the blue one. The violinist drinks orange juice. The fox is in a house next to that of the physician. The horse is in a house next to that of the diplomat. [*Hint:* Make a table where the rows represent the men and columns represent the color of their houses, their jobs, their pets, and their favorite drinks and use logical reasoning to determine the correct entries in the table.]

1.2 PROPOSITIONAL EQUIVALENCES

Introduction An important type of step used in a mathematical argument is the replacement of a statement with another statement with the same truth value. Because of this, methods that produce propositions with the same truth value as a given compound proposition are used extensively in the construction of mathematical arguments. Note that we will use the term "compound proposition" to refer to an expression formed from propositional variables using logical operators, such as $p \wedge q$.

We begin our discussion with a classification of compound propositions according to their possible truth values.

proposition that is always true, no matter what the truth values of the proposi-

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