AE 332: Modeling and Analysis Lab II

LAB REPORT-02

by

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Problem 1 A long, rectangular bar has dimensions of L by W, is shown in Figure. The bar is initially heated to a temperature of T_3. Subsequently, its surfaces are subjected to the constant temperatures of T_1, T_2, T_5, and T_4 as depicted in Figure, It is required to compute the transient solution where the governing equation is

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

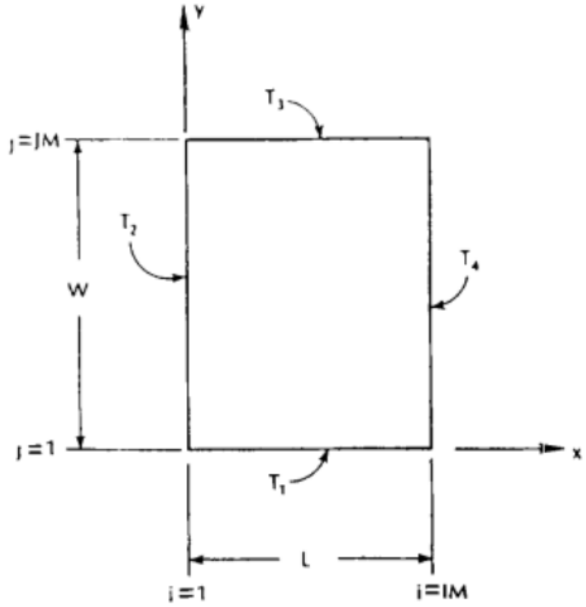


Figure 1: figure 1.

The bar is composed of copper with a thermal conductivity of 380 W/(m°C) and a thermal diffusivity of 11.234×10^{-5} m^2/sec, both assumed constant for this problem. The rectangular bar has dimensions of $L = 0.3$ m. and $W = 0.4$ m. The computational grid is specified by IMAX = 31 and 3MAX = 41.

Use the FTCS explicit scheme with time steps of 0.2 sec- and 1.0 sec. to compute the transient solution. The initial and boundary conditions are specified as: $T_0 = 0.0°\mathrm{C}, \mathrm{T}_1 = 40.0°\mathrm{C}, T_2 = 0.0°\mathrm{C}, T_3 = 10.0°\mathrm{C}$, and $T_4 = 0.0°\mathrm{C}$ (1) Plot the solutions at intervals of 0.05 m in the x-direction and all y-locations at $t = 10.0\mathrm{sec}, t = 40.0\mathrm{sec}$, and steady state. Assume the solution has reached a steady state If the total variation in temperature from one-time level to the next is less than CONSS, where CONSS is specified as 0.01°C. The total variation is determined as

T V=$\sum_{\substack{i=2 \\ j=2}}^{\substack{j=JMM_1 \\ i=IMM_1}} ABS\left(T_{i,j}^{n+1} - T_{i,j}^{n}\right)$

(II) Compare the steady-state solution obtained in (I) to the analytical solution. The analytical solution is

$$T = T_A + T_B$$

where

$$T_A = T_2 \cdot 2.0 \sum_{n=1}^{\infty} \frac{1-\cos(m\pi)}{m\pi} \frac{\sinh\left(\frac{m\pi(W-y)}{L}\right)}{\sinh \frac{m\pi W}{L}} \sin \frac{m\pi x}{L}$$

and

$$T_B = T_3 \cdot 2.0 \sum_{m=1}^{\infty} \frac{1-\infty\cos(m x)}{m\pi} \frac{\sinh \frac{m x y}{L}}{\sinh \frac{m x W}{L}} \sin \frac{m\pi x}{L}$$

(III) Plot the transient solution for the following locations: $(0.1, 0.05), (0.15, 0.10)$, and $(0.1, 0.3)$.
(IV) Plot the heat transfer distributions along the sides $y = 0, 0, y = W$, and $x = 0.0$ for the steady state solution.

MATLAB CODE:

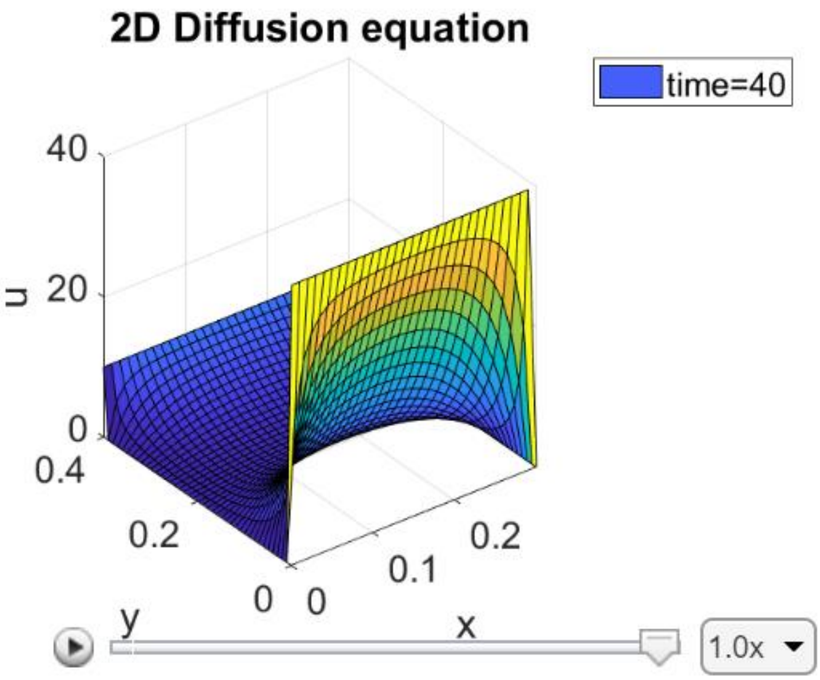
```
clear all
clc
nx =31;% number of points in x direction
ny = 41;
alpha=11.234*10^(-5);
%nt=2050; %for steady state and choose dt=0.2 for steady state
nt =50;% t=10 and dt=0.2
%nt=200 %for t=40 and dt=0.2
%nt=40 %for t=40 and dt=1
%nt=10 %for t=10 and dt=1
dx = 0.3./(nx-1);%grid size
dy = 0.4./(ny-1);%grid size
sigma = .2;%courant number
dt = 0.2;
%dt=1;
x=0:dx:0.3;
y=0:dy:0.4;


u=zeros(nx,ny);
un=zeros(nx,ny);
v=zeros(nx,ny);
vn=zeros(nx,ny);
u(1,:)=0;
u(:,1)=40;
u(nx,:)=0;
u(:,ny)=10;
[X, Y] = meshgrid(x, y);


surf(X,Y,u');


for it =1:nt
    un=u;
   % vn=v;
    for j=2:ny-1;
         for i=2:nx-1;
     u(i,j)=  un(i,j)+alpha*dt/dx^2*(un(i+1,j)-2*un(i,j)+un(i-
1,j))+alpha*dt/dy^2*(un(i,j+1)-2*un(i,j)+un(i,j-1));%diffusion equation
     % u(i,j) =un(i,j)-(un(i,j)dt/dx(un(i,j)-un(i-1,j)))-
(vn(i,j)dt/dx(un(i,j)-un(i,j-1)));
         end
    end
    surf(x,y,u');
        xlabel('x')
    ylabel('y')
    zlabel('u')
        set(gca,'FontSize',18)
title('2D Diffusion equation')
    legend(['time=' num2str(it*dt)])
```

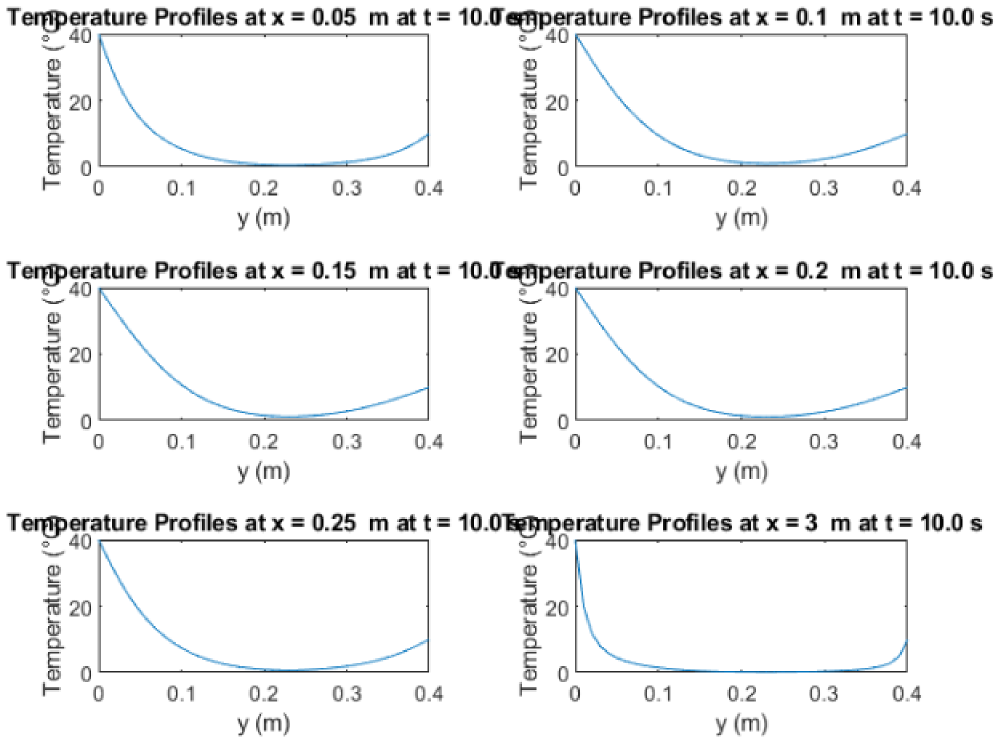
```
    pause(0.04)
end
figure
nexttile
plot(y,u(0.05/dx,:))
xlabel('y (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles at x = 0.05  m at t = 10.0 s ');
nexttile;
plot(y,u(0.1/dx,:))
xlabel('y (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles at x = 0.1  m at t = 10.0 s ');
nexttile;
plot(y,u(0.15/dx,:))
xlabel('y (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles at x = 0.15  m at t = 10.0 s ');
nexttile;
plot(y,u(0.2/dx,:))
xlabel('y (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles at x = 0.2  m at t = 10.0 s ');
nexttile;
plot(y,u(0.25/dx,:))
xlabel('y (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles at x = 0.25  m at t = 10.0 s ');
nexttile;
plot(y,u(0.3/dx,:))
xlabel('y (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles at x = 3  m at t = 10.0 s ');


% for part 4
figure
nexttile;
plot(x,u(:,1))
xlabel('x (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles along the sides y =0 m at steady state');
nexttile;
plot(x,u(:,ny))
xlabel('x (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles along the sides y =W m at steady state');
nexttile;
plot(y,u(1,:))
xlabel('y (m)');
ylabel('Temperature (°C)');
title('Temperature Profiles along the sides x=0 m at steady state');
```

```
%Code for part II
% Given Parameters
T1 = 40.0;   % Initial condition at T1
T3 = 10.0;   % Initial condition at T3
L = 0.3;     % Length of the bar (m)
W = 0.4;     % Width of the bar (m)
x = 0.05;    % x-coordinate (m)
y = 0.05;     % y-coordinate (m)


% Number of terms to include in the summation
N = 100;  % You can adjust this number based on accuracy requirements


% Initialize T_a and T_b
T_a = 0;
T_b = 0;


% Calculate T_a
for m = 1:N
    T_a = T_a + (1 - cos(m*pi)) / (m*pi) * sinh(m*pi * (W - y) / L) /
sinh(m*pi * W / L) * sin(m*pi * x / L);
end
T_a = T1 * 2 * T_a;


% Calculate T_b
for m = 1:N
    T_b = T_b + (1 - cos(m*pi)) / (m*pi) * sinh(m*pi * y / L) / sinh(m*pi * W
/ L) * sin(m*pi * x / L);
end
T_b = T3 * 2 * T_b;


T =T_a+T_b
```

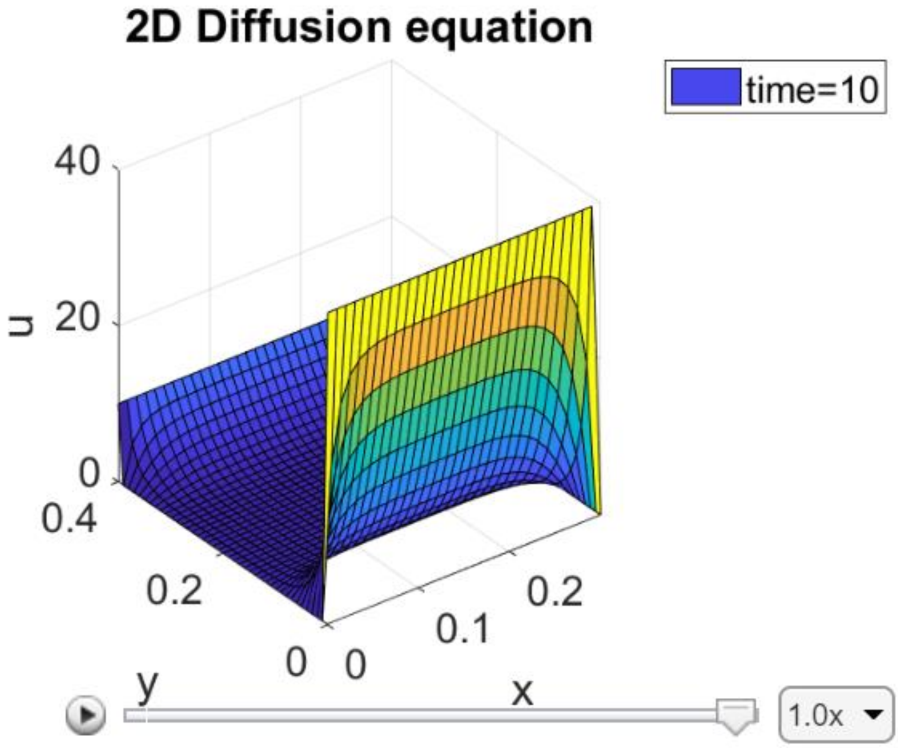
PLOT:

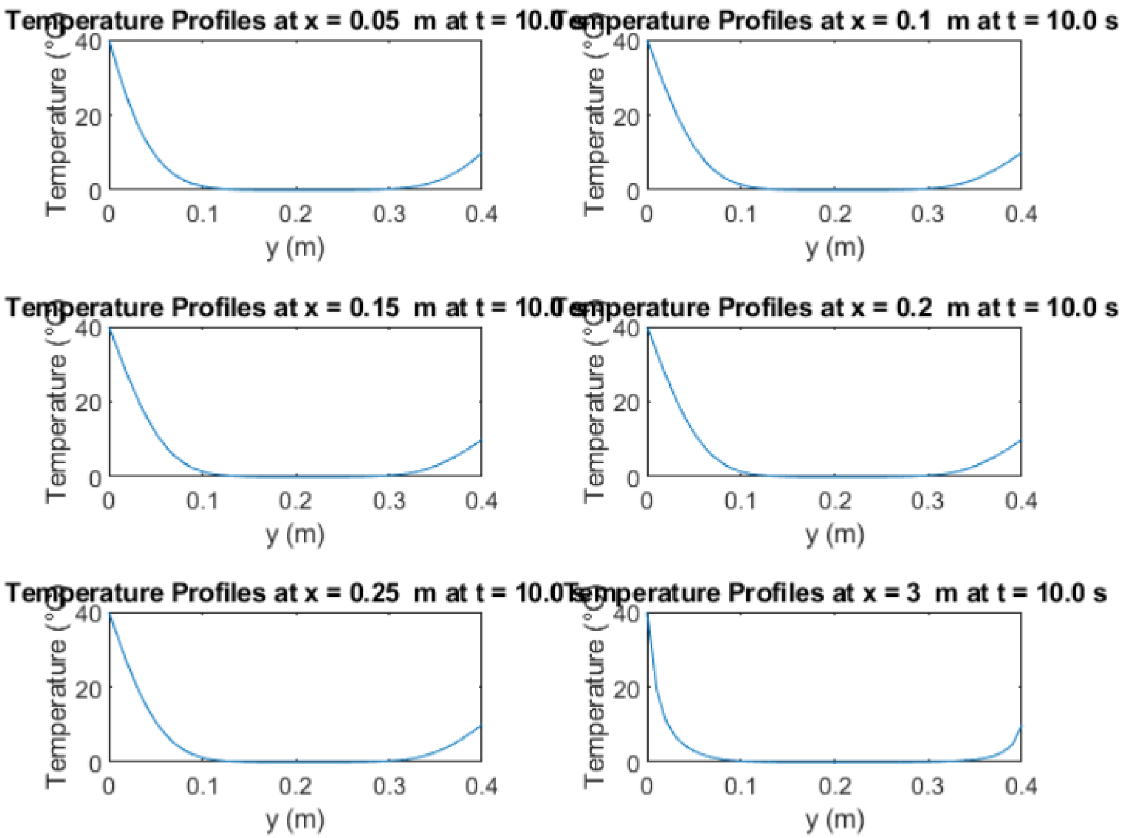
- **t=40 sec and dt=0.2**



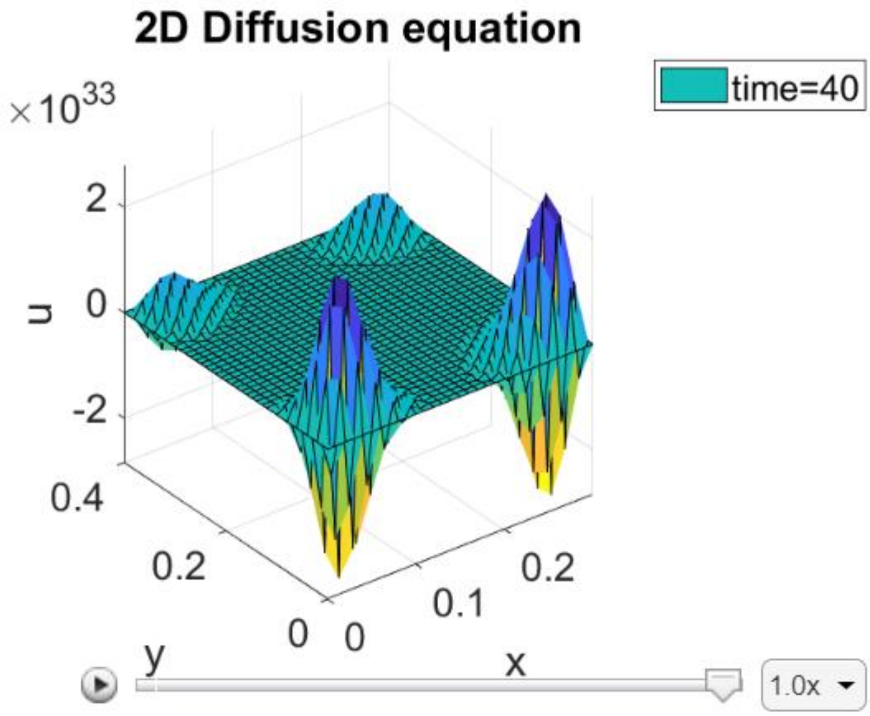


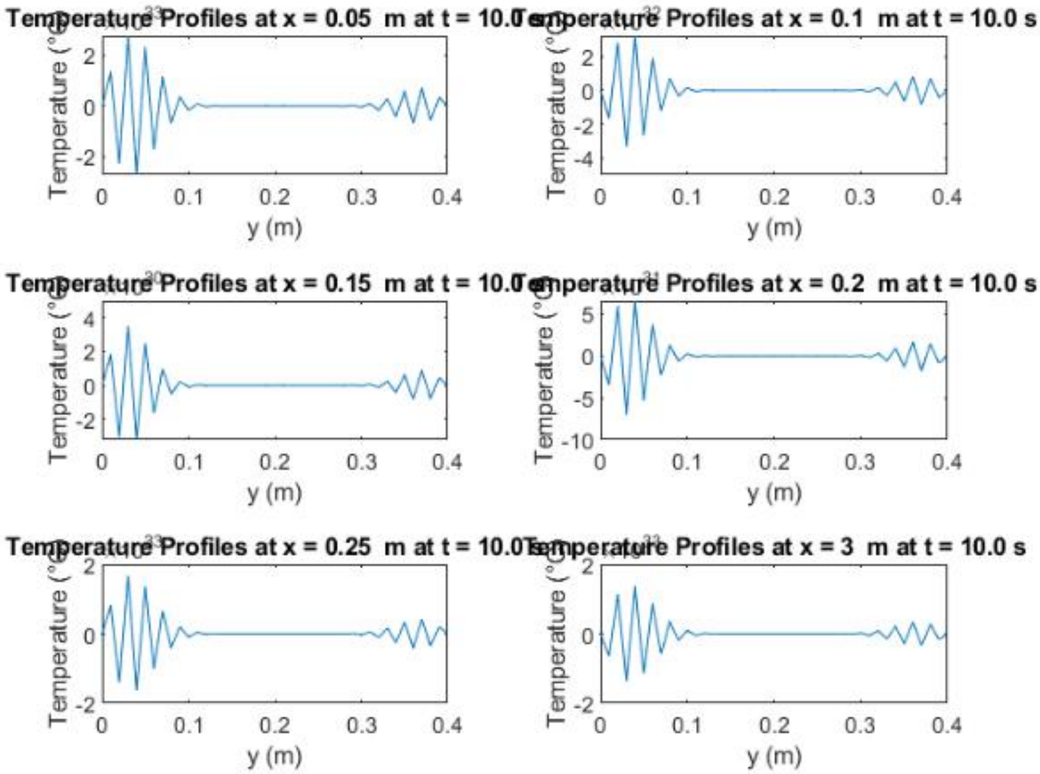
- **t=10 and dt=0.2**


2D Diffusion equation
time=10
40
20
0
u
0.4
0.2
0
y
0
0.1
0.2
x
1.0x

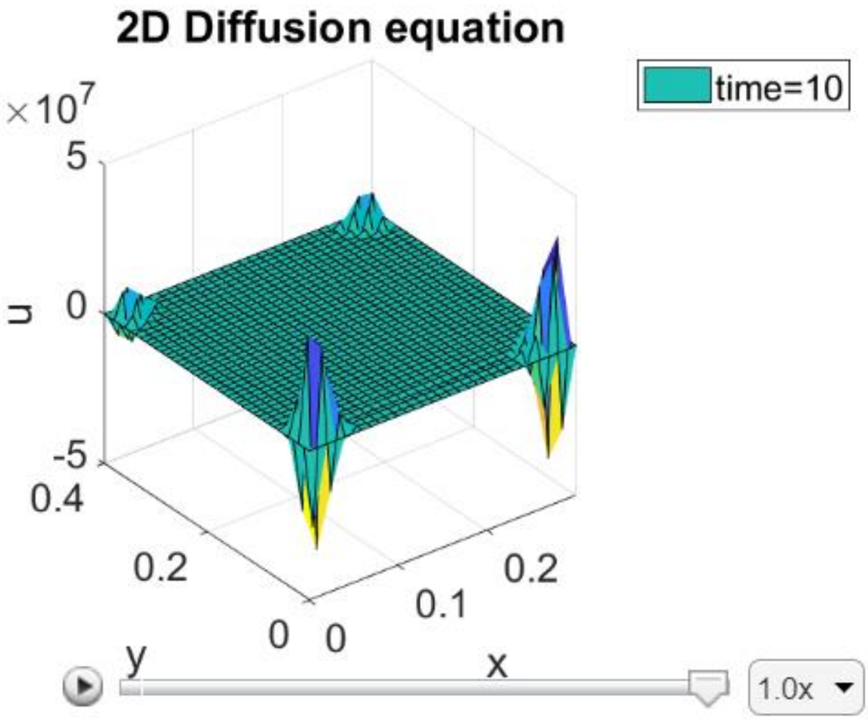

Temperature Profiles at x = 0.05 m at t = 10.0
Temperature Profiles at x = 0.1 m at t = 10.0 s
Temperature Profiles at x = 0.15 m at t = 10.0
Temperature Profiles at x = 0.2 m at t = 10.0 s
Temperature Profiles at x = 0.25 m at t = 10.0
Temperature Profiles at x = 3 m at t = 10.0 s
Temperature (°C)
y (m)
0
0.1
0.2
0.3
0.4
0
20
40

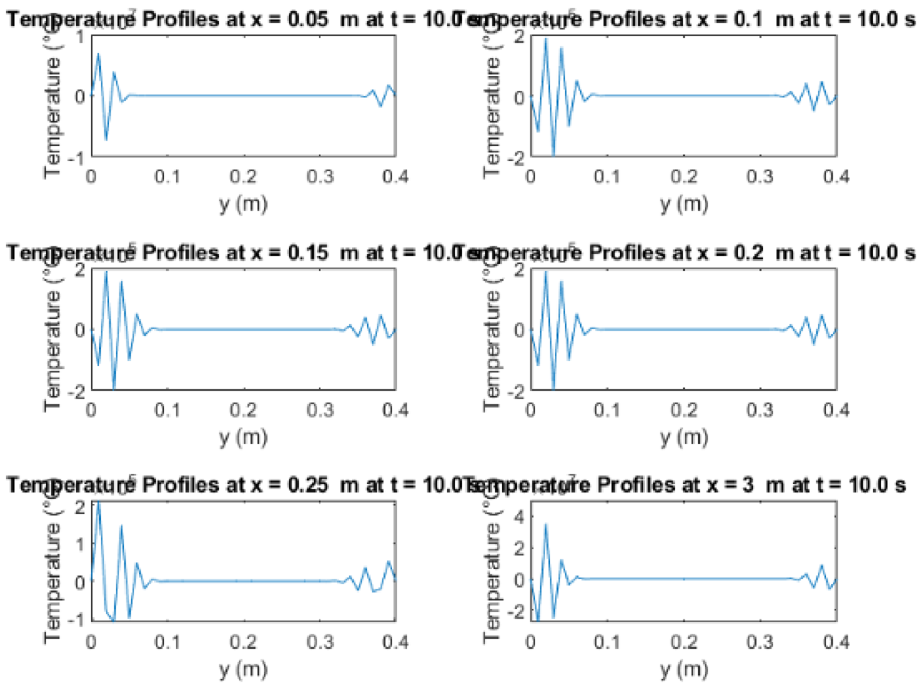
- **t=40 sec and dt=1sec**





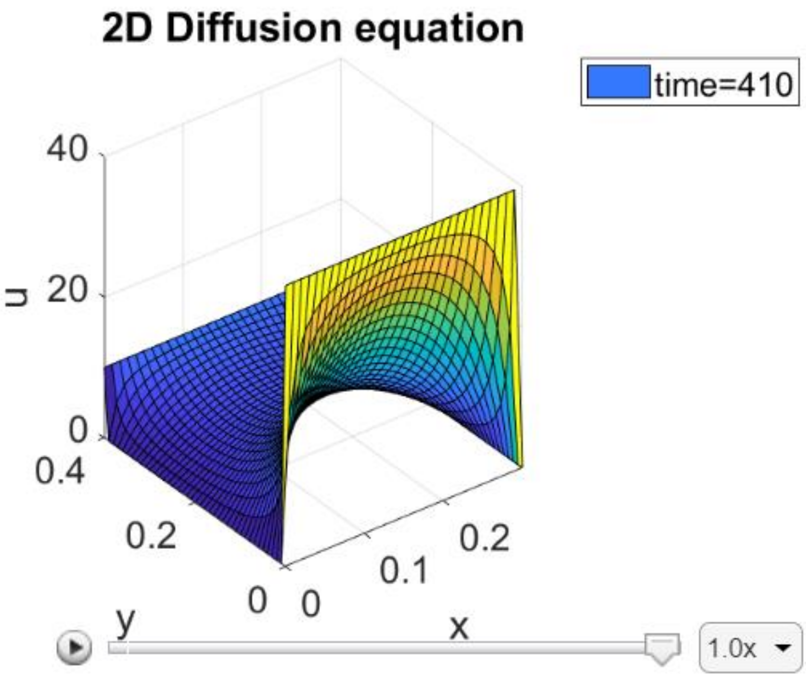
- **t=10 sec and dt=1 sec**

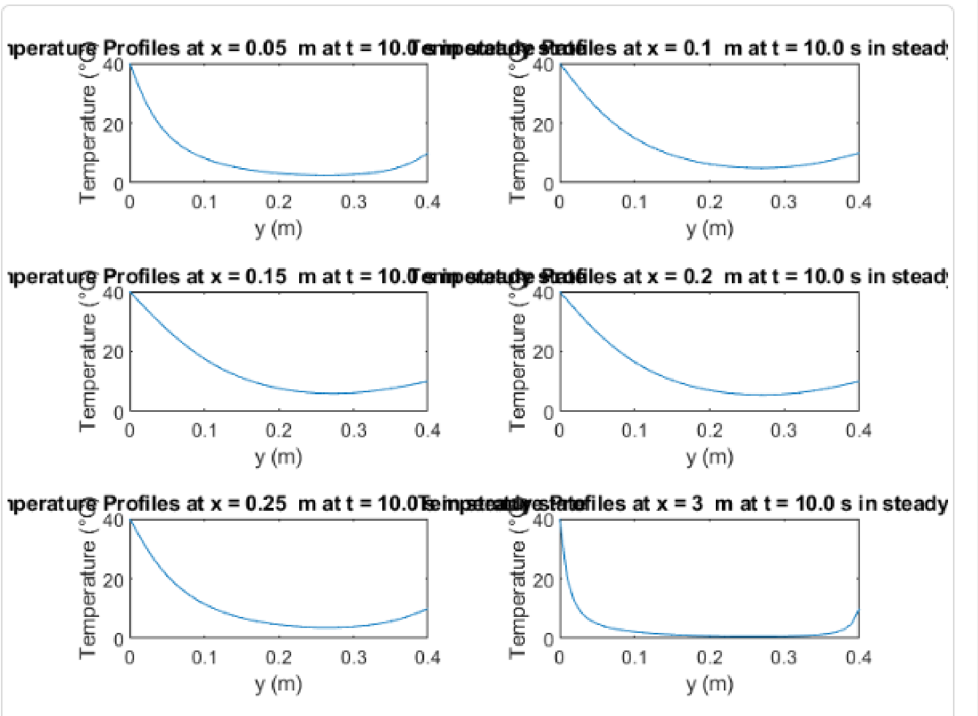




STEADY STATE

- **t=410 sec and dt=0.2 sec at STEADY STATE achieved**





- **Value of numerical solutions at u(5,5) at different time.**

  We want difference 0.01 for steady state so at t=410 sec

Time (sec)	u(5,5)	Difference
200	19.2701	
220	19.2919	0.0218
100	19.0137	
110	19.073	0.0593
400	19.3178	
410	19.3179	0.0001

Steady State occurs at time = 410 sec

II. Compare the steady-state solution obtained in (I) to the analytical solution

$$T_A = T_1 * 2.0 \sum_{m=1}^{\infty} \frac{1-\cos(m\pi)}{m\pi} \frac{\sinh\left(\frac{m\pi(W-y)}{L}\right)}{\sinh\frac{m\pi W}{L}} \sin\frac{m\pi x}{L}$$

$$T_B = T_3 * 2.0 \sum_{m=1}^{\infty} \frac{1-\cos(m\pi)}{m\pi} \frac{\sinh\frac{m\pi y}{L}}{\sinh\frac{m\pi W}{L}} \sin\frac{m\pi x}{L}$$

$$T = T_A + T_B$$

For x = 0.05 , y = 0.05

Analytical Solution : T = 18.9373

Numerical Solution : T = 19.3178

Difference = 0.3805

III. Plot the transient solution for the following locations: ⟨0.1, 0.05),(0.15, 0.10), and (0.1, 0.3).

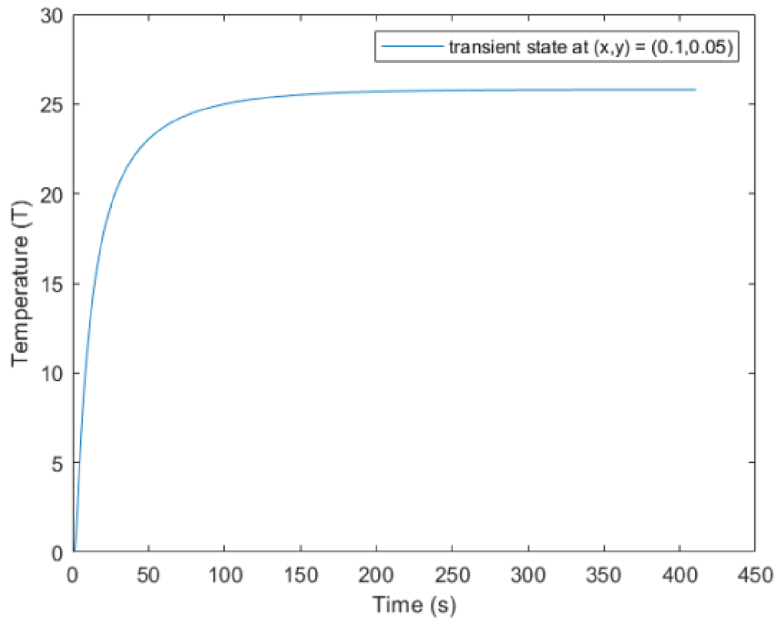


Figure 5.1: Transient solution at (0.1,0.05)

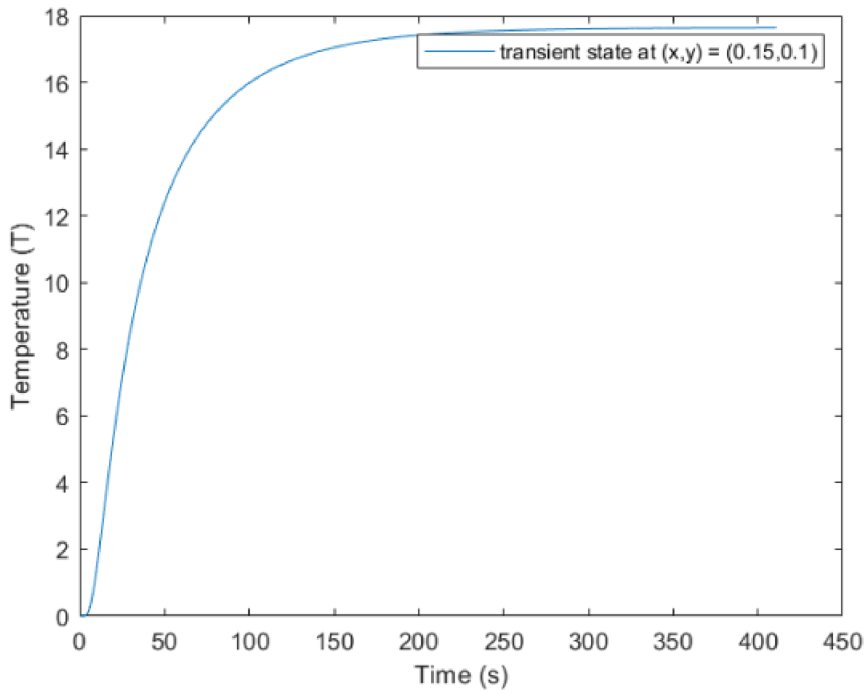


Figure 5.2: Transient solution at (0.1,0.05)

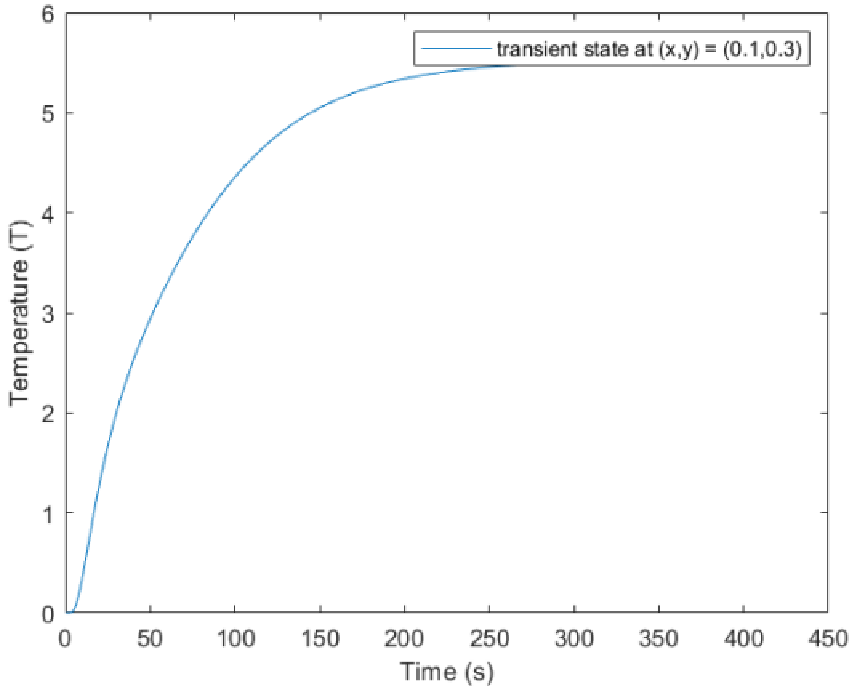
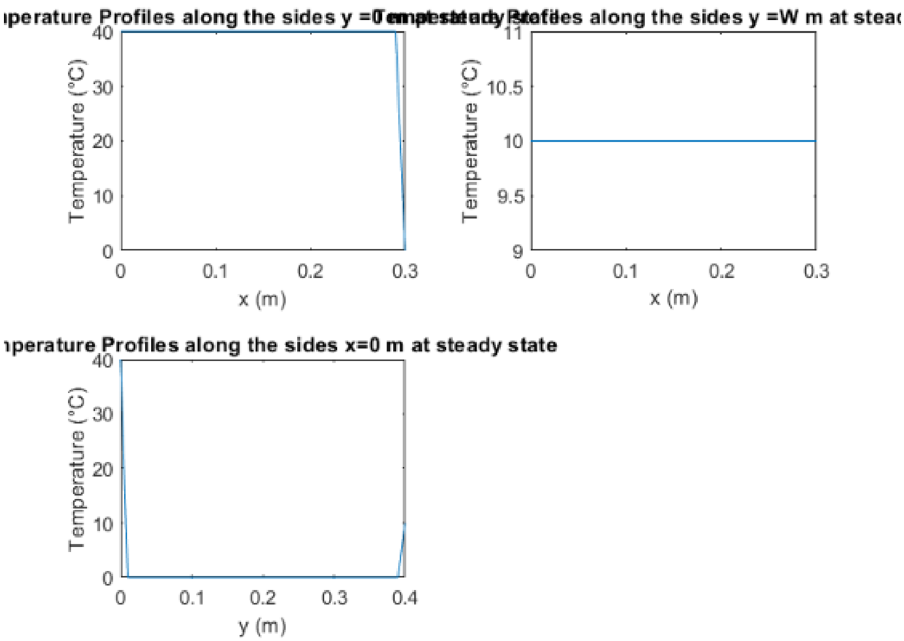
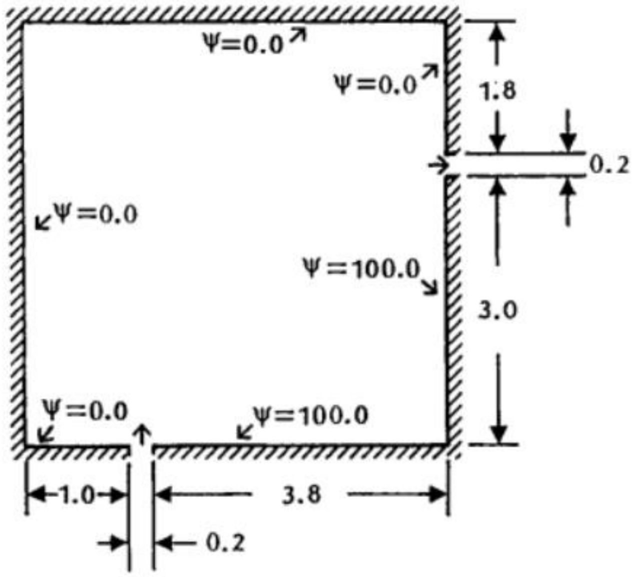


Figure 5.3: Transient solution at (0.1,0.05)

IV. Plot the heat transfer distributions along the sides y = 0, 0, y = W , and x = 0.0 for the steady state solution.



Problem 2 A two-dimensional inviscid, incompressible fluid is flowing steadily through a chamber between the inlet and the outlet, as shown in Figure. It is required to determine the streamline pattern within the chamber. For a two-dimensional, incompressible flow, the continuity equation



The goal in this problem is to obtain the solution of this elliptic partial differential equation. The solution will provide the streamline pattern within the chamber.

Since the chamber walls are streamlines, i.e., lines of constant Ψ, we will assign values for these streamlines. The assignment of these values is totally arbitrary as long as continuity is satisfied. Note that, for this application, the boundary conditions are the Dirichlet type, i.e., the values of the dependent variable are specified.

Assume that the inlet and the outlet are 0.2ft. (per unit depth), and the chamber is 5ft. by 5ft. The locations of the inlet and the outlet are shown in Figure.

The step sizes are specified as:

$$\Delta x = 0.2, \quad \Delta y = 0.2, \text{ and ERRORMAX } = 0.01$$

Print the converged solution for each scheme for all y locations at $x = 0.0, 1.0, 2.0, 3.0, 4.0,$ and 5.0. Use initial data distribution of $\Psi = 0.0$.

Plot: (a) The streamline pattern, i.e., lines of constant Ψ s. Only one plot is sufficient since, as you will notice, solutions by various schemes are very similar; (b) The relaxation parameter versus the number of iterations for PSOR and LSOR schemes. In addition, the following tasks are to be investigated, I. The effect of the direction for which the finite difference formulation (i.e., LGS and

LSOR) is applied on the convergence. II. The effect of the initial data on convergence. For this investigation use the PSOR scheme with optimum value of the relaxation parameter. Suggested values of initial data are (a) 0.0 , (b) 25.0, (c) 50.0, and (d) 100.0. In addition you may consider a non-uniform initial data distribution.

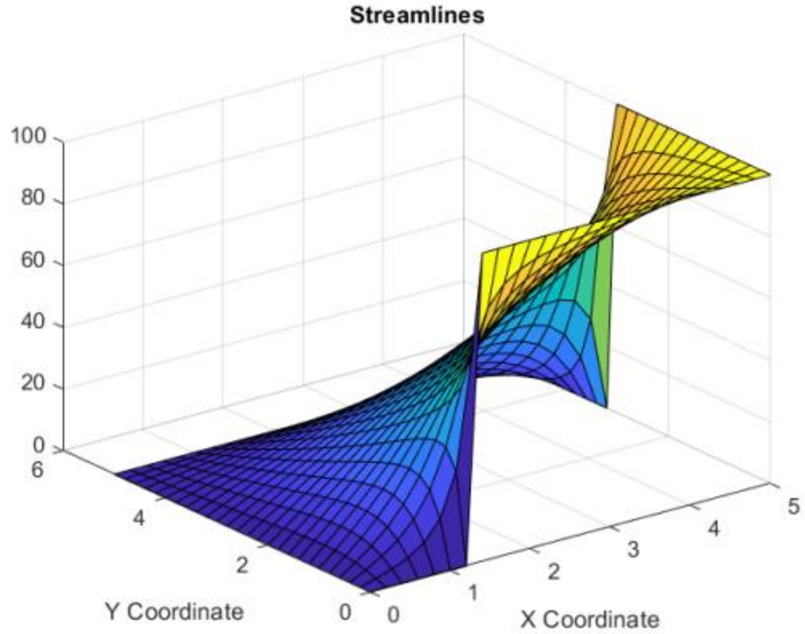
MATLAB CODE:

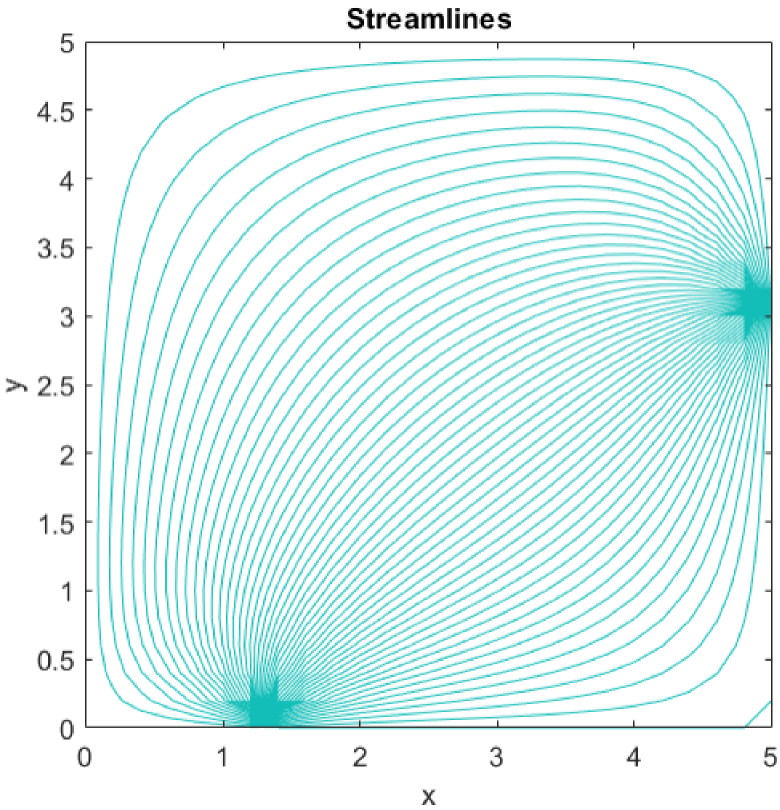
```
clear;
clc;


% Domain parameters
W = 5;
L = 5;
dx = 0.2;
dy = 0.2;
nx = W / dx;
ny = L / dy;
x = 0:dx:W;
y = 0:dy:L;


% Relaxation factor and convergence criterion
w = 1;
TV = 10;


% Initialize stream function
psi = zeros(nx+1, ny+1);
psi(1:7, 1) = 0;
psi(8:end, 1) = 100;
psi(end, 1:16) = 100;
psi(end, 17:end) = 0;
psi(1, :) = 0;
psi(:, end) = 0;


[X, Y] = meshgrid(x, y);


while TV > 0.01
    psin = psi;
    TV = 0;
    for i = 2:nx
        for j = 2:ny
            psi(i, j) = (1 - w) * psin(i, j) + 0.25 * w * (psin(i + 1, j) +
psin(i - 1, j) + psin(i, j + 1) + psin(i, j - 1));
            TV = TV + abs(psi(i, j) - psin(i, j));
        end
    end
    psin = psi;
end
```

```
% Plot streamlines
surf(X', Y', psi);
ylabel("Y Coordinate");
xlabel("X Coordinate");
title("Streamlines");


figure(2);
hold on;
plot(psi(1, :), y);
plot(psi(6, :), y);
plot(psi(11, :), y);
plot(psi(16, :), y);
plot(psi(21, :), y);
plot(psi(26, :), y);


legend('x=0', 'x=1', 'x=2', 'x=3', 'x=4', 'x=5');
hold off;


figure(3);
for C = 0:2:100
    contour(Y, X, psi - C, [0 0]);
    drawnow;
    axis equal;
    xlabel('x');
    ylabel('y');
    title('Streamlines');
    hold on;
end
```

PLOT:

The plots are as follows: The streamline plot is as follows:





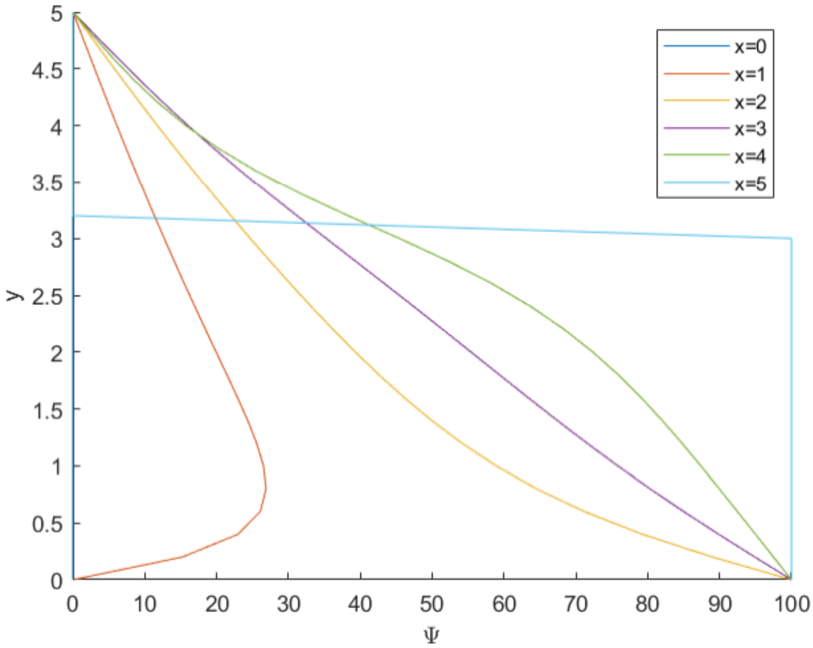


Figure 8.2: Values of Ψ for different values of y and fixed x locations