4D to 3D reduction of Seiberg duality for SU(N) susy gauge theories with adjoint matter: a partition function approach

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1 | Introduction

2 Physics

2.1 Introduction

Supersymmetric quantum field theories enjoy an enlarged group of symmetries compared to other field theories. Since the symmetry group is a non trivial combination of internal and spacetime symmetries, they have many unexpected features and new techniques were found to study them. On top of that *superstring theory* provided many insights and explanations that were not clear from a field theory perspective only. Almost all of the new tools found are available only for supersymmetric field theories, making them the theater for exciting discoveries in physics.

Renormalization (and non-renormalization)

A remarkable feature of supersymmetry is the constraints that imposes on the renormalization properties of the theories.

One of the first aspects that brought attention to supersymmetry was that divergences coming from loop diagrams were milder because of the cancellation between diagrams with bosons and fermions running in the loops. Even if this looks like a very promising feature, now it is a property that is exploited by even more powerful theorems about the renormalization properties of the theory.

Renormalization

Being a symmetry between bosons and fermions, supersymmetry imposes that states need be organized in multiplets containing different representations of the *Lorentz group* such as scalars and spinors for example. Different multiplets exist and their properties depend on their explicit construction and on the number of supercharges of the theory.

In order to preserve supersymmetry, the renormalization process has to preserve the Hilbert Space structure i.e. for example the wave function renormalization of different *particles* inside a multiplet must be the same. With the same reasoning, the masses of different particles of the same multiplet must be the same (supersymmetry algebra requires this explicitly).

2.2 Four dimensional dualities

BUUUUU

Seiberg Duality

This is the duality originally found by Seiberg [1].
POSSO DIRE ALTRO!!
The electric theory is a $SU(N_c)$ supersymmetric non-chiral gauge theory with N_f flavours global symmetry group $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$. The charges of the matter content of the theory are summarized in the table below.

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
Q	N_c	N_F	1	1	$\frac{N_f - N_c}{N_f}$
$\tilde{Q}$	$\overline{N_c}$	1	$\overline{N_F}$	-1	$\frac{N_f - N_c}{N_f}$

Table 2.1: Charge of matter content of the electric theory

The *R-Charge* is fixed by requiring that the *R-Symmetry* is non anomalous.
Da spiegare meglio.
What really happens is that $U(1)_A$ symmetry (which is anomalous) mixes with the classical (anomalous) $U(1)_{R'}$ R-symmetry. Their mixing result in a non anomalous $U(1)_R$ R-symmetry and the disappearance of $U(1)_A$.
The triangular graph corresponding to this anomaly constrains the R-charge of the quarks imposing

$$R_{gaugino} T(\mathrm{Ad}) + \sum_{fermions\ f} (R_f - 1) T(r) = 0$$

$$N_c + \frac{1}{2} 2N_f (R_Q - 1) = 0 \quad \rightarrow \quad R_Q = \frac{N_f - N_c}{N_f}$$

The magnetic theory is a theory with the same global symmetries as the electric theory, but the gauge group is now $SU(N_f - N_c)$ and in addition there are N_f^2 fields, that we will call mesons. Dual quarks will be represented as $q, \tilde{q}$ and mesons as $M^i_{\tilde{j}}$ The charges for the magnetic theory are given by

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
q	N_c	N_F	1	$\frac{N_c}{N-f-N_c}$	$\frac{N_c}{N_f}$
$\tilde{q}$	$\overline{N_c}$	1	$\overline{N_F}$	$-\frac{N_c}{N-f-N_c}$	$\frac{N_c}{N_f}$
$M^i_{\tilde{j}}$	1	N_f	$\overline{N_f}$	0	$2\frac{N_f - N_c}{N_f}$

Table 2.2: Charge of matter content of the magnetic theory

Kutasov-Schwimmer duality

2.3 3D dualities

Aharony duality

Kutasov-Schwimmer duality

3 Math

4 | My work

5 conclusions

6 | Appendix

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