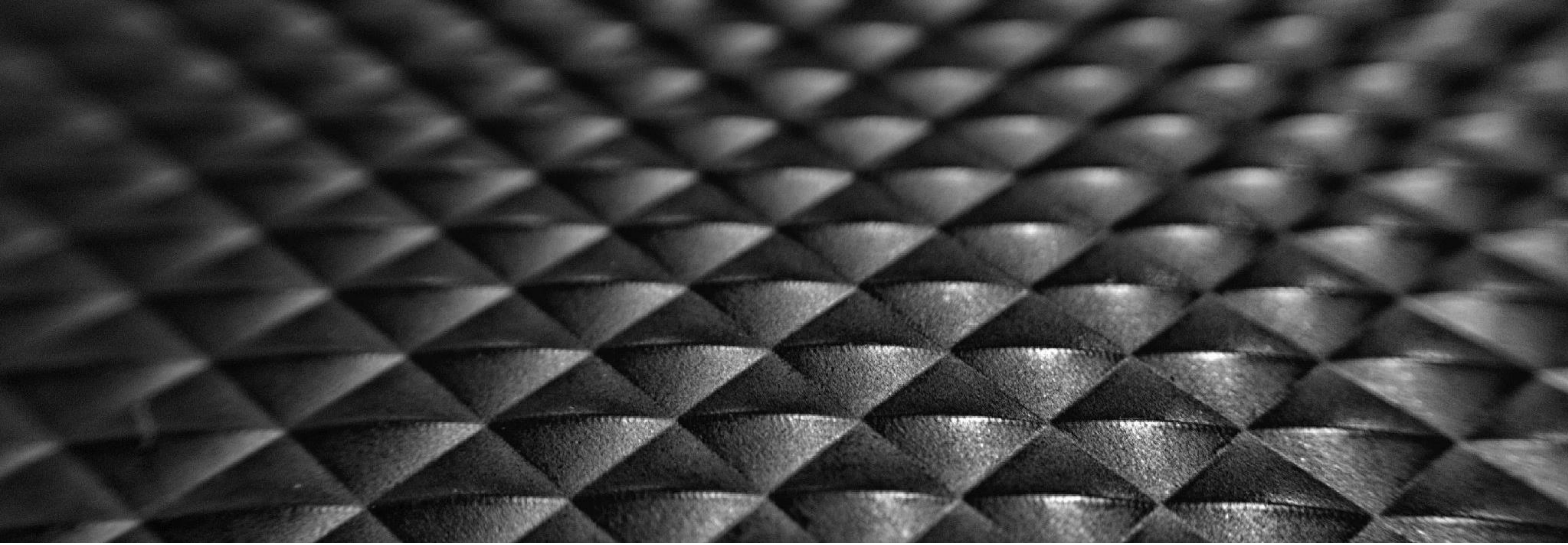


Bending and Buckling analysis of Composite plate.

Why are composite materials Important?

- Composite plates are a big deal in aerospace because they're super strong but also lightweight.
- They help make airplanes lighter, which means they use less fuel and can fly farther.
- Lighter airplanes also need smaller engines, which saves even more fuel and reduces emissions.
- They're used in everything from airplane wings to the body of the plane itself.

Our Material

Material Chosen: AS4-3501/6 Graphite/Epoxy Composite

Widely Used in Aerospace: Key material in aircraft structures due to its exceptional properties.

Common Applications: Utilized in aircraft wings, fuselages, engine components, and interior structures.

Advantages Over Other Composites:

- High Strength-to-Weight Ratio: Provides robustness without adding unnecessary weight, enhancing fuel efficiency.
- Excellent Stiffness: Ensures structural integrity under aerodynamic forces, enhancing flight performance.
- Corrosion Resistance: Maintains durability in harsh environments, reducing maintenance needs.
- Tailorable Properties: Can be customized through fiber orientation, offering versatility in design and performance optimization.

- Preferred Choice: AS4-3501/6 is favored for its balance of strength, lightweight, and adaptability, making it a top pick for aerospace applications.

Material Properties and Q Values

Equations

- Constants:
  - Angle:
    $\theta = [0 \quad 90 \quad 90 \quad 0]$
  - Number of layers: $k = 4$
  - Thickness:
    thickness $= 1.34 \times 10^{-4}$ meters
  - Density: ρ_m=1580 kg/m^3
- Derivation of the Q matrix:

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$E_1 = 142 \times 10^9 \text{ Pa}$$

$$E_2 = 9.8 \times 10^9 \text{ Pa}$$

$$\nu_{12} = 0.3$$

$$\nu_{21} = \frac{E_2 \nu_{12}}{E_1}$$

$$G_{12} = 6 \times 10^9 \text{ Pa}$$

$$Q_{12} = \frac{\nu_{21} E_1}{1 - \nu_{12}\nu_{21}}$$

$$Q_{21} = Q_{12}$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

Q Transformation and D Value Formulations

Stacking Sequence: [0 90 90 0]

$$
\begin{aligned}
m &= \cos(\theta) \\
n &= \sin(\theta) \\
\bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4) \\
\bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12})m^2n^2 + Q_{66}(m^2 - n^2)^2
\end{aligned}
$$

$$
D_{ij} = \frac{1}{3}\sum_{k=1}^{n}(\bar{Q}_{ij})k\left(tk^3 - t_{k-1}^3\right)
$$

Results

Given material properties:

$$\begin{aligned} D_{21} &= 0.037964, & D_1 &= D_{11} = 1.6202, \\ D_2 = D_{22} &= 0.33993, & D_3 &= D12 + 2 * D66 = 0.19195 \\ a &= 2m, & b &= 1m \end{aligned}$$

Natural Frequency

$$\omega_{mn} = \frac{\pi^2}{\sqrt{\rho_m h}}\left[D_1\left(\frac{m}{a}\right)^4 + 2D_3\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_2\left(\frac{n}{b}\right)^4\right]^{1/2}$$

$$\omega_{mn} = \frac{\pi^2}{\sqrt{1580(0.536)_1}}\Bigg[(1.6202\ \mathrm{Nm})\left(\frac{1}{2}\right)^4 +$$
$$2(0.19195\ \mathrm{Nm})\left(\frac{1}{2}\right)^2\left(\frac{1}{1}\right)^2 + (0.33993\ \mathrm{Nm})\left(\frac{1}{1}\right)^4\Bigg]^{1/2}$$

ω_{mn}= 1.251 Hz

DATE

$\Rightarrow$

$$w_{mn} = \frac{16 q_0 mn}{\pi^2 ab}\left[D_{11}\left(\frac{m}{a}\right)^4 + D_{22}\left(\frac{n}{b}\right)^4 + 2\left(D_{12} - 2D_{66}\right)\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2\right]^{-1}$$

$m = \text{odd}$

$n = \text{odd}.$

$$\Rightarrow +D_{11}\frac{\partial^4 w}{\partial x^4} + D_{22}\frac{\partial^4 w}{\partial y^4} + (2D_{12} - 4D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} = q$$

$$w_{mn} = \sum w_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)$$

$$q_{mn} = \sum q_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \Rightarrow q_{mn} = 16 q_0 \frac{mn\pi^2}{ab}$$

$$\Rightarrow D_{11} w_{mn}\left(\frac{m\pi}{a}\right)^4 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right) + D_{22} w_{mn}\left(\frac{n\pi}{b}\right)^4 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$+ 2(D_{12} - 2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) = 16 q_0 \frac{mn\pi^2}{ab}\sin\left(\frac{m\pi x}{a}\right)$$

Thin plate Theory :-

CLT :-

$$\{M\} = [B]\{\varepsilon^{\circ}\} + [D]\{\chi\}$$

$$\{N\} = [A]\{\varepsilon^{\circ}\} + [B]\{\chi\}$$

$$\cancel{M_{xx}} = M_x = D_{11}\chi_x + D_{12}K_y + D_{16}\chi_{xy}$$

$$M_y = D_{21}\chi_x + D_{22}K_y + D_{26}K_{xy}$$

$$M_{xy} = D_{61}\chi_x + D_{62}\chi_y + D_{66}K_{xy}$$

$q \Rightarrow$ pressure load. (N/m^2)

$$K_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

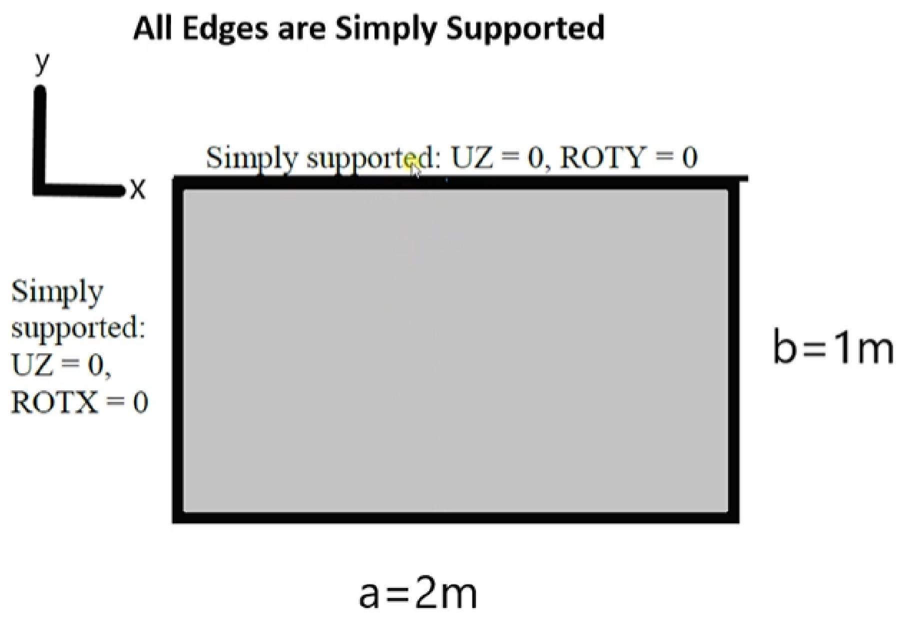
$$\Rightarrow \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + q = 0$$

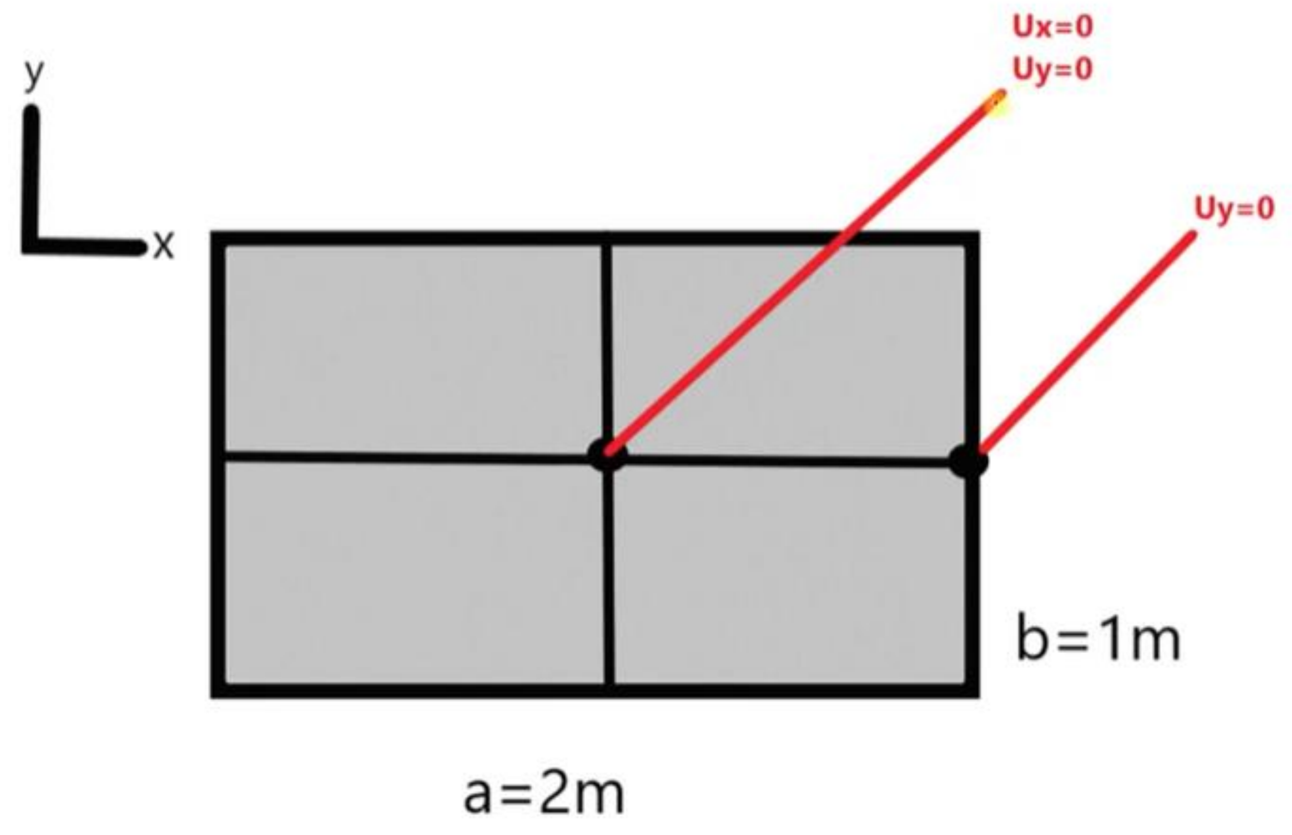
$$\frac{\partial^2 M_x}{\partial x^2} = -D_{11}\frac{\partial^4 w}{\partial x^4} - D_{12}\frac{\partial^4 w}{\partial x^2 \partial y^2} - 2D_{16}\frac{\partial^4 w}{\partial x^3 \partial y}$$

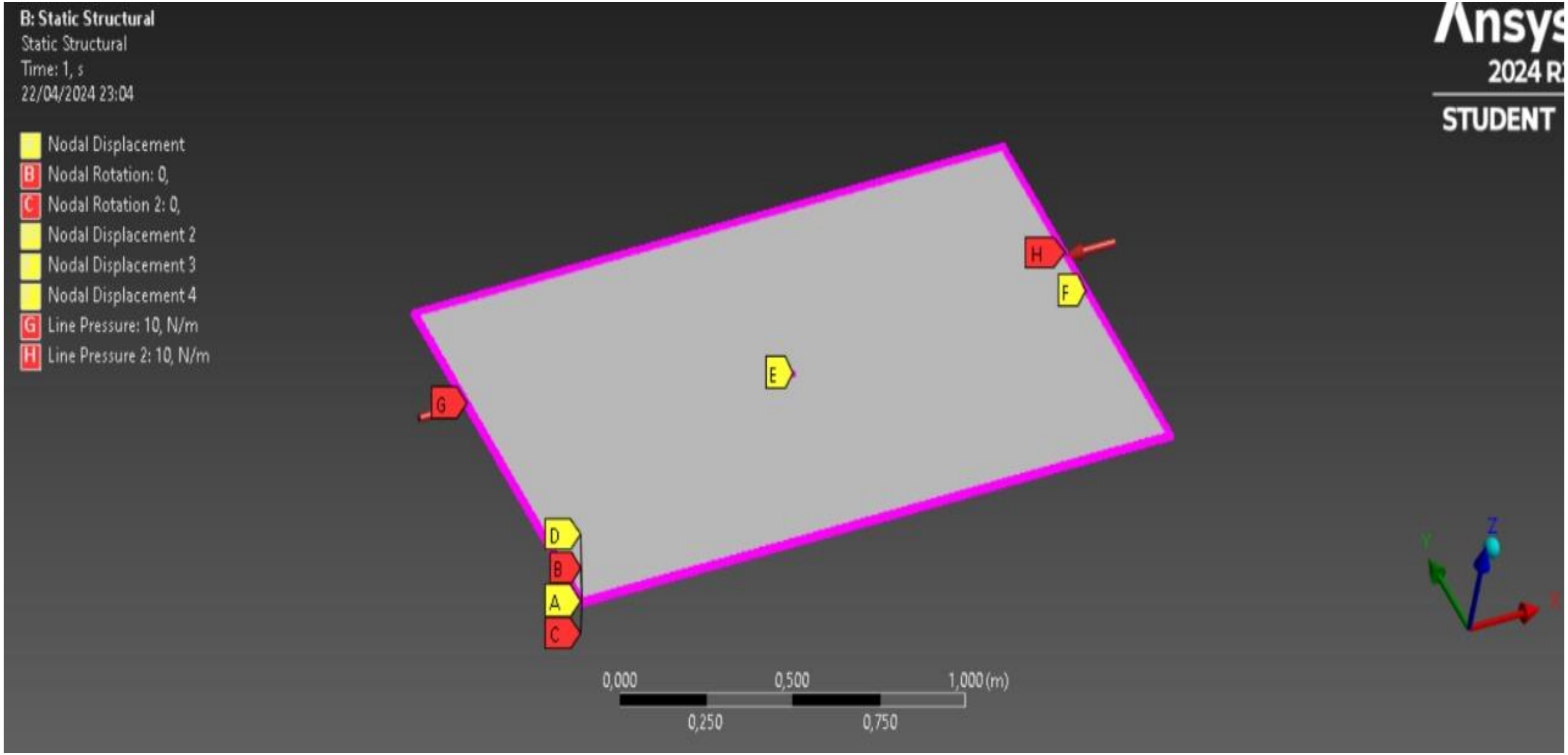
$$\frac{\partial^2 M_y}{\partial y^2} = -D_{21}\frac{\partial^4 w}{\partial x^2 \partial y^2} - D_{22}\frac{\partial^4 w}{\partial y^4} - 2D_{26}\frac{\partial^4 w}{\partial x \partial y^3}$$

$$-2\frac{\partial^2 M_{xy}}{\partial x \partial y} = 2D_{61}\frac{\partial^4 w}{\partial x^3 \partial y} + 2D_{62}\frac{\partial^4 w}{\partial x \partial y^3} + 4D_{66}\frac{\partial^4 w}{\partial x^2 \partial y^2}$$

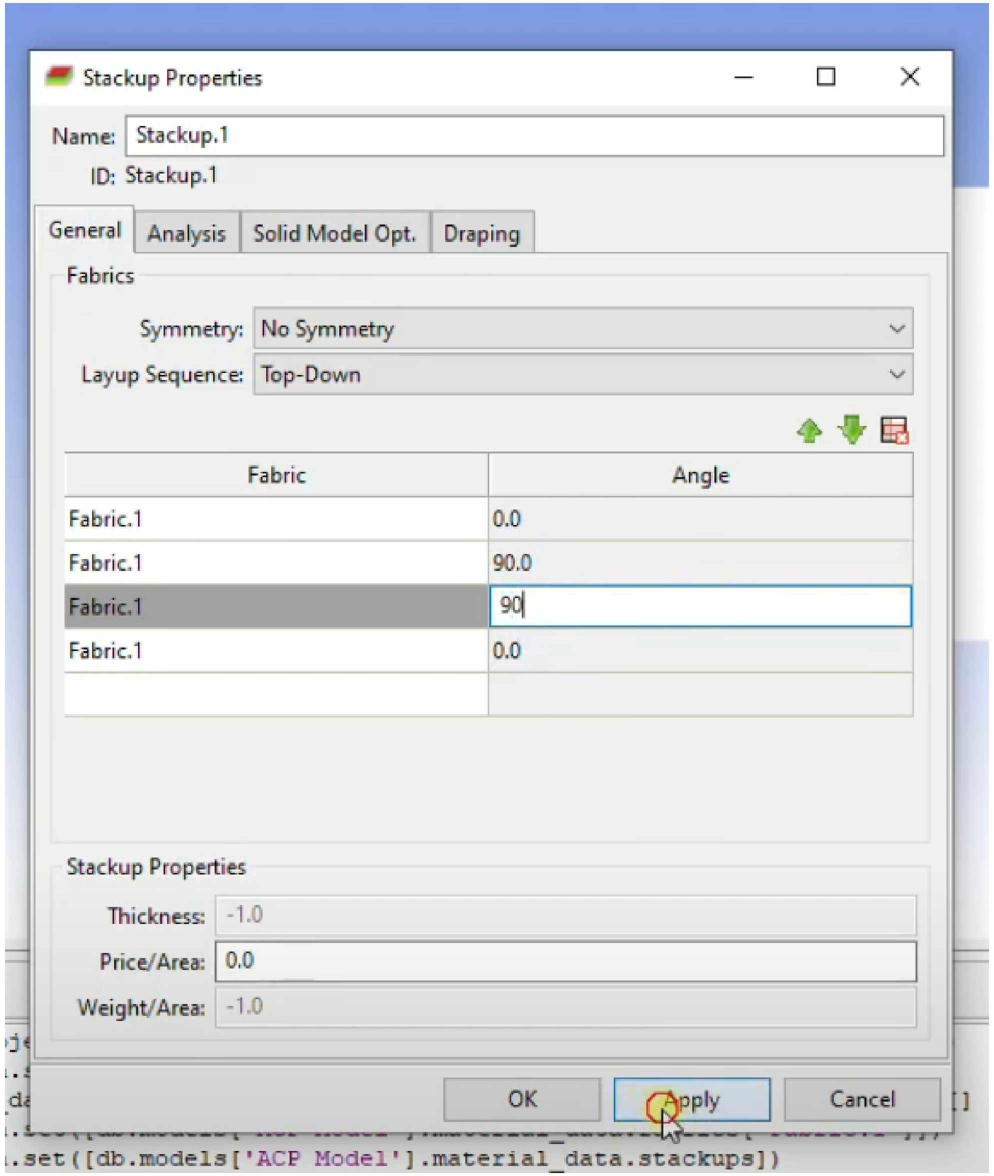
Geometry and Boundary Conditions

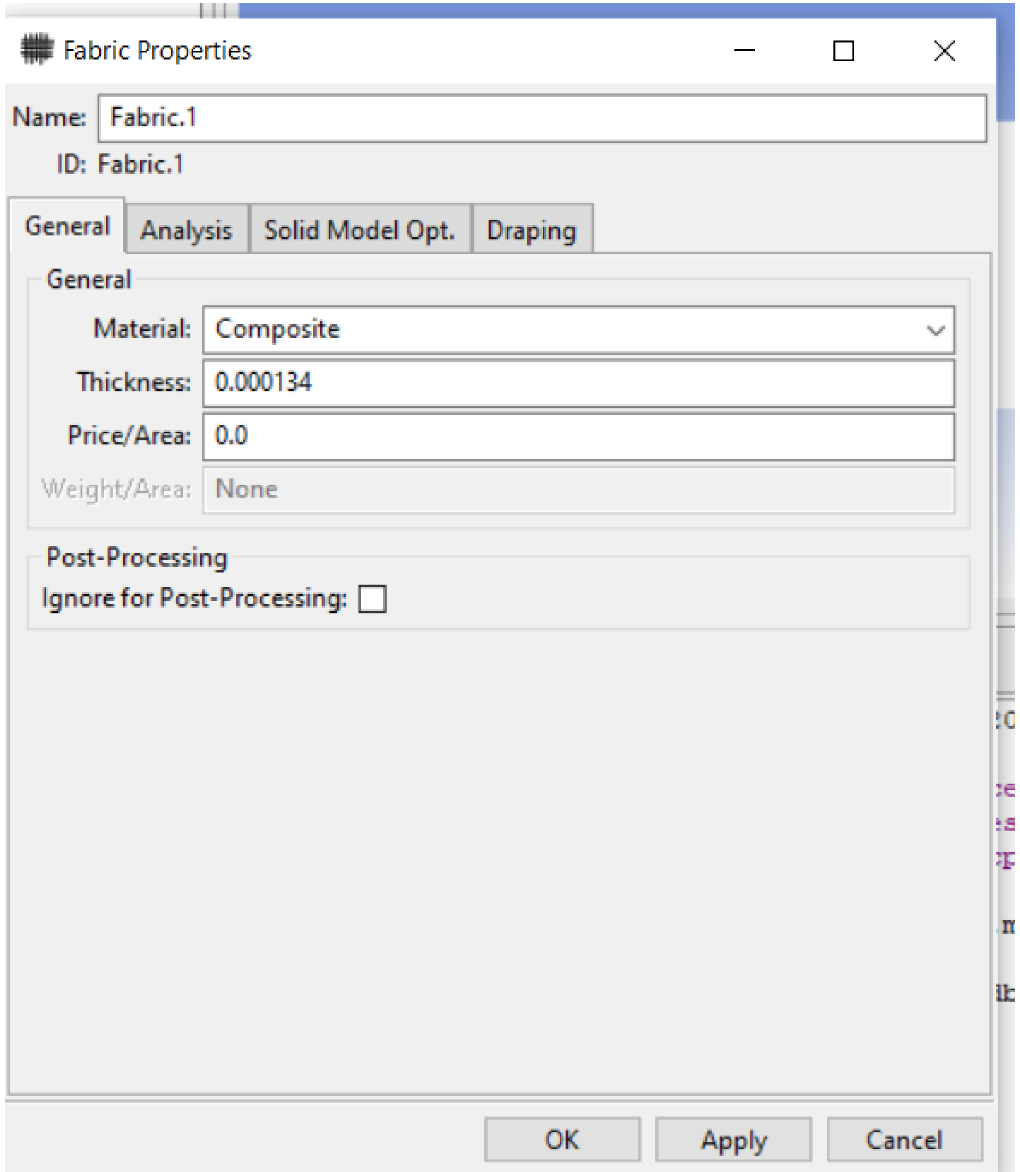


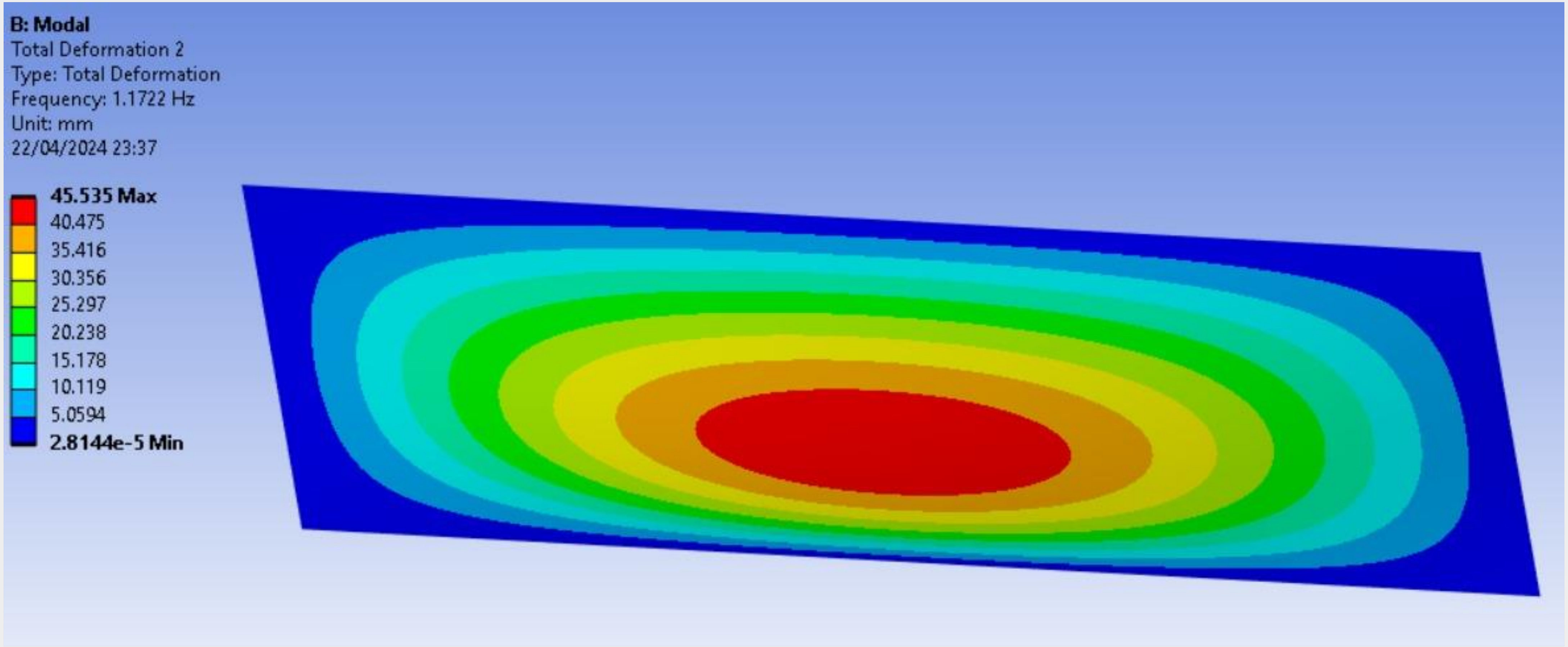

y
x
Ux=0
Uy=0
Uy=0
b=1m
a=2m

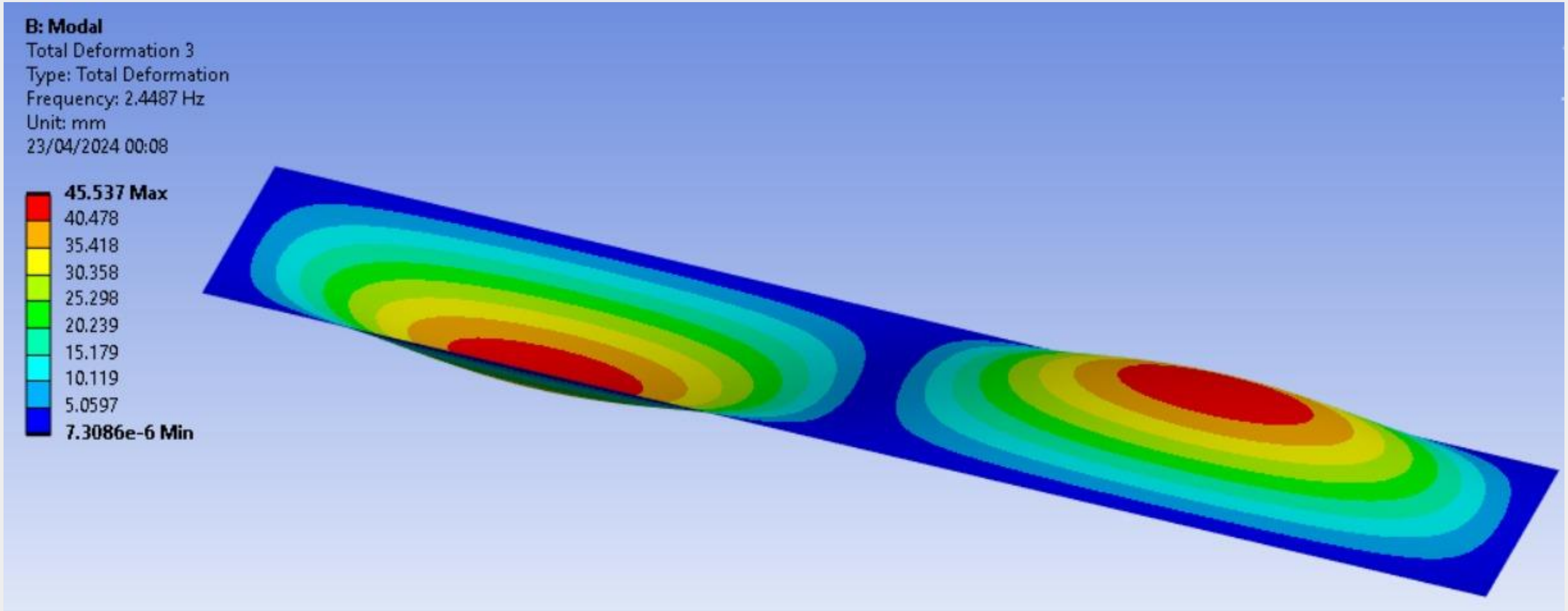

B: Static Structural
Static Structural
Time: 1, s
22/04/2024 23:04
Nodal Displacement
Nodal Rotation: 0,
Nodal Rotation 2: 0,
Nodal Displacement 2
Nodal Displacement 3
Nodal Displacement 4
Line Pressure: 10, N/m
Line Pressure 2: 10, N/m
Ansys
2024 R
STUDENT
0,000
0,500
1,000 (m)
0,250
0,750

	A	B	C	D	E
1	Property	Value	Unit		
2	Material Field Variables	Table			
3	Density	1800	kg m^-3		
4	Orthotropic Elasticity				
5	Young's Modulus X direction	142	GPa		
6	Young's Modulus Y direction	9.8	GPa		
7	Young's Modulus Z direction	8.9	GPa		
8	Poisson's Ratio XY	0.3			
9	Poisson's Ratio YZ	0.0559			
10	Poisson's Ratio XZ	0.3			
11	Shear Modulus XY	6	GPa		
12	Shear Modulus YZ	2.9	GPa		
13	Shear Modulus XZ	4.5	GPa		

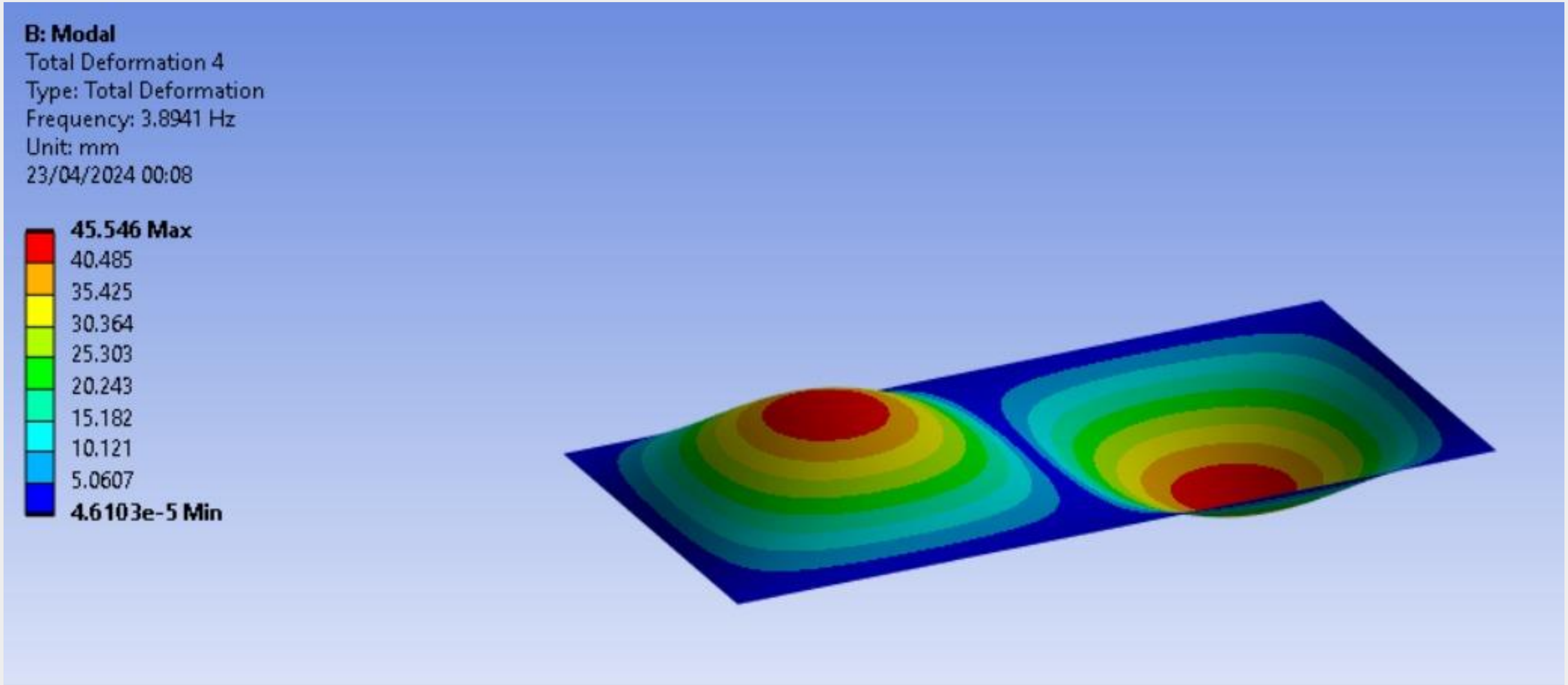

Stackup Properties
Name: Stackup.1
ID: Stackup.1
General
Analysis
Solid Model Opt.
Draping
Fabrics
Symmetry: No Symmetry
Layup Sequence: Top-Down
Fabric
Angle
Fabric.1 0.0
Fabric.1 90.0
Fabric.1 90
Fabric.1 0.0
Stackup Properties
Thickness: -1.0
Price/Area: 0.0
Weight/Area: -1.0
OK
Apply
Cancel
.set([db.models['ACP Model'].material_data.stackups])


Fabric Properties
Name: Fabric.1
ID: Fabric.1
General
Analysis
Solid Model Opt.
Draping
General
Material: Composite
Thickness: 0.000134
Price/Area: 0.0
Weight/Area: None
Post-Processing
Ignore for Post-Processing:
OK
Apply
Cancel


B: Modal
Total Deformation 2
Type: Total Deformation
Frequency: 1.1722 Hz
Unit: mm
22/04/2024 23:37
45.535 Max
40.475
35.416
30.356
25.297
20.238
15.178
10.119
5.0594
2.8144e-5 Min



B: Modal
Total Deformation 3
Type: Total Deformation
Frequency: 2.4487 Hz
Unit: mm
23/04/2024 00:08
45.537 Max
40.478
35.418
30.358
25.298
20.239
15.179
10.119
5.0597
7.3086e-6 Min


B: Modal
Total Deformation 4
Type: Total Deformation
Frequency: 3.8941 Hz
Unit: mm
23/04/2024 00:08
45.546 Max
40.485
35.425
30.364
25.303
20.243
15.182
10.121
5.0607
4.6103e-5 Min

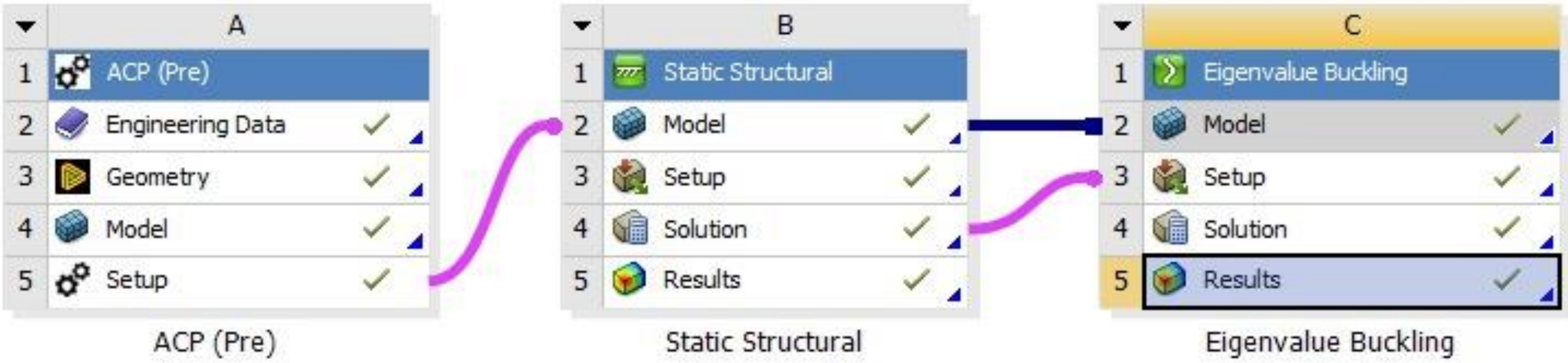
M	n	Theoretical Frequency	ANSYS Result
1	1	1.251	1.17
2	1	2.6133	2.4487
1	2	4.1545	3.8941
2	2	5.0041	4.6901
3	1	5.2341	4.9059
3	2	7.05	6.6145

$$N_{x\,cr} = -\frac{\pi^2 a^2}{m^2}\left[D_1\left(\frac{m}{a}\right)^4 + 2D_3\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_2\left(\frac{n}{b}\right)^4\right]^1$$

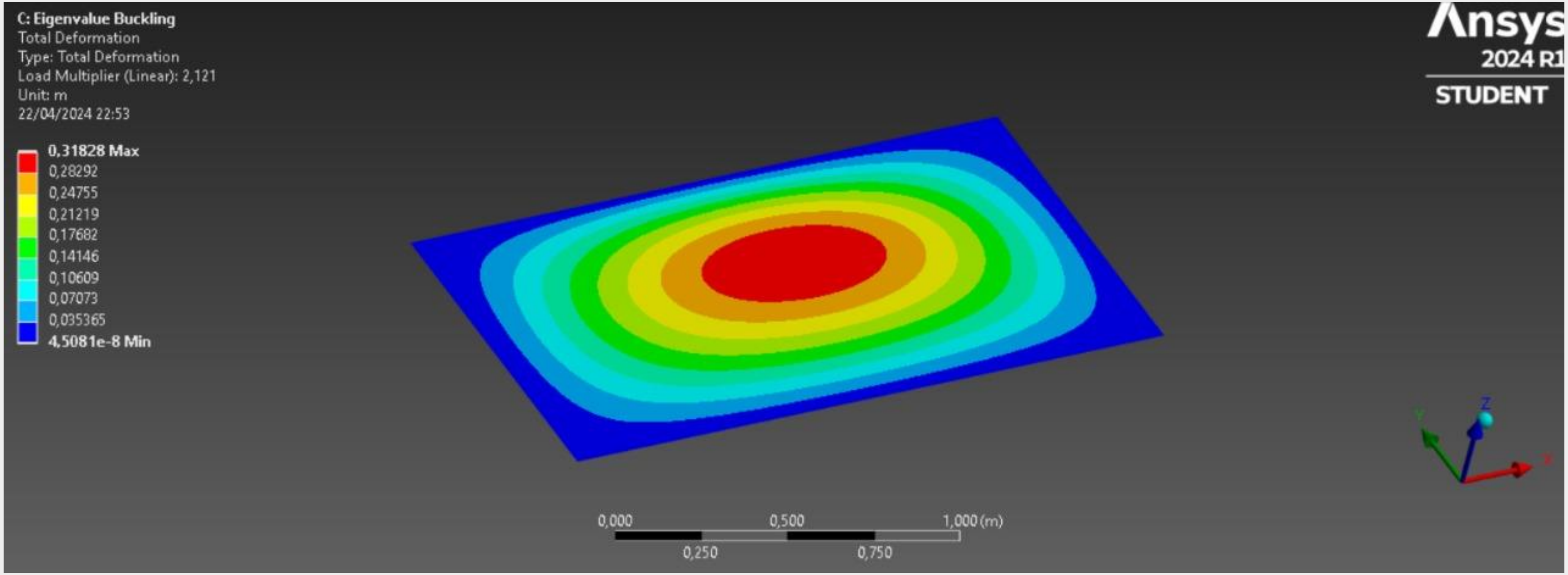
$$N_{x\,cr} = -\frac{\pi^2 2^2}{1^2}\left[(1.6202\ \mathrm{Nm})\left(\frac{1}{2}\right)^4 + 2(0.19195\ \mathrm{Nm})\left(\frac{1}{2}\right)^2\left(\frac{1}{1}\right)^2 + (0.33993\ \mathrm{Nm})\left(\frac{1}{1}\right)^4\right]^1$$

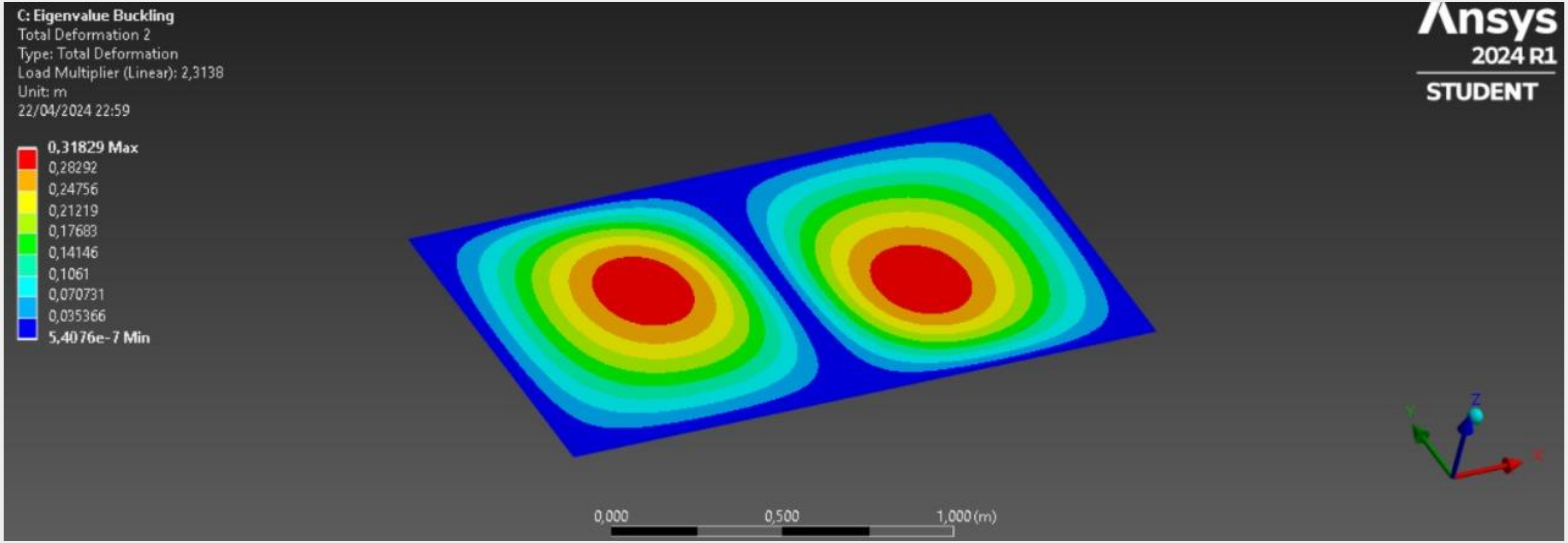
$N_{x\,cr}$ =21.2 N/m

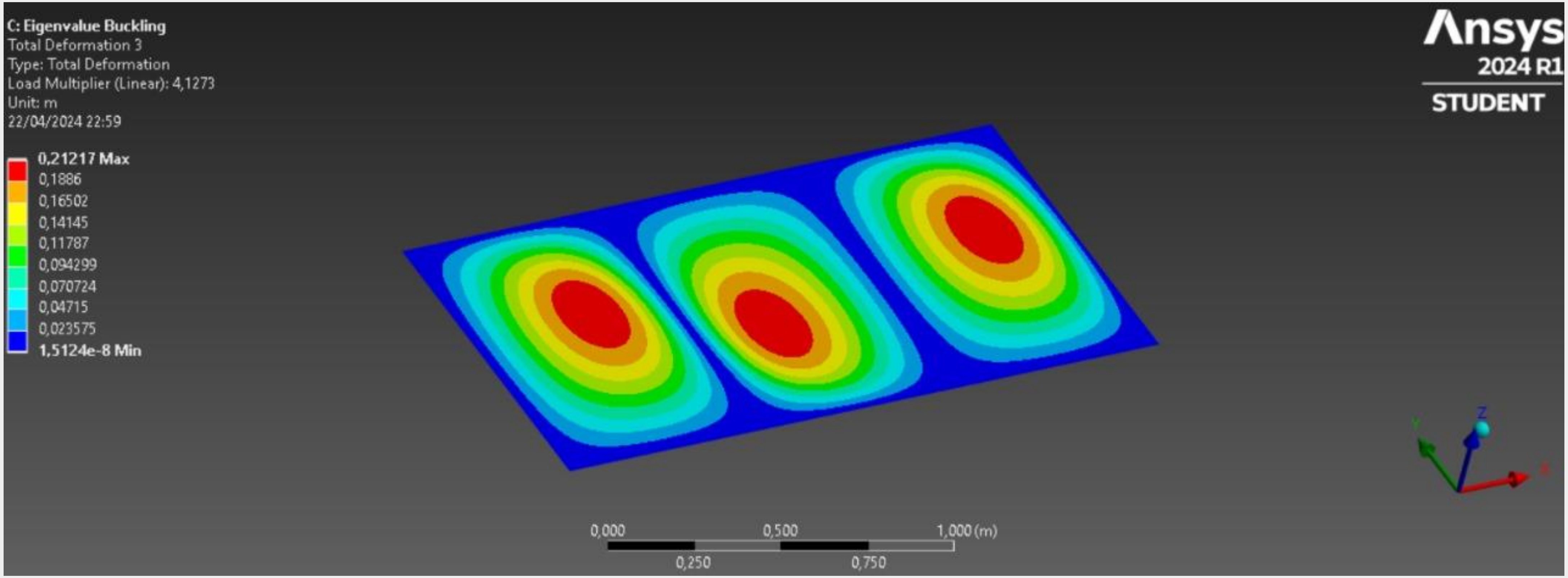
Load Factor = $\frac{N_{x\,cr}}{\text{Applied Load}} = \frac{21.2}{10}$ = 2.12

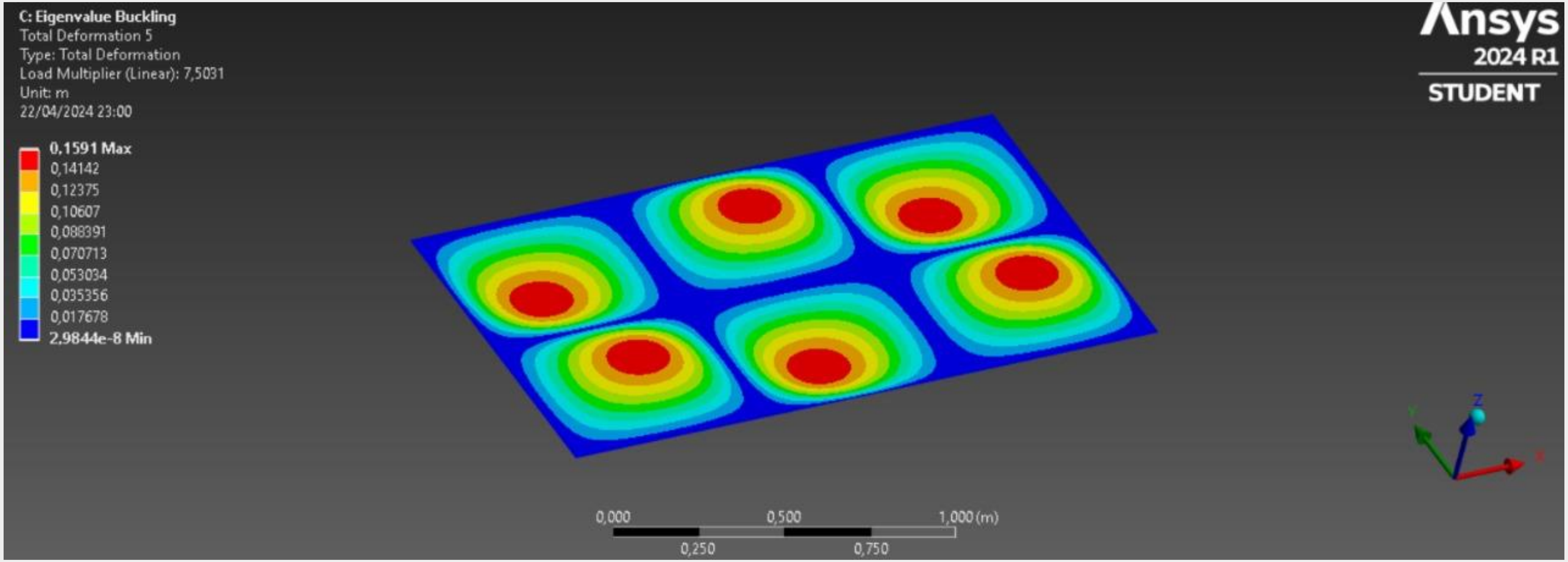

A
1 ACP (Pre)
2 Engineering Data
3 Geometry
4 Model
5 Setup
ACP (Pre)
B
1 Static Structural
2 Model
3 Setup
4 Solution
5 Results
Static Structural
C
1 Eigenvalue Buckling
2 Model
3 Setup
4 Solution
5 Results
Eigenvalue Buckling

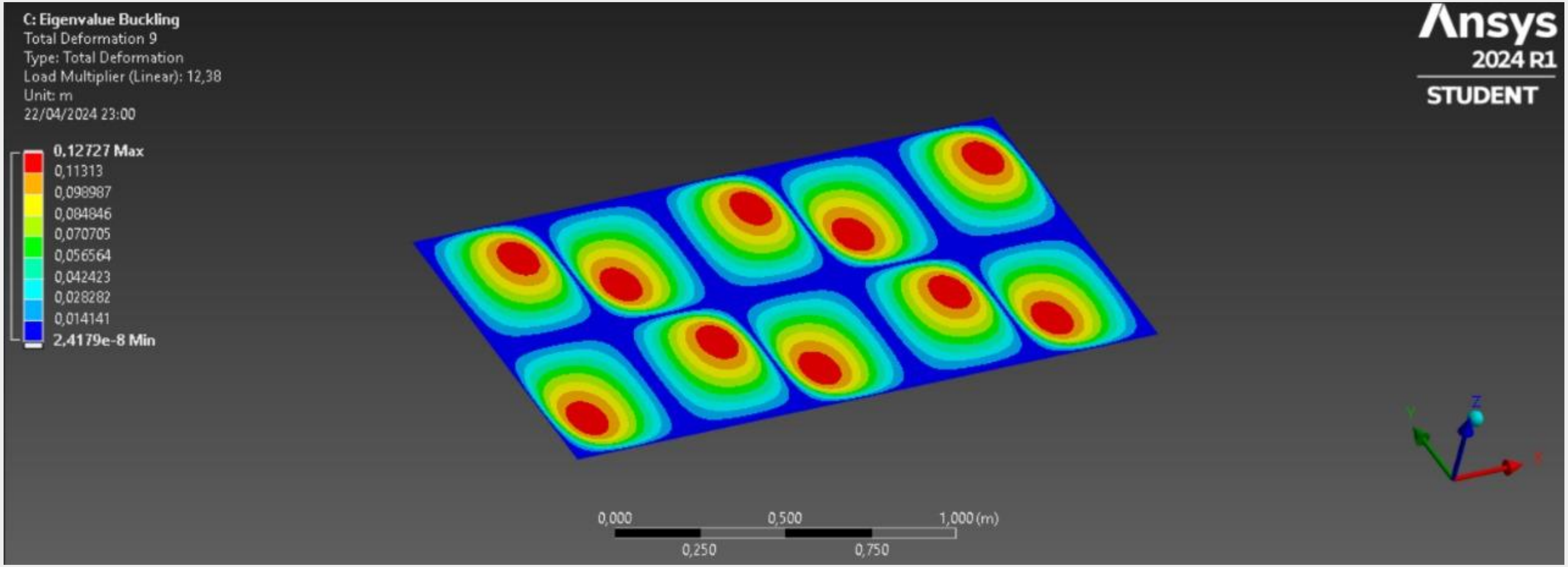
Buckling


C: Eigenvalue Buckling
Total Deformation
Type: Total Deformation
Load Multiplier (Linear): 2,121
Unit: m
22/04/2024 22:53
0,31828 Max
0,28292
0,24755
0,21219
0,17682
0,14146
0,10609
0,07073
0,035365
4,5081e-8 Min
Ansys
2024 R1
STUDENT
0,000
0,500
1,000 (m)
0,250
0,750


C: Eigenvalue Buckling
Total Deformation 2
Type: Total Deformation
Load Multiplier (Linear): 2,3138
Unit: m
22/04/2024 22:59
0,31829 Max
0,28292
0,24756
0,21219
0,17683
0,14146
0,1061
0,070731
0,035366
5,4076e-7 Min
Ansys
2024 R1
STUDENT
0,000
0,500
1,000 (m)
Z
Y
X


C: Eigenvalue Buckling
Total Deformation 3
Type: Total Deformation
Load Multiplier (Linear): 4,1273
Unit: m
22/04/2024 22:59
0,21217 Max
0,1886
0,16502
0,14145
0,11787
0,094299
0,070724
0,04715
0,023575
1,5124e-8 Min
Ansys
2024 R1
STUDENT
0,000
0,500
1,000 (m)
0,250
0,750


C: Eigenvalue Buckling
Total Deformation 5
Type: Total Deformation
Load Multiplier (Linear): 7,5031
Unit: m
22/04/2024 23:00
0,1591 Max
0,14142
0,12375
0,10607
0,088391
0,070713
0,053034
0,035356
0,017678
2,9844e-8 Min
Ansys
2024 R1
STUDENT
0,000
0,500
1,000 (m)
0,250
0,750


C: Eigenvalue Buckling
Total Deformation 9
Type: Total Deformation
Load Multiplier (Linear): 12,38
Unit: m
22/04/2024 23:00
0,12727 Max
0,11313
0,098987
0,084846
0,070705
0,056564
0,042423
0,028282
0,014141
2,4179e-8 Min
Ansys
2024 R1
STUDENT
0,000
0,500
1,000 (m)
0,250
0,750

m	n	Load Multiplier(ANSYS wala)	Load(Ansys wala)	Load (Theoretical)
1	1	2.121	21.21	21.207
2	1	2.3138	23.138	23.135
3	1	4.1273	41.273	41.26
4	1	6.8632	68.632	68.592
5	1	10.437	104.37	104.27
3	2	7.5031	75.031	74.993
4	2	9.2595	92.595	92.54
2	2	8.4883	84.883	84.826
5	2	12.38	123.8	123.69
6	1	14.828	148.28	148.08

Bending Calculation

For bending, we have the following expressions: taking mode (1,1)
p_0=10 pascal

$$B_{mn} = \frac{P_0}{mn\pi^2}\left[1-(-1)^m\right]\left[1-(-1)^n\right]$$

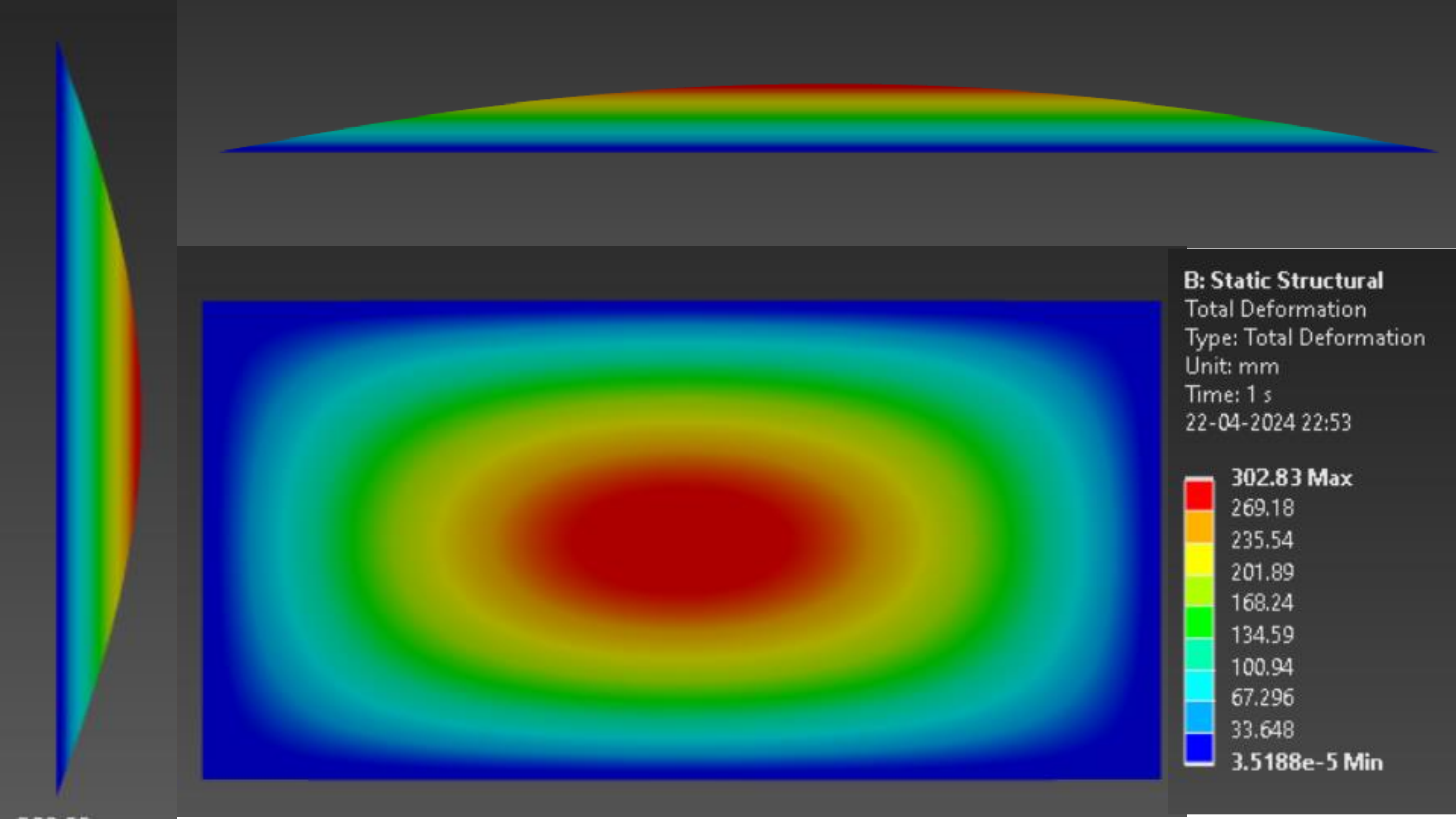
$$A_{mn} = \frac{B_{mn}}{D_1(\frac{m\pi}{a})^4 + 2D_3(\frac{m\pi}{a})^2(\frac{n\pi}{b})^2 + D_2(\frac{n\pi}{b})^4}$$

Note: All edges are simply supported.
The bending deflection $w(x, y)$ is given by:

$$w(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

Bending deflection: 302.8545 **mm**

Bending


B: Static Structural
Total Deformation
Type: Total Deformation
Unit: mm
Time: 1 s
22-04-2024 22:53
302.83 Max
269.18
235.54
201.89
168.24
134.59
100.94
67.296
33.648
3.5188e-5 Min