4D to 3D reduction of Seiberg duality for SU(N) susy gauge theories with adjoint matter: a partition function approach

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1 | Physics

——— INTRODUCTION OUTLINE———
~ More symmetry = more tools for studying t\heories
~ State structure: multiplet & superspace
~ Milder divergences
~ Renormalization constraints
~ Non renormalization theorems (perturbative)
~ Holomorphicity, couplings as background fields
(important smoothness of weak coupling limits, e.g. classic imit g in well defined.
~ Exact results (superpotential, exact beta function)
~Moduli space

1.1 Introduction

Supersymmetric quantum field theories enjoy an enlarged group of symmetries compared to other field theories. Since the symmetry group is a non trivial combination of internal and spacetime symmetries, they have many unexpected features and new techniques were found to study them. Almost all of the new tools found are available only for supersymmetric field theories, making them the theater for many advances in physics.

A more technical introduction on supersymmetry and its representation on fields can be found in appendix A.

In this section we will analyse more advanced features of supersymmetric field theories that has been used intensively in the discovery and in the analysis of electric magnetic duality and its generalisations.

1.1.1 General renormalization properties

A remarkable feature of supersymmetry is the constraint that the additional symmetry imposes on the renormalization properties of the theories.

One of the first aspects that brought attention to supersymmetry was that divergences of loop diagrams were milder because of the cancellation between diagrams with bosons and fermions running in the loops.

Nowadays we know powerful theorems that restrict the behaviour of supersymmetric field theories during renormalization. In order to preserve supersymmetry, the renormalization process has to preserve the Hilbert space structure. For example the wave function renormalization of different *particles* inside a multiplet must be the same, otherwise the renormalized lagrangian is not supersymmetric invariant anymore.

Moreover, in the supersymmetry algebra P^2 is still a Casimir operator i.e. it commutes with every operator in the algebra: particles in the same multiplet must have the same mass. Renormalization cannot break this condition, otherwise it would break supersymmetry.

For a *Super Yang Mills* theory with $\mathcal{N} = 1$ we have the additional requirement that gV, where g is the coupling and V is the vector superfield, cannot be renormalized by symmetry considerations.

Adding more supersymmetry the wave function renormalization of the various field are even more constrained by symmetry. For example, for $\mathcal{N} = 4$ *SYM* the fields and the coupling are not renormalized at all.

Beta function for SYM and SQCD

Another nice feature of supersymmetric field theories is that some quantities can be calculated exactly. The first object of this kind that we encounter is the β function of four dimensional $\mathcal{N} = 1$ *Super Yang Mills* and *Super QCD* theories.

It is given by the *NSVZ β function*

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\left[3\,T(Adj) - \sum_i T(R_i)(1-\gamma_i)\right]\left(1 - \frac{\alpha\,T(Adj)}{2\pi}\right)^{-1} \qquad \alpha = \frac{g^2}{4\pi} \tag{1.1}$$

where γ_i are the anomalous dimensions of the matter fields and $T(R_i)$ are the dynkin indices of their representation.
The anomalous dimensions are defined as

$$\gamma_i = -\frac{d\,\log(Z_i)}{d\,\log(\mu)} \tag{1.2}$$

where Z_i is the wave function renormalization coefficient. For example for gauge group $SU(N)$ we have

$$T(N) = \frac{1}{2} \qquad T(Adj) = N$$

The *NSVZ β function* was first calculated using instanton methods in [1]. Over the years it has been calculated in other ways using the fact that the action is holomorphic in the complexified coupling

$$\frac{1}{g_h^2} = \frac{1}{g^2} + i\frac{\theta}{8\pi^2} \tag{1.3}$$

Using the holomorphic coupling the action for the vector field is written as

$$\mathcal{L}_h(V_h) = \frac{1}{16}\int d^2\theta \frac{1}{g_h^2} W^a(V_h) W^a(V_h) \tag{1.4}$$

whereas with the canonical normalization for the vector field is

$$\mathcal{L}_c(V_c) = \frac{1}{16}\int d^2\theta \left(\frac{1}{g_c^2} + i\frac{\theta}{8\pi^2}\right) W^a(g_c V_c) W^a(g_c V_c) \tag{1.5}$$

Using the canonical normalization g_cV_c is a real superfield, imposing that g_c is real. For this reason with the canonical normalization the lagrangian is not holomorphic in the combination $\frac{1}{g^2} + i\frac{\theta}{8\pi^2}$. Thanks to holomorphy, the holomorphic coupling is only renormalized at one-loop and the β function can be computed but its expression is different from *NSVZ β function*. In fact, the *NSVZ β function* is defined using the canonical (or physical) coupling constant and receives contribution from all orders in perturbation theory.

At first sight, one should expect that the expressions should match since the first two orders in α of the β function are scheme independent. The reason why the two expressions differ is that the Jacobian of the transformation between canonical and holomorphic normalization is anomalous. Once the anomaly is taken into account the two expressions for the β function agree. An explicit calculation can be found in [2].

1.1.2 Superpotential: holomorphy and non-renormalization

Other than renormalization constraints, supersymmetry provides non-renormalization theorems for certain objects, such as the superpotential. In [3] it has been demonstrated that the superpotential is tree-level exact, i.e. it does not receive correction in perturbation theory. However it usually receive contributions from non perturbative dynamics.

Perturbative calculations can be done using supergraphs, i.e. Feynman diagrams with superfields. The advantage of this approach is that supersymmetry is explicit and many semplification occur naturally. The demonstration is based on the fact that for general supersymmetric field theories, supergraph loops diagrams yield a term that can be written in the form

$$\int d^4x_1 \dots d^4x_n d^2\theta d^2\bar{\theta}\; G(x_1, \dots, x_n)\, F_1(x_1, \theta, \bar{\theta}) \dots F_n(x_n, \theta, \bar{\theta}) \tag{1.6}$$

where $G(x_1, \dots, x_n)$ is translationally invariant function.
The importance of this result is that all contribution from Feynman diagrams are given by a single integral over full superspace ($d^2\theta d^2\bar{\theta}$) whereas the superpotential must be written as an integral in half-superspace($d^2\theta$ only) of chiral fields. Exploiting the fact that a product of chiral fields is a chiral field, the most general form of a superpotential is

$$W(\lambda, \Phi) = \sum_{n=1}^{\infty} \left(\int d^2\theta\, \lambda_n \Phi^n \; + \; \int d^2\bar{\theta}\, \lambda_n^\dagger \bar{\Phi}^n \right) \tag{1.7}$$

The second term of the superpotential is added in order to give a real lagrangian after the integration in superspace. From the definition, we can see that the superpotential is holomorphic in the fields and in the coupling constants.

Fifteen years later, Seiberg [4] provided a proof of this theorem using a different approach. He noted that the coupling constants λ_n can be treated as background fields, i.e. chiral superfields with no dynamics.

Using this observation we can assign transformation laws to the coupling constants, making the lagrangian invariant under a larger symmetry. Fields and coupling constants are charged under this symmetry and only certain combinations of them can appear in the superpotential. In addition, in a suitable weak coupling limit the effective superpotential must be identical with the tree-level one. These conditions, taken together, fix the expansion of the superpotential to the expression of the tree-level potential. A more detailed discussion can be found in [5] and [6].

1.1.3 Moduli space

Supersymmetric field theories have a larger set of vacua compared to ordinary field theories. This is related to the fact that chiral fields, which represent matter, contain a scalar field.

Lorentz invariance of the vacuum forbids fields with spin different from zero to acquire a vacuum expectation value. With the same reasoning, derivatives of scalar fields must be set to zero because of translational invariance of the vacuum. The scalar potential is the only term in the lagrangian that can differ from zero and in fact it is the only object that can be different from zero in the Hamiltonian. As a result, the minimal of the scalar potential are in one-to-one correspondence with the states of minimal energy of the theory.

Appendices

A | Supersymmetry and superfields

A.1 Supersymmetry algebra

The supersymmetry algebra is an extension of the Poincarè group involving anticommutators together with commutators. Since it is not a ordinary Lie algebra, Coleman-Mandula theorem does not apply for theories that are invariant under it.

The supersymmetry algebra is divided into two subalgebras, the bosonic and fermionic part. The bosonic part contains Poincarè Lie algebra ($M_{\mu\nu}, P_\mu$) while fermionic subalgebra is generated by the *supercharges* ($Q^I_\alpha, \bar{Q}^I_{\dot{\alpha}}$) with $I = 1, \dots, \mathcal{N}$. When more than one pair of supercharges is present we refer to extended supersymmetry.

The supercharges sit in spinorial representations of the Lorentz group, respectively $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$.

We will not repeat the bosonic subalgebra, since is given by the Poincarè Lie algebra. The fermionic generators satisfy anticommutation rules between themselves and commutation rules with bosonic generators. For this reason, the supersymmetry algebra is defined in mathematical literature as a graded Lie algebra with grade one.
The (anti)commutation rules in four dimensions are

$$[P_\mu, Q^I_\alpha] = 0 \tag{A.1}$$

$$[P_\mu, \bar{Q}^I_{\dot{\alpha}}] = 0 \tag{A.2}$$

$$[M_{\mu\nu}, Q^I_\alpha] = i(\sigma_{\mu\nu})^\beta_\alpha Q^I_\beta \tag{A.3}$$

$$[M_{\mu\nu}, \bar{Q}^I_{\dot{\alpha}}] = i(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \bar{Q}^I_{\dot{\beta}} \tag{A.4}$$

$$\{Q^I_\alpha, \bar{Q}^J_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^{IJ} \tag{A.5}$$

$$\{Q^I_\alpha, Q^J_\beta\} = \epsilon_{\alpha\beta} Z^{IJ} \qquad Z^{IJ} = -Z^{JI} \tag{A.6}$$

$$\{Q^I_{\dot{\alpha}}, Q^J_{\dot{\beta}}\} = \epsilon_{\dot{\alpha}\dot{\beta}} \left(Z^{IJ}\right)^* \tag{A.7}$$

This set of commutation rules can be found using symmetry arguments and enforcing the consistency of the algebra using the graded Jacobi identity.
It is important to stress the fact that Z^{IJ} are operators that span an invariant subalgebra: they are *central charges*. They play an important role especially in massive representations.

There is an additional symmetry that is not present in the previous commutation rules: R-symmetry. It is an automorphism of the algebra that act on the supercharges. For generic $\mathcal{N}$ the R-Symmetry group is $U(\mathcal{N})$.

A.2 Representations

Since the supercharges do not commute with Lorentz generators, their action on a state will result in a state with different spin: they generate a symmetry between bosons and fermions.
Representations of supersymmetry contain particle with different spin but same mass and they are organized in supermultiplets. The mass of particles in the same multiplet must be the same because P^2 is still a Casimir operator of the supersymmetry algebra, while the Pauli-Lubanski operator W^2 isn't anymore.

Moreover, the supersymmetry algebra imposes that every state must have positive energy and that every supermultiplet must contain the same number of bosonic and fermionic degrees of freedom *on-shell* .

Various supermultiplets exist and their properties depend on the number of supercharges of the theory and on what they represent e.g matter, glue or gravity.

Massless supermultiplet are typically shorter than massive multiplet because in the massless case half of the supercharges are represented trivially. Massive representation of extended supersymmetry can be shortened in case some of the central charges of the algebra are equal to twice the mass of the multiplet. These states are usually called (ultra)short multiplet or BPS states.

We will introduce the multiplets that can be defined for $4d$ $\mathcal{N} = 1$ theories and only later we will explain the differences with $3d$ $\mathcal{N} = 2$ theories. For four dimensional theories, we can define two different multiplet that are invariant under supersymmetry transformations. The matter or chiral multiplet contains a complex scalar (*squark*) and a Weyl fermion (*quark*). It identifies the matter content of the theory. The vector or gauge multiplet contains a Weyl fermion (*gaugino*) and a vector (*gluon or photon*). Particles in the same multiplet transform in the same representation of global or gauge symmetries. For this reason the gaugino cannot represent matter.

A representation of these multiplets on fields can be easily found using the *superspace* formalism that we will introduce in the next section. In this formalism it is possible to represent fields that are *off-shell*, in contrast with multiplets that we introduced previously that are *on-shell* since they represent states in Hilbert space.

A.2.1 Superfields and superspace

Supersymmetry representations on fields can be found more sistematically using the formulation of *superspace* instead of acting directly with supercharges and verifying that the algebra closes.

A simple formulation of superspace exist for theories with 4 supercharges while for theories with a bigger number of supercharges its definition is much more complex. We will be interested only in theories with 4 supercharges such as theories in 4D with $\mathcal{N} = 1$ or 3D with $\mathcal{N} = 2$.

Superspace can be seen as the extension of Minkowsky space with *fermionic coordinates* i.e. *Grassman numbers* θ^α , $\bar{\theta}^{\dot{\alpha}}$. They commute with bosonic object and anticommute between fermionic objects

$$\{\theta^\alpha, \theta^\beta\} = 0 \quad \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0 \quad \{\theta^\alpha, \bar{\theta}^{\dot{\beta}}\} = 0 \qquad \alpha, \dot{\alpha} = 1, 2 \tag{A.8}$$

We will use the following conventions for derivation and integration in superspace

$$\partial_\alpha = \frac{\partial}{\partial\theta_\alpha} \quad \partial^\alpha = -\epsilon^{\alpha\beta}\partial_\beta \quad \bar{\partial}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \quad \bar{\partial}^{\dot{\alpha}} = -\epsilon^{\dot{\alpha}\dot{\beta}}\partial_{\dot{\beta}} \quad \partial_\alpha\theta^\beta = \delta^\beta_\alpha \quad \partial_\alpha\bar{\theta}^{\dot{\alpha}} = 0 \tag{A.9}$$

$$\int d\theta = 0 \quad \int d\theta\, \theta = 1 \quad d^2\theta = \frac{1}{2}d\theta^1 d\theta^2 \quad \int d^2\theta = \frac{1}{4}\epsilon^{\alpha\beta}\partial_\alpha\partial\beta \tag{A.10}$$

For a more detailed introduction on Grassmann numbers and their properties see [7].

Using Grassmann numbers we can write the anticommutators in the supersymmetry algebra as commutators defining $\theta Q = \theta^\alpha Q_\alpha$ and $\bar{\theta}\bar{Q} = \bar{\theta}^{\dot{\alpha}}\bar{Q}_{\dot{\alpha}}$

$$[\theta Q, \bar{\theta}\bar{Q}] = 2\theta\sigma^\mu\bar{\theta}P_\mu \quad , \quad [\theta Q, \theta Q] = [\bar{\theta}\bar{Q}, \bar{\theta}\bar{Q}] = 0 \tag{A.11}$$

Using this trick we are able to represent the supersymmetry algebra as a Lie algebra. The advantage of this is that we can represent a generic element of the superPoincarè group as

$$G(x, \theta, \bar{\theta}, \omega) = \exp\left(ixP + i\theta Q + i\bar{\theta}\bar{Q} + \frac{1}{2}i\omega M\right) \tag{A.12}$$

The superspace can be represented as the 4+4 dimension group coset

$$M_{4|1} = \frac{\text{SuperPoincarè}}{\text{Lorentz}} \tag{A.13}$$

in analogy to Minkowsky space that can be seen as a coset between Poincarè and Lorentz groups.

A generic point in superspace is parametrized by $(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$. A superfields is a field in superspace i.e. function of the superspace coordinates. Since θ coordinates anticommute, the expansion of a superfield in fermionic coordinates is finite. The most general superfield $Y = Y(x, \theta, \bar{\theta})$ is given by

$$\begin{aligned} Y(x, \theta, \bar{\theta}) = f(x) + \theta\psi_1(x) + \bar{\theta}\bar{\psi}_2(x) + \theta\theta g_1(x) + \bar{\theta}\bar{\theta}g_2(x) \\ + \theta\sigma^\mu\theta v_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\rho(x) + \theta\theta\bar{\theta}\bar{\theta}s(x) \end{aligned} \tag{A.14}$$

fields with uncontracted θ such as $\psi_1, \psi_2, \lambda, \rho$ are spinors while v_μ is a vector.

Supercharges can be represented as differential operators that act on superfield. Their expression is

$$\begin{cases} Q_\alpha &= -i\partial_\alpha - \sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \\ \bar{Q}_{\dot{\alpha}} &= +i\dot{\partial}_{\dot{\alpha}} + \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\bar{\theta}^{\dot{\beta}}\partial_\mu \end{cases} \tag{A.15}$$

An infinitesimal supersymmetry transformation on a superfield is given by

$$\delta_{\epsilon,\bar{\epsilon}}Y = \left(i\epsilon Q + i\bar{\epsilon}\bar{Q}\right)Y \tag{A.16}$$

The powerfulness of the superfield formalism is due to the fact that an integral in full superspace coordinates of a superfield is supersymmetric invariant.

$$\delta_{\epsilon,\bar{\epsilon}} \int d^4x\, d^2\theta\, d^2\bar{\theta}\; Y = \int d^4x\, d^2\theta\, d^2\bar{\theta}\; \delta_{\epsilon,\bar{\epsilon}} Y = 0 \tag{A.17}$$

The first equality holds because the Grassmann measure is inviariant under translation. Using the explicit expression of the supercharges, we can see that the variation of the superfield is either killed by the integration in the θ variables or is proportional to a spacetime derivative that does not contribute after integration in space.

Using this fact we can construct supersymmetric invariant lagrangians by integrating superfields in superspace. Clearly, in order to find a physically significant lagrangian we should choose the superfield we wish to integrate wisely. More importantly, we want to use irreducible representation of supersymmetry i.e. the supermultiplets we introduced before. We need to find conditions that can be imposed on a general superfield that are invariant under a supersymmetry transformation.

Chiral superfield

One way to achieve this goal is to find an operator that commute with the supercharges and annihilate the superfield An example of such operator is the *covariant derivative*

$$\begin{cases} D_\alpha = & \partial_\alpha + i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \\ \bar{D}_{\dot{\alpha}} = & \dot{\partial}_{\dot{\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\bar{\theta}^{\dot{\beta}}\partial_\mu \end{cases} \tag{A.18}$$

We can define a (anti)chiral superfield Φ

$$\bar{D}_{\dot{\alpha}}\Phi = 0 \quad \text{chiral} \qquad\qquad D_\alpha\Psi = 0 \quad \text{anti-chiral} \tag{A.19}$$

This condition reduces the number of components of the superfield. It can be easily demonstrated that if Ψ is chiral, then $\bar{\Psi}$ is anti-chiral. As a result a chiral field cannot be real.

The expansion of a chiral fields in components is give by

$$\Phi(x,\theta,\bar{\theta}) = \phi(x) + \sqrt{2}\theta\psi(x) + i\theta\sigma^\mu\bar{\theta}\partial\mu\phi(x) - \theta\theta F(x) - \frac{i}{\sqrt{2}}\theta\theta\partial\mu\psi(x)\sigma^\mu\bar{\theta} - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}(x) \tag{A.20}$$

We can see that a chiral field is composed by three fields: two complex scalars (ϕ and F) and a spinor (ψ).

The chiral superfield identifies the matter multiplet we introduced previously. It contains an additional bosonic field ($F(x)$) that is present because superfields provide an *off-shell* representation of supersymmetry and it is needed in order to close the algebra. It is called *auxiliary field* because it will not have kinetic terms in every Lagrangian that can be constructed.

Real or Vector Field

We can impose that the superfield is real. In this way we find the *real* or *vector* multiplet. Its general expression in component is messy and a simplification can be made noting that $\Phi + \bar{\Phi}$ is a vector superfield if Φ is chiral. Choosing an appropriate chiral field, the real superfield can be put in what is called Wess-Zumino gauge. In this gauge the vector superfield can be written as

$$V_{WZ}(x, \theta, \bar{\theta}) = \theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \tag{A.21}$$

The vector superfields represents the vector multiplet (which contains radiation) and similarly to the chiral superfields contains an auxiliary field ($D(x)$).

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