Semester Break Problem Solving Competition

Contest 4

September 28

Problem A - Moosick

Everyone knows that cows love to listen to all forms of music. Almost all forms, that is -- the great cow composer Wolfgang Amadeus Moozart once discovered that a specific chord tends to make cows rather ill. This chord, known as the ruminant seventh chord, is therefore typically avoided in all cow musical compositions.

Farmer John, not knowing the finer points of cow musical history, decides to play his favorite song over the loudspeakers in the barn. Your task is to identify all the ruminant seventh chords in this song, to estimate how sick it will make the cows.

The song played by FJ is a series of N (1 <= N <= 20,000) notes, each an integer in the range 1..88. A ruminant seventh chord is specified by a sequence of C (1 <= C <= 10) distinct notes, also integers in the range 1..88. However, even if these notes are transposed (increased or decreased by a common amount), or re-ordered, the chord remains a ruminant seventh chord! For example, if "4 6 7" is a ruminant seventh chord, then "3 5 6" (transposed by -1), "6 8 9" (transposed by +2), "6 4 7" (re-ordered), and "5 3 6" (transposed and re-ordered) are also ruminant seventh chords.

A ruminant seventh chord is a sequence of C consecutive notes satisfying the above criteria. It is therefore uniquely determined by its starting location in the song. Please determine the indices of the starting locations of all of the ruminant seventh chords.

PROBLEM NAME: moosick

INPUT FORMAT:

* Line 1: A single integer: N.

* Lines 2..1+N: The N notes in FJ's song, one note per line.

* Line 2+N: A single integer: C.

* Lines 3+N..2+N+C: The C notes in an example of a ruminant seventh chord. All transpositions and/or re-orderings of these notes are also ruminant seventh chords.

SAMPLE INPUT (file moosick.in):

```
6
1
8
5
7
```

```
9
10
3
4
6
7
```

INPUT DETAILS:

FJ's song is 1,8,5,7,9,10. A ruminant seventh chord is some transposition/re-ordering of 4,6,7.

OUTPUT FORMAT:

* Line 1: A count, K, of the number of ruminant seventh chords that appear in FJ's song. Observe that different instances of ruminant seventh chords can overlap each-other.

* Lines 2..1+K: Each line specifies the starting index of a ruminant seventh chord (index 1 is the first note in FJ's song, index N is the last). Indices should be listed in increasing sorted order.

SAMPLE OUTPUT :

```
2
2
4
```

OUTPUT DETAILS:

Two ruminant seventh chords appear in FJ's song (and these occurrences actually overlap by one note). The first is 8,5,7 (transposed by +1 and reordered) starting at index 2, and the second is 7,9,10 (transposed by +3) starting at index 4.

Problem B - Median

Farmer John has lined up his N (1 <= N <= 100,000) cows in a row to measure their heights; cow i has height H_i (1 <= H_i <= 1,000,000,000) nanometers--FJ believes in precise measurements! He wants to take a picture of some contiguous subsequence of the cows to submit to a bovine photography contest at the county fair.

The fair has a very strange rule about all submitted photos: a photograph is only valid to submit if it depicts a group of cows whose median height is at least a certain threshold X (1 <= X <= 1,000,000,000).

For purposes of this problem, we define the median of an array A[0...K] to be A[ceiling(K/2)] after A is sorted, where ceiling(K/2) gives K/2 rounded up to the nearest integer (or K/2 itself, it K/2 is an integer to begin with). For example the median of {7, 3, 2, 6} is 6, and the median of {5, 4, 8} is 5.

Please help FJ count the number of different contiguous subsequences of his cows that he could potentially submit to the photography contest.

PROBLEM NAME: median

INPUT FORMAT:

* Line 1: Two space-separated integers: N and X.

* Lines 2..N+1: Line i+1 contains the single integer H_i.

SAMPLE INPUT :

```
4 6
10
5
6
2
```

INPUT DETAILS:

FJ's four cows have heights 10, 5, 6, 2. We want to know how many contiguous subsequences have median at least 6.

OUTPUT FORMAT:

* Line 1: The number of subsequences of FJ's cows that have median at least X. Note this may not fit into a 32-bit integer.

SAMPLE OUTPUT :

7

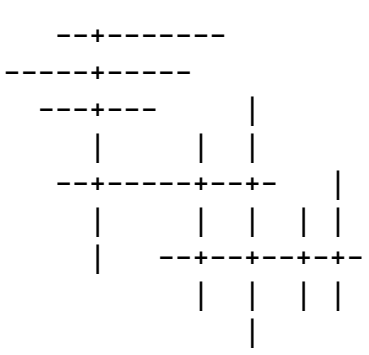
OUTPUT DETAILS:

There are 10 possible contiguous subsequences to consider. Of these, only 7 have median at least 6. They are {10}, {6}, {10, 5}, {5, 6}, {6, 2}, {10, 5, 6}, {10, 5, 6, 2}.

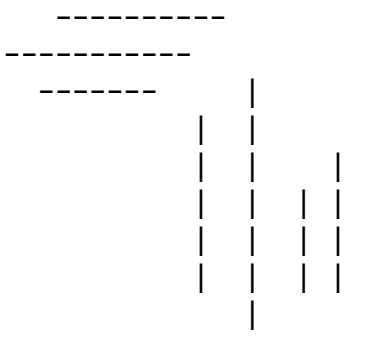
Problem C - Steeple

Farmer John has a brilliant idea for the next great spectator sport: Cow Steeplechase! As everyone knows, regular steeplechase involves a group of horses that race around a course filled with obstacles they must jump over. FJ figures the same contest should work with highly-trained cows, as long as the obstacles are made short enough.

In order to design his course, FJ makes a diagram of all the N (1 <= N <= 250) possible obstacles he could potentially build. Each one is represented by a line segment in the 2D plane that is parallel to the horizontal or vertical axis. Obstacle i has distinct endpoints (X1_i, Y1_i) and (X2_i, Y2_i) (1 <= X1_i, Y1_i, X2_i, Y2_i <= 1,000,000,000). An example is as follows:



FJ would like to build as many of these obstacles as possible, subject to the constraint that no two of them intersect. Starting with the diagram above, FJ can build 7 obstacles:



Two segments are said to intersect if they share any point in common, even an endpoint of one or both of the segments. FJ is certain that no two horizontal segments in the original input diagram will intersect, and that similarly no two vertical segments in the input diagram will intersect.

Please help FJ determine the maximum number of obstacles he can build.

PROBLEM NAME: steeple

INPUT FORMAT:

* Line 1: A single integer: N.

* Lines 2..N+1: Line i+1 contains four space-separated integers
  representing an obstacle: X1_i, Y1_i, X2_i, and Y2_i.

SAMPLE INPUT :

```
3
4 5 10 5
6 2 6 12
8 3 8 5
```

INPUT DETAILS:

There are three potential obstacles. The first is a horizontal segment connecting (4, 5) to (10, 5); the second and third are vertical segments connecting (6, 2) to (6, 12) and (8, 3) to (8, 5).

OUTPUT FORMAT:

* Line 1: The maximum number of non-crossing segments FJ can choose.

SAMPLE OUTPUT :

```
2
```

OUTPUT DETAILS:

The optimal solution is to choose both vertical segments.