AE332-MODELLING AND ANALYSIS LAB

SESSION 6

Rigid Body Motion

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1)Simulate the rotation of a rigid body with and without moments, and verify Energy is conserved.

CASE A: MOMENT NOT EQUAL TO 0;

```
%moment of inertia about body fixed refrence frame
i_1=0.5;
i_2=1;
i_3=1.5;
i_fixed=[i_1 0 0;
         0 i_2 0;
         0 0 i_3];
m=5;
% let the rotation is about z axis
u=[0 0 1];
theta=45*pi/180;
q_0=cos(theta/2);
q_1=sin(theta/2)*u(1);
q_2=sin(theta/2)*u(2);
q_3=sin(theta/2)*u(3);
rotation_matrix=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
                 2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
                 2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
i_abs=rotation_matrix*i_fixed*rotation_matrix.';
%defining intial conditions
x_0=0;
y_0=0;
z_0=0;
x_dot0=0;
y_dot0=0;
z_dot0=0;
w_x=1;
w_y=2;
w_z=3;
%Defining force and moments
F=[1;1;1];
M=[5;5;5];%for 2nd case and 3rd case it become zero
intial_conditions=[x_0,
y_0,z_0,q_0,q_1,q_2,q_3,x_dot0,y_dot0,z_dot0,w_x,w_y,w_z,0,0];
```

```
tolerance=odeset('RelTol',1e-14,'AbsTol',1e-14);
[t,z]=ode45(@(t,z)f(t,z,F,M),(0:0.01:10),intial_conditions,tolerance);
j=1;
Ang_mom=zeros;
RKE=zeros;
for i=(0:0.01:10)
    q_0=z(j,4);
    q_1=z(j,5);
    q_2=z(j,6);
    q_3=z(j,7);
    Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
       2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
       2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[i_1 0 0;
          0 i_2 0;
          0 0 i_3];
    I_abs=Q*I_fix*Q.';
    w=[z(j,11);z(j,12);z(j,13)];
    L_0=I_abs*w;
    Ang_mom(j,1)=L_0(1,1);
    Ang_mom(j,2)=L_0(2,1);
    Ang_mom(j,3)=L_0(3,1);
    RKE(j,1)=0.5*Ang_mom(j,:)*w;
    j=j+1;
end
%calculating energy and energy difference
Total_energy=0.5*m*(z(:,8).^2+z(:,9).^2+z(:,10).^2)+RKE-z(:,14)-z(:,15);
change_in_energy=max(Total_energy)-min(Total_energy);
L_x=Ang_mom(:,1);
L_y=Ang_mom(:,2);
L_z=Ang_mom(:,3);
change_in_ang_momx=max(L_x)-min(L_x);
change_in_ang_momy=max(L_y)-min(L_y);
change_in_ang_momz=max(L_z)-min(L_z);
%Angular velocity conservation;
change_in_wx=max(z(:,11))-min(z(:,11));
change_in_wy=max(z(:,12))-min(z(:,12));
change_in_wz=max(z(:,13))-min(z(:,13));
```

```
function zdot=f(~,z,F,M)
m=5;
i_1=0.5;
i_2=1;
i_3=1.5;
q_0=z(4,1);
q_1=z(5,1);
q_2=z(6,1);
q_3=z(7,1);
zdot(1,1)=z(8,1);
zdot(2,1)=z(9,1);
zdot(3,1)=z(10,1);
zdot(4,1)=-0.5*(q_1*z(11,1)+q_2*z(12,1)+q_3*z(13,1));
```

```
zdot(5,1)=0.5*(q_0*z(11,1)+q_3*z(12,1)-q_2*z(13,1));
zdot(6,1)=0.5*(q_1*z(13,1)+q_0*z(12,1)-q_3*z(11,1));
zdot(7,1)=0.5*(q_2*z(11,1)-q_1*z(12,1)+q_0*z(13,1));
mass=[m 0 0;
      0 m 0;
      0 0 m];
Linear_acceleration=mass\F;
zdot(8,1)=Linear_acceleration(1,1);
zdot(9,1)=Linear_acceleration(2,1);
zdot(10,1)=Linear_acceleration(3,1);
Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
   2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
   2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[i_1 0 0;
           0 i_2 0;
           0 0 i_3];
    I_abs=Q*I_fix*Q.';
    w=[z(11,1);z(12,1);z(13,1)];
    w_1=I_abs*w;
 Angular_acceleration=I_abs\(M-cross(w,w_1));
  zdot(11,1)=Angular_acceleration(1,1);
zdot(12,1)=Angular_acceleration(2,1);
zdot(13,1)=Angular_acceleration(3,1);
zdot(14,1)=F'*[z(8,1);z(9,1);z(10,1)];
zdot(15,1)=M'*w;
end
```

CHANGE IN ENERGY:

=9.345e-11

Change in energy is very small so we can consider that energy is constant.

CASE B: MOMENT EQUAL TO 0;

```
%moment of inertia about body fixed refrence frame
i_1=0.5;
i_2=1;
i_3=1.5;
i_fixed=[i_1 0 0;
         0 i_2 0;
         0 0 i_3];
m=5;
% let the rotation is about z axis
u=[0 0 1];
theta=45*pi/180;
q_0=cos(theta/2);
q_1=sin(theta/2)*u(1);
q_2=sin(theta/2)*u(2);
q_3=sin(theta/2)*u(3);
```

```
rotation_matrix=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
                 2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
                 2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
i_abs=rotation_matrix*i_fixed*rotation_matrix.';
%defining intial conditions
x_0=0;
y_0=0;
z_0=0;
x_dot0=0;
y_dot0=0;
z_dot0=0;
w_x=1;
w_y=2;
w_z=3;
%Defining force and moments
F=[1;1;1];
M=[0;0;0];%for 2nd case and 3rd case it become zero
intial_conditions=[x_0,
y_0,z_0,q_0,q_1,q_2,q_3,x_dot0,y_dot0,z_dot0,w_x,w_y,w_z,0,0];
tolerance=odeset('RelTol',1e-14,'AbsTol',1e-14);
[t,z]=ode45(@(t,z)f(t,z,F,M),(0:0.01:10),intial_conditions,tolerance);
j=1;
Ang_mom=zeros;
RKE=zeros;
for i=(0:0.01:10)
    q_0=z(j,4);
    q_1=z(j,5);
    q_2=z(j,6);
    q_3=z(j,7);
    Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
       2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
       2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[i_1 0 0;
          0 i_2 0;
          0 0 i_3];
    I_abs=Q*I_fix*Q.';
    w=[z(j,11);z(j,12);z(j,13)];
    L_0=I_abs*w;
    Ang_mom(j,1)=L_0(1,1);
    Ang_mom(j,2)=L_0(2,1);
    Ang_mom(j,3)=L_0(3,1);
    RKE(j,1)=0.5*Ang_mom(j,:)*w;
    j=j+1;
end
%calculating energy and energy difference
Total_energy=0.5*m*(z(:,8).^2+z(:,9).^2+z(:,10).^2)+RKE-z(:,14)-z(:,15);
change_in_energy=max(Total_energy)-min(Total_energy);
L_x=Ang_mom(:,1);
L_y=Ang_mom(:,2);
L_z=Ang_mom(:,3);
change_in_ang_momx=max(L_x)-min(L_x);
change_in_ang_momy=max(L_y)-min(L_y);
change_in_ang_momz=max(L_z)-min(L_z);
```

```
%Angular velocity conservation;
change_in_wx=max(z(:,11))-min(z(:,11));
change_in_wy=max(z(:,12))-min(z(:,12));
change_in_wz=max(z(:,13))-min(z(:,13));
```

```
function zdot=f(~,z,F,M)
m=5;
i_1=0.5;
i_2=1;
i_3=1.5;
q_0=z(4,1);
q_1=z(5,1);
q_2=z(6,1);
q_3=z(7,1);
zdot(1,1)=z(8,1);
zdot(2,1)=z(9,1);
zdot(3,1)=z(10,1);
zdot(4,1)=-0.5*(q_1*z(11,1)+q_2*z(12,1)+q_3*z(13,1));
zdot(5,1)=0.5*(q_0*z(11,1)+q_3*z(12,1)-q_2*z(13,1));
zdot(6,1)=0.5*(q_1*z(13,1)+q_0*z(12,1)-q_3*z(11,1));
zdot(7,1)=0.5*(q_2*z(11,1)-q_1*z(12,1)+q_0*z(13,1));
mass=[m 0 0;
      0 m 0;
      0 0 m];
Linear_acceleration=mass\F;
zdot(8,1)=Linear_acceleration(1,1);
zdot(9,1)=Linear_acceleration(2,1);
zdot(10,1)=Linear_acceleration(3,1);
Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
   2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
   2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[i_1 0 0;
           0 i_2 0;
           0 0 i_3];
    I_abs=Q*I_fix*Q.';
    w=[z(11,1);z(12,1);z(13,1)];
    w_1=I_abs*w;
 Angular_acceleration=I_abs\(M-cross(w,w_1));
  zdot(11,1)=Angular_acceleration(1,1);
zdot(12,1)=Angular_acceleration(2,1);
zdot(13,1)=Angular_acceleration(3,1);
zdot(14,1)=F'*[z(8,1);z(9,1);z(10,1)];
zdot(15,1)=M'*w;
end
```

CHANGE IN ENERGY:

= 1.847e-13

Change in energy is very small so we can consider that energy is constant.

2)Simulate the rotation of a rigid body without moments, and verify when there is no moment, angular momentum is conserved.

```
%moment of inertia about body fixed refrence frame
i_1=0.5;
i_2=1;
i_3=1.5;
i_fixed=[i_1 0 0;
         0 i_2 0;
         0 0 i_3];
m=5;
% let the rotation is about z axis
u=[0 0 1];
theta=45*pi/180;
q_0=cos(theta/2);
q_1=sin(theta/2)*u(1);
q_2=sin(theta/2)*u(2);
q_3=sin(theta/2)*u(3);
rotation_matrix=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
                 2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0
;
                 2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-
2*q_1^2];
i_abs=rotation_matrix*i_fixed*rotation_matrix.';
%defining intial conditions
x_0=0;
y_0=0;
z_0=0;
x_dot0=0;
y_dot0=0;
z_dot0=0;
w_x=1;
w_y=2;
w_z=3;
%Defining force and moments
F=[1;1;1];
M=[0;0;0];%for 2nd case and 3rd case it become zero
```

```
intial_conditions=[x_0,
y_0,z_0,q_0,q_1,q_2,q_3,x_dot0,y_dot0,z_dot0,w_x,w_y,w_z,0,0];
tolerance=odeset('RelTol',1e-14,'AbsTol',1e-14);
[t,z]=ode45(@(t,z)f(t,z,F,M),(0:0.01:10),intial_conditions,tolerance);
j=1;
Ang_mom=zeros;
RKE=zeros;
for i=(0:0.01:10)
    q_0=z(j,4);
    q_1=z(j,5);
    q_2=z(j,6);
    q_3=z(j,7);
    Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
       2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
       2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[i_1 0 0;
          0 i_2 0;
          0 0 i_3];
    I_abs=Q*I_fix*Q.';
    w=[z(j,11);z(j,12);z(j,13)];
    L_0=I_abs*w;
    Ang_mom(j,1)=L_0(1,1);
    Ang_mom(j,2)=L_0(2,1);
    Ang_mom(j,3)=L_0(3,1);
    RKE(j,1)=0.5*Ang_mom(j,:)*w;
    j=j+1;
end
%calculating energy and energy difference
Total_energy=0.5*m*(z(:,8).^2+z(:,9).^2+z(:,10).^2)+RKE-z(:,14)-z(:,15);
change_in_energy=max(Total_energy)-min(Total_energy);
L_x=Ang_mom(:,1);
L_y=Ang_mom(:,2);
L_z=Ang_mom(:,3);
change_in_ang_momx=max(L_x)-min(L_x);
change_in_ang_momy=max(L_y)-min(L_y);
change_in_ang_momz=max(L_z)-min(L_z);
%Angular velocity conservation;
change_in_wx=max(z(:,11))-min(z(:,11));
change_in_wy=max(z(:,12))-min(z(:,12));
change_in_wz=max(z(:,13))-min(z(:,13));
```

```
function zdot=f(~,z,F,M)
m=5;
i_1=0.5;
i_2=1;
i_3=1.5;
q_0=z(4,1);
q_1=z(5,1);
q_2=z(6,1);
q_3=z(7,1);
zdot(1,1)=z(8,1);
zdot(2,1)=z(9,1);
```

```
zdot(3,1)=z(10,1);
zdot(4,1)=-0.5*(q_1*z(11,1)+q_2*z(12,1)+q_3*z(13,1));
zdot(5,1)=0.5*(q_0*z(11,1)+q_3*z(12,1)-q_2*z(13,1));
zdot(6,1)=0.5*(q_1*z(13,1)+q_0*z(12,1)-q_3*z(11,1));
zdot(7,1)=0.5*(q_2*z(11,1)-q_1*z(12,1)+q_0*z(13,1));
mass=[m 0 0;
      0 m 0;
      0 0 m];
Linear_acceleration=mass\F;
zdot(8,1)=Linear_acceleration(1,1);
zdot(9,1)=Linear_acceleration(2,1);
zdot(10,1)=Linear_acceleration(3,1);
Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
   2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
   2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[i_1 0 0;
           0 i_2 0;
           0 0 i_3];
    I_abs=Q*I_fix*Q.';
    w=[z(11,1);z(12,1);z(13,1)];
    w_1=I_abs*w;
 Angular_acceleration=I_abs\(M-cross(w,w_1));
  zdot(11,1)=Angular_acceleration(1,1);
zdot(12,1)=Angular_acceleration(2,1);
zdot(13,1)=Angular_acceleration(3,1);
zdot(14,1)=F'*[z(8,1);z(9,1);z(10,1)];
zdot(15,1)=M'*w;
end
```

Change in angular momentum:

In x direction:2.1532e-13

In y direction:5.1070e-14

In z direction:4.1744e-14

In this case the change in angular momentum is very small hence in the case of 0 moment angular momentum is constant.

3)Simulate the rotation of a rigid body when there is no moment and when the initial angular velocity is along one of the principal inertia directions, angular velocity is conserved.

```
%moment of inertia about body fixed refrence frame
```

```
i_1=0.5;
i_2=1;
i_3=1.5;
i_fixed=[i_1 0 0;
         0 i_2 0;
         0 0 i_3];
m=5;
% let the rotation is about z axis
u=[0 0 1];
theta=45*pi/180;
q_0=cos(theta/2);
q_1=sin(theta/2)*u(1);
q_2=sin(theta/2)*u(2);
q_3=sin(theta/2)*u(3);
rotation_matrix=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
                 2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0
;
                 2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-
2*q_1^2];
i_abs=rotation_matrix*i_fixed*rotation_matrix.';
%defining intial conditions
x_0=0;
y_0=0;
z_0=0;
x_dot0=0;
y_dot0=0;
z_dot0=0;
w_x=0;
w_y=0;
w_z=3;
%Defining force and moments
F=[1;1;1];
M=[0;0;0];%for 2nd case and 3rd case it become zero
intial_conditions=[x_0,
y_0,z_0,q_0,q_1,q_2,q_3,x_dot0,y_dot0,z_dot0,w_x,w_y,w_z,0,0];
```

```
tolerance=odeset('RelTol',1e-14,'AbsTol',1e-14);
[t,z]=ode45(@(t,z)f(t,z,F,M),(0:0.01:10),intial_conditions,tolerance);
j=1;
Ang_mom=zeros;
RKE=zeros;
for i=(0:0.01:10)
    q_0=z(j,4);
    q_1=z(j,5);
    q_2=z(j,6);
    q_3=z(j,7);
    Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
       2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
       2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[i_1 0 0;
          0 i_2 0;
          0 0 i_3];
    I_abs=Q*I_fix*Q.';
    w=[z(j,11);z(j,12);z(j,13)];
    L_0=I_abs*w;
    Ang_mom(j,1)=L_0(1,1);
    Ang_mom(j,2)=L_0(2,1);
    Ang_mom(j,3)=L_0(3,1);
    RKE(j,1)=0.5*Ang_mom(j,:)*w;
    j=j+1;
end
%calculating energy and energy difference
Total_energy=0.5*m*(z(:,8).^2+z(:,9).^2+z(:,10).^2)+RKE-z(:,14)-z(:,15);
change_in_energy=max(Total_energy)-min(Total_energy);
L_x=Ang_mom(:,1);
L_y=Ang_mom(:,2);
L_z=Ang_mom(:,3);
change_in_ang_momx=max(L_x)-min(L_x);
change_in_ang_momy=max(L_y)-min(L_y);
change_in_ang_momz=max(L_z)-min(L_z);
```

```
%Angular velocity conservation;
change_in_wx=max(z(:,11))-min(z(:,11));
change_in_wy=max(z(:,12))-min(z(:,12));
change_in_wz=max(z(:,13))-min(z(:,13));


function zdot=f(~,z,F,M)
m=5;
i_1=0.5;
i_2=1;
i_3=1.5;
q_0=z(4,1);
q_1=z(5,1);
q_2=z(6,1);
q_3=z(7,1);
zdot(1,1)=z(8,1);
zdot(2,1)=z(9,1);
zdot(3,1)=z(10,1);
zdot(4,1)=-0.5*(q_1*z(11,1)+q_2*z(12,1)+q_3*z(13,1));
zdot(5,1)=0.5*(q_0*z(11,1)+q_3*z(12,1)-q_2*z(13,1));
zdot(6,1)=0.5*(q_1*z(13,1)+q_0*z(12,1)-q_3*z(11,1));
zdot(7,1)=0.5*(q_2*z(11,1)-q_1*z(12,1)+q_0*z(13,1));
mass=[m 0 0;
      0 m 0;
      0 0 m];
Linear_acceleration=mass\F;
zdot(8,1)=Linear_acceleration(1,1);
zdot(9,1)=Linear_acceleration(2,1);
zdot(10,1)=Linear_acceleration(3,1);
Q=[1-2*q_2^2-2*q_3^2 2*q_1*q_2-2*q_3*q_0 2*q_1*q_3+2*q_2*q_0;
   2*q_1*q_2+2*q_3*q_0 1-2*q_3^2-2*q_1^2 2*q_3*q_2-2*q_1*q_0 ;
   2*q_1*q_3-2*q_2*q_0 2*q_3*q_2+2*q_1*q_0 1-2*q_2^2-2*q_1^2];
    I_fix=[i_1 0 0;
          0 i_2 0;
          0 0 i_3];
```

```
    I_abs=Q*I_fix*Q.';
    w=[z(11,1);z(12,1);z(13,1)];
    w_1=I_abs*w;
 Angular_acceleration=I_abs\(M-cross(w,w_1));
  zdot(11,1)=Angular_acceleration(1,1);
zdot(12,1)=Angular_acceleration(2,1);
zdot(13,1)=Angular_acceleration(3,1);
zdot(14,1)=F'*[z(8,1);z(9,1);z(10,1)];
zdot(15,1)=M'*w;
end
```

Change in angular velocity:

1)In x direction:0

2)In y direction:0

3)In z direction:0

In this case change in angular momentum is also absolute 0 in all 3 directions.