# CS231n Lecture 4


- BOAZ 10기 박성현
- BOAZ 11기 김태희
- BOAZ 11기 홍지민

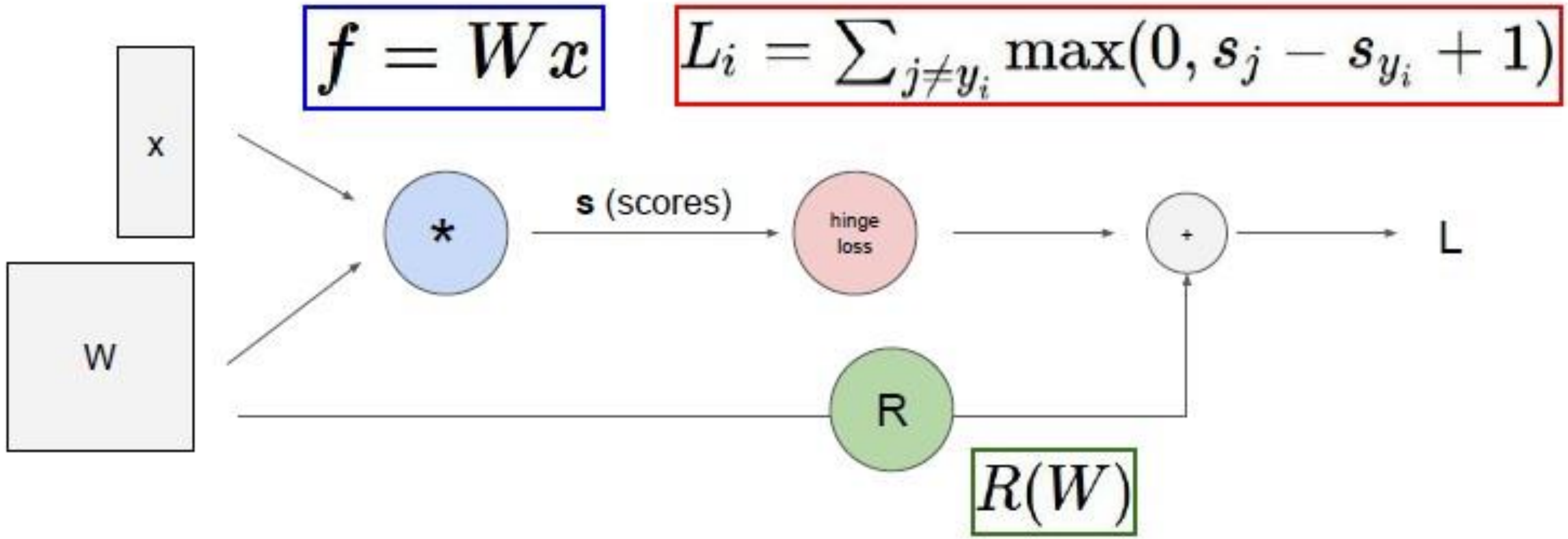
# Gradient Descent

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

**Numerical gradient**: slow :(, approximate :(, easy to write :)
**Analytic gradient**: fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient

# 02 Computational graphs



# Backpropagation example

Backpropagation: a simple example

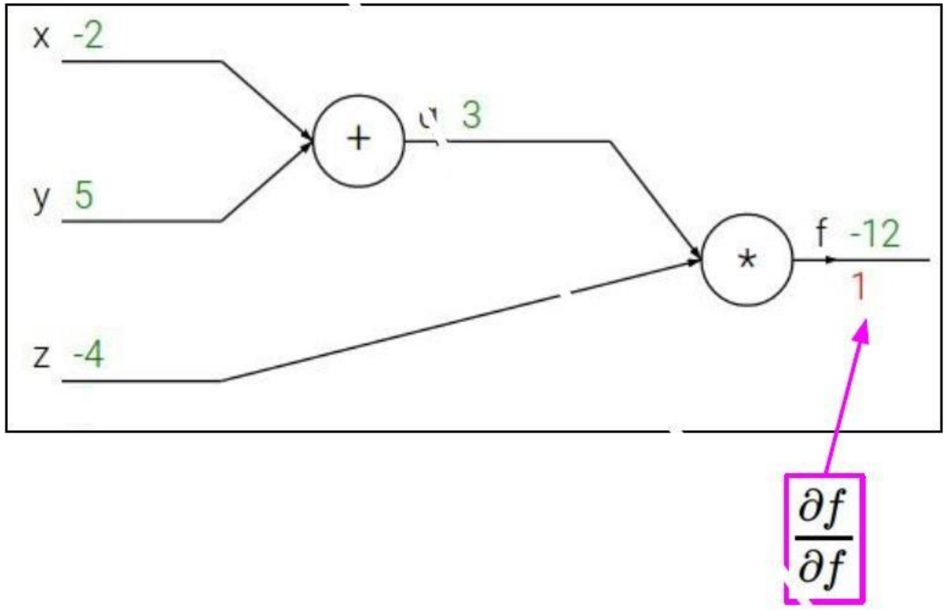
$$f(x,y,z) = (x+y)z$$

e.g. x = -2, y = 5, z = -4

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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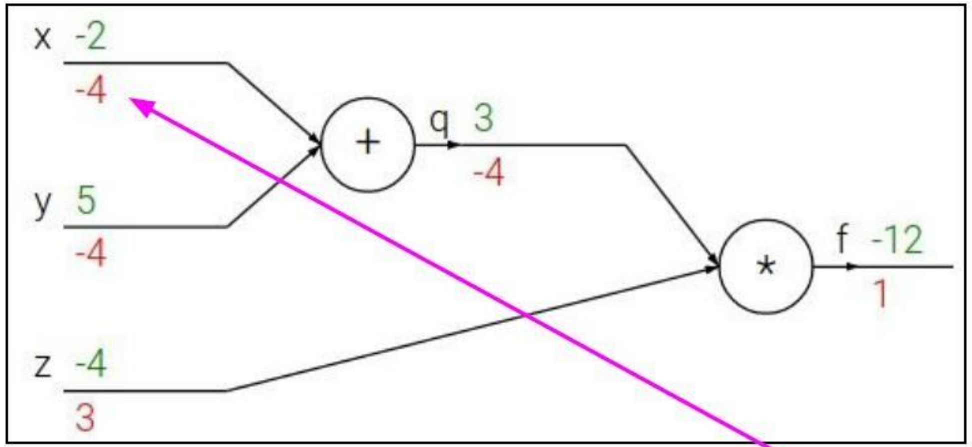



$$\boxed{\frac{\partial f}{\partial x}} = -4$$

Chain rule:

$$\boxed{\frac{\partial f}{\partial x} = \underbrace{\frac{\partial f}{\partial q}}_{= z} \underbrace{\frac{\partial q}{\partial x}}_{= 1}}$$

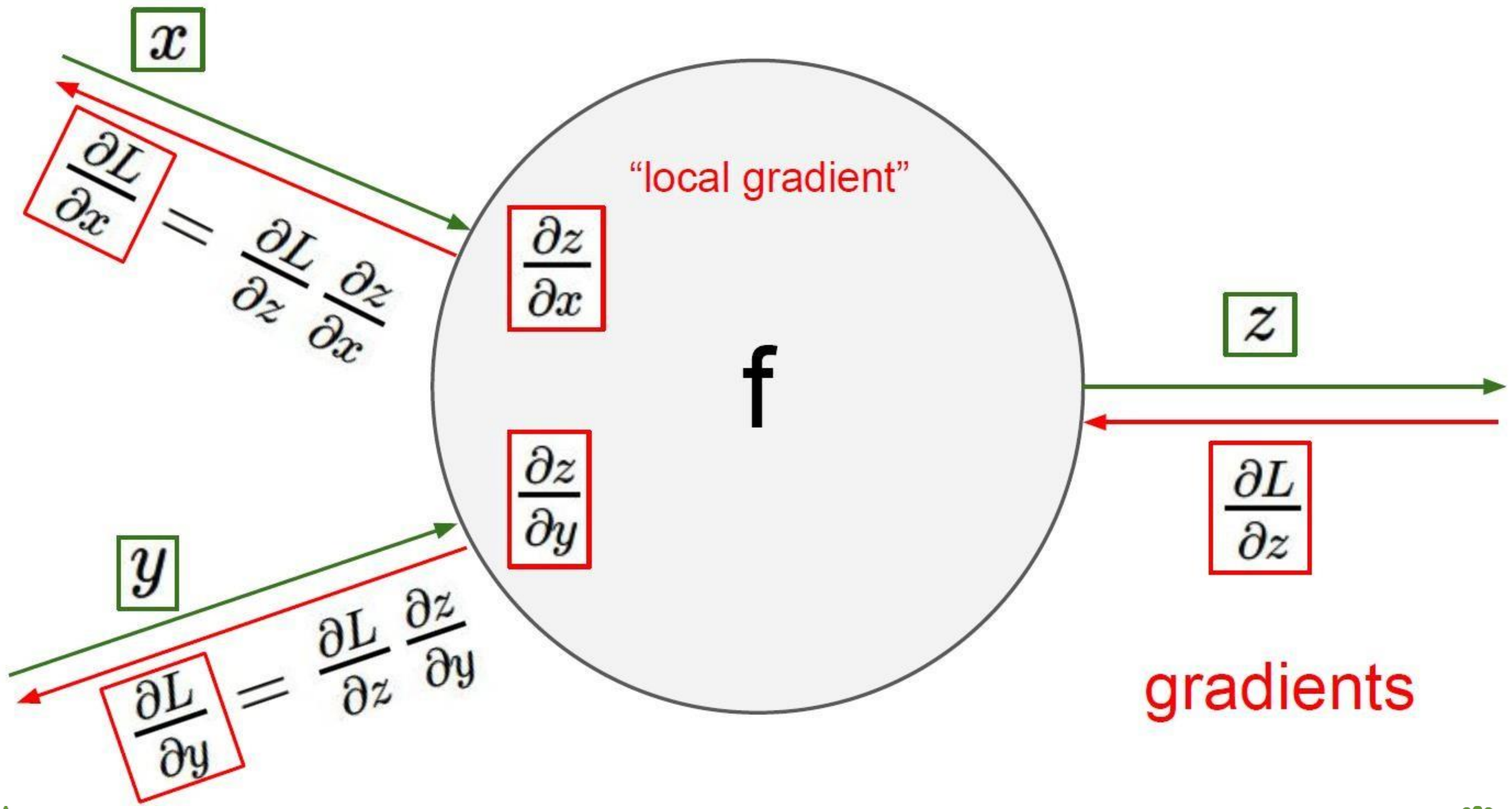
# Chain Rule

$$f'(x) = (\ g(h(x))\ )' = g'(\ h(x)\ )\ h'(x)$$

- keep the inside
- take derivative of outside

multiply by derivative of the inside

# 05 Local gradient



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# What is Jacobian Matrix?

The Jacobian matrix of the function $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^4$ with components

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= 5x_3 \\ y_3 &= 4x_2^2 - 2x_3 \\ y_4 &= x_3 \sin x_1 \end{aligned}$$

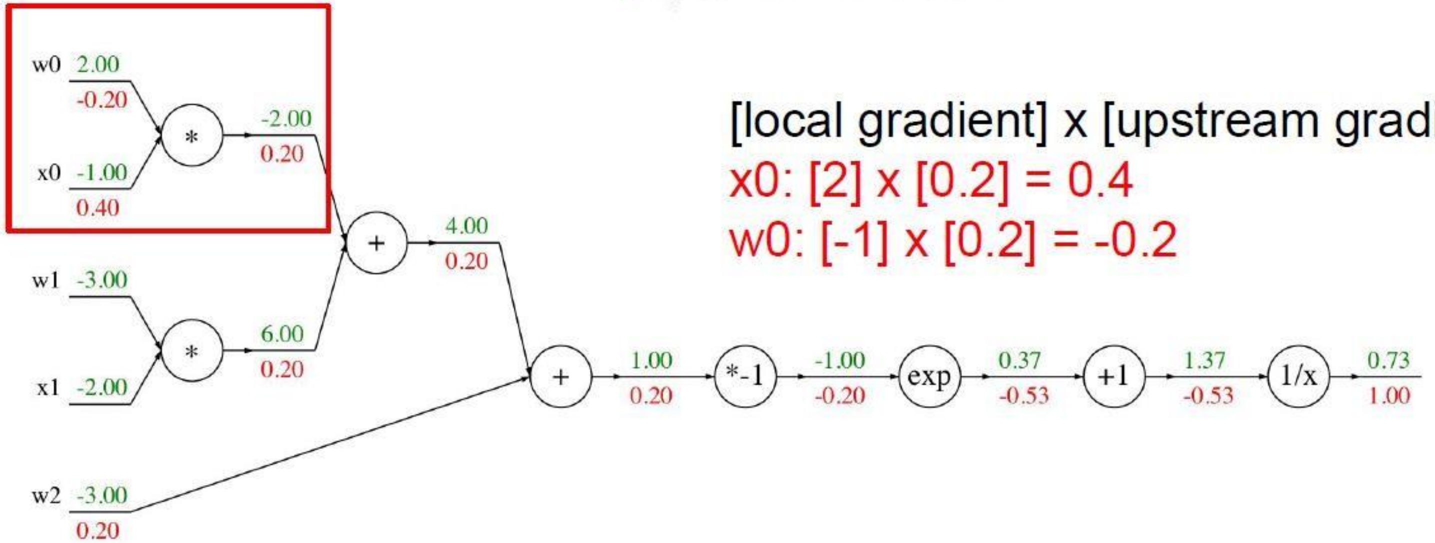
is

$$\mathbf{J_F}(x_1, x_2, x_3) = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \frac{\partial y_2}{\partial x_3} \\ \frac{\partial y_3}{\partial x_1} & \frac{\partial y_3}{\partial x_2} & \frac{\partial y_3}{\partial x_3} \\ \frac{\partial y_4}{\partial x_1} & \frac{\partial y_4}{\partial x_2} & \frac{\partial y_4}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 5 \\ 0 & 8x_2 & -2 \\ x_3 \cos x_1 & 0 & \sin x_1 \end{bmatrix}.$$

This example shows that the Jacobian need not be a square matrix.

# Another Backpropagation Example

## Another example: $f(w,x) = \dfrac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$




[local gradient] x [upstream gradient]

x0: [2] x [0.2] = 0.4

w0: [-1] x [0.2] = -0.2

| | |
|---|---|
| $f(x) = e^x \quad \rightarrow \quad \dfrac{df}{dx} = e^x$ | $f(x) = \dfrac{1}{x} \quad \rightarrow \quad \dfrac{df}{dx} = -1/x^2$ |
| $f_a(x) = ax \quad \rightarrow \quad \dfrac{df}{dx} = a$ | $f_c(x) = c + x \quad \rightarrow \quad \dfrac{df}{dx} = 1$ |

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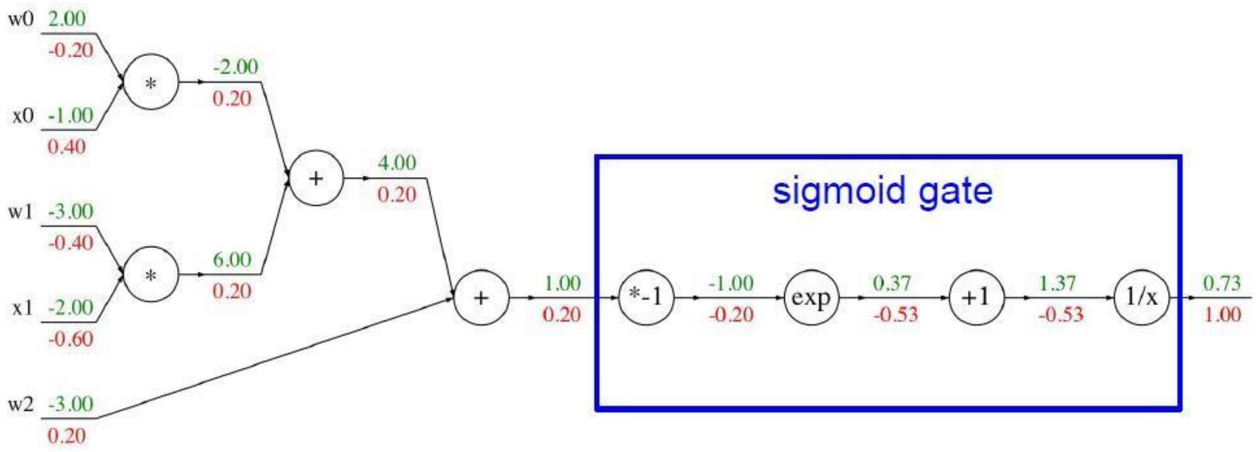
# Derivative of sigmoid function

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$

sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\sigma(x)$$

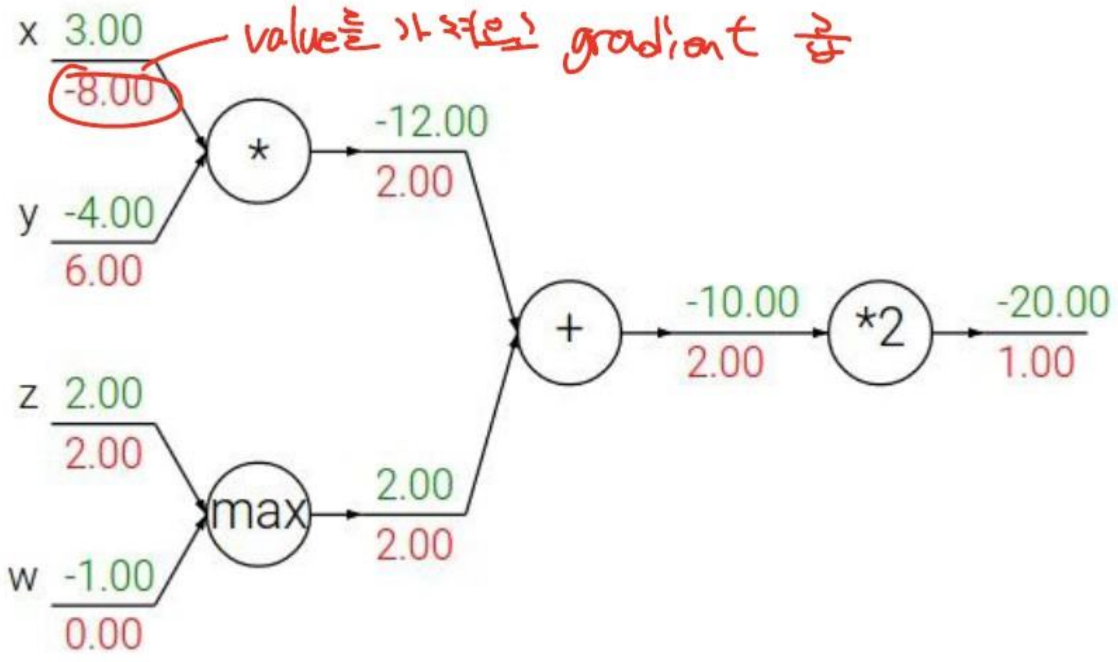


# Patterns in derivative of various gates

**add** gate: gradient distributor

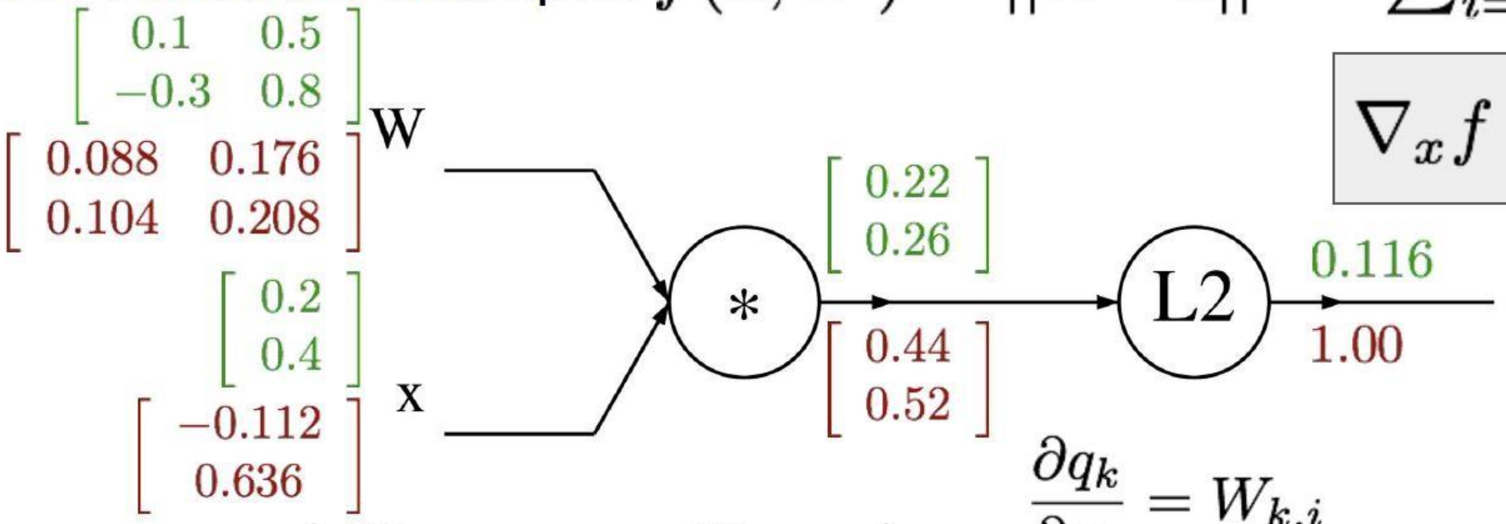
**max** gate: gradient router

**mul** gate: gradient switcher



# Backpropagation with Vector

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$




$$\nabla_x f = 2W^T \cdot q$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

$$\frac{\partial q_k}{\partial x_i} = W_{k,i}$$

$$\frac{\partial f}{\partial x_i} = \sum_k \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial x_i}$$

$$= \sum_k 2 q_k W_{k,i}$$

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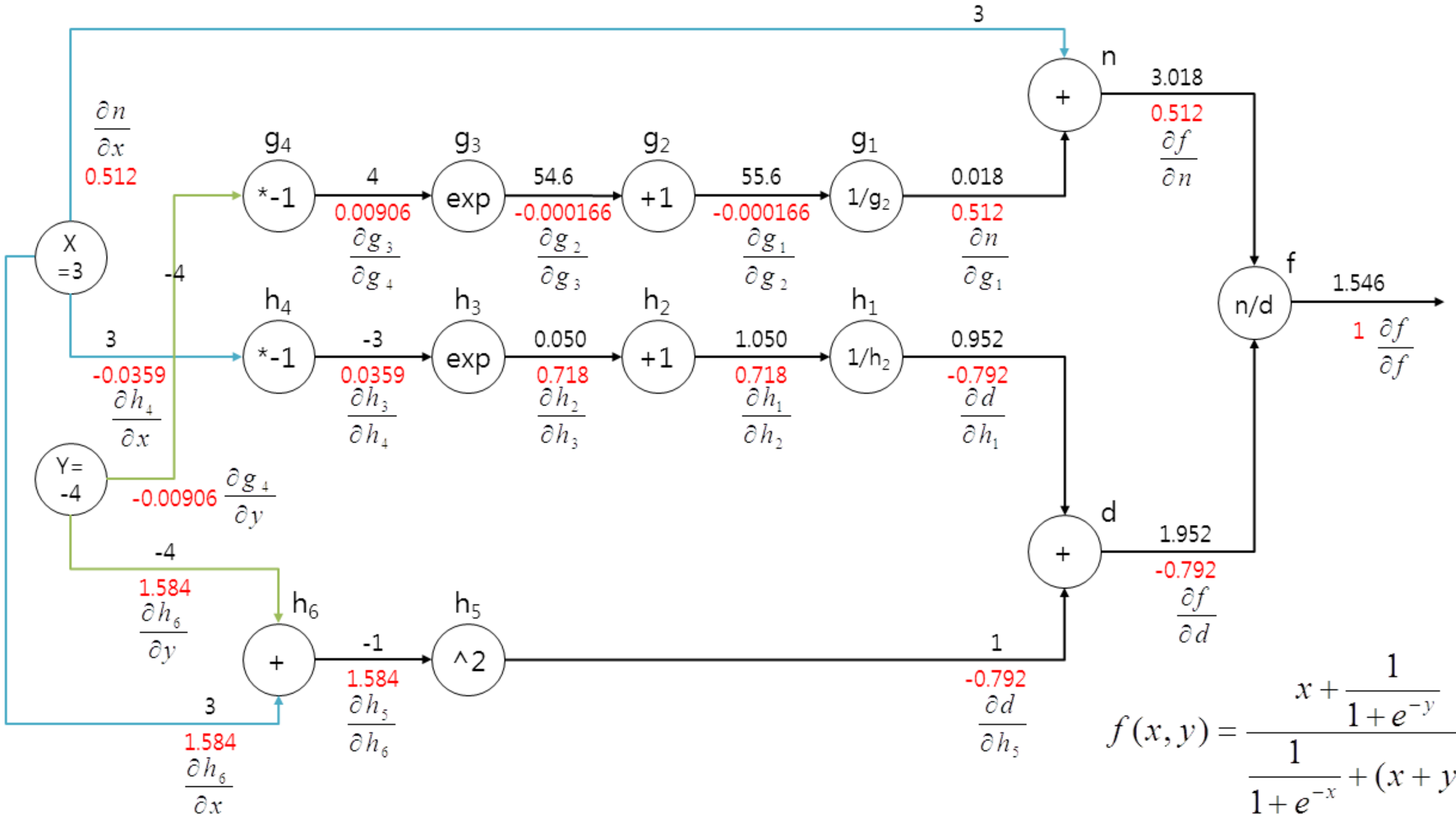
# Backpropagation with Vector




$$f(x, y) = \frac{x + \dfrac{1}{1+e^{-y}}}{\dfrac{1}{1+e^{-x}} + (x+y)^2}$$

End

# 참고 자료

CS231n : http://cs231n.stanford.edu/syllabus.html

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