

# Outline

Airlines Graph

Knight Transposition

Seven Bridges of Königsberg

# Airlines Graph

Consider a small country with five cities:  $A, B, C, D, E$ .

There are six flights:

$A - B, A - C, A - E,$   
 $B - D, C - D, C - E.$

Is there a direct flight from  $A$  to  $D$ ?

With one stop?

With two stops?

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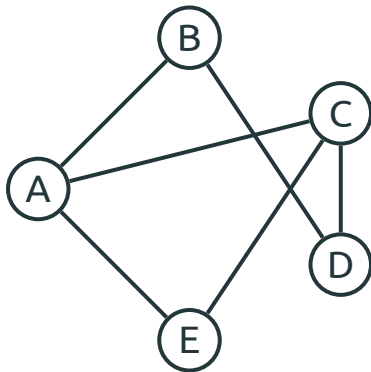
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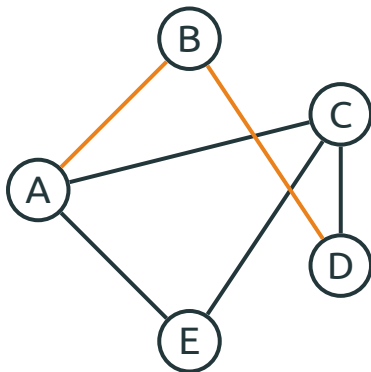
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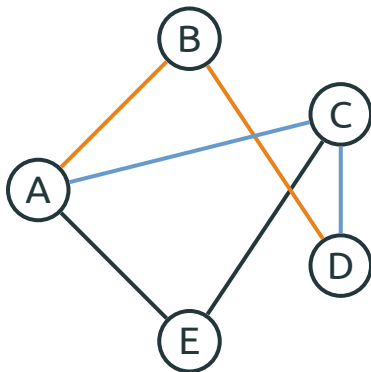
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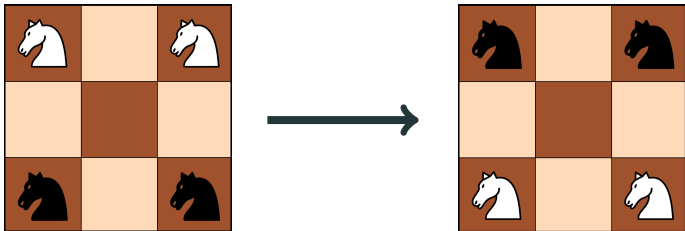
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**Knight Transposition**

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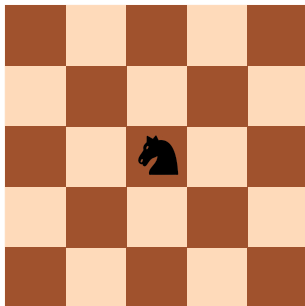
# Guarini's Puzzle



Exchange the places of the white knights and the black knights.

# Chess Knight

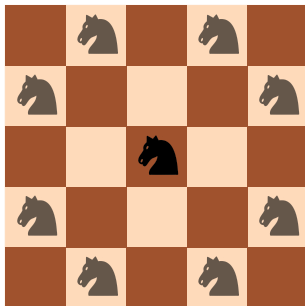
A chess knight can move in an **L** shape in any direction



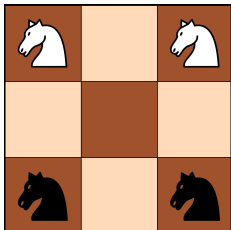


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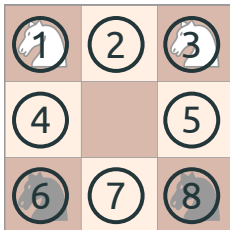


# Guarini's Puzzle. Solution



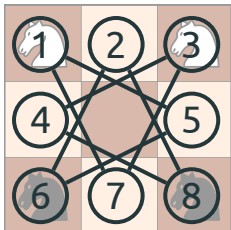
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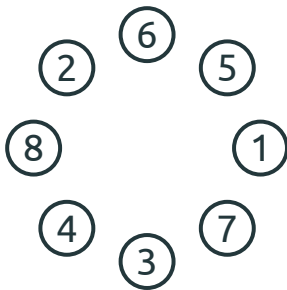
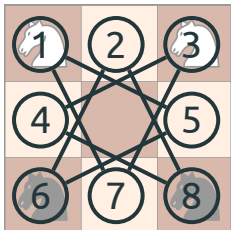
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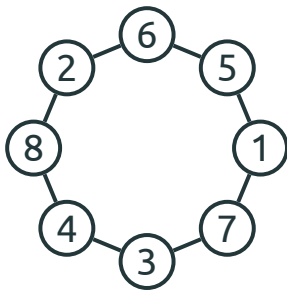
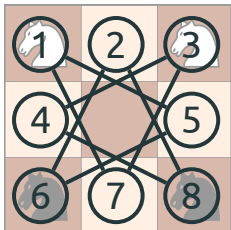
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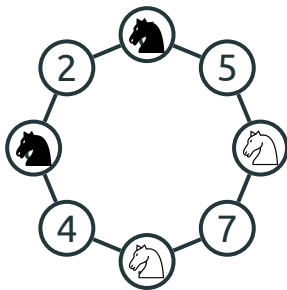
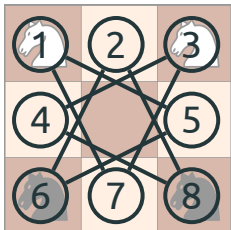
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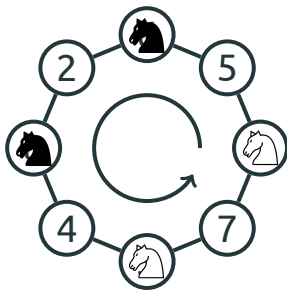
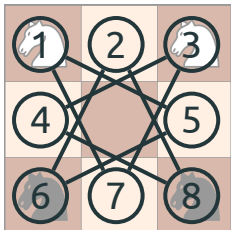
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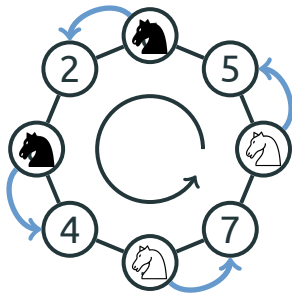
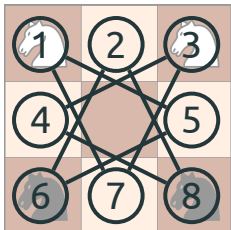
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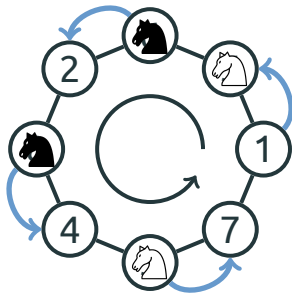
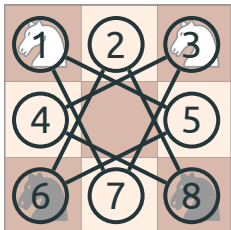


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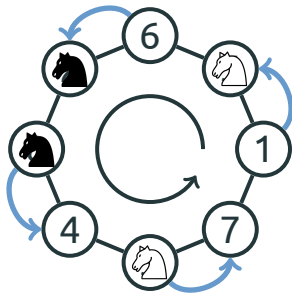
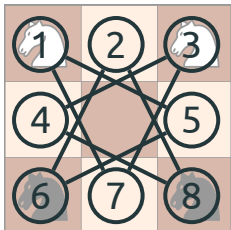
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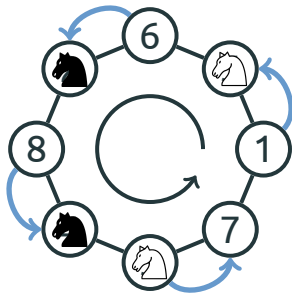
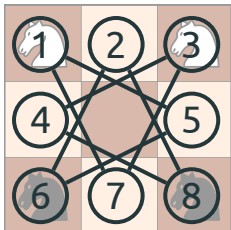
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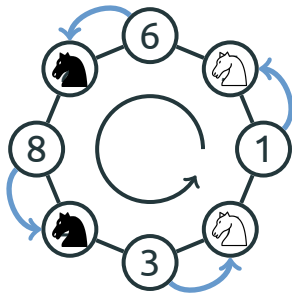
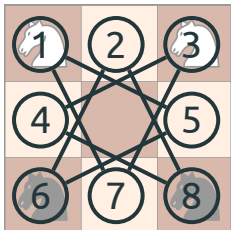
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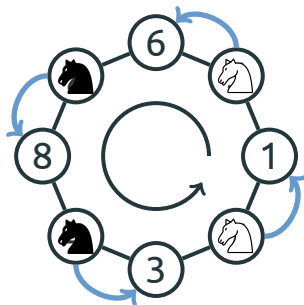
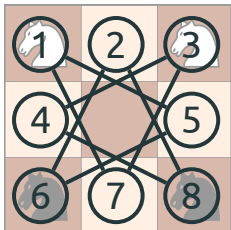
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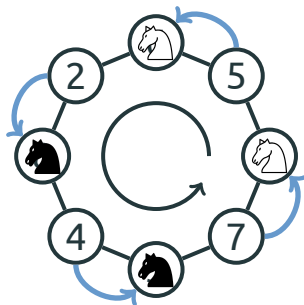
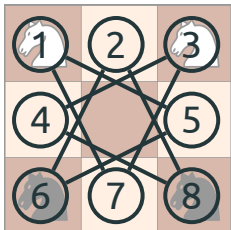
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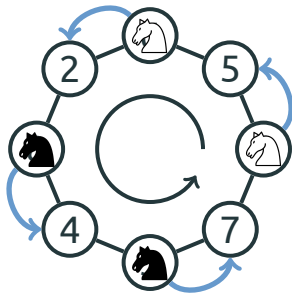
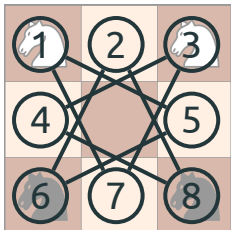
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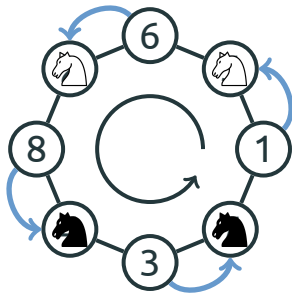
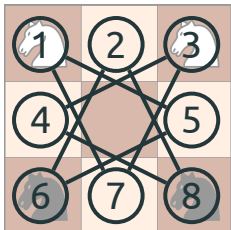
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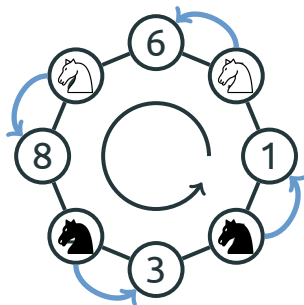
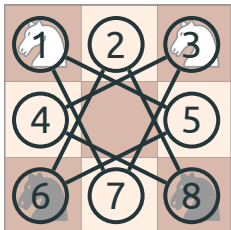


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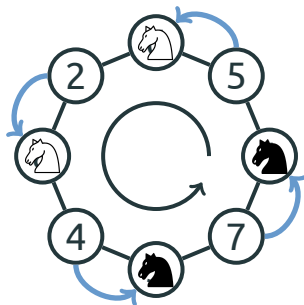
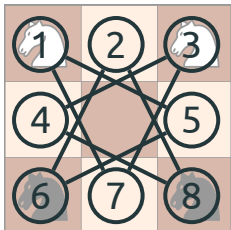
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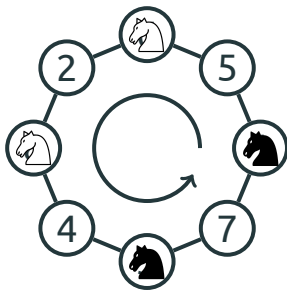
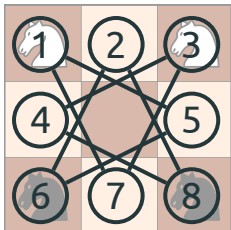
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Königsberg, Prussia, 1735



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# Walk through Königsberg

Cross each bridge  
exactly once



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Leonhard Euler





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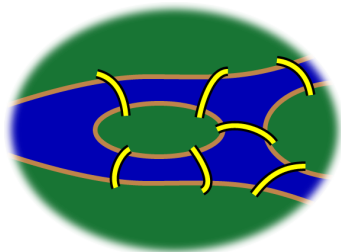
Impossible!



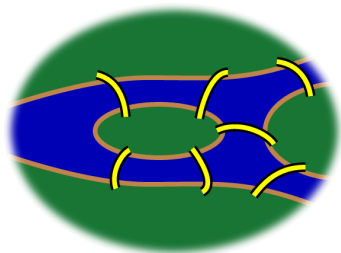
## Bridges of Königsberg. Graph



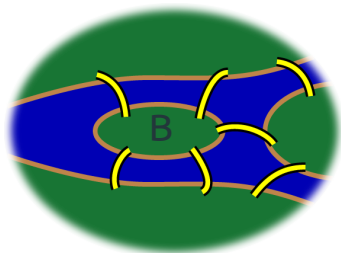
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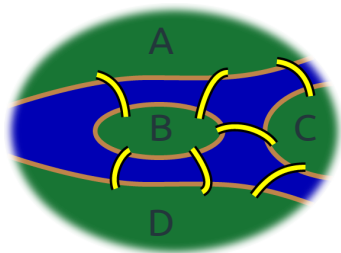
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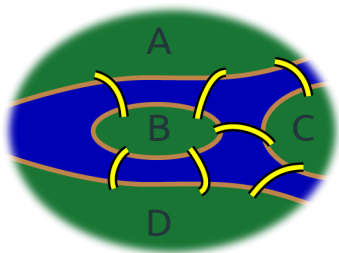
# Bridges of Königsberg. Graph

A

B

D

C

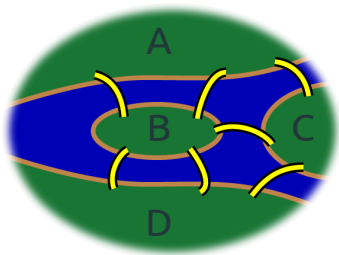


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A

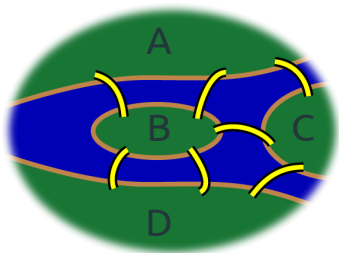
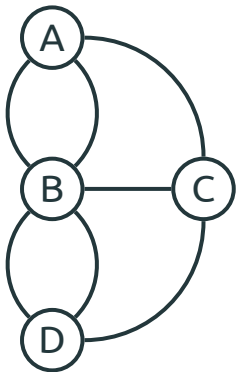
B — C

D

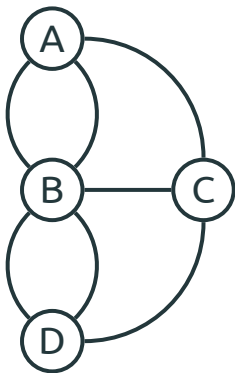




# Bridges of Königsberg. Graph



# Bridges of Königsberg. Graph



Is there a path which visits every edge exactly once?

Eulerian Path

# Eulerian Path

START

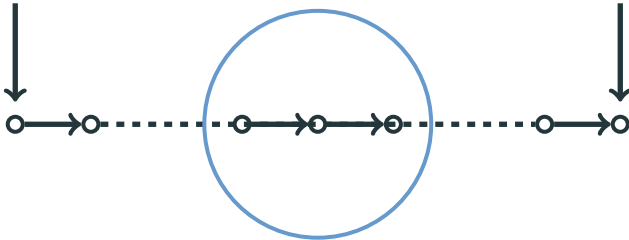
FINISH



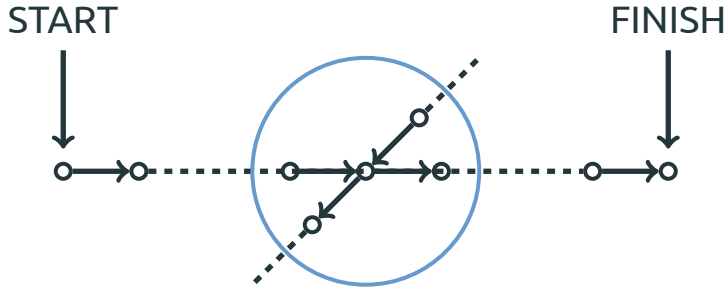
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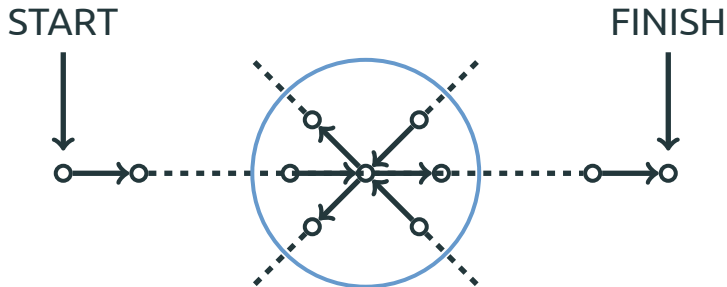
FINISH



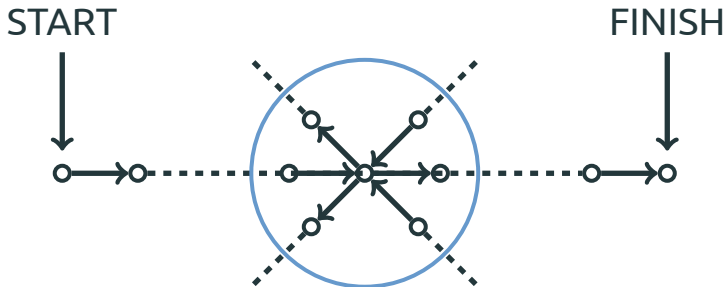
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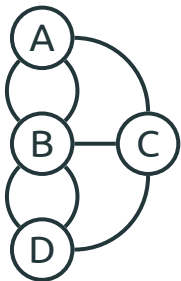


# Eulerian Path



All but START and FINISH vertices  
have **even** number of neighbors

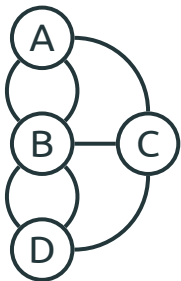
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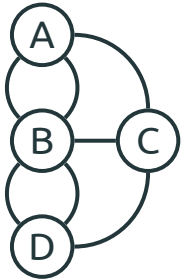
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All four vertices have  
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All but START and FINISH vertices  
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# Eulerian Path

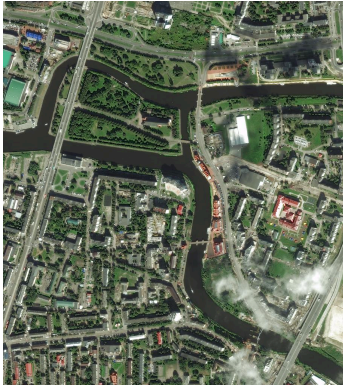


All four vertices have  
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**Impossible!**

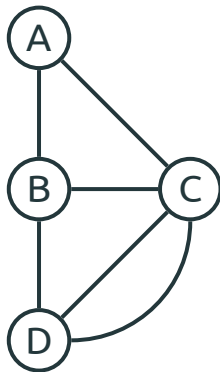
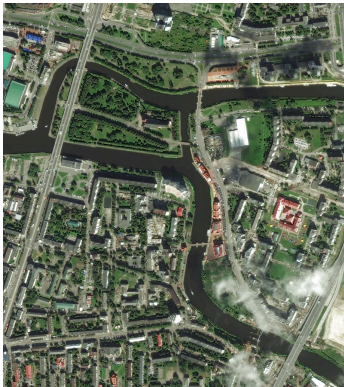


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# Six Bridges of Kaliningrad



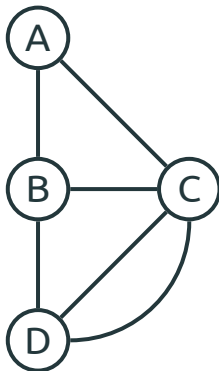
# Six Bridges of Kaliningrad



# Six Bridges of Kaliningrad

B and D have **odd**  
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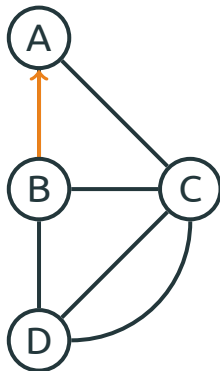
If there exists an Eulerian path, B and D must be START and FINISH



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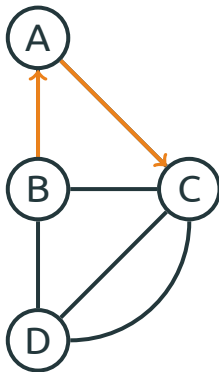
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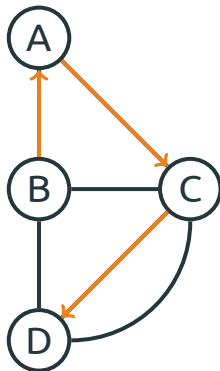
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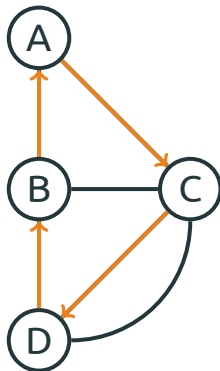




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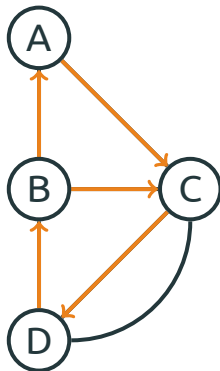
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