AE 332: Modeling and Analysis Lab II

LAB REPORT-01

by

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Contents

Chapter 1

One dimensional wave propogation

1.1 Problem To be Slove

Consider the one-dimensional advection equation with the following form:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \tag{1.1}$$

where c is the propagation speed and is selected to be 300 m/s (near the speed of sound).

At $t = 0$, a half-sinusoidal disturbance has been created.

The initial condition is specified as:

$$u(x,0) = u(x,0) = \begin{cases} 0 & \text{for } 0 < x < 50 \\ 100\sin\left(\frac{\pi(x-50)}{60}\right) & \text{for } 50 < x < 110 \\ 0 & \text{for } 110 < x < 300 \end{cases}$$

The boundary conditions are $u(0,t) = 0$ and $u(L,t) = 0$. We want to find and plot the solution at $t = 0.45$ seconds using the FTBS method.

To solve the problem, we'll consider three different step sizes: $\Delta x = 5$, $\Delta x = 2$, and $\Delta x = 1$, while ensuring that $\Delta t < \frac{\Delta x}{c}$. We will compare the solutions for these step sizes.

1.2 Matlab code

```
%Linear convection problem
% FTBS (Forward in time Backward in space
% case 1:Delta_x=5
clear all
clc
Lx=300; % Length of the domain
nx =61;% number of points in x direction
dx = 300./(nx-1);%grid size
x=0:dx:Lx;
dt = 0.01;
nt =46;% number of time steps
c=300; % convection velocity

% Define initial condition
for i=1:nx
    if x(i)>=50 && x(i)<110
```

```
            u(i)= 100*sin(pi*(x(i)-50)/60);
        else
            u(i)=0;
        end
end
for it =1:nt
    un=u;
    u(1) = u(nx-1)==0;
    for i=2:nx-1
      u(i) =un(i)-(c*dt/dx)*(un(i)-un(i-1));%linear convection problem
    end
    plot(x,un)
    xlabel('x')
    ylabel('u')
    legend(['time=' num2str(it*dt)])
    pause(0.04)
end
```

Listing 1.1: Example MATLAB Code

1.3 Results

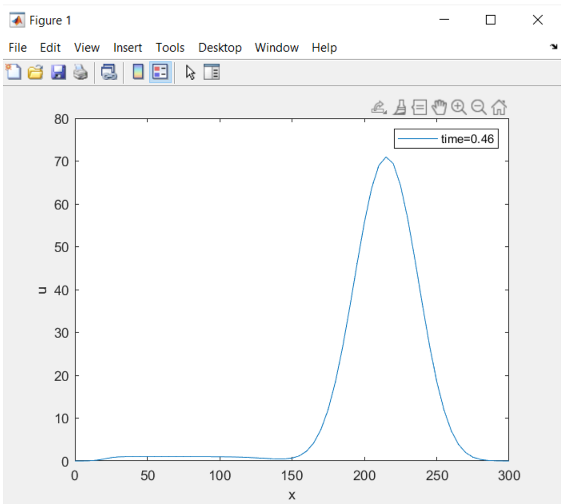

Figure 1.1: $\delta x = 5$

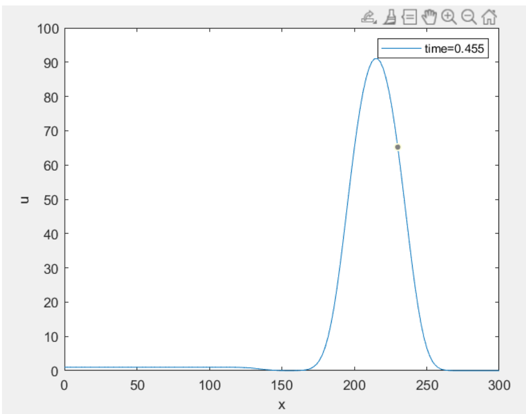

Figure 1.2: $\delta x = 2$

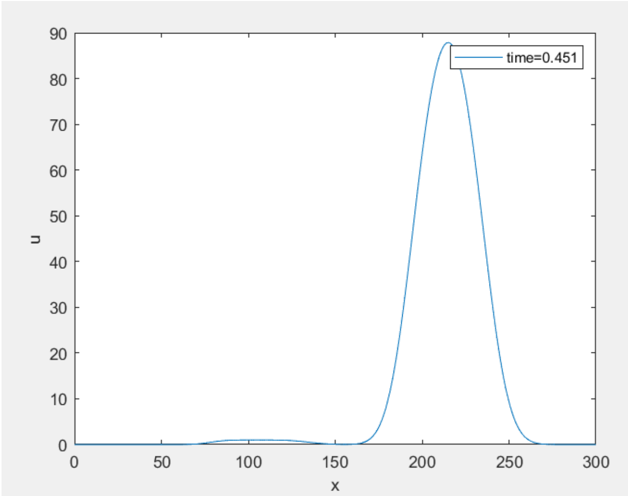

Figure 1.3: $\delta x = 1$

Graphs for the same are below. We can see that as the step size is reduced, the solution becomes more stable resembling the nature of Wave equation hyperbolic. The initial condition moves along the x-axis, resembling linear convection.

Chapter 2

Nonlinear Convection equation

2.1 Problem

We consider the nonlinear convection equation with the following form:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \quad (2.1) \tag{2.1}$$

The initial velocity distribution representing a compression wave is given by:

$$u(x,0) = \begin{cases} 1 & \text{for } x < 0.25 \\ 1.25 - x & \text{for } 0.25 < x < 1.25 \\ 0 & \text{for } x > 1.25 \end{cases}$$

We are solving the nonlinear convection equation for wave propagation within a domain of $0.0 < x < 4.0$ up to $t = 6.0$ seconds. The initial wave is compressed (steepens) with time and subsequently forms a shock wave. Within the specified time and space intervals, no shock reflection occurs. Therefore, the boundary conditions are simply specified as:

$$u(0.0, t) = 1.0 \quad \text{and} \quad u(4.0, t) = 0.0$$

We will use the FTBS (Forward in Time, Backward in Space) method with a spatial step of 0.05 meters to obtain the solutions for the following cases.

2.2 Matlab code

```
clear all
clc
 Lx =4; % Length of the domain
 dx =0.05;
 nx =( Lx / dx ) +1; % grid size
 x =0: dx : Lx ;
 dt = 0.01;
% dt = 0.025; % case-2
% dt = 0.05; % case-3
% dt = 0.1; % case-4
 nt =600;
 % number of time steps
% nt=240; % case-2
% nt=120; % case-3
```

```
% nt=60; % case-4

 % Define initial condition
 for i =1: nx
 if x ( i ) <0.25
 u ( i ) =1;
 else if x ( i ) >=0.25 && x ( i ) <=1.25
 u ( i ) =1.25 - x ( i ) ;
 else
 u ( i ) =0;
 end
 end
 end
 for it =1: nt
 un = u ;
 for i =2: nx -1
 u ( i ) = un ( i ) -( un ( i ) * dt / dx ) *( un ( i ) - un (i -1) ) ;%non linearconvection problem
 end
 plot(x,un)
    xlabel('x')
    ylabel('u')
    legend(['time=' num2str(it*dt)])
    pause(0.04)
end
```

Listing 2.1: Example MATLAB Code

2.3 Results

2.3.1 Results for $\Delta t = 0.01$ s

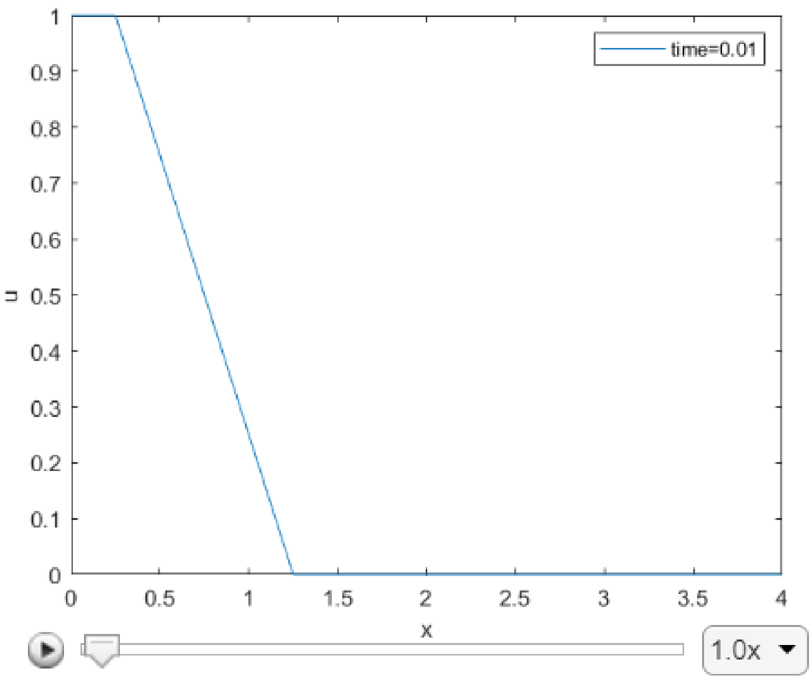

Figure 2.1: Result for $\delta t = 0.01$ s at $t = 0$ s

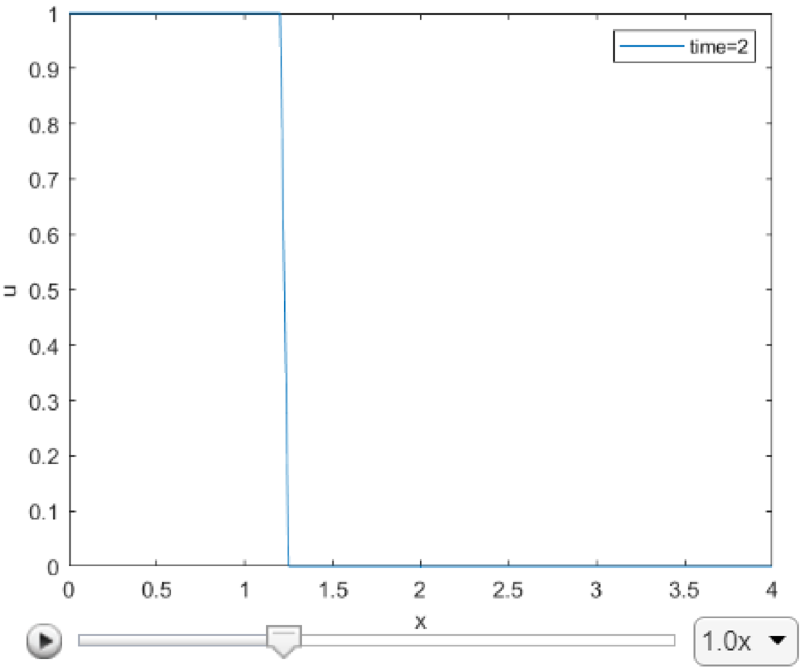

Figure 2.2: Result for $\delta t = 0.01$ s at $t = 2$ s

2.3.2 Results for $\delta t = 0.025$ s

2.3.3 Results for $\delta t = 0.05$ s

2.3.4 Results for $\delta t = 0.1$ s

In the last case, our case blows up due to instability

In this case, the code experiences instability, and we cannot plot the solution for $t = 0$, $t = 2$, $t = 4$, or $t = 6$ seconds.

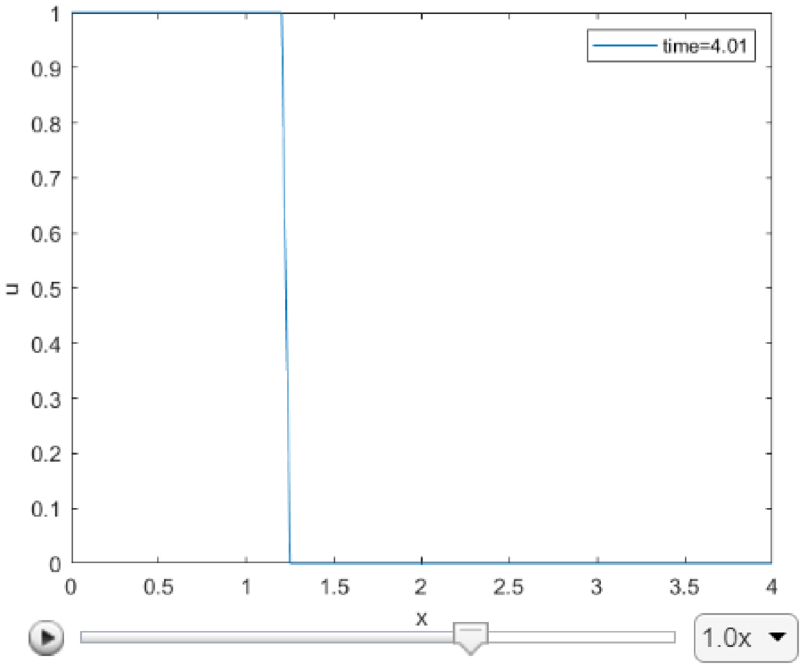

Figure 2.3: Result for $\delta t = 0.01$ s at $t = 4$ s

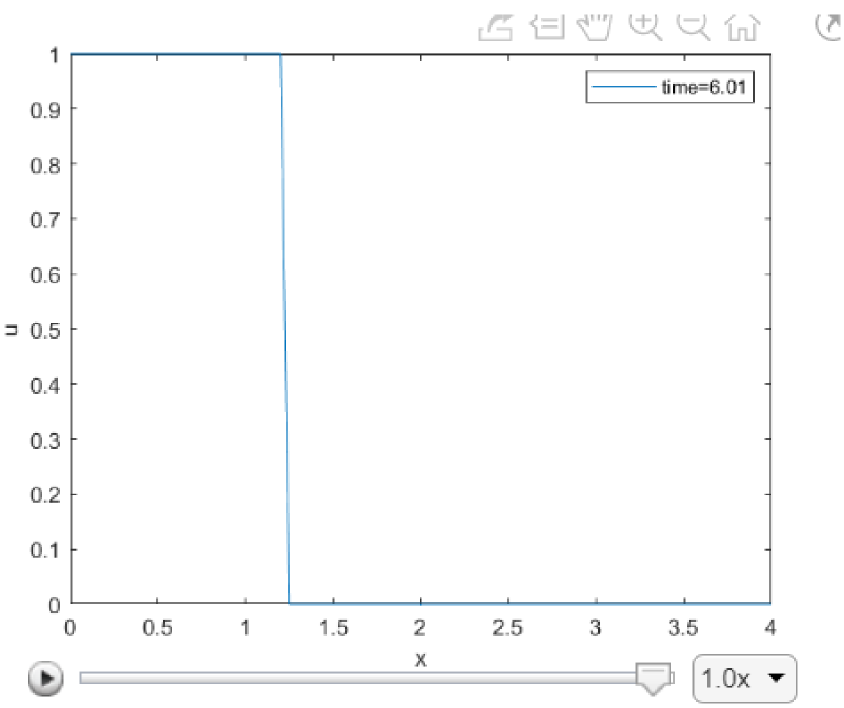

Figure 2.4: Result for $\delta t = 0.01$ s at $t = 6$ s

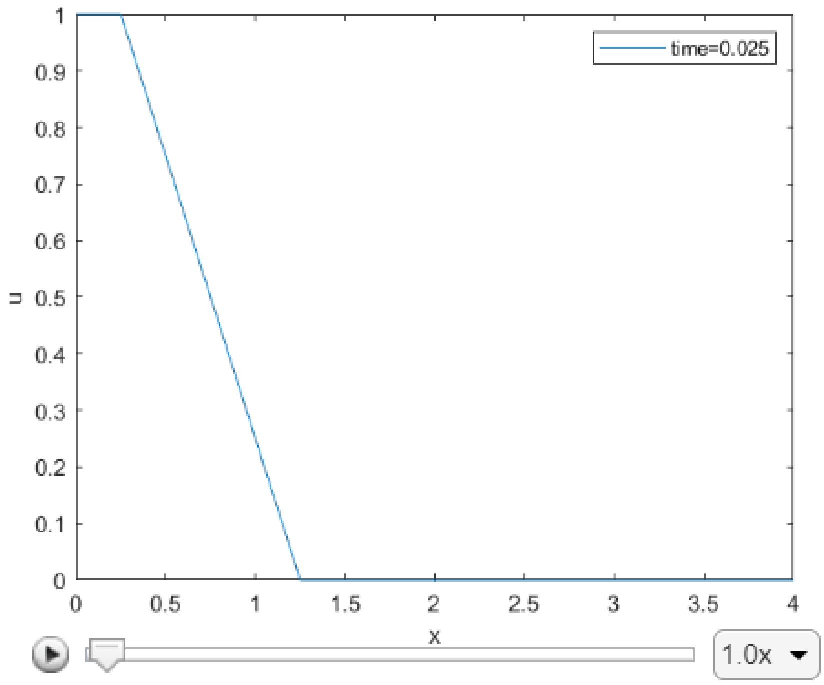

Figure 2.5: Result for $\delta t = 0.025$ s at $t = 0$ s

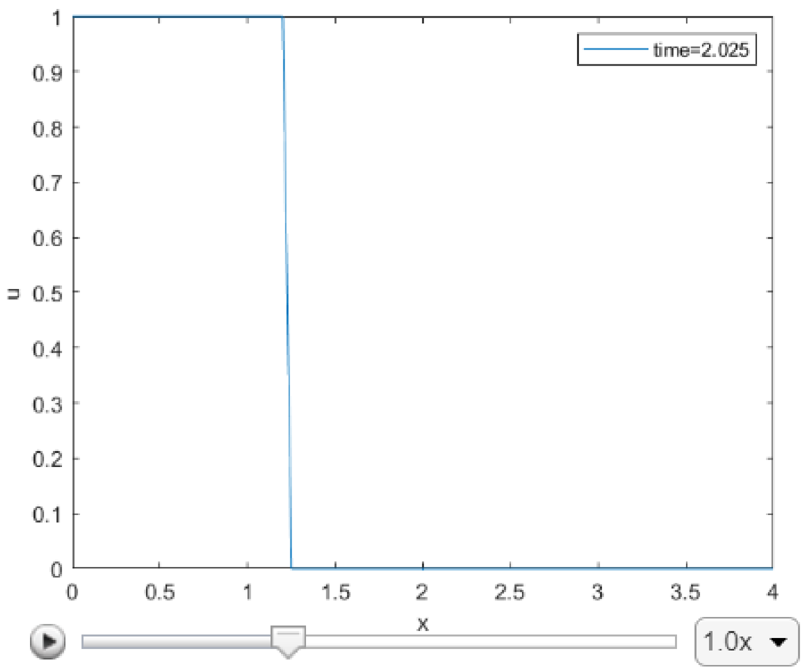

Figure 2.6: Result for $\delta t = 0.025$ s at $t = 2$ s

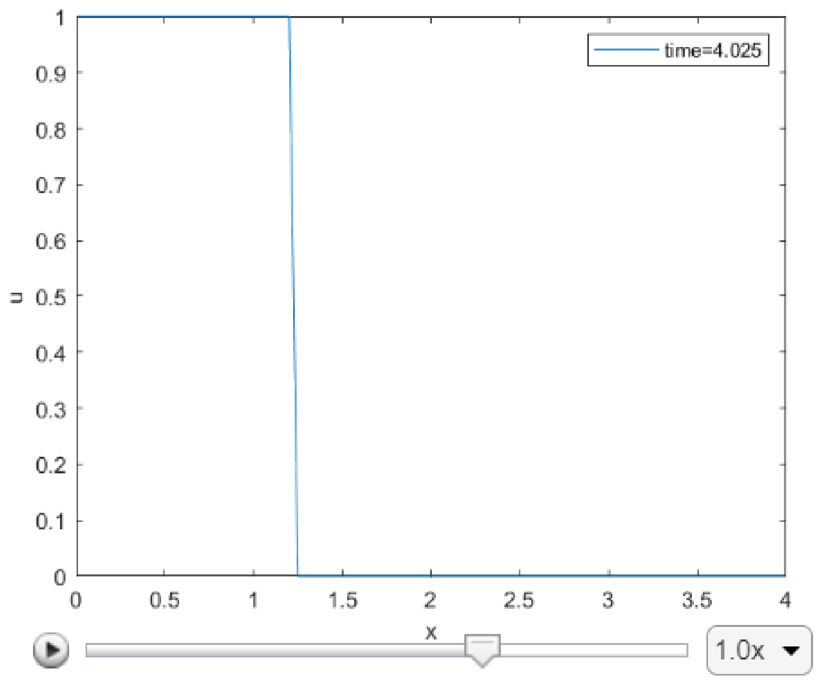

Figure 2.7: Result for $\delta t = 0.025$ s at $t = 4$ s

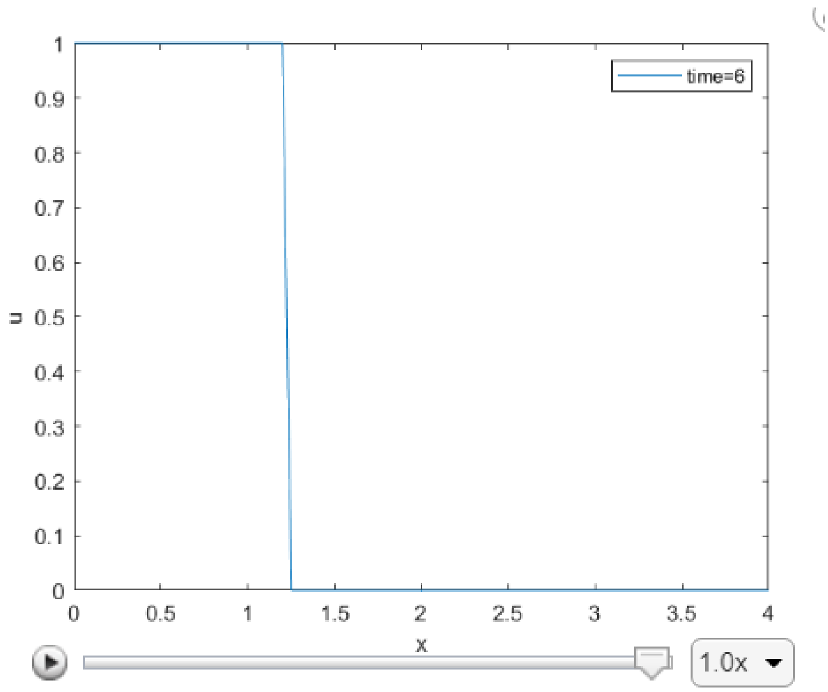

Figure 2.8: Result for $\delta t = 0.025$ s at $t = 6$ s

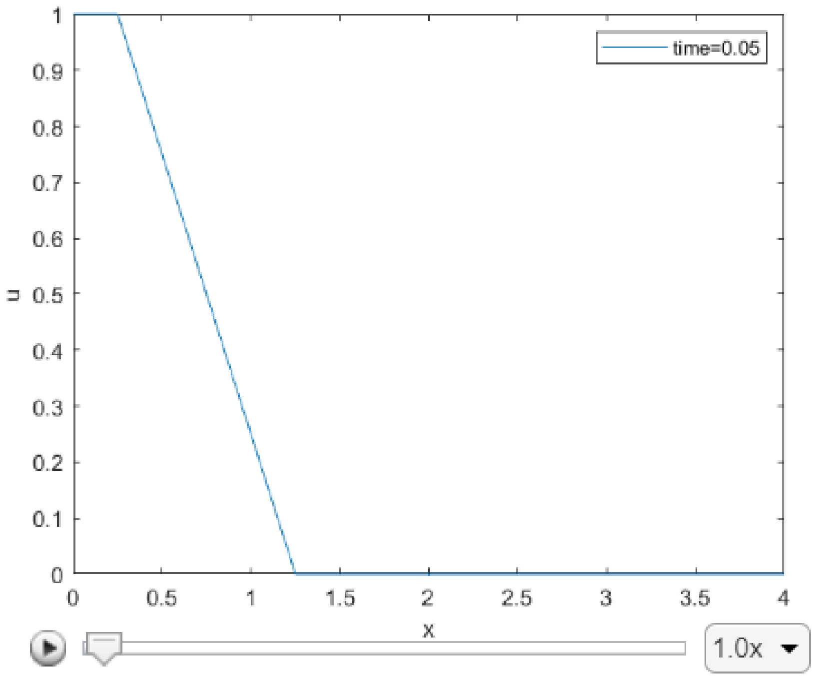

Figure 2.9: Result for $\delta t = 0.05$ s at $t = 0$ s

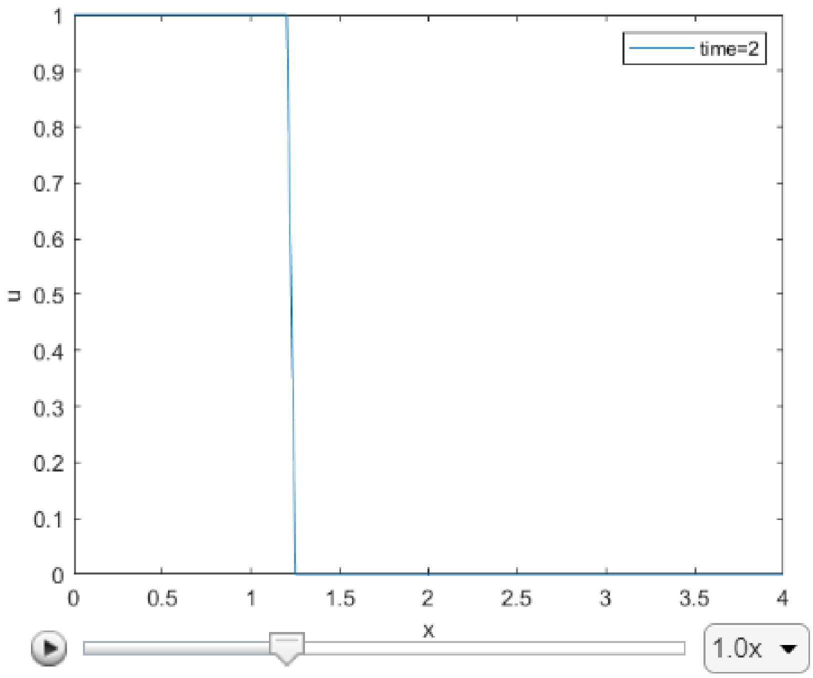

Figure 2.10: Result for $\delta t = 0.05$ s at $t = 2$ s

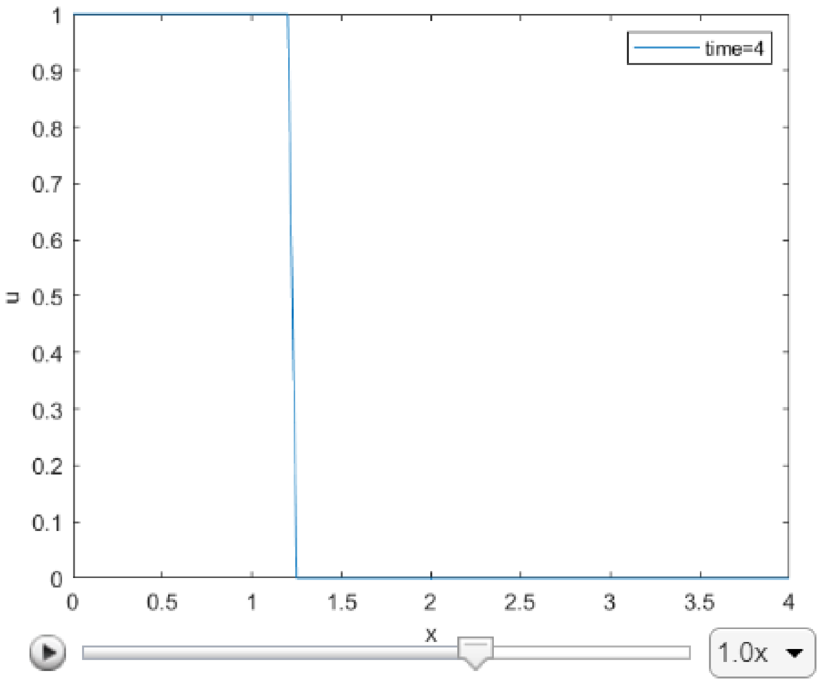

Figure 2.11: Result for $\delta t = 0.05$ s at $t = 4$ s

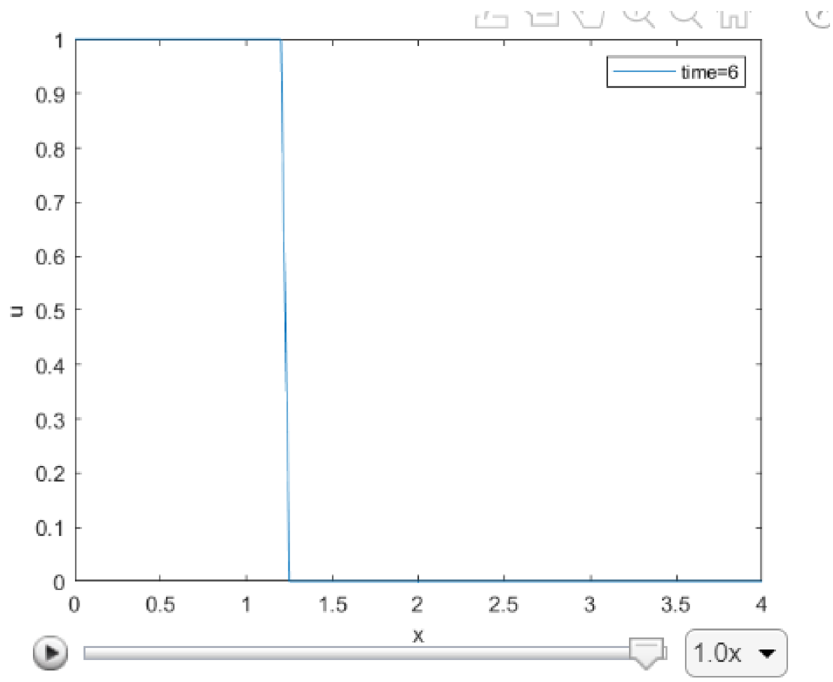

Figure 2.12: Result for $\delta t = 0.05$ s at $t = 6$ s

Chapter 3

Heat diffusion Equation

3.1 Problem

A plate of length L = 1.0 cm and thermal diffusivity v = 0.01cm2/s is heated to an initial temperature distribution T(x, 0) as specified below. Suddenly the heat source is turned off. Solve the heat diffusion equation using the FDM method. Plot the temperature distribution of the plate at t = 0.5, 1, 2, 3, 4, 5, and 10 s.
The equation to solve:

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2} \tag{3.1}$$

$$\text{T(x, 0)} = \begin{cases} 200x & \text{if } 0 \leq x < 0.5 \\ 200(1-x) & \text{if } 0.5 \leq x < 1 \end{cases}$$

3.2 Matlab code

```
% Linear convection problem
% FTBS ( Forward in time Backward in space
 clear all
 clc
 Lx =1; % Length of the domain
 dx =0.01;
 nx =( Lx / dx ) +1; % grid size
 x =0: dx : Lx ;
 dt = 0.004;
 %nt =2500;% number of time steps for 10 seconds
 %nt =1250:% 5 seconds
 %nt =750:% 3 seconds

 %nt =500:% 2 seconds
 %nt =250:% 1 seconds
 %nt =125:% 0.5 seconds
 nt =1000; % nt =40 number of time steps for 4 seconds
 c =0.01; % convection velocity (nue)


 %cdt/dx ^2 <0.5

 % Define initial condition
 for i =1: nx
 if x ( i ) <0.5
```

```
u ( i ) =200* x ( i ) ;
else
u ( i ) =200*(1 - x ( i ) )
end
end
for it =1: nt
un = u ;
for i =2: nx -1;
u ( i ) = un ( i ) + c * dt / dx ^2*( un ( i +1) -2* un ( i ) + un (i -1) ) ;%non linear
    convection problem
end
plot(x,un)
   xlabel('x')
   ylabel('u')
   legend(['time=' num2str(it*dt)])
   pause(0.04)
end
```

Listing 3.1: Example MATLAB Code

3.3 Results for $\delta t = 0.025s$

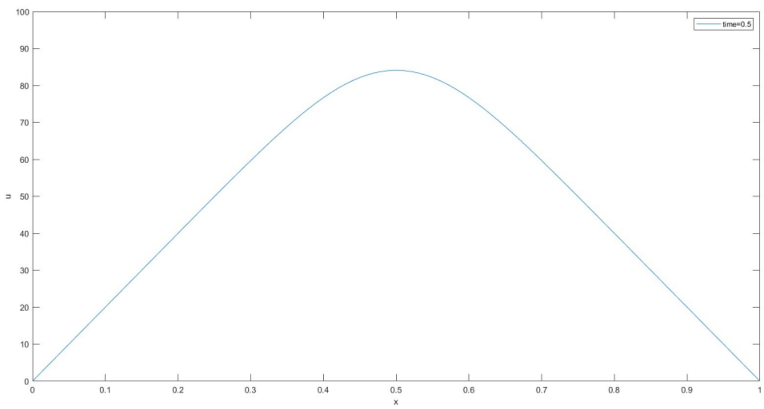

Figure 3.1: t=0.1 seconds

The analytical solution can be compared to the values obtained from the graph from which we can conclude the FEM yields good approximates.Moreover, as time passes the graph spreads out in the domain, validating the diffusion equation.

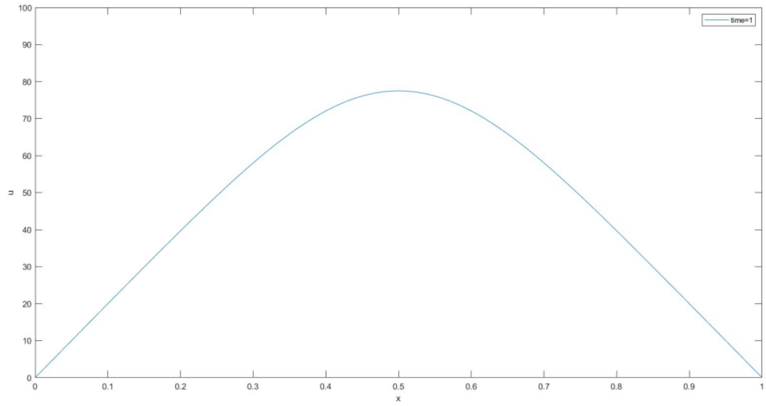

Figure 3.2: t=1 seconds

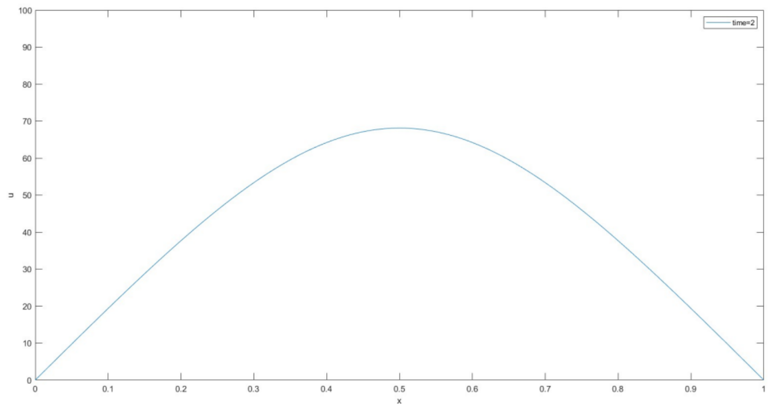

Figure 3.3: t=2 seconds

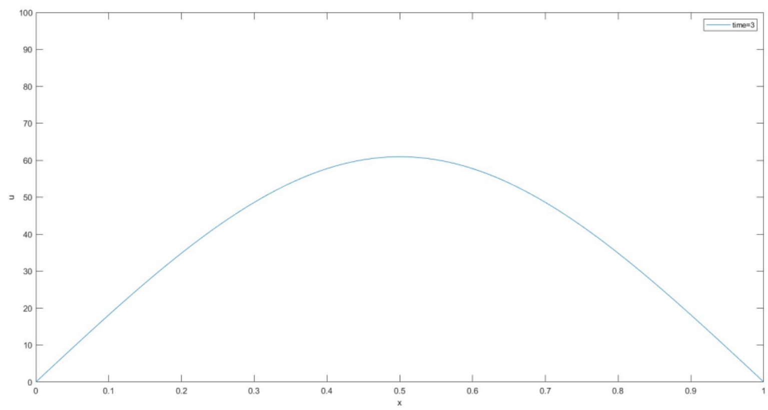

Figure 3.4: t=3 seconds

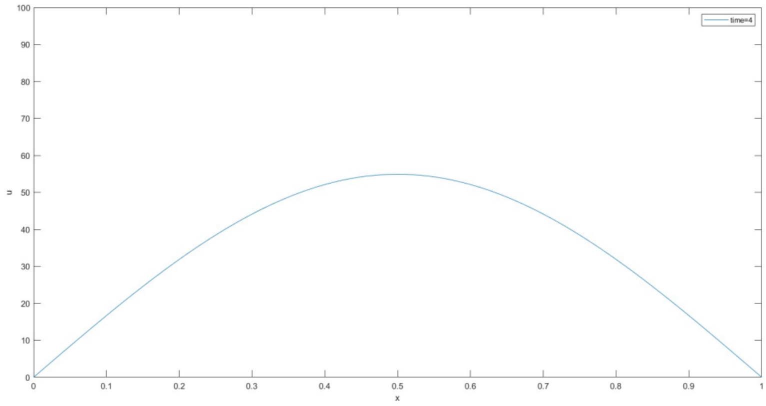

Figure 3.5: t=4 seconds

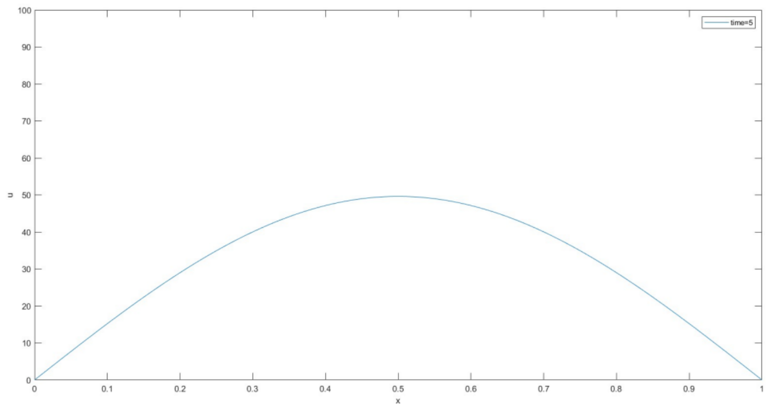

Figure 3.6: t=5 seconds

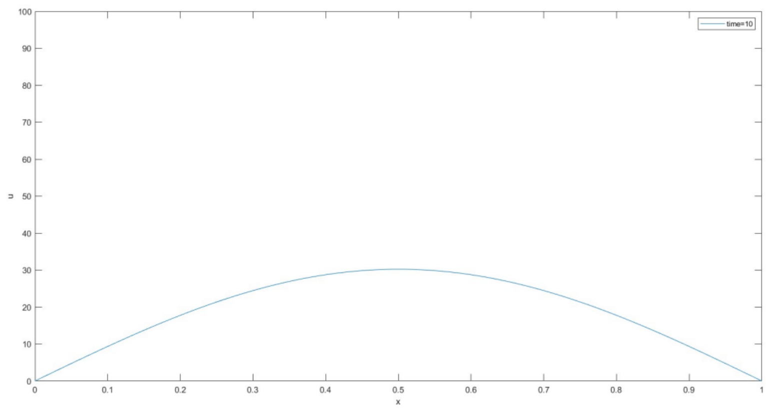

Figure 3.7: t=10 seconds

Chapter 4

Motion of fluid between 2 infinite plates

4.1 Problems

Two parallel plates extended to infinity are a distance of h apart. The fluid within the plates has a kinematic viscosity of 0.000217 m2/s and a density of 800kg/m3 . The upper plate is stationary and the lower plate is suddenly set in motion with a constant velocity of 40 m/s. The spacing h is 4 cm. A constant streamwise pressure gradient of dp/dx is imposed within the domain at the instant motion starts. A spatial size of 0.001 m is specified. Recall that the governing equation is reduced from the Navier-Stokes equation and is given by:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{4.1}$$

(a) Use the FTBS explicit scheme with a time step of 0.002 sec. to compute the velocity within the domain for dp/dx = 0.0,(II)dp/dx = 20000.0N/m2,(III)Dp/dx = 30000.0N/m2.(I) Print the solutions at time levels of 0.0, 0.18, 0.36, 0.54, 0.72, 0.9, and 1.08 seconds. Plot the velocity profiles at time levels of 0.0, 0.18, and 1.08 seconds.

4.2 Matlab code

```
% Linear convection problem
% FTBS ( Forward in time Backward in space
 clear all
 clc
 Lx =0.04; % Length of the domain
 dx =0.001;
 nx =( Lx / dx ) +1; % grid size
 x =0: dx : Lx ;
 dt = 0.002;
 %nt =540;% 1.08 seconds
 nt =450; % 0.9 seconds
 %nt =360 0.72 seconds
 %nt =270 0.54


 nu =0.000217; % convection velocity (nue )

 % dpdx = 0000.0;
 % dpdx =20000;
 dpdx = -30000;
```

```
density =800;
c = - dpdx / density ;
 u = zeros ;
% Define initial condition
for i =1: nx
if x ( i ) ==0
u ( i ) =40;
else
u ( i ) =0;
end
end
for it =1: nt
un = u ;
for i =2: nx -1
u ( i ) = un ( i ) + nu * dt / dx ^2*( un ( i +1) -2* un ( i ) + un (i -1) ) + c * dt ;% lineardiffusion problem
end
plot(x,un)
    xlabel('x')
    ylabel('u')
    legend(['time=' num2str(it*dt)])
    pause(0.04)
end
```

Listing 4.1: Example MATLAB Code

4.3 Results for dp/dx=0

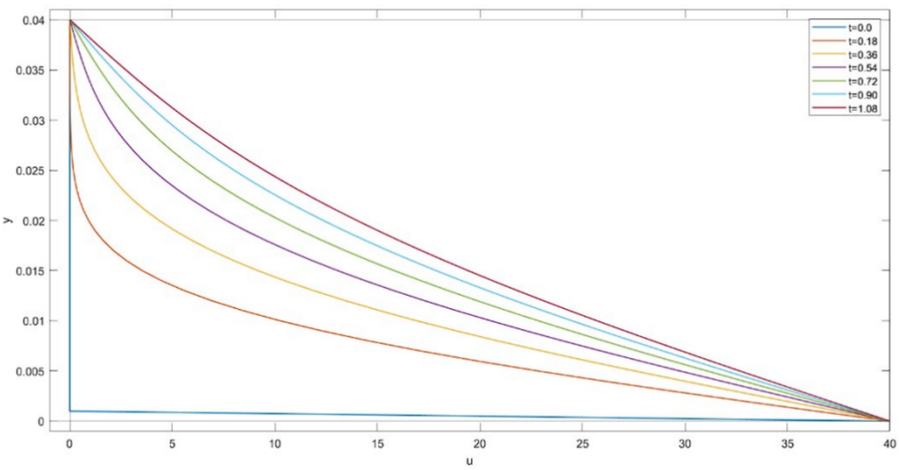

Figure 4.1: Result for dp/dx=0

4.4 Result for dp/dx=20000

4.5 Result for dp/dx=-30000

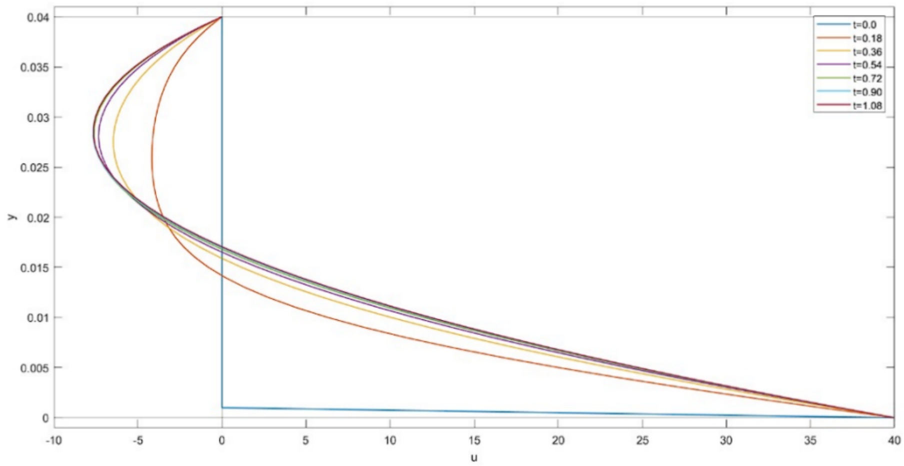

Figure 4.2: Result for dp/dx=20000

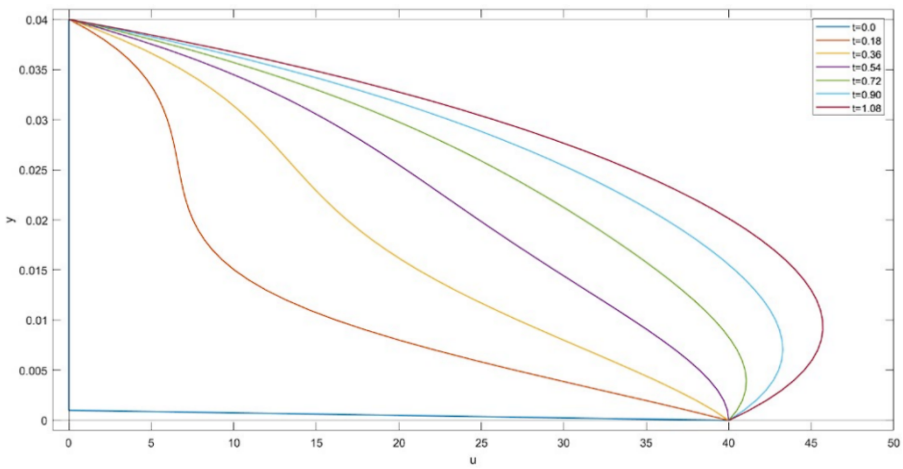

Figure 4.3: Result for dp/dx=-30000