

4D to 3D reduction of Seiberg duality for $SU(N)$ susy
gauge theories with adjoint matter: a partition
function approach

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1 | Introduction

2 | Physics

2.1 General features of Seiberg-like dualities

2.2 Four dimensional dualities

Seiberg Duality

This is the duality originally found by Seiberg [1].

POSSO DIRE ALTRO!!

The electric theory is a $SU(N_c)$ supersymmetric non-chiral gauge theory with N_f flavours global symmetry group $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$. The charges of the matter content of the theory are summarized in the table below.

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
Q	N_c	N_F	1	1	$\frac{N_f - N_c}{N_f}$
\tilde{Q}	$\overline{N_c}$	1	$\overline{N_F}$	-1	$\frac{N_f - N_c}{N_f}$

Table 2.1: Charge of matter content of the electric theory

The R -Charge is fixed by requiring that the R -Symmetry is non anomalous.

Da spiegare meglio.

What really happens is that $U(1)_A$ symmetry (which is anomalous) mixes with the classical (anomalous) $U(1)_{R'}$ R-symmetry. Their mixing result in a non anomalous $U(1)_R$ R-symmetry and the disappearance of $U(1)_A$.

The triangular graph corresponding to this anomaly constrains the R-charge of the quarks imposing

$$R_{gaugino}T(\text{Ad}) + \sum_{\text{fermions } f} (R_f - 1)T(r) = 0$$

$$N_c + \frac{1}{2} 2N_f(R_Q - 1) = 0 \quad \rightarrow \quad R_Q = \frac{N_f - N_c}{N_f}$$

The magnetic theory is a theory with the same global symmetries as the electric theory, but the gauge group is now $SU(N_f - N_c)$ and in addition there are N_f^2 fields, that we will call mesons. Dual quarks will be represented as q, \tilde{q} and mesons as M_j^i . The charges for the magnetic theory are given by

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$
q	N_c	N_F	1	$\frac{N_c}{N-f-N_c}$	$\frac{N_c}{N_f}$
\tilde{q}	$\overline{N_c}$	1	$\overline{N_F}$	$-\frac{N_c}{N-f-N_c}$	$\frac{N_c}{N_f}$
M_j^i	1	N_f	$\overline{N_f}$	0	$2\frac{N_f-N_c}{N_f}$

Table 2.2: Charge of matter content of the magnetic theory

Kutasov-Schwimmer duality

3 | Physics

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Kutasov-Schwimmer duality

3.3 3D dualities

Aharony duality

Kutasov-Schwimmer duality

4 | Math

5 | My work

6 | conclusions

7 | Appendix

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