Lecture 9:-

FA → TG → RE

Kleene theorem III

"Every RE can be represented by an FA".

$\underbrace{(a+b)^{*}(a+b)^{*}}_{\gamma_1}\ \underbrace{(a)(b+a)^{*}}_{\gamma_2}\ \underbrace{\text{------}}_{\gamma_3}\ \underbrace{\text{--------}}_{\gamma_4}$

$FA_1 \quad FA_2 \quad FA_3 \quad FA_4$.

- How to combine.
  - Union, Sum, +.
  - Concatenation.
  - Closure, *

- Union, Sum, +.

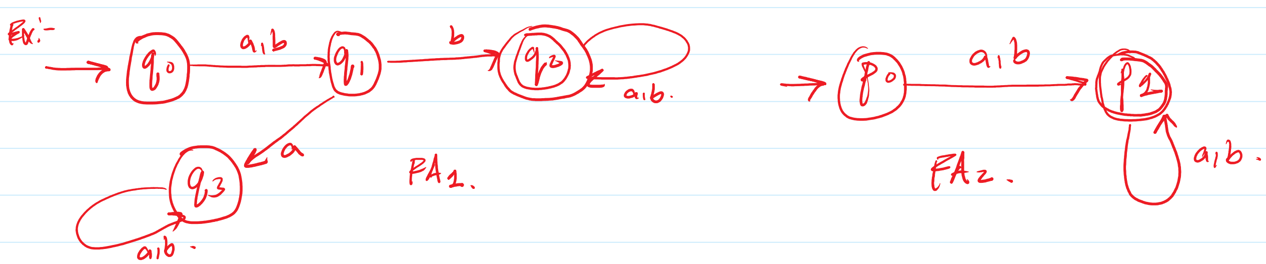
γ_1, γ_2, then $\gamma_3 = \gamma_1 + \gamma_2$ is also RE.

$FA_1, FA_2 \quad FA_3 = FA_1 + FA_2$

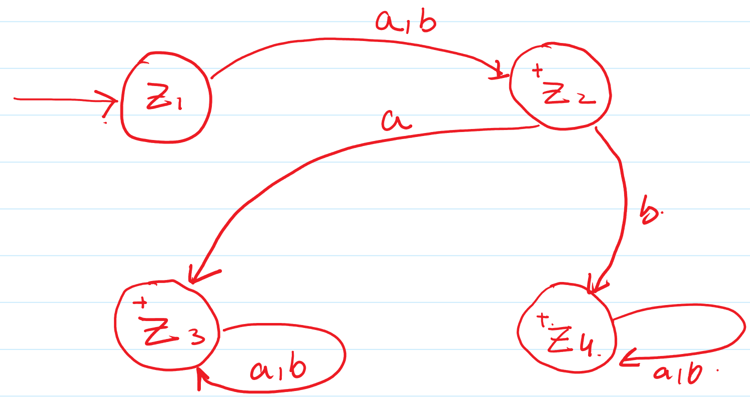
Algorithm :- 1- Start by taking both FA's initial state. & traverse on the respective inputs.

2- During the process. Any state encountered final, the resultant state will be final.

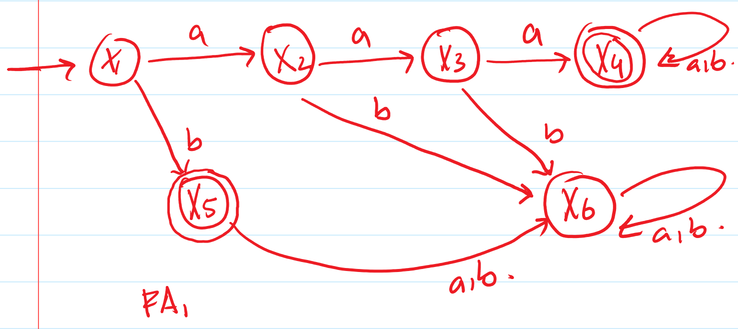
Ex:-



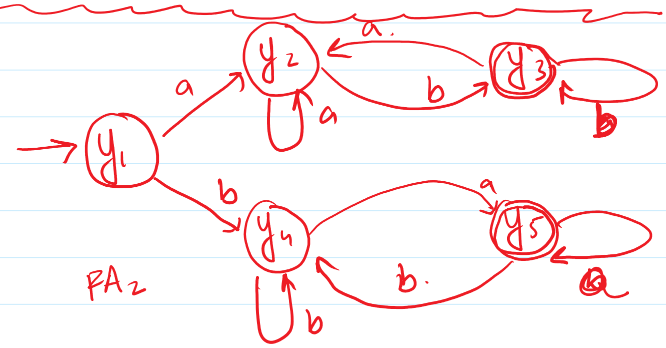
Old State	transition at a	transition at b.
$Z_1 \equiv (q_0, p_0)$.	$Z_2^+ \equiv (q_1, p_1)$	$Z_2^+ \equiv (q_1, p_1)$
$Z_2^+ \equiv (q_1, p_1)$.	$Z_3^+ \equiv (q_3, p_1)$	$Z_4^+ \equiv (q_2, p_1)$.
$Z_3^+ \equiv (q_3, p_1)$.	$Z_3^+ \equiv (q_3, p_1)$	$Z_3^+ \equiv (q_3, p_1)$
$Z_4^+ \equiv (q_2, p_1)$	$Z_4^+ \equiv (q_2, p_1)$	$Z_4^+ \equiv (q_2, p_1)$.



$FA_3 = FA_1 + FA_2$.



FA_1



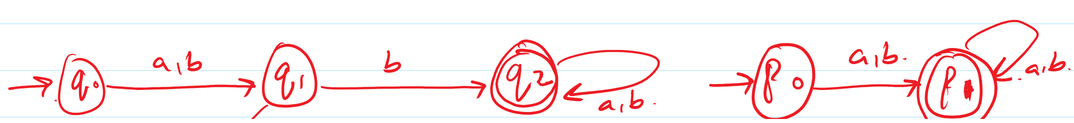
FA_2

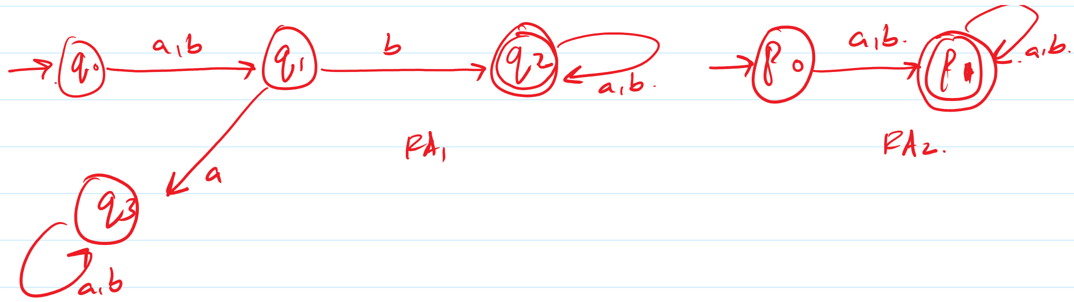
old states	transition 'a'	transition 'b'
$Z_1 \equiv (x_1, y_1)$.	$Z_2 \equiv (X_2, y_2)$	$Z_3^+ \equiv (X_5, y_4)$
$Z_2 \equiv (x_2, y_2)$.	$Z_4 \equiv (X_3, y_2)$	$Z_5^+ \equiv (X_6, y_3)$
$Z_3^+ \equiv (X_5, y_4)$.	$Z_6^+ \equiv (X_6, y_5)$	$Z_7 \equiv (X_6, y_4)$
$Z_4 \equiv (X_3, y_2)$	$Z_8^+ \equiv (X_4, y_2)$	$Z_5^+ \equiv (X_6, y_3)$
$Z_5^+ \equiv (X_6, y_3)$.	$Z_9 = (X_6, y_2)$	$Z_5^+ = (X_6, y_3)$
⋮	⋮	⋮

Concatenation :- $r_3 = r_1 r_2$

$FA_3 = FA_1 FA_2$

1- Start by taking FA_1 & traverse its states.
2- Initial state = FA_1 Initial state.
3- During process, any state encountered final the resultant state will be final, the second FA's initial will be concatenated with the final of FA_1.





old Status	transition 'a'	transition 'b'
$Z_1 \equiv q_0$	$Z_2 \equiv q_1$	$Z_2 \equiv q_1$.
$Z_2 \equiv q_1$.	$Z_3 \equiv q_3$	$Z_4^+ \equiv (q_2, p_0)$
$Z_3 \equiv q_3$.	$Z_3 \equiv q_3$	$Z_3 \equiv q_3$.
$Z_4^+ \equiv (q_2, p_0)$.	$Z_5^+ \equiv (q_2, p_0, p_1)$.	$Z_5^+ \equiv (q_2, p_0, p_1)$
$Z_5^+ \equiv (q_2, p_0, p_1)$.	$\equiv (q_2, p_0, p_1, p_1)$. $Z_5^+ \equiv (q_2, p_0, p_1)$	$\equiv (q_2, p_0, p_1, p_1)$. $Z_5^+ \equiv (q_2, p_0, p_1)$.

Draw FA.