

## lecture 22:-

CFG.

Ex:-

$$S \rightarrow aSb.$$

①

$$S \rightarrow \lambda.$$

②

$\{\lambda, ab, aabb, aaabbb, \dots\}$ .

$$= \{a^n b^n \mid n \geq 0, \dots\}.$$

Non Regular.

$aSb$

$aaSbb$

$aaasbbbb$

Simplification of Rules.

Example:-

$$S \rightarrow aSa$$

$$S \rightarrow aBa$$

$$B \rightarrow bB$$

$$B \rightarrow b.$$

$$S \rightarrow aSa \mid aBa$$

$$B \rightarrow bB \mid b.$$

the left sides of the production must be same.

$$L = \{a^n b^m a^n \mid n \geq 0, m \geq 0\}.$$

Ex:-

$$S \rightarrow aXb \mid bXa \mid \lambda.$$

$$X \rightarrow aX \mid bX \mid \lambda.$$

HW.

A Grammar that generates even length of strings  $\{a, b\}$ .

$$S \rightarrow aO \mid bO \mid \lambda.$$

$$O \rightarrow aS \mid bS.$$

$\{\lambda, aa, ab, ba, bb, \dots\}$ .

$aO$

$aas$

$aa\lambda.$

$aa$

$aO$

$abs$

$ab\lambda$

$ab$

$bO$

$bas$

$ba\lambda$

$ba$

$bO$

$bbs$

$bb\lambda$

$bb$

$aO$

$aas$

$a^0$   
 $a^1S$   
 $a^2a^0$   
 $a^3a^1S$   
 $a^4a^2h$   
 $a^5a^3a$

Ex:-

$$\begin{aligned}
 S &\rightarrow \underline{AB} \\
 A &\rightarrow aA \mid a \\
 B &\rightarrow bB \mid h
 \end{aligned}$$

$$\begin{aligned}
 S &\rightarrow as \mid aB \\
 B &\rightarrow bB \mid h
 \end{aligned}$$

$a^+b^+$

Multiple Grammars may exist for a certain language.

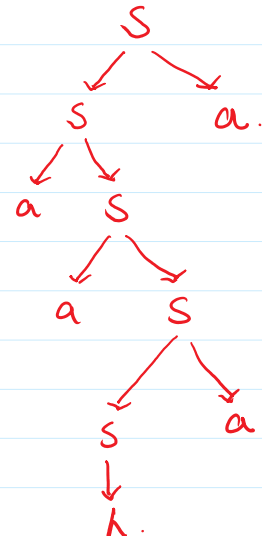
$a^+b^+$

PARSE TREE :- (Syntax Tree, Generation Tree, Production Tree)  
Derivation

$$\begin{aligned}
 S &\rightarrow Sa & ① \\
 S &\rightarrow aS & ② \\
 S &\rightarrow h & ③
 \end{aligned}$$

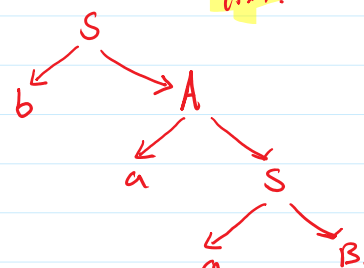
$aaaa$

$$\begin{aligned}
 S &\rightarrow Sa & - ① \checkmark \\
 &\rightarrow aSa & - ② \checkmark \\
 &\rightarrow aasa & - ② \checkmark \\
 &\rightarrow aasaa & - ① \checkmark \\
 &\rightarrow aahaa & - ③ \checkmark \\
 &\rightarrow aaaa
 \end{aligned}$$



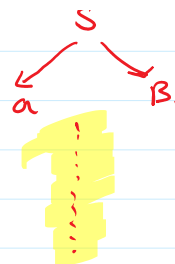
$baaabbbab$

1-	$S \rightarrow h$	$S \rightarrow bA$	② ✓
2-	$S \rightarrow bA$	$\rightarrow baS$	⑤ ✓
3-	$S \rightarrow aB$	$\rightarrow baab$	③ ✓
4-	$A \rightarrow a$	$\rightarrow baaaBB$	⑨
5-	$A \rightarrow aS$	$\rightarrow baaaBb$	⑦



- 4-  $A \rightarrow a \rightarrow baaaBB.$  (9)
- 5-  $A \rightarrow aS \rightarrow baaaBb$  (7)
- 6-  $A \rightarrow bAA \rightarrow baaabSb.$  (8)
- 7-  $B \rightarrow b \rightarrow baaabbaB.$  (2)
- 8-  $B \rightarrow bS \rightarrow baaabbaB.$  (4)
- 9-  $B \rightarrow aBB$

Tree.

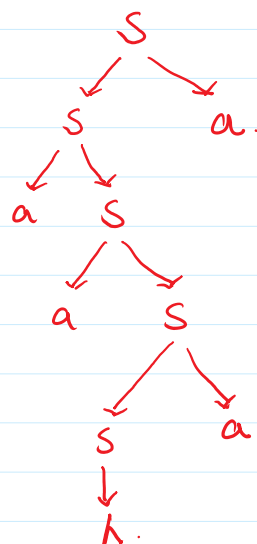


Ambiguous

"A CFG is called ambiguous if  $\exists$  word which has two different parse trees."

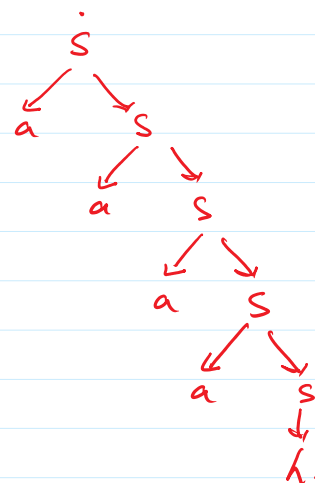
Ex:-  $S \rightarrow aS | Sa | \lambda.$

$S \rightarrow Sa$  — (1) ✓  
 $\rightarrow aSa$  — (2) ✓  
 $\rightarrow aasa$  — (2) ✓  $\Leftrightarrow$   
 $\rightarrow aasaa$  — (1) ✓  
 $\rightarrow aahaa$  — (3) ✓  
 $\rightarrow aaaa$



$S \rightarrow aS | Sa | \lambda.$

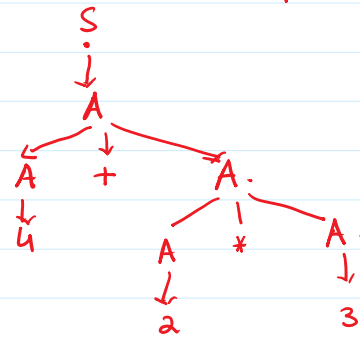
$S \rightarrow aS$   
 $\rightarrow aas$   
 $\rightarrow aaas$   
 $\rightarrow aaaaS$   
 $\rightarrow aaaaah$   
 $\rightarrow aaaa.$



Ex:- Arithmetic Expression.

$S \rightarrow A$   
 $A \rightarrow \text{Integer} | A+A | A-A | A * A | A / A | (A)$

$\uparrow =$  Terminals.

$$\begin{aligned} S &\rightarrow A. \\ &\rightarrow A + A. \\ &\rightarrow A + A * A. \\ &\rightarrow 4 + 2 * 3. \end{aligned}$$

$$\begin{aligned} S &\rightarrow A \\ &\rightarrow A * A \\ &\rightarrow A + A * A \\ &\rightarrow 4 + 2 * 3. \end{aligned}$$

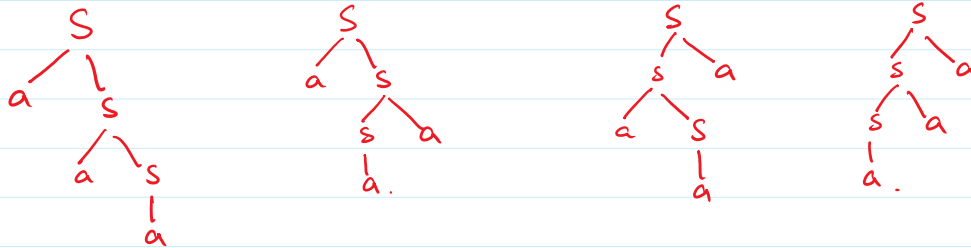
⇒ Parse Tree (H.W.)

This creates Ambiguity.

Ex:-

$$S \rightarrow aS \mid Sa \mid a.$$

Derive  $a^3$  using 4 Different Derivations.  
 $aaa$ .

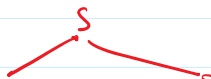


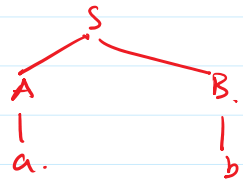
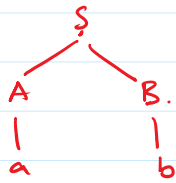
Write the derivations. (16/18)

Ex:-

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow a \\ B &\rightarrow b. \end{aligned}$$

ab.

$$S \rightarrow AB$$
$$aB$$
$$ab.$$
$$\begin{aligned} S &\rightarrow AB. \\ &\rightarrow Ab \\ &\rightarrow ab. \end{aligned}$$




if Derivations are different but ST are same  
So the word is not Ambiguous.

Left Most Derivation.

Right Most Derivation.

↓ = Terminals.

$S \rightarrow E$   
 $E \rightarrow T \downarrow + E \mid T \downarrow - E \mid T$   
 $T \rightarrow P \downarrow * T \mid P \downarrow / T \mid F$   
 $P \rightarrow Integer \mid (E) \downarrow .$   
                   ↑.           ↑   ↑.

4+2\*3.

$S \rightarrow E$   
 $\rightarrow T + E$   
 $\rightarrow P + E.$   
 $\rightarrow 4 + E$   
 $\rightarrow 4 + T$   
 $\rightarrow 4 + P * T$   
 $\rightarrow 4 + 2 * T$   
 $\rightarrow 4 + 2 * F$   
 $\rightarrow 4 + 2 * 3.$

$S \rightarrow E$   
 $E \rightarrow T \downarrow + E \mid T \downarrow - E \mid T$   
 $T \rightarrow P \downarrow * T \mid P \downarrow / T \mid F$   
 $P \rightarrow Integer \mid (E) \downarrow .$   
                   ↑.           ↑   ↑.

$S \rightarrow E$   
 $\rightarrow T + E$   
 $\rightarrow T + T$   
 $\rightarrow T + P * T$   
 $\rightarrow T + P * P$   
 $\rightarrow T + P * 3$   
 $\rightarrow T + 2 * 3.$   
 $\rightarrow P + 2 * 3.$   
 $\rightarrow 4 + 2 * 3.$



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