

$$EX = G'_X(1) \quad (1)$$

$$VarX = G''_X(1) + G'_X(1) - G'_X(1)^2 \quad (2)$$

$$\begin{cases} \kappa_1 = \mu_1 = 3 \\ \kappa_2 = \mu_2 - \mu_1^2 = 0 \\ \kappa_3 = \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3 = 0 \end{cases} \quad (3)$$

$$(coin) \begin{cases} EX = \sum_{k=1}^m \tilde{A}_{(k)}[A_{(k)} = A^{(k)}] \\ VX = (EX)^2 - \sum_{k=1}^m (2k-1) \tilde{A}_{(k)}[A_{(k)} = A^{(k)}] \end{cases} \quad (4)$$

$$(5)$$

欧拉求和公式:

$$\begin{aligned} \sum_{a \leq k < b} f(k) &= \int_a^b f(x) dx - \frac{1}{2} f(x) \Big|_a^b + \sum_{k=1}^m \frac{B_{2k}}{(2k)!} f^{(2k-1)}(x) \Big|_a^b \\ &\quad + O((2\pi)^{-2m}) \int_a^b |f^{(2m)}(x)| dx \end{aligned} \quad (6)$$

伯努利数:

$$\begin{array}{c|cccccccccccccc} n & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \hline B_n & 1 & \frac{-1}{2} & \frac{1}{6} & 0 & \frac{-1}{30} & 0 & \frac{1}{42} & 0 & \frac{-1}{30} & 0 & \frac{5}{66} & 0 & \frac{-691}{2730} \end{array} \quad (7)$$

对数形式斯特林近似

$$\ln n! = n \ln n - n + \frac{\ln n}{2} + \frac{\ln 2\pi}{2} + \frac{1}{12n} - \frac{1}{360n^3} + \frac{\varphi_{2,n}}{1260n^5} \quad (8)$$

钟形求和

$$\Theta_n = \sum_k e^{-k^2/n} = \sqrt{\pi n} + O(n^{-M}) \quad (9)$$