Hastings 4.1

$$\frac{dN}{dt} = rN\left[1 - \left(\frac{N}{K}\right)^{\theta}\right] \qquad \theta > 0$$

(a) $\frac{dN}{dt} = 0 = F(\hat{N})$

$$= r\hat{N}\left[1 - \left(\frac{\hat{N}}{K}\right)^{\theta}\right]$$

$\hat{N} = 0, K$

Stability analysis

$\left.\frac{dF}{dN}\right|_{N=\hat{N}} < 0$?

$$F(N) = rN\left[1 - \left(\frac{N}{K}\right)^{\theta}\right] = rN - \frac{rN^{\theta+1}}{K^{\theta}}$$

$$\frac{dF(N)}{dN} = r - \frac{r}{K^{\theta}}\left[(\theta+1)N^{\theta}\right]$$

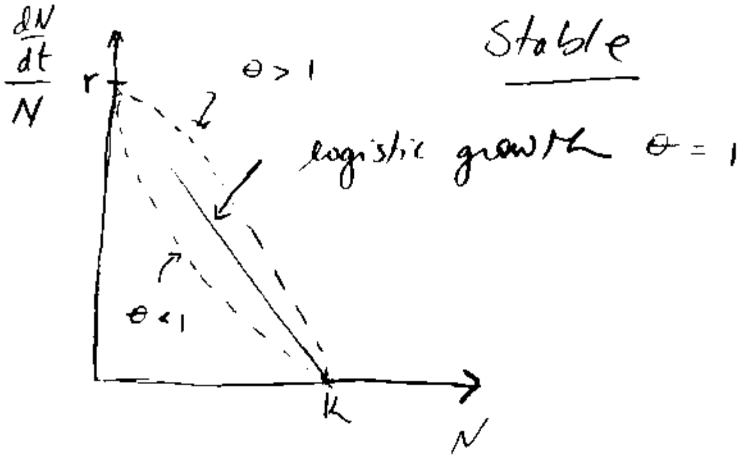
$\hat{N} = 0$, $\frac{dF(N)}{dN} = r$ <u>unstable</u>

$\hat{N} = K$, $\frac{dF}{dN} = r - \frac{r}{\cancel{K^{\theta}}}\left[(\theta+1)\cancel{K^{\theta}}\right] = r - r\theta - r$

$= -r\theta$

<u>Stable</u>

(b) $\frac{dN/dt}{N} = r - r\left(\frac{N}{K}\right)^{\theta}$



Hastings 4.3

$$\frac{dN}{dt} = rN(N-a)\left[1-\frac{N}{k}\right]$$

(a) $$\frac{dN}{dt} = 0 = r\hat{N}(\hat{N}-a)\left[1-\frac{\hat{N}}{k}\right]$$

$$\hat{N} = 0,\ a,\ K$$

(b) $$\frac{dF}{dN} = \frac{d}{dN}\left[rN(N-a)\left(1-\frac{N}{k}\right)\right]$$

$$= \frac{d}{dN}\left[(rN^2 - rNa)\left(1-\frac{N}{k}\right)\right]$$

$$= \frac{d}{dN}\left[rN^2 - rNa - \frac{rN^3}{K} + \frac{rN^2a}{k}\right]$$

$$= 2rN - ra - \frac{3rN^2}{k} + \frac{2rNa}{k}$$

$$\left.\frac{dF}{dN}\right|_{\hat{N}=0} = -ra \quad [\text{Stable}]$$

$$\left.\frac{dF}{dN}\right|_{\hat{N}=a} = 2ra - ra - \frac{3ra^2}{k} + \frac{2ra^2}{k}$$

$$= ra - \frac{ra^2}{k}$$

Note that a < K by definition.
Thus this equilibria is unstable.

$$= ra\left(1-\frac{a}{k}\right) \quad [\text{stable for } a > K]$$

$$\left.\frac{dF}{dN}\right|_{\hat{N}=K} = 2rK - ra - \frac{3rK^2}{K} + \frac{2rKa}{K} = -rk + ra$$

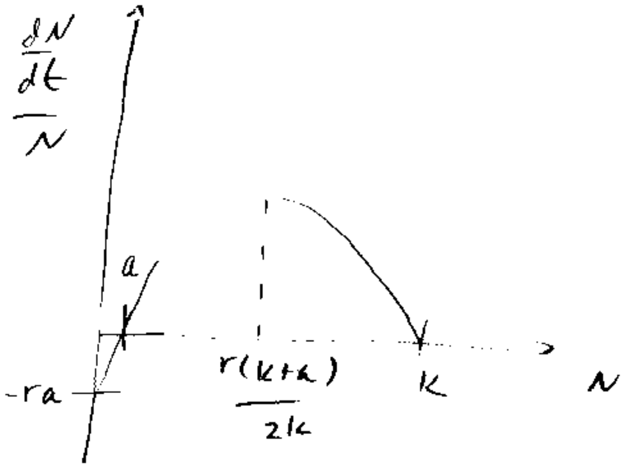
$$= r(a-k)$$

[stable for $K > a$]

4.3 (c)

$$\frac{\frac{dN}{dt}}{N} = r(N-a)\left(1-\frac{N}{K}\right)$$

$$= -ra + rN\left(1+\frac{a}{K}\right) - \frac{r}{K}N^2$$



(d) see notes from class discussion

This model is fundamentally different from the logistic model because the modeled population does not show a monotonic decline in per capita growth with increasing density. Instead per capita growth rates increase with population density at low levels and decline only at much higher population levels. One biological reason for the ascending part of the curve at low density is that species may require conspecifics for protection from predators or from climatic extremes; other species may forage more effectively in groups than alone. In sexual species, individuals may have a difficult time finding mates at low densities, so mating rates increase with population density. The result of these types of effects is that positive per capita growth rates might not even be possible until the population reaches some threshold size and per capita growth rates then increase with population density - at least up to a point. Only when population densities are far above this size might the negative effects of crowding become evident.