

# Internal Migration and Productivity Growth in China

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PRELIMINARY AND INCOMPLETE

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## **Abstract**

Between 1998 and 2007, China's economy underwent substantial growth in aggregate productivity. This paper investigates how labor market frictions influenced the nation's total factor productivity (TFP) growth. By developing a model incorporating Schumpeterian growth theory and internal migration dynamics, the paper quantifies the impact of China's migration costs. These costs were initially high in 2000 but declined thereafter. From 2000 to 2005 alone, reducing migration costs enabled over 20 million workers to relocate from western to eastern China. The calibrated model illustrates that this migration, facilitated by lower migration costs, accounted for 52% of the rise in China's TFP growth during this period.

# 1 Introduction

Between 1998 and 2007 (after joining the WTO but before the financial crisis), China’s economy experienced significant growth in aggregate productivity. [Brandt et al. \(2012\)](#) estimates that in 1998-2002, the average annual firm-level total factor productivity (TFP) growth rate in China was 6.0%; in 2002-2007, this rate increased to 12.7%. Such an acceleration in growth rate has provoked extensive discussion. One potential explanation is China’s WTO accession, which reduced trade uncertainty and encouraged Chinese firms to invest in R&D, boosting productivity growth ([Handley and Limão \(2017\)](#)). Another is the mitigation of capital and labor misallocation ([Hsieh and Klenow \(2009\)](#)). In this paper, I consider the faster growth from a new perspective: dramatic internal economic integration in the labor market. From 2000 to 2005, worker flows across Chinese regions represented the largest migration in human history, with over 20 million migrating, predominantly from western to eastern China. Migration policy reforms in 2003 are considered the main driver of this migration wave ([Poncet \(2006\)](#); [Cai et al. \(2008\)](#)), decreasing migration costs by 40% ([Tombe and Zhu \(2019\)](#)). Before this, China had substantial policy-induced migration costs. Could this migration wave explain China’s rapid growth? Using a Schumpeterian growth model, I analyze the migration’s effects to address this question. The calibrated model finds that migration driven by reduced migration costs can explain 52% of the increase in China’s productivity growth rate, echoing [Tombe and Zhu \(2019\)](#)’s conclusion that internal migration importantly contributed to China’s productivity growth.

China’s innovation geography motivates this analysis. Figure [1a](#) delineates China’s east and west regions. Per the National Bureau of Statistics, the red area depicts the eastern re-

gion along China's eastern coast, while the blue denotes the western inland region. Despite occupying less land area and having a smaller population, the east accounted for 41.5% of total employment in 2002. Figure 1b displays the logarithmic number of invention patents in both regions, reflecting their innovation levels. The east leads substantially in innovation: in 2002, it generated 68.7% of China's total invention patents. In a sense, the east is more efficient at innovation – producing more patents with a smaller population <sup>1</sup>. This raises a key question: Could migration that reallocates population from the less innovative west to the more innovative east explain the increase in China's TFP growth rate? If the migration prompts greater innovation gains in the east versus losses in the west, it would accelerate overall growth. But will the east sustain its higher innovation levels with more population?

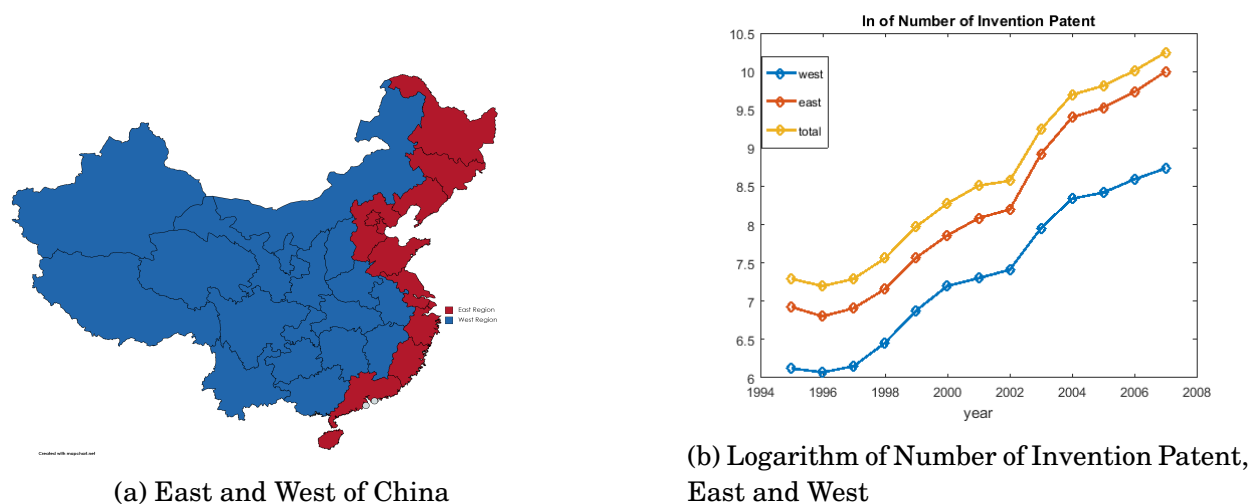


Figure 1: Invention Patent, China, 2002

To answer the question, I build a two-region framework featuring innovation-driven growth.

<sup>1</sup>Several factors could explain the east's higher innovation efficiency, including superior institutions, greater trade openness, and a more developed financial system. This paper does not examine the roots of differing innovation efficiencies but instead takes a reduced form approach.

I assume Ricardian trade between the two regions. In both regions, final-good firms produce output by combining labor and a set of intermediate goods, sourced from home and foreign producers. In each intermediate sector, a home and a foreign firm compete for global market shares and invest in R&D to improve the quality of their products. Following [Ahlfeldt et al. \(2015\)](#) and [Redding \(2016\)](#), I assume households can migrate across the regions, subject to some utility cost. Finite amenity in each region provides a force of dispersion. The model features an increasing return to scale in production which provides a force of agglomeration. The two forces shape the equilibrium. <sup>2</sup>

The open-economy dimension of the model defines firms' incentives to innovate. In a standard Schumpeterian model, two factors drive firms to do more innovation: *larger market size*, and *stronger competition*. With larger market size, firms make more profit, and hence the marginal return of R&D expenditure increases. Therefore firms invest more in R&D. For the competition effect, in each product line firms from both regions compete to serve the home and foreign market. Innovation generates a ranking of the product lines based on the quality difference between the home firm and the foreign firm. When the two firms are close to each other (hence a stronger competition), the leading firm will increase innovation efforts to escape from the competitor; in this case the laggard firm will also do more innovation because they know the quality gap is small and once surpass the leading firms they will win the market.

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<sup>2</sup>For simplicity, I assume households are homogeneous. I do not consider demographic heterogeneity such as working experience and education level in this preliminary model setting. In reality, people with higher education levels tend to have more contributions to innovation. Meanwhile, we can observe selection bias in the migration—majority of the migrants is young people. The assumption of homogeneity is certainly lack of generality. I will improve this in future.

This setting implies an increasing return to scale in production. As a region grows larger, firms in this region do more innovation because of market size effect. And hence in general those firms will have higher quality. With Ricardian trade, this means the region will produce more varieties. As I will show later, the increasing return to scale in production provides a force of agglomeration, together with reduction in migration costs, driving the migration wave. <sup>3</sup>

Meanwhile, this makes the effect of migration on innovation ambiguous. As people move into the east region, firms in the east region tend to do more innovation because of larger market size. But at the same time, firms in the west region will do less innovation because of smaller market size, which implies that firms in the east are facing less competition. Due to the competition effects, firms in the east will do less innovation. Hence with migration from the west region to the east, if the market size effect is stronger, the east region will do more innovation; if the competition effect is stronger, we will have an opposite result. Quantitative analysis will be done to figure out the effect of migration on innovation of both regions, and hence the effect on aggregate growth rate.

By assuming China was on a balanced growth path in 1998-2002, I calibrate the model to match key trade, population and growth data. The average annual growth rate is 6.0%. With the calibrated parameters, I find the east region is way more efficient in innovation. As people move from west to east, market size effect dominates and firms in the east do more innovation. This compensates the loss in the west, and hence the growth rate increases.

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<sup>3</sup>With iceberg cost, import is costly. Larger region produces more varieties, and hence people benefit from being in a large region.

I solve for the new BGP by decreasing the migration cost by 40%. The new growth rate is 9.48%. Assume China was on a balanced growth path in 2002-2007 (after the migration cost reduction policy), with average annual growth rate 12.7%. Thus the migration can explain 52% of the increase in the growth rate.

**Literature Review** This paper is closely related to two branches of literature. First is trade and endogenous growth. The endogenous technical change framework that I use as the backbone of our framework is a model of growth through step-by-step innovation as in [Aghion et al. \(2001\)](#), [Aghion et al. \(2005\)](#), [Acemoglu and Akcigit \(2012\)](#) and [Akcigit et al. \(2018\)](#), which analyze the strategic interaction among firms. This paper features endogenous comparative advantages as in [Grossman and Helpman \(1990\)](#) and [Eaton and Kortum \(2001\)](#). The main contribution is to consider economics scale and firms' innovation efforts, which provides a new perspective to consider effects of migration.

Second, this paper is related to literature linking trade flows with the spatial distribution of labor and economic activity, such as [Coşar and Fajgelbaum \(2016\)](#); [Dix-Carneiro and Kovak \(2017\)](#); [Allen and Arkolakis \(2014\)](#) and [Redding \(2016\)](#). For the internal migration in China, [Tombe and Zhu \(2019\)](#) investigates the effects of labor market integration, which mitigates misallocation, on aggregate productivity. The main contribution is to combine migration with endogenous growth, which provides an additional channel to quantify the aggregate productivity loss due to misallocation of labor.

The rest of the paper is organized as follows. Section 2 discusses migration policies in China around 2000 and how they subsequently changed. Section 3 introduces the theo-

retical framework. Section 4 is the numerical exercise to study the properties of the model. Section 5 shows the calibration results, and the counterfactual analysis. Section 6 concludes.

## 2 Background

This section discusses migration policies in China around 2000 and how they subsequently changed.

In 1958, the Chinese government formally instituted a household registration system to control population mobility. Each Chinese citizen is assigned a "hukou" (registration status), in a specific administrative unit that is at or lower than the county or city level. Individuals need approvals from local governments to change the location of hukou, and it is extremely difficult to obtain such approvals. Before the economic reform started in 1978, working outside one's hukou location was prohibited. This prohibition was relaxed in the 1980s but, prior to 2003, workers without local hukou still had to apply for a temporary residence permit. The permit was also difficult to get. Without such a permit, migrate workers have very limited access to local health and public services. Therefore, the migration cost was very high before 2003. As [Meng \(2012\)](#) estimates, moving across provinces is equivalent to shrinking one's income by a factor of ten.

The large regional income inequality is a strong evidence of the high migration cost. I use the province level real income in 2000 calculated by [Tombe and Zhu \(2019\)](#). In general, the provinces in the east region (coastal) had substantially higher levels of real GDP per worker than provinces in the west regions.



Figure 2: Real Income, China, 2000

Figure 2 shows the province level real income. The darker the color is, the richer the province is. On average, the real income of the east region is 2.3 times as high as that of the west region <sup>4</sup>.

The large income gap implies that workers cannot migrate due to high cost, though they have strong incentives to do so. By 2002, the total number of inter-provincial migrants is 26.5 million, the majority of which is those working in the army, state owned firms, and government.

There was a nationwide administrative reform in 2003 that greatly streamlined the process

<sup>4</sup>Xinjiang is an outsider. It has the highest real income among provinces in the west. Some literature attributes this to a high subsidy.



for getting a temporary residence permit in other provinces. These policy changes made it much easier for a worker to leave their hukou location and work somewhere else (hence lower migration costs). By 2005, the total number of inter-provincial migrants increased to 49 million, almost doubled comparing to that of 2002. The majority of the new migrants are those leaving hometown in the west to look for jobs in the east region ([Chan \(2013\)](#)).

As a summary, before the migration cost reduction policy, the migration cost was prohibitively high; after that, a large amount of labor resources were reallocated from the west region of China to the east.

### **3 Model**

I present a two-region framework featuring innovation-driven growth, based on Schumpeterian innovation model. I assume Ricardian trade between the two regions. In both regions, final-good firms produce output by combining labor and a set of intermediate goods, sourced from home and foreign producers. In each intermediate sector, a home and a foreign firm compete for global market shares and invest in R&D to improve the quality of their products. Households can migrate across the regions, subject to some utility cost. Finite amenity in each region provides a force of dispersion. The model features an increasing return to scale in production which provides a force of agglomeration. The two forces shape the equilibrium.

### 3.1 Household Preferences

Consider the following economy in continuous time. There are two regions in the economy, indexed by  $k \in \{A, B\}$ , representing the east and west region of China. Each region is endowed with  $\bar{H}_k$  unit of residential housing. There is a continuum of infinitesimally small households of unit mass in total. Households can move across regions, subject to some utility cost. Hence the measure of households in region  $k$ ,  $L_{kt}$ , will be determined endogenously.

Households are infinitely lived and maximize their discounted utility. At each moment  $t$ , after the realization of a idiosyncratic taste shock of living in each region, households decide which region to live in, consumption and housing. If households decide to migrate, they will bear a moving cost, which is a utility cost. We write the households' problem in the following recursive form:

$$V_t^H(k, \epsilon_{kt}, \epsilon_{k't}) = \max\{V_t^s(k) + \epsilon_{kt}, V_t^m(k, k') + \epsilon_{k't} - \psi_{kk't}\} \quad (1)$$

At time  $t$ , given the realization of the taste shock of living in region  $k$  and  $k'$  as  $\epsilon_{kt}$  and  $\epsilon_{k't}$ , a household in region  $k$  decides where to live to maximize the utility—to stay or to move. The moving cost from region  $k$  to region  $k'$  is  $\psi_{kk't}$ . If the household decides to stay, then in a small time interval  $\Delta t$ , the value function is:

$$V_t^s(k) = u(c_{kt}, h_{kt})\Delta t + (1 - \rho\Delta t)E_\epsilon[V_{t+\Delta t}^H(k, \epsilon_{kt+\Delta t}, \epsilon_{k't+\Delta t})] \quad (2)$$

If the household decides to move from region  $k$  to  $k'$ , the value function is:

$$V_t^m(k, k') = u(c_{kt}, h_{kt})\Delta t + (1 - \rho\Delta t)E_\epsilon[V_{t+\Delta t}^H(k', \epsilon_{kt+\Delta t}, \epsilon_{k't+\Delta t})] \quad (3)$$

We assume the taste shock,  $\epsilon_{kt}$ , is i.i.d. across households, regions, and time, and follows a

type I extreme value distribution <sup>5</sup>. This would give us a nice analytical expression of the number of households moving from one region to the other.

The period utility depends on consumption and housing with the following explicit form:

$$u(c_{kt}, h_{kt}) = \ln[(c_{kt})^\alpha (h_{kt})^{1-\alpha}] \quad (4)$$

where  $c_{kt}$  is consumption of final goods,  $h_{kt}$  is the unit of housing, and  $\alpha$  is the share of income spent on final goods. The budget constraint of a household in region  $k$  at time  $t$  is

$$P_{kt}c_{kt} + R_{kt}^H h_{kt} = w_{kt} + (r_{kt}A_{kt} - \dot{A}_{kt})/L_{kt} + T_{kt}/L_{kt} \quad (5)$$

LHS of Equation (5) is the expenditure, where  $P_{kt}$  is the price of final good in region  $k$ , and  $R_{kt}^H$  is the housing price.

RHS of Equation (5) is the income. The first term is labor income. We assume each household supplies one unit of labor inelastically, and  $w_{kt}$  is the wage. The second term is benefits from assets. Households in region  $k$  own all the firms in the region; therefore, the asset market clearing condition requires that the asset holdings have to be equal to the sum of firm values

$$A_{kt} = \int_0^1 V_{kjt} dj \quad (6)$$

The third term is a lump sum transfer from the government. We assume that the government in region  $k$  owns the residential housing. Households pay rent to the government; and the government rebates it to the households as a lump sum transfer.

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<sup>5</sup>This is incorrect in that we do not have i.i.d. or a type I extreme value distribution in continuous time. The idea is that, when solving for the continuous time model, we are actually discretizing time into small time intervals. In each unit time interval  $\Delta t$ , the taste shock is i.i.d. and follows a type I extreme value distribution.

Obviously, households in region  $k$  have same income. Hence each household consumes  $\frac{\bar{H}_k}{L_{kt}}$  unit of housing. It implies that though housing is free, it still provides a force of dispersion.

No borrowing and lending is allowed. Hence at each time  $t$ , trade between the two regions is balanced.

## 3.2 Production and Innovation

The trade between the two regions is of Ricardian setting. There are a unit continuum of intermediate goods, indexed by  $j \in [0, 1]$ . Each region has one potential producer of each good  $j$  with varying levels of quality<sup>6</sup>. Variety  $j$  from the two regions are perfectly substitutable. Intermediate good firms make investment to improve their qualities.

### 3.2.1 Final Good

The final good, which is to be used for consumption and R&D expenditure, and as an input in the intermediate good production, is produced in perfectly competitive markets in both regions according to the following technology:

$$Y_{kt} = \frac{1}{1-\beta} L_{kt}^\beta \left[ \int_0^1 (q_{Ajt}^{\frac{\beta}{1-\beta}} m_{Ajt} + q_{Bjt}^{\frac{\beta}{1-\beta}} m_{Bjt})^{1-\beta} dj \right] \quad (7)$$

The Cobb-Douglas technology combines labor,  $L_{kt}$  and a composition of intermediate goods to produce final goods. Here,  $q_{kjt}$  is the quality of good  $j$  sourced from region  $k$ ,  $m_{kjt}$  is the quantity sourced from region  $k$ , and  $\beta$  is the labor share. Assume that the elasticity of substitution of the intermediate goods is equal to  $\frac{1}{\beta}$ , which simplifies the calculation. The final good is non-tradable.

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<sup>6</sup>As in Ricardian trade model, if region  $k$  exports good  $j$  to region  $k'$ , then region  $k'$  does not produce good  $j$ .

Each variety can be sourced from both regions. Trade across regions is subject to iceberg cost. Therefore final good producers will always buy their inputs from the firm that offers a higher quality of the same variety (after adjusting for relative prices).

### 3.2.2 Intermediate Goods

As we mentioned in the beginning, there is a unit continuum of intermediate goods, indexed by  $j \in [0, 1]$ . Each region has one potential producer of each good  $j$ . The two firms—one from each region—compete for the production of variety  $j$  à la Bertrand. We will use  $j$  to denote the firm that produce it from now on.

All the intermediate good firms have a same technology:

$$m_{kjt} = \eta y_{kjt}, \quad \forall k \forall j \quad (8)$$

i.e. firms can use  $\eta$  units of local final good to produce one unit of output, which implies that firms in a region have same marginal costs.

On the other hand, firms are heterogeneous in terms of their quality,  $q_{kjt}$ . The quality  $q_{Ajt}$  improves through successive innovations by firm  $j$  in A, or spillovers from B. To clarify, the spillover is not a designed channel of growth <sup>7</sup>, as we will show more details later. Firms invest in R&D to do innovation with following cost function:

$$R_{kjt} = \alpha_k \frac{x_{kjt}^2}{2} q_{kjt}$$

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<sup>7</sup>For example, Konig et. al. (2020) shows that spillover is an important channel of growth in that laggard firms can imitate leading firms and have faster growth. Here we do not have such a channel. We assume that the quality gap between two firms is bounded. If a leading firm with largest possible gap makes successful innovation, we have to let the laggard firm also improve, or the quality gap would go beyond the boundary. Hence we actually have a passive spillover channel.

firm  $j$  in region  $k$  spends  $R_{kjt}$  unit of final goods to receive innovation at Poisson rate  $x_{kjt}$ . Also notice that the cost function is linear in quality  $q_{kjt}$ , which means innovation is more costly for more advanced firms.

When firm  $j$  in region  $k$  has a successful innovation in a small time interval  $\Delta t$ , its quality improves in a fashion of quality ladder:

$$q_{kj(t+\delta t)} = \lambda^{d_{kjt}} q_{kjt} \quad (9)$$

where  $\lambda$  is the step length of improvement, and  $d_{kjt} \in \mathbb{N}$  is a random variable with stands for the number of steps that firm  $j$  improves—we allow for multiple steps improvement. We will specify  $d_{kjt}$  later.

Let us denote by  $N_{kjt} = \int_0^t d_{kjs} ds$  the number of quality improves up to time  $t$ . Hence, the quality of a firm at time  $t$  is  $q_{kjt} = \lambda^{N_{kjt}}$ . Then the quality gap of firms producing variety  $j$  from region A and B,  $n_{Ajt}$ , can be defined as:

$$n_{Ajt} = \log_{\lambda} \left( \frac{q_{Ajt}}{q_{Bjt}} \right) = d_{Ajt} - d_{Bjt}$$

As we shall see,  $n_{kjt}$  is a sufficient statistic for firm values. We assume that there is a sufficiently large but exogenously given limit in the quality gap,  $\bar{N}$ , such that the gap between two firms is  $n_{kjt} \in \{-\bar{N}, \dots, 0, \dots, \bar{N}\}$  <sup>8</sup>.

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<sup>8</sup>Following Akcigit et al. (2021), we set  $\bar{N} = 17$ . As we will see later, the quality gap,  $n_{kjt}$  is the state variable of firms' value function. The assumption of upper bound guarantees that  $n_{kjt}$  is taken from a compact set, which supports Blackwell sufficient condition and makes the computation quite feasible.

### 3.2.3 Innovation and Step Size

Recall that upon innovation success, firm  $j$  in region  $k$  improves its quality by  $d_{kjt}$  steps, where  $d_{kjt} \in \mathbb{N}$  is a random variable. Here we specify the distribution of it.

For both regions, the number of steps of improvement upon innovation success has a probability mass distribution  $\mathbb{F}_n(\cdot)$ , where  $n$  is the current quality gap. Due to the upper bound of quality gap, the support of the distribution depends on firms' current quality position. For example, if a firm is  $\bar{N}$  steps laggard, upon success it may improve by  $\{1, 2, \dots, 2\bar{N}\}$  steps. And if a firm is one step leading, upon success it may improve by  $\{1, 2, \dots, \bar{N} - 1\}$  steps.

We follow the assumption of [Akcigit et al. \(2018\)](#) to set  $\mathbb{F}_n(\cdot)$ . For firms at position  $-\bar{N}$ , i.e. the most laggard ones, upon success they may improve by  $\{1, 2, \dots, 2\bar{N}\}$  steps. We assume  $\mathbb{F}_{-\bar{N}}(\cdot)$  has the following distribution:

$$\mathbb{F}_{-\bar{N}}(x) = c_0 x^{-\phi} \quad \forall x \in \{1, 2, \dots, 2\bar{N}\} \quad (10)$$

The shifter  $c_0$  ensures that the sum of mass adds up to one. Parameter  $\phi > 0$  determines the curvature: larger values of  $\phi$  means less likely of faster growth.

Based on this we can define  $\mathbb{F}_n(\cdot)$  for  $-\bar{N} < n < \bar{N}$ .

$$\mathbb{F}_n(x) = \begin{cases} \sum_1^{n+\bar{N}+1} \mathbb{F}_{-\bar{N}}(s) & \text{if } x = 1 \\ \mathbb{F}_{-\bar{N}}(x + n + \bar{N}) & \text{if } x \in \{2, 3, \dots, \bar{N} - n\} \end{cases} \quad (11)$$

This setting implies that more advanced firms are less likely to improve by multiple steps. Instead they are more likely to grow by one step. This mechanism provides a faster growth channel for laggard firms: comparing to more advanced competitors, they are growing faster.

As for  $n = \bar{N}$ , i.e. firms that lead the most, we assume that they growth by one step with probability 1 upon success.

Thus, during a small time interval  $\Delta t \rightarrow 0$ , the law of motion of the quality level of firm  $j$  from region A can be summarized as follows. For  $-\bar{N} < n \leq \bar{N}$ :

$$q_{Aj(t+\Delta t)} = \begin{cases} \lambda^x q_{Ajt} & \text{w. p. } x_{Ajt} \mathbb{F}_n(x) \Delta t & \text{if } x \in \{1, \dots, \bar{N} - n\} \\ q_{Ajt} & \text{w. p. } 1 - x_{Ajt} \Delta t \end{cases} \quad (12)$$

For  $n = -\bar{N}$ :

$$q_{Aj(t+\Delta t)} = \begin{cases} \lambda^x q_{Ajt} & \text{w. p. } x_{Ajt} \mathbb{F}_{-\bar{N}}(x) \Delta t & \text{if } x \in \{1, \dots, 2\bar{N}\} \\ q_{Ajt} & \text{w. p. } 1 - (x_{Ajt} + x_{Bjt}) \Delta t \\ \lambda q_{Ajt} & \text{w. p. } x_{Bjt} \Delta t \end{cases} \quad (13)$$

In this situation, we see the passive spillover mentioned before. For a most laggard firm, it may improve by one step because of its competitor's innovation success—we assume the quality gap is bounded.

### 3.3 Equilibrium

In this section, we solve for the Markov Perfect Equilibrium of the model. We first take the qualities as given, in order to solve for the static equilibrium. Then we set up value function of firms so as to find the innovation decision. Thus we can characterize the evolution of the economy.



### 3.3.1 Households

By solving the utility maximization problem of households, we can get the Euler Equation

$$r_{kt} = g_{kt} + \rho \quad (14)$$

where  $r_{kt}$  is the interest rate, and  $g_{kt}$  is the growth rate in region k.

Given consumption and housing in region k, and the moving cost, the assumption that the taste shock follows a type I extreme value distribution yields a analytical expression for the number of households migrating. The number of households moving from region A to B is:

$$L_{ABt} = \frac{(\exp[V^m(A,B) - \psi_{ABt}])^\kappa}{(\exp[V^m(A,B) - \psi_{ABt}]^\kappa + (\exp[V^s(A)])^\kappa} L_{At} \quad (15)$$

where  $\kappa$  is the scale parameter of the type I extreme value distribution. We can get a similar result for the number of households moving from region B to A.

### 3.3.2 Final Good Production

Using the production function [Equation \(7\)](#) of final good firms, and given price of final good  $P_{kt}$ , we can derive the demand for each variety j:

$$m_{kjt} = p_{kjt}^{-\frac{1}{\beta}} P_{kt}^{\frac{1}{\beta}} L_{kt} q_{kjt} \quad (16)$$

Here we first forget about where the final good firms in region k source variety j. Let  $p_{kjt}$  denote the price at which final good firms in k purchase variety j. The demand of variety j in region k,  $m_{kjt}$ , is linear in population in k,  $L_{kt}$ , and in the quality of variety j they purchase,  $q_{kjt}$ .

### 3.3.3 Intermediate Good Production

Now we consider the intermediate-good producers' problem. In our Ricardian trade setting, with iceberg costs, a intermediate good firm in  $k$  may export, i.e. serve both region  $k$  and  $k'$ , may only serve region  $k$ , and may shut down. Hence in order to solve intermediate good firms' problem, we need first figure out the Ricardian comparative advantages.

Consider firm  $j$  in region A. Recall that variety  $j$  from the two regions are perfectly substitutable. Hence if firm  $j$  from A exports to B, its quality adjust by price must be better than that of B:

$$\frac{q_{Ajt}^{\frac{\beta}{1-\beta}}}{\eta P_{At}(1 + \tau_{AB})} > \frac{q_{Bjt}^{\frac{\beta}{1-\beta}}}{\eta P_{Bt}} \quad (17)$$

Intermediate good firms can use  $\eta$  units of final goods to produce one unit of intermediate good, hence  $\eta P_{kt}$  in the denominator is the marginal cost. For firm  $j$  in A, since it is exporting, it is also subject to an iceberg cost,  $\tau_{AB}$ .

Similarly, if the following condition is satisfied, firm  $j$  in region A would only serve region A:

$$\frac{q_{Bjt}^{\frac{\beta}{1-\beta}}}{\eta P_{Bt}(1 + \tau_{BA})} \leq \frac{q_{Ajt}^{\frac{\beta}{1-\beta}}}{\eta P_{At}} \leq \frac{q_{Bjt}^{\frac{\beta}{1-\beta}}(1 + \tau_{AB})}{\eta P_{Bt}} \quad (18)$$

And is firm  $j$  in region shuts down:

$$\frac{q_{Ajt}^{\frac{\beta}{1-\beta}}}{\eta P_{At}} < \frac{q_{Bjt}^{\frac{\beta}{1-\beta}}}{\eta P_{Bt}(1 + \tau_{BA})} \quad (19)$$

Thus we can define two thresholds of quality gap for both regions to characterize the com-

parative advantage<sup>9</sup>:

$$n_{At}^E = \frac{1-\beta}{\beta} \log_{\lambda} \left[ \frac{P_{At}(1+\tau_{AB})}{P_{Bt}} \right]$$

$$n_{At}^I = \frac{1-\beta}{\beta} \log_{\lambda} \left[ \frac{P_{At}}{P_{Bt}(1+\tau_{BA})} \right]$$

That is, given the quality gap of firm  $j$  in region A,  $n_{Ajt} = \log_{\lambda} \frac{q_{Ajt}}{q_{Bjt}}$ , firm  $j$  will

- export if  $n_{Ajt} \geq n_{At}^E$
- only serve region A if  $n_{At}^I < n_{Ajt} < n_{At}^E$
- shut down is  $n_{Ajt} \leq n_{At}^I$

We can do exactly the same for firms in region B. And it's easy to show that:

$$n_{At}^E = -n_{Bt}^I, \quad n_{At}^I = -n_{Bt}^E$$

Next we solve for intermediate good firms' profit maximization problem. Before jumping into it, we first make an assumption about prices. The two firms—one from each region—that produce variety  $j$ , are competing à la Bertrand. In [Bernard et al. \(2003\)](#) the laggard firm would push the leading firm to set a limiting price, which is the marginal cost of the laggard firm. This is complicated. To keep everything simple, we follow the assumption of [Akcigit et al. \(2018\)](#):

**Assumption** *For each variety, firms can choose to enter a two-stage game or not. If enter, each firm pays an arbitrarily small fee  $\epsilon > 0$  in the first stage in order to bid prices in the second stage.*

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<sup>9</sup>the thresholds are same for all firms in A because they all have same marginal cost and trade cost

This assumption guarantee that only the firm with better quality adjusted by cost would bid, and hence firms can set optimal prices without any constraint of limiting prices.

For an exporting firm  $j$  in A, i.e.  $n_{Ajt} \geq n_{At}^E$ , the profit is:

$$\Pi(q_{Ajt}) = \max_{p_{Ajt}, p_{Bjt}} \{ (p_{Ajt} - \eta P_{At}) p_{Ajt}^{-\frac{1}{\beta}} P_{At}^{\frac{1}{\beta}} L_{At} q_{Ajt} + (p_{Bjt} - (1 + \tau_{AB}) \eta P_{At}) p_{Bjt}^{-\frac{1}{\beta}} P_{Bt}^{\frac{1}{\beta}} L_{Bt} q_{Ajt} \}$$

where  $p_{kjt}$  is the price for the two regions set by firm  $j$ . We can solve for the optimal quantity and price:

$$p_{Ajt} = \frac{\eta P_{At}}{1 - \beta}, \quad m_{Ajt} = \left( \frac{\eta}{1 - \beta} \right)^{-\frac{1}{\beta}} L_{At} q_{Ajt} \quad (20)$$

$$p_{Bjt} = \frac{\eta(1 + \tau_{AB}) P_{At}}{1 - \beta}, \quad m_{Bjt} = \left( \frac{\eta(1 + \tau_{AB}) P_{At}}{1 - \beta} \right)^{-\frac{1}{\beta}} P_{Bt}^{\frac{1}{\beta}} L_{Bt} q_{Ajt} \quad (21)$$

For firm  $j$  that only serves region A, we know

$$p_{Ajt} = \frac{\eta P_{At}}{1 - \beta}, \quad m_{Ajt} = \left( \frac{\eta}{1 - \beta} \right)^{-\frac{1}{\beta}} L_{At} q_{Ajt} \quad (22)$$

Define  $\mu_{Ant}$  as the set of firms in region a whose quality gaps equal to  $n$ . Then the aggregate export of region A is:

$$\begin{aligned} EX_{At} &= \sum_{n=n_{At}^E}^{\bar{N}} \sum_{j \in \mu_{Ant}} \left( \frac{\eta(1 + \tau_{AB}) P_{At}}{1 - \beta} \right)^{1 - \frac{1}{\beta}} P_{Bt}^{\frac{1}{\beta}} L_{Bt} q_{Ajt} \\ &= \left( \frac{\eta(1 + \tau_{AB}) P_{At}}{1 - \beta} \right)^{1 - \frac{1}{\beta}} P_{Bt}^{\frac{1}{\beta}} L_{Bt} \left( \sum_{n=n_{At}^E}^{\bar{N}} \int_{j \in \mu_{Ant}} q_{Ajt} dj \right) \end{aligned} \quad (23)$$

Define  $Q_{Ant}$  as aggregate quality of firms in region A whose quality gaps are  $n$ , i.e.  $Q_{Ant} =$

$\int_{j \in \mu_{Ant}} q_{Ajt} dj$ . then we can rewrite the aggregate export as:

$$EX_{At} = \sum_{n=n_{At}^E}^{\bar{N}} \left( \frac{\eta(1+\tau_{AB})P_{At}}{1-\beta} \right)^{1-\frac{1}{\beta}} P_{Bt}^{\frac{1}{\beta}} L_{Bt} Q_{Ant} \quad (24)$$

Using the equilibrium conditions derived previously, total output is given as

$$P_{At}Y_{At} = \sum_{n=-\bar{n}}^{n_{At}^I} \left( \frac{\eta(1+\tau_{BA})P_{Bt}}{(1-\beta)} \right)^{-\frac{1-\beta}{\beta}} \frac{P_{At}^{\frac{1}{\beta}} L_{At} Q_{B(-n)t}}{1-\beta} + \sum_{n=n_{At}^I+1}^{\bar{n}} \left( \frac{\eta}{(1-\beta)} \right)^{-\frac{1-\beta}{\beta}} \frac{P_{At} L_{At} Q_{Ant}}{1-\beta} \quad (25)$$

The RHS is total value of intermediate good divided by  $1-\beta$ —the production function of final good firms is Cobb-Douglas with intermediate good share equal to  $1-\beta$ . The first term is the value of imported goods, and the second term is the contribution of domestic goods.

Exactly the same analysis can be done for region B.

### 3.3.4 Innovation

I set up the value function for region A's firms. The state variables are quality gap,  $n$ , and quality,  $q$ . I show the value function for interior value of  $n$ , i.e.  $-\bar{N} < n < \bar{N}$ :

$$\begin{aligned} r_{At}V_{At}(n, q) - \dot{V}_{At}(n, q) = \max_{x_{Ant}} \bigg\{ & \Pi_{At}(n)q - \alpha_A \frac{x_{Ant}^2}{2} P_{At}q \\ & + x_{Ant} \sum_{z=1}^{\bar{N}-n} \mathbb{F}_n(z) [V_{At}(n+z, \lambda^z q) - V_{At}(n, q)] \\ & + x_{B(-n)t} \sum_{z=1}^{\bar{N}+n} \mathbb{F}_{-n}(z) [V_{At}(n-z, q) - V_{At}(n, q)] \bigg\} \quad (26) \end{aligned}$$

where  $\Pi_{At}(n)$  is defined as:

$$\Pi_{At}(n) = \begin{cases} \beta \left( \frac{\eta}{1-\beta} \right)^{1-\frac{1}{\beta}} P_{At} L_{At} + \beta \left( \frac{\eta(1+\tau_{AB})P_{At}}{1-\beta} \right)^{1-\frac{1}{\beta}} P_{Bt}^{\frac{1}{\beta}} L_{Bt} & n_{At}^E \leq n \leq \bar{N} \\ \beta \left( \frac{\eta}{1-\beta} \right)^{1-\frac{1}{\beta}} P_{At} L_{At} & n_{At}^I < n < n_{At}^E \\ 0 & -\bar{N} \leq n \leq n_{At}^I \end{cases} \quad (27)$$

Please see Appendix A.1 for the value function when  $n = -\bar{N}$  and when  $n = \bar{N}$ .

The first line on RHS is profit minus R&D expenditures. The second line denotes the expected gains from innovation. This expectation is over potential new positions. The exact position is determined probabilistically by the step size of innovation. The last line explains the changes as a result of innovation in the foreign country.

For the two state variables, the quality gap is defined in a compact set by assumption, while the quality is unbounded. To deal with this, one way is to assume a sufficient large upper bound for the quality, which would increase computational burden. However by observing the profit and R&D costs are both linear in quality, we can prove that the quality can be separated from the value function:

**Lemma 1** *The value functions are linear in quality such that  $V_{At}(n, q) = V_{At}(n)q$  for  $n \in \{-\bar{N}, \dots, \bar{N}\}$ .*

**Proof** Please see Appendix A.2

With the value functions defined, innovation decisions can be derived by F.O.C:

$$x_{Ant} = \begin{cases} \frac{V_{At}(n, \lambda q) - V_{At}(n, q)}{\alpha_A P_{At}} & n = \bar{N} \\ \frac{\sum_{z=1}^{\bar{N}-n} \mathbb{F}_n(z) [V_{At}(n+z, \lambda^z q) - V_{At}(n, q)]}{\alpha_A P_{At}} & -\bar{N} \leq n < \bar{N} \end{cases} \quad (28)$$

In a standard Schumpeterian model, firms do more innovation if they are facing larger market size, or if the competition is strong. As we can see, the innovation decision depends on the level of the value function, and also on the change in the value function if the quality

gap opens up. This can be understood in a Schumpeterian way. Larger market size means larger value of the value function. Stronger competition means the marginal benefit from one more step growth is larger.

### 3.3.5 Closing the Model

To close the model, final good market is cleared in both region at each moment  $t$ :

$$Y_{kt} = c_{kt}L_{kt} + R_{kt} + I_{kt} \quad (29)$$

Final good is used for consumption, innovation input and intermediate goods' production.

$R_{kt}$  is the aggregate innovation input:

$$R_{kt} = \sum_{n=-\bar{N}}^{\bar{N}} \alpha_k \frac{x_{knt}^2}{2} Q_{knt} \quad (30)$$

and  $I_{kt}$  is the aggregate intermediate good production input:

$$I_{kt} = \sum_{n=n_{kt}^I+1}^{\bar{N}} \eta \left( \frac{\eta}{1-\beta} \right)^{-\frac{1}{\beta}} L_{kt} Q_{knt} + \sum_{n=n_{kt}^E}^{\bar{N}} \eta \left( \frac{\eta(1+\tau_{kk'})P_{kt}}{1-\beta} \right)^{-\frac{1}{\beta}} P_{k't}^{\frac{1}{\beta}} L_{k't} Q_{knt} \quad (31)$$

This implies that trade between the two regions must be balanced at each moment  $t$ :

$$EX_{At} = EX_{Bt} \quad (32)$$

where

$$EX_{kt} = \sum_{n=n_{kt}^E}^{\bar{N}} \left( \frac{\eta(1+\tau_{kk'})P_{kt}}{1-\beta} \right)^{1-\frac{1}{\beta}} P_{k't}^{\frac{1}{\beta}} L_{k't} Q_{knt} \quad (33)$$

The law of motion of population in the two regions:

$$\dot{L}_{At} = \nu_{BA} L_{Bt} - \nu_{AB} L_{At} \quad (34)$$

and

$$\dot{L}_{Bt} = -\dot{L}_{At} \quad (35)$$

where  $v_{kk't}$  represents the share of population migrating from region  $k$  to  $k'$ :

$$v_{kk't} = \frac{(\exp[V^m(k, k') - \psi_{kk't}])^\kappa}{(\exp[V^m(k, k') - \psi_{kk't}])^\kappa + (\exp[V^s(k)])^\kappa} \quad (36)$$

The law of motion of quality, for  $-\bar{N} < n < \bar{N}$  will be:

$$\begin{aligned} \dot{Q}_{knt} = & \sum_{s=-\bar{N}}^{n-1} \mathbb{F}_s(n-s)x_{kst}\lambda^{n-s}Q_{kst} \\ & + \sum_{s=n+1}^{\bar{N}} \mathbb{F}_{-s}(-s+n)x_{k'(-s)t}Q_{kst} \\ & - [x_{knt} + x_{k'(-n)t}]Q_{knt} \end{aligned} \quad (37)$$

On the RHS, the first line is the quality that increases to gap  $n$  from lower position. The second line is the quality that decreases to gap  $n$  from a higher position. And the third line is the quality that does not change. For  $n = -\bar{N}$  and  $n = \bar{N}$  please see Appendix A.3.

With initial quality and population given, we can solve for the equilibrium, which is defined as following:

**Equilibrium** Let the economy consist of two regions  $k \in \{A, B\}$ . A Markov Perfect Equilibrium is an allocation:

$$\{r_{kt}, w_{kt}, P_{kt}, c_{kt}, h_{kt}, T_{kt}, L_{kt}, \{V_t^H, V_t^s, V_t^m\}, m_{kjt}, p_{kjt}, x_{kjt}, Y_{kt}, R_{kt}, I_{kt}, \{V_{kt}(n), Q_{knt}\}_{n=-\bar{N}}^{\bar{N}}\}_{k \in \{A, B\}, j \in [0, 1], t \in [0, \infty)}$$

such that

- (1) interest rates  $r_{kt}$  satisfies [Equation \(14\)](#)



- (2) *households' budget constraints are satisfied.*
- (3) *government's budget constraint satisfied.*
- (4)  $\{V_t^H, V_t^s, V_t^m\}$  *solve household's problem [Equation \(1\)](#)*
- (5) *the sequence of prices and quantities  $p_{kjt}$ ,  $m_{kjt}$  satisfy [Equation \(20\)](#), [Equation \(21\)](#), [Equation \(22\)](#) and maximize the operating profits of intermediate good firms.*
- (6) *the innovation decision  $x_{knt}$  satisfies [Equation \(28\)](#)*
- (7) *the total output  $Y_{kt}$  satisfies [Equation \(25\)](#)*
- (8) *final good market clearing condition [Equation \(37\)](#) is satisfied.*
- (9) *the prices of final good  $P_{kt}$  satisfy trade balance condition [Equation \(32\)](#)*
- (10) *value functions  $V_{kt}(n)$  solve the intermediate good firms' problem as [Equation \(26\)](#)*
- (11) *populations  $L_{kt}$  are consistent with [Equation \(34\)](#)*
- (12)  $\{Q_{knt}\}_{n=-\bar{N}}^{\bar{N}}$  *are consistent with innovation decisions as [Equation \(37\)](#)*

## Balanced Growth Path

In this project, we focus on the balanced growth path of the economy <sup>10</sup>. We can prove the existence and uniqueness of the balanced growth path:

**Proposition 1** *There exists a unique balanced growth path such that:*

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<sup>10</sup>To make the population fixed, we need the moving costs grow at a same rate as the value function of household. See Appendix A.4 for details.

1. *All aggregate variables grow at a same rate.*
2. *Interest rates are same in the two regions.*
3. *Prices are constant.*
4. *Populations are constant*

**Proof** Please see Appendix A.4

Thus we finish the model setting.

## 4 Numerical Exercise

In this section, we will do some numerical exercises to study the quantitative implications of our theoretical framework.

In our model setting, finite housing in each region provides a force of dispersion. We will show that with iceberg costs higher than some level, trade and endogenous growth provide a force of agglomeration. The force of agglomeration can dominate the force of dispersion.

We are also interested in the growth rates on the balanced growth paths. Because of market size effect and competition effect, the growth rate varies with sizes of the two regions.

### 4.1 Computational Strategy

Here we discuss the strategy for solving the balanced growth path. In a nutshell, I solve the equilibrium with two conditions: trade balance, and migration balance (on BGP the

population in each region is fixed). We first take population as given, and solve the relative price with trade balance condition. Then we find the population such that the migration is balanced.

First, solve the BGP without migration. That is, take the population in each region as given, and solve the BGP. The algorithm is as following:

1. Make an initial guess of  $\{P_B, r_A, r_B\}$ .
2. Solve intermediate good firms' problem defined as Equation (26), and compute the innovation decisions as Equation (28).
3. Take the innovation decisions as given and set  $Q_{knt} = 1, \forall k, \forall n$ . Iterate  $Q_{knt}$  as Equation (37) until growth rates stabilize. Denote the growth rates as  $\{g_A, g_B\}$ , and qualities as  $\{Q_{kn}\}_{n=-\tilde{N}}^{\tilde{N}}$ .
4. Update interest rate as  $r_k = \rho + g_k$
5. Check if the trade balance condition Equation (32) is satisfied. If so, we are done. If not, update relative price  $P_B$  by the trade balance condition and go back to step 2.

Thus with population given, we solve the BGP. Immediately, we can solve the value function of household's problem defined as Equation (1).

Second, we find the populations such that migration is balanced, i.e. migration from A to B should be equal to that from B to A. After the first step, we know the value function of household's problem with given population, and hence we can calculate the number of households moving from one region to the other by Equation (15) under each given pair of

populations. We just need to find the populations such that the migration is balanced. This is the population on BGP.

## **4.2 Parameters**

Parameters for the numerical exercises are listed in [Table 1](#):

Table 1: Parameters for numerical exercise

Description	Parameter	Value
Labor share	$\beta$	0.6
Consumption share	$\alpha$	0.8
Discount Factor	$\rho$	0.01
R&D cost scale	$\alpha_A, \alpha_B$	(2,2)
Innovation step length	$\lambda$	1.06
Upper bound of quality gap	$\bar{N}$	17
Marginal cost of intermediate good	$\eta$	1
Moving cost	$(\phi_{AB}, \phi_{BA})$	(1,1)
Elasticity of Migration	$\kappa$	2
Housing endowment ratio	$H_A/H_B$	1
Iceberg cost	$\tau$	1, 2, $\infty$

The two regions are symmetric in that parameters are same. We will try different values of iceberg cost: free trade, finite iceberg cost, and autarky.

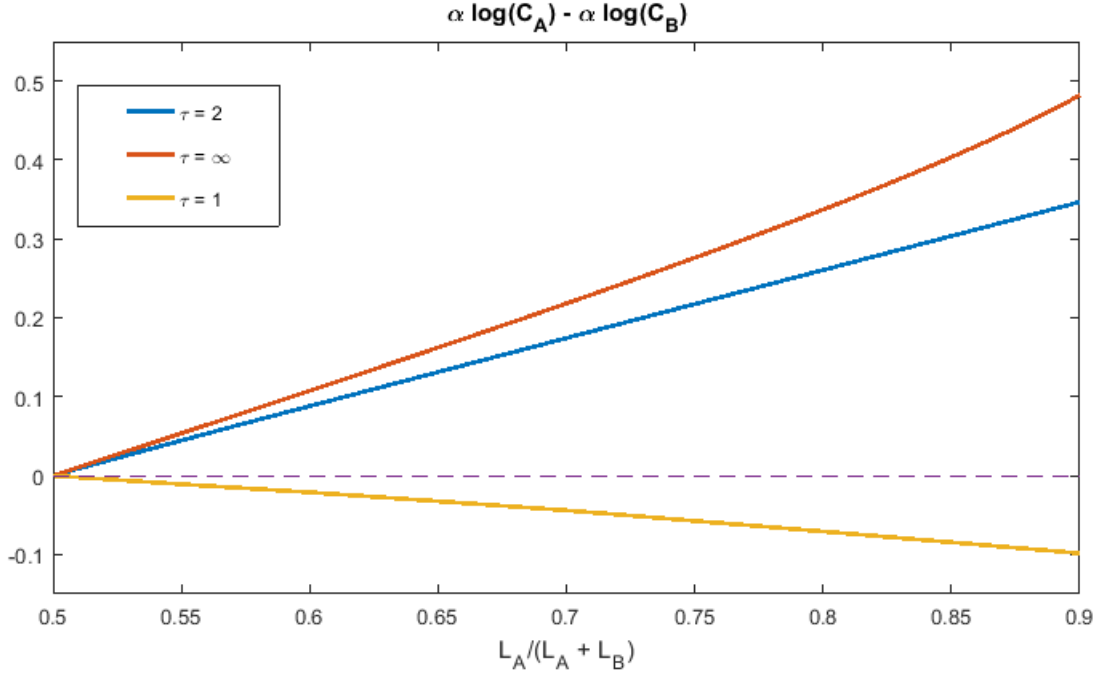
### 4.3 Trade and Agglomeration

Before the BGP, we first show that trade and endogenous growth can provide a force of agglomeration.

In Figure 3, we show the difference of period utility between the two regions with given population, i.e. assume no migration. The x-axis is the share of population in region A, which starts from 0.5 because we assume symmetry. As for the y-axis, recall that the period utility  $u(\cdot)$  depends on consumption and housing. Here to focus on the effect of trade and endogenous growth, we just consider the utility from consumption. Hence the y-axis is the utility difference just from consumption. On BGP, the value function of households, defined as Equation (1), is a function of the period utility  $u(\cdot)$ . Hence the difference in period utility

reflects the difference in value function of households.

Figure 3: Trade and Agglomeration



There are three curves in the figure: free trade, finite iceberg cost, and autarky. As we can see, there is a force of dispersion when trade is free. This is because when trade is free, households in both regions consume exactly a same set of goods with same price. As region A get larger, the relative price in region B will rise, and hence households' income in region B <sup>11</sup>. As a result when trade is free, households benefit from being in the smaller region.

But when there is some iceberg costs, things are different. In the cases where  $\tau = 2$  and autarky, there is a force of agglomeration. This is because when trade is costly, firms in the

<sup>11</sup>The smaller region will have a higher price level. The idea is as following: assume region B is smaller than region A. If they have a same price, when trade is free, firms in the two regions will be same, in that half varieties are produced in A, and the other half produced in B, and the aggregate qualities  $\{Q_{knt}\}_{n=-\bar{N}}^{\bar{N}}$  will be same. Then since region B is smaller, if price equals, region B will have a trade surplus. Hence to keep trade balanced, smaller region must have a higher price.

larger region tend to do more innovation than firms in the smaller region because of the market size effect. Hence in general firms in the larger region tend to have higher qualities, and hence under Ricardian trade setting, the larger region is producing more varieties. With iceberg cost, import will be costly. By living in a larger region, households can consume more local varieties, which is cheaper than imported goods. Hence, households in the larger region get better off. And this benefit from being in a larger region will be stronger with higher trade cost. Hence in the graph, the curve of autarky is above the curve with  $\tau = 2$

Trade may provide a force to agglomerate while housing for sure provides a force to disperse. Then we put things together to figure out the BGP.

## 4.4 Balanced Growth Path

We show the BGP with different trade costs.

### 4.4.1 Free Trade

In Figure 4, we show the BGP under free trade.

In Figure 4a, x-axis is the share of population in region A. Y-axis is the difference in the period utility. There are two curves. The solid blue curve is the difference of the period utility ( $U_A - U_B$ ) with given population, i.e. solve for the equilibrium with given population in each region, and no migration. It is decreasing with the population in region A because under free trade both housing and trade provide force of dispersion. The red dash line is the

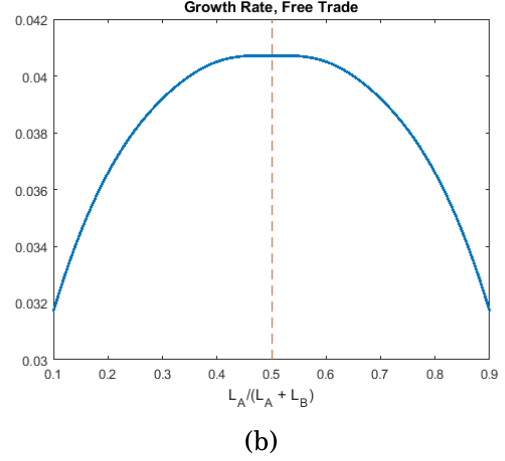
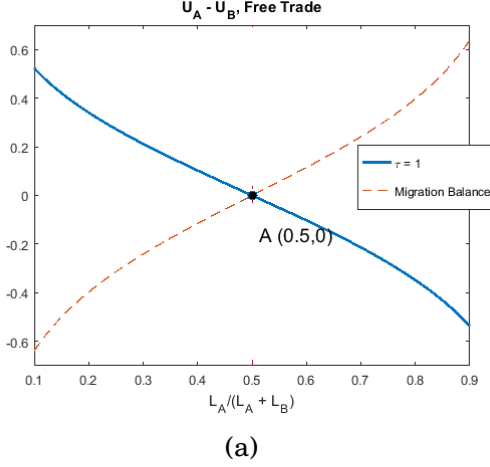


Figure 4: BGP under Free Trade

difference of period utility and population such that migration is balanced <sup>12</sup>. By Equation (34), the number of households moving across regions is a function of the difference in the value function of households in the two regions. On BGP, the value function of households is a function of the period utility. Therefore I use the difference in period utility as y-axis. The intersection is the equilibrium, where the two regions are equally populated with same period utility.

Figure 4b shows the growth rate with given population, i.e. if no migration. On BGP, the growth rate is 4.1%, where the two regions are equally populated. What's more, we can see that the growth rate is highest at this point. As households move into one region, i.e. unequally populated, the growth rate decreases.

We can understand the decreasing growth rate from a Schumpeterian view, i.e. firms do more innovation for larger market size and stronger competition effect. Here because of free trade, all firms are facing the whole population, and hence no market size effect. On

<sup>12</sup>It's easy to show that the value function of households is linear in the period utility on BGP. Hence given the period utility, we can calculate the number of households moving from one region to the other.



the other hand, as we mentioned, the smaller region has a higher price. As region A gets larger, price in region B gets higher, which means firms in region B are less competitive than before. Firms in region A are facing weaker competition, and hence do less innovation. As a larger region, region A has stronger impact on aggregate growth rate. Therefore, as population gets unequally distributed, the economy grows slower.

#### 4.4.2 Costly Trade

When trade is costly, things will be different.

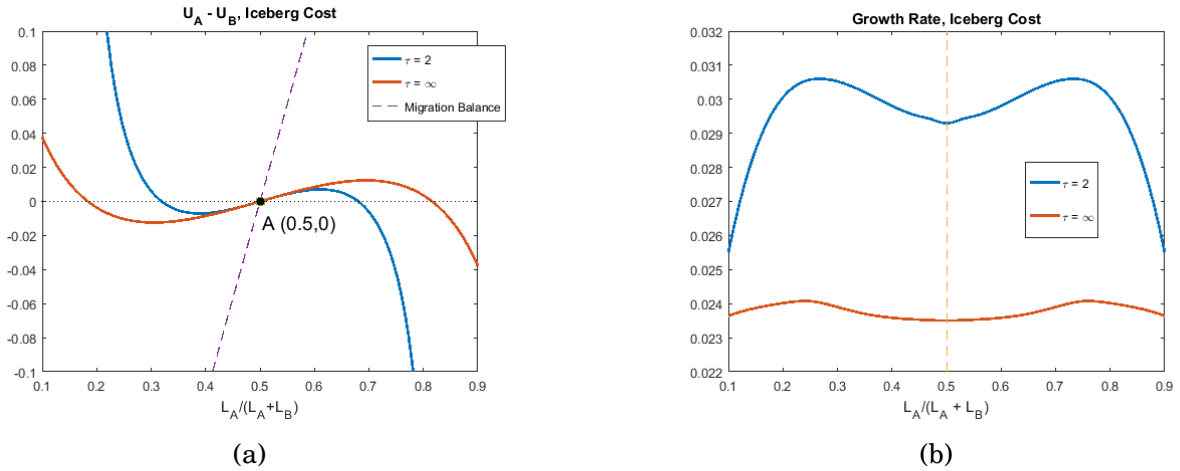


Figure 5: BGP under Costly Trade

In Figure 5a, as before the x-axis is the share of population in region A, and the y-axis is the period utility difference. There are three curves. The two solid curves are the difference of period utility with given population, one with finite iceberg cost, one autarky. In this case, trade and endogenous growth provide a force of agglomeration. Hence starting from equal population, as households move into region A, the utility difference first increases. This is because the force of agglomeration dominates the force of dispersion by housing. But as the

two region get too unequally populated, the force of dispersion will dominate again. The force of agglomeration is stronger with higher trade costs, therefore we see as region A get larger, the curve of autarky is above the curve of finite iceberg cost. And the dash line is the utility difference and population such that migration is balanced. The intersection is the equilibrium.

Figure 5b shows the growth rates of finite iceberg cost and autarky with given population.

We first look at the blue curve—growth rate with  $\tau = 2$ . Starting from equal population, as households move into region A, the growth rate first increases, and then decreases. It first increases because of market size effect. With iceberg cost, local market size matters<sup>13</sup>. As region A gets larger, firms in region A do more innovation. Firms in region B do less innovation because of smaller market size, and hence firms in A are facing weaker competition. But at this time, the market size effect dominates the competition, and hence we see the overall growth rate first increases. However, as too many households move into region A, the competition from firms in region B get too weaker. Then the competition effect dominates, and we see the growth rate decreases in the end.

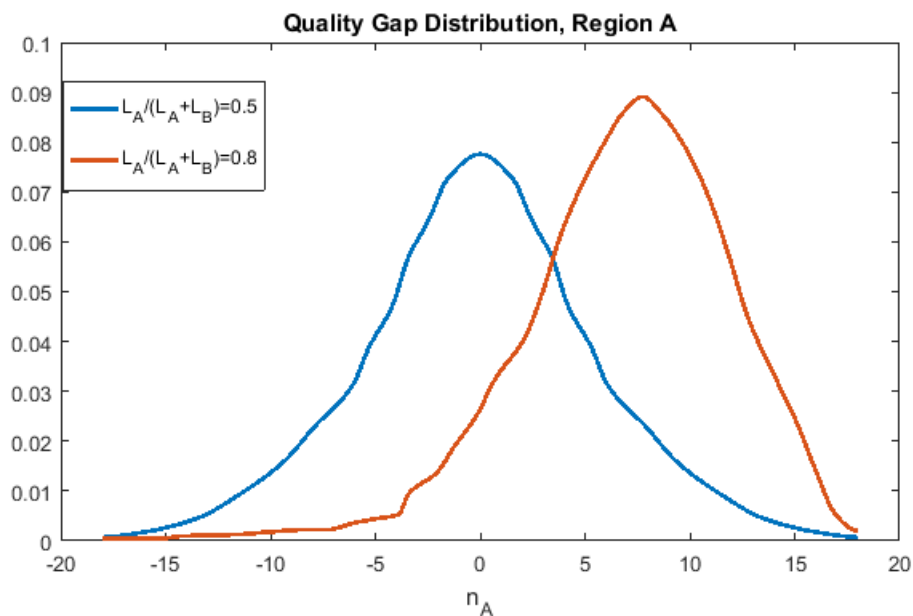
Besides the market size and competition, our model setting also provides a force to slow down the economy when population is unequally distributed. Recall the setting of innovation step size, Equation (11). With same innovation efforts, leading firms growth slower than laggard firms. And the more firms lead, the slower they grow. Here as households move into region A, because of market size effect, firms in region A do more innovation while firms

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<sup>13</sup>Similar to a home bias. With iceberg cost, firms in larger region make more profit.

in region B do less innovation. As a result firms in region A are generally leading firms. And as population gets more unequally distributed, firms in region A will lead more.

Figure 6: Quality Gap Distribution of Firms in Region A



We can see this from Figure 6. The x-axis is the quality gap, and the y-axis is the density. Blue curve is the distribution when the two regions are equally populated. But when the share of population in region A increases to 0.8, the distribution moves rightward to the red curve. Hence as region A get larger, firms in region A lead more. With same innovation efforts, they growth slower. This also explains why as region A gets too large, the growth rate decreases.

We can apply a very same theory to the case of autarky. In this case there's no competition. Hence the growth rate is lower than the case of finite cost uniformly. As region A gets larger, the rate first increases because of larger market size. But then decreases because firms in region A lead too much.

As a summary, in this model, trade and endogenous growth may provide a force of agglomeration while finite housing provides a force of dispersion. Growth rate depends the distribution of population.

## 5 Calibration and Counterfactual

In this section, by assuming China is on a BGP in period 1998-2002, we calibrate the model to match data. After that, we do counterfactual analysis by decreasing the migration cost.

### 5.1 Calibration

Assume China is on a BGP in 2002. The east and west region of China correspond to the two regions in the model. We try to keep the least amount of heterogeneity across regions. The two regions share symmetric technologies except the scale parameters of R&D cost. This leaves us with the following 12 structural parameters to be determined:

$$\{\beta, \alpha, \rho, \eta, \tau, \kappa, \lambda, \alpha_{east}, \alpha_{west}, \psi_{EW}, \psi_{WE}, \frac{H_{east}}{H_{west}}\}$$

#### 5.1.1 External Calibration

External calibration results are listed in Table 2:

We set time discount factor  $\rho = 0.01$ . Labor share of China is around 0.6. The average fraction of household spending on housing is about 20%, hence we set consumption share to 0.8. We estimate housing unit with aggregate residential area. Interestingly, we find

Table 2: External Calibration

Parameter	Value	Source
Labor share $\beta$	0.6	Data
Consumption share $\alpha$	0.8	Data
Housing endowment $H_{east}/H_{west}$	1.56	Data
Discount Factor $\rho$	0.01	Literature
Marginal cost $\eta$	1	Akcigit et al. (2021)
Iceberg cost $\tau$	2.67	Tombe and Zhu (2019)
Migration Elasticity $\kappa$	2	Tombe and Zhu (2019)

$H_{east}/H_{west} = 1.56$ , even though the west region is way larger than the east in terms of land area. Marginal cost of intermediate good firms is set to one. [Tombe and Zhu \(2019\)](#) estimates province level iceberg cost of China. Here the iceberg is the weighted average. For migration elasticity we also follows the result estimated by [Tombe and Zhu \(2019\)](#).

### 5.1.2 Internal Calibration

We have five remaining parameters to estimate:  $\{\lambda, \alpha_{east}, \alpha_{west}, \psi_{EW}, \psi_{WE}\}$ . Without loss of generality, we assume  $\psi_{EW} = 0$ , i.e. migration from the east region to west is frictionless. Given utility in the two regions, we just need calibrate  $\psi_{WE}$  to match the population data.

We have four targets to match. First is growth rate. We will match to results calculated by [Brandt et al. \(2012\)](#). Second is the population of the two regions. We will use number of employment to estimate. The other two are the R&D expenditure to GDP ratio of the two regions, which is collected from the first innovation census of China.

The targeted moments and the model performance in matching these moments are summarized in Table 3:

Table 3: Internal Calibration

Parameter	Estimate	
Research cost east, $\alpha_{east}$	0.518	
Research cost west, $\alpha_{west}$	14.5	
Step length of innovation, $\lambda$	1.06	
Moving cost, $\phi_{WE}$	2.64	
Target Moments	Data	Model
R&D/GDP, east	1.98%	1.98%
R&D/GDP, west	1.15%	1.15%
Growth Rate	6.0%	6.01%
Empl East/Total Empl	0.415	0.415

The model matches the data well. As we can see, the scale of research cost of east,  $\alpha_{east}$ , is much lower than that of west, implying that the east region is more efficient in innovation.

## 5.2 Counterfactual

We will next decrease the migration cost, and solve for the new BGP, so that we can find the answer to the question that whether the migration from west to east can explain the growth in productivity.

[Tombe and Zhu \(2019\)](#) estimates that on average the migration decreases by 40% after the migration cost reduction policy. We will follow this result. Considering the consistency of the model setting, we will decrease  $exp(\psi_{WE})$  by 40%.

Figure 7 shows the BGP before and after the policy. In Figure 7a, the blue curve is the difference of period utility given population. The red dash line is the migration balance condition before the policy, i.e. with high migration cost. Hence the intersection, point A,

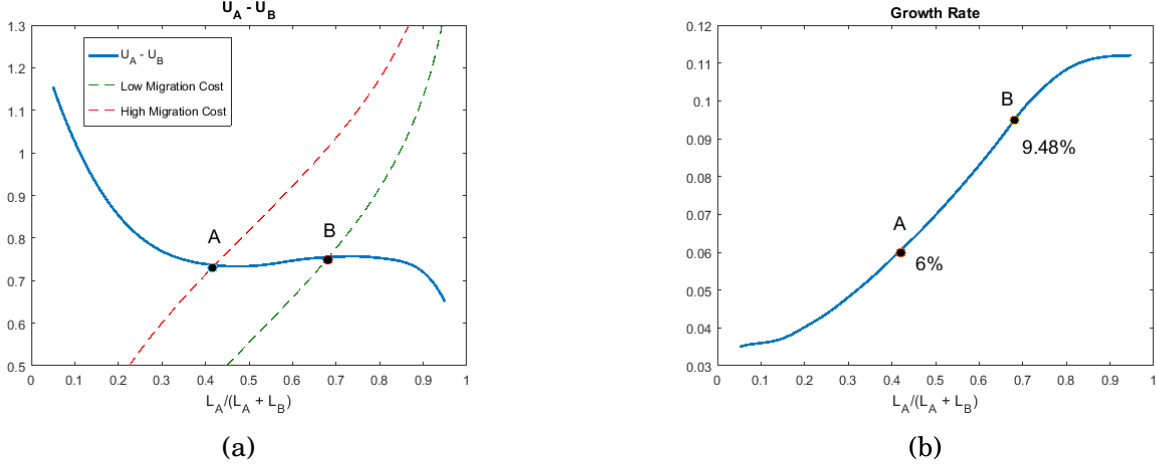


Figure 7: BGP Before and After the Cost Reduction Policy

is the equilibrium before the policy. Figure 7b is the growth rate with given population. At equilibrium A, the growth rate is 6%.

Then we decrease  $\exp(\psi_{WE})$  by 40%. The new migration balance condition in Figure 7a is the green dash line. Now the equilibrium after the policy is at point B. And from Figure 7b, the growth rate at point B is 9.48%.

Hence after the reallocation of labor from the west to east of China, the growth rate increases from 6% to 9.48%. This is because the east region is more efficient in innovation. After the migration, the larger market size makes firms in the east do more innovation. Competition from the west is weaker, but is dominated by the market size effect. Therefore in aggregate the growth rate increases. Furthermore, Brandt et al. (2012) estimates that in the period of 1998-2002, the average firm level TFP growth rate is 6.0%; In the period of 2002-2007, the average firm level TFP growth rate is 12.7%. We can see that the migration can explain 52% of the increase in TFP growth rate.

## 6 Conclusion

China experienced rapid growth between 1998 and 2007. Many attribute this to WTO accession or mitigation of misallocation. Internal policy reforms, which reduced migration costs, undertaken by the Chinese government during the same period have not received as much attention. Using a model featuring Schumpeterian growth and internal migration, I quantify the effect of changes in migration costs on China's aggregate productivity growth. Assume China was on BGP in the period of 1998-2002 with average annual growth rate 6.0%. With the calibrated model, by decreasing the migration costs by 40%, the growth rate is 9.48% on the new BGP. The average growth rate in 2002-2007 was 12.7%. Hence the reduction in migration costs accounts for 52% of the increase in productivity growth rate.

Next step will be to consider a richer setting. First in this version, I assume households are homogeneous. However, demographic heterogeneity matters when considering innovation and migration. In reality, people with higher education tend to have more contributions to innovation. Meanwhile, we can observe selection bias in the migration—majority of the migrants is young people. Second, I do not consider international trade here. Third, considering the increasing internal production chain in China, a Ricardian setting may not be able to capture related feature. I will try to accommodate these improvements in the future version.



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# Appendix

## Appendix A.1

When  $n = \bar{N}$ , the value function for firms in region  $k$  is:

$$\begin{aligned} r_{kt}V_{kt}(n, q) - \dot{V}_{kt}(n, q) = & \max_{x_{knt}} \left\{ \Pi_{kt}(n)q - \alpha_k \frac{x_{knt}^2}{2} P_{kt}q \right. \\ & + x_{knt} [V_{kt}(n, \lambda q) - V_{kt}(n, q)] \\ & \left. + x_{k'(-n)t} \sum_{z=1}^{\bar{N}+n} f(z|(-n)) [V_{kt}(n-z, q) - V_{kt}(n, q)] \right\} \end{aligned}$$

Different from a internal value of  $n$ , at the largest quality gap, firms cannot open up gaps through innovation. They can only improve their quality.

When  $n = -\bar{N}$ , the value function for firms in region  $k$  is:

$$\begin{aligned} r_{kt}V_{kt}(n, q) - \dot{V}_{kt}(n, q) = & \max_{x_{knt}} \left\{ \Pi_{kt}(n)q - \alpha_k \frac{x_{knt}^2}{2} P_{kt}q \right. \\ & + x_{knt} \sum_{z=1}^{\bar{N}-n} f(z|n) [V_{kt}(n+z, \lambda^z q) - V_{kt}(n, q)] \\ & \left. + x_{k'(-n)t} [V_{kt}(n, \lambda q) - V_{kt}(n, q)] \right\} \end{aligned}$$

Similarly, in this case competitors cannot open up gaps through innovation.

## Appendix A.2

**Lemma 1** *The value functions are linear in quality such that  $V_{At}(n, q) = V_{At}(n)q$  for  $n \in \{-\bar{N}, \dots, \bar{N}\}$ .*

**Proof:** We prove this lemma by guess and verify. Consider the value function for internal value of  $n$ . The boundary is just similar. Assume  $V_{At}(n, q) = V_{At}(n)q$ . Substitute this into [Equation \(26\)](#) and we have:

$$\begin{aligned} r_{At}V_{At}(n)q - \dot{V}_{At}(n)q = \max_{x_{Ant}} \bigg\{ & \Pi_{At}(n)q - \alpha_A \frac{x_{Ant}^2}{2} P_{At}q \\ & + x_{Ant} \sum_{z=1}^{\bar{N}-n} \mathbb{F}_n(z) [V_{At}(n+z)\lambda^z q - V_{At}(n)q] \\ & + x_{B(-n)t} \sum_{z=1}^{\bar{N}+n} \mathbb{F}_{-n}(z) [V_{At}(n-z)q - V_{At}(n)q] \bigg\} \end{aligned}$$

Cancel  $q$  on both sides and we can get the desired results.

## Appendix A.3

When  $n = \bar{N}$ , the law of motion of the aggregate quality is:

$$\begin{aligned} \dot{Q}_{knt} = & \sum_{s=-\bar{N}}^{n-1} f(n-s|s)x_{kst}\lambda^{n-s}Q_{kst} \\ & + x_{knt}(\lambda - 1)Q_{knt} - x_{k'(-n)t}Q_{knt} \end{aligned}$$

When  $n = -\bar{N}$ , the law of motion of the aggregate quality is:

$$\begin{aligned}\dot{Q}_{knt} = & \sum_{s=n+1}^{\bar{N}} f(-s+n|-s)x_{k'(-s)t}Q_{kst} \\ & - x_{knt}Q_{knt} + (\lambda - 1)x_{k'(-n)t}Q_{knt}\end{aligned}$$

## Appendix A.4

**Proof:** To prove the existence and uniqueness of the balanced growth path, first we observe that, if the aggregate quality,  $Q_{knt}$ , can be written as:

$$Q_{knt} = Q_{kn}e^{gt}, \quad \forall k, \forall n$$

that is,  $\forall k, \forall n$ ,  $Q_{knt}$  grows at a same rate. Then take population of the two regions as given, with the trade balance condition [Equation \(33\)](#), we can uniquely pin down the relative price, which is constant.

Then with [Equation \(25\)](#), we know the aggregate output,  $Y_k$  is also growing at rate  $g$ . Similarly we can show that other aggregate variables, consumption  $c_k$ , innovation cost  $R_k$  and production input  $I_k$  are all growing at rate  $g$ .

With [Equation \(14\)](#), we know the two regions have a same interest rate  $r = \rho + g$ .

For the population, note that the consumption  $c_k$  grows at rate  $g$ , hence the period utility  $u(\cdot) = a + bt$ , and so does the value function of the households. We further assume that the moving costs,  $\psi_{kk't} = c + dt$ . Then by [Equation \(34\)](#) we can solve for  $L_A$  and  $L_B$  such that the number of households moving from east to west is equal to that from west to east. Then

we have the population fixed.

Hence we can see that, to prove the existence and uniqueness of the BGP, it is sufficient to prove that  $\forall k, \forall n, Q_{knt}$  grows at a same rate.

We first show that for each region  $k$ ,  $Q_{knt}$  grows at a same rate for all  $n$ . To prove this, consider the law of motion of the aggregate quality,  $Q_{knt}$ , as [Equation \(37\)](#), and boundary case in Appendix A.3.  $Q_{knt}$  satisfies the differential equation:

$$\dot{Q}_{kt} = T_k Q_{kt}$$

where  $T$  is the transition matrix defined by the law of motion. The solution is of the form  $Q_{kt} = a_0 e^{g_k t}$ , where  $g_k$  is the eigenvalue of the matrix  $T$ . Since  $T$  may have multi eigenvalues, to be more specific, multi positive eigenvalues, we may have multi solution for  $Q_{kt}$ , and hence growth rate. However, recall that the number of positive eigenvalues equals to the number of positive pivots. By observing, the only positive pivot of  $T_k$  is the last pivot. Hence  $T_k$  only has one positive eigenvalue. And hence for each region  $k$ ,  $Q_{knt}$  grows at a same rate  $g_k$  for all  $n$ .

Next we need to prove  $g_k = g_{k'}$ . In this version, I do not have a nice proof. The idea is that, because of the fast catch-up mechanism, and the upper boundary of the quality gap, the growth rate of the two regions cannot be unequal. If the two regions grow at a different rate, the quality gap must go beyond the boundary.

Thus we preliminary prove that

$$Q_{knt} = Q_{kn}e^{gt}, \quad \forall k, \forall n$$

And hence we prove the existence and uniqueness of the BGP.