

DP	Flow	Greedy	Approximation
100	100	100	100
200	200	200	200
300	300	300	300
400	400	400	400
500	500	500	500

Subject 1

100

What are the steps you use to solve a dynamic programming problem?

Question
Answer

Done!
Home

Describe recursive structure, define array, state subproblems in array, write iterative code, and if needed, retrace the code to find answer (ex path). Steps do not need to be exactly in this order, or exactly these, but this is the overall process.

(<https://mcs.utm.utoronto.ca/~zingarod/373/lecture4/lecture4.pdf>)

Write out the code for returning an array of the first n Fibonacci numbers.

```
def fib(n):  
    lst = [0,1]  
    for i in (2...n-1):  
        lst[i] = lst[i-1] + lst[i-1]  
    return lst
```

Robot Knapsack: Suppose a robot has capacity of I , and wants to maximise the value of its loot. It is located on a checkerboard. The robot can move one of three ways, and must pick up the loot from the position it moves to: - Up and to the right - Up and to the left - Directly upwards. State the basic recurrence and dimensions for the DP array that would solve this problem.

We create $m + 2 * n + 2$ array, DP:

$DP[i][j] = -\text{inf}/0/\text{etc}$ if $i = 0$, or if $j = 0$, or if $i=m+1$ or $j = n+1$ (fill in a "border" square around the checkerboard)

$DP[i][j] = \max DP[i+1][j+1], DP[i+1][j-1], DP[i][j] + v_{i,j}$ where $i \leq m$ and $j \leq n$

The Longest Increasing Subsequence (LIS) problem is to find the length of the longest subsequence of a given sequence such that all elements of the subsequence are sorted in increasing order.

Input : $\text{arr}[] = 3, 10, 2, 1, 20$

Output : Length of LIS = 3

The longest increasing subsequence is 3, 10, 20

Let $\text{arr}[0..n-1]$ be the input array and $L(i)$ be the length of the LIS ending at index i such that $\text{arr}[i]$ is the last element of the LIS :

$$L(i) = 1 + \max(L(j)) \text{ where } 0 \leq j < i \text{ and } \text{arr}[j] < \text{arr}[i]; \text{ or}$$
$$L(i) = 1, \text{ if no such } j \text{ exists.}$$

Woody the woodcutter will cut a given log of wood, at any place you choose, for a price equal to the length of the given log. Suppose you have a log of length L , marked to be cut in n different locations labeled $1, 2, \dots, n$. For simplicity, let indices 0 and $n + 1$ denote the left and right endpoints of the original log of length L . Let d_i denote the distance of mark i from the left end of the log, and assume that $0 = d_0 < d_1 < d_2 < \dots < d_n < d_{n+1} = L$. The wood-cutting problem is the problem of determining the sequence of cuts to the log that will cut the log at all the marked places and minimize your total payment. Give an efficient algorithm to solve this problem.

$c(i, j) = \min_{i < k < j} c(i, k) + c(k, j) + (d_j d_i)$ where $c(i, j)$ is the min cost of cutting a log with left endpoint i and right endpoint j at all its marked locations. Start with $c(i, i + 1)$ (consecutive cuts) and move outwards to $c(i, i + 2), c(i, i + 3)$ until the maximal distance $c(1, n)$, which gives the optimal score for the whole wood. Remember pointers to k that gave max score at each step, and trace back pointers to construct optimal solution. Each iteration takes $O(n)$ (linear search between i and j) and there are $O(n^2)$ entries to fill. (a triangle, not really a square). from <https://courses.csail.mit.edu/6.006/fall11/rec/rec24.pdf>

Subject 1

100

Define what the cut, minimum cut, capacity, flow, and max flow are in any max flow problem.

Question
Answer

Done!
Home

Subject 1

100

Question
Answer

Done!
Home

What is the method that we used to determine what the max possible flow is? In 3/4 short steps, list out what this method does.

Subject 2

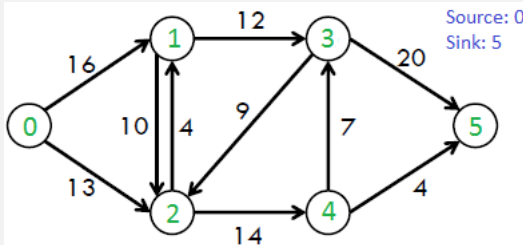
200

What is the method that we used to determine what the max possible flow is? In 3/4 short steps, list out what this method does.

Question
Answer

Done!
Home

What is the max possible flow on this network?



Question
Answer

Done!
Home

Subject 3

300

Question
Answer

Done!
Home

Consider a set of mobile computing clients in a certain town who each need to be connected to one of several possible base stations. Well suppose there are n clients, with the position of each client specified by its (x, y) coordinates in the plane. There are also m base stations, each of whose position is specified by (x, y) coordinates as well.

We wish to connect each client to exactly one base station. Our choice of connections is constrained in the following ways: There is a range parameter r . A client can only be connected to a base station that is within distance r of that client. There is also a load parameter L . For any given base station, no more than L clients can be connected to it.

How would you represent this problem as a flow network?

Let $\text{arr}[0..n-1]$ be the input array and $L(i)$ be the length of the LIS ending at index i such that $\text{arr}[i]$ is the last element of the LIS :

$$L(i) = 1 + \max(L(j)) \text{ where } 0 \leq j < i \text{ and } \text{arr}[j] < \text{arr}[i]; \text{ or}$$
$$L(i) = 1, \text{ if no such } j \text{ exists.}$$

Outline an algorithm for finding bottleneck edges (where increasing the capacity of this edge would increase the max) given the residual network.

$c(i, j) = \min_{i < k < j} c(i, k) + c(k, j) + (d_j d_i)$ where $c(i, j)$ is the min cost of cutting a log with left endpoint i and right endpoint j at all its marked locations. Start with $c(i, i + 1)$ (consecutive cuts) and move outwards to $c(i, i + 2), c(i, i + 3)$ until the maximal distance $c(1, n)$, which gives the optimal score for the whole wood. Remember pointers to k that gave max score at each step, and trace back pointers to construct optimal solution. Each iteration takes $O(n)$ (linear search between i and j) and there are $O(n^2)$ entries to fill. (a triangle, not really a square). from <https://courses.csail.mit.edu/6.006/fall11/rec/rec24.pdf>

Subject 1

100

What does Dijkstras algorithm find? Describe the algorithm.

Question
Answer

Done!
Home

Subject 1

100

Question
Answer

Done!
Home

How can you find MST using max heap?

Question
Answer

Done!
Home

By deleting all safe edges until MST is found.

Not just any greedy approach to the activity-selection problem produces a maximum-size set of mutually compatible activities. Give an example to show that the approach of selecting the activity of least duration from among those that are compatible with previously selected activities does not work.

Subject 3

300

Question
Answer

Done!
Home

Graph Coloring Problem. Create a greedy algorithm. Is it optimal? What are its bounds (num of colors)?

No. Max vertex degree $+1$.

If negative edges are only adjacent to the source, can Dijkstra's algorithm produce a correct shortest path tree?

Subject 5

500

Yes.

**Question
Answer**

**Done!
Home**

Subject 1

100

Define approximation ratio.

Question
Answer

Done!
Home

Subject 1

100

Question
Answer

Done!
Home

Give an example of a graph for which APPROX-VERTEX-COVER always yields a suboptimal solution.

Subject 2

200

Question
Answer

Done!
Home

Subject 3

300

Consider each of the following words as a set of letters: farid; dash; drain; heard; lost; nose; shun; slate; snare; threadg. Show which set cover GREEDY-SET-COVER produces when we break ties in favor of the word that appears first in the dictionary.

Question
Answer

Done!
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Subject 3

300

Question
Answer

Done!
Home

GREEDY-SET-COVER can return a number of different solutions, depending on how we break ties when picking a set. Give a procedure $\text{BAD-SET-COVER-INSTANCE}(n)$ that returns an n -element instance of the set-covering problem for which, depending on how we break ties while picking a set, GREEDY-SET-COVER can return a number of different solutions that is exponential in n .

Subject 4

400

Question
Answer

Done!
Home

For the travelling salesman problem, prove that the following algorithm produces a 2-approximation: APPROX-TSP-TOUR (G) Select a vertex $r \in G:V$ to be a root vertex Compute a minimum spanning tree T for G from root r using MST-PRIM (G, r) Let H be a list of vertices, ordered according to when they are first visited in a preorder tree walk of T
Return the hamiltonian cycle H

A full walk of T lists the vertices when they are first visited and also whenever they are returned to after a visit to a subtree. Since the full walk traverses every edge of T exactly twice, we have $c(W) = 2 * c(T) \rightarrow c(W) \leq 2 * c(H^*)$. Unfortunately, the full walk W is generally not a tour, since it visits some vertices more than once. By the triangle inequality, however, we can delete a visit to any vertex from W and the cost does not increase.