11.) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-z^2 \le 1$, $0 \le z \le 3$

$x = ar\cos\theta \qquad y = br\sin\theta$

$$dV = \left|\frac{\partial(x,y,z)}{\partial(r,\theta,z)}\right| = \begin{vmatrix} a\cos\theta & b\sin\theta & 0 \\ -ar\sin\theta & br\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$= a\cos\theta \begin{vmatrix} br\cos\theta & 0 \\ 0 & 1 \end{vmatrix} - b\sin\theta \begin{vmatrix} -ar\sin\theta & 0 \\ 0 & 1 \end{vmatrix} =$$

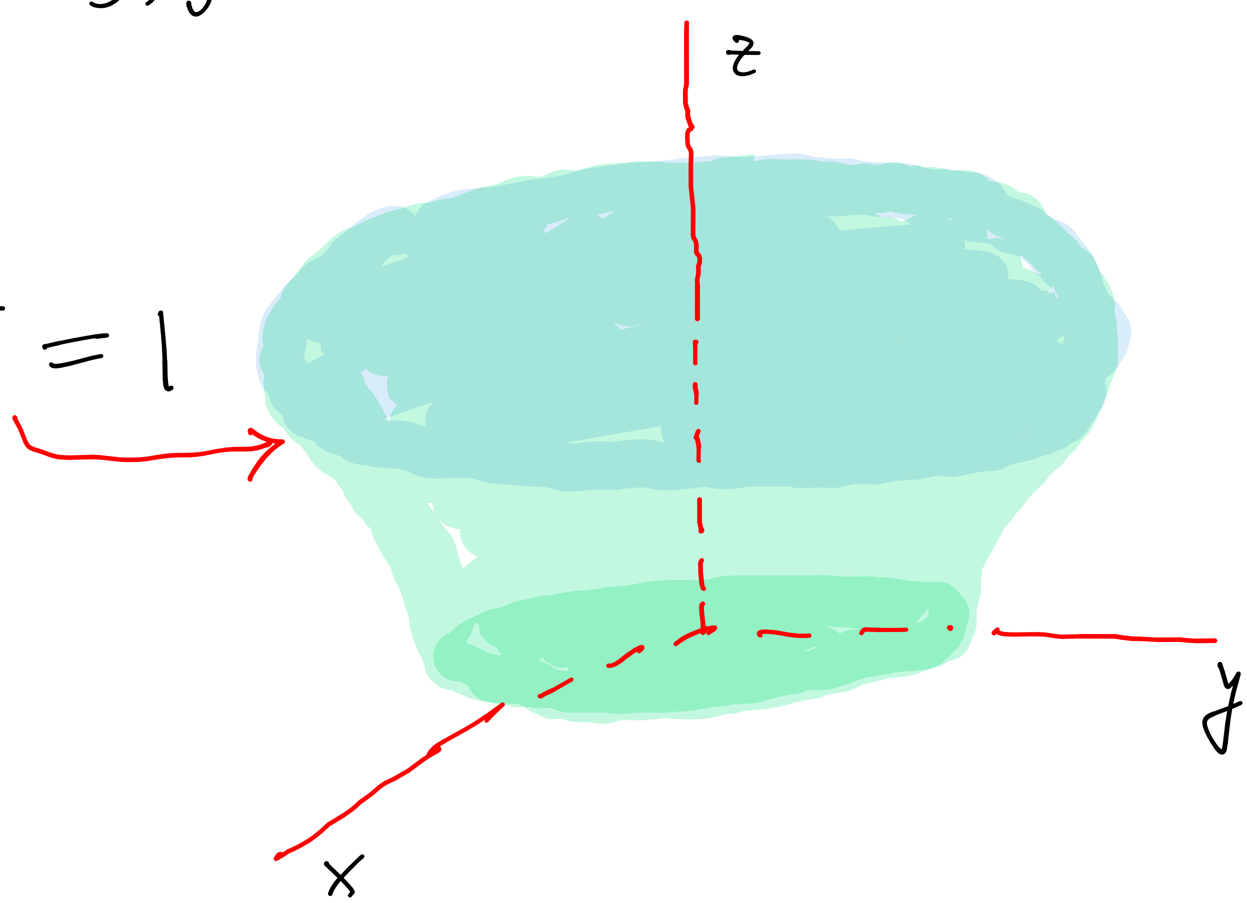
$$= abr\cos^2\theta + abr\sin^2\theta = abr$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-z^2 = r^2-z^2 = 1 \Rightarrow r = \sqrt{1+z^2}$$

$$V = \int_0^{2\pi} d\theta \int_0^3 dz \int_0^{\sqrt{1+z^2}} abr\,dr = 2\pi ab \int_0^3 \frac{1+z^2}{2}\,dz =$$

$$= \pi ab\left(z+\frac{z^3}{3}\right)\Big|_0^3 = 12\pi ab$$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-z^2 = 1$



12.) D: $z^2 \geq \frac{1}{3}(x^2+y^2)$, $z^2 \leq 3(x^2+y^2)$, $x^2+y^2+z^2 \leq a^2$

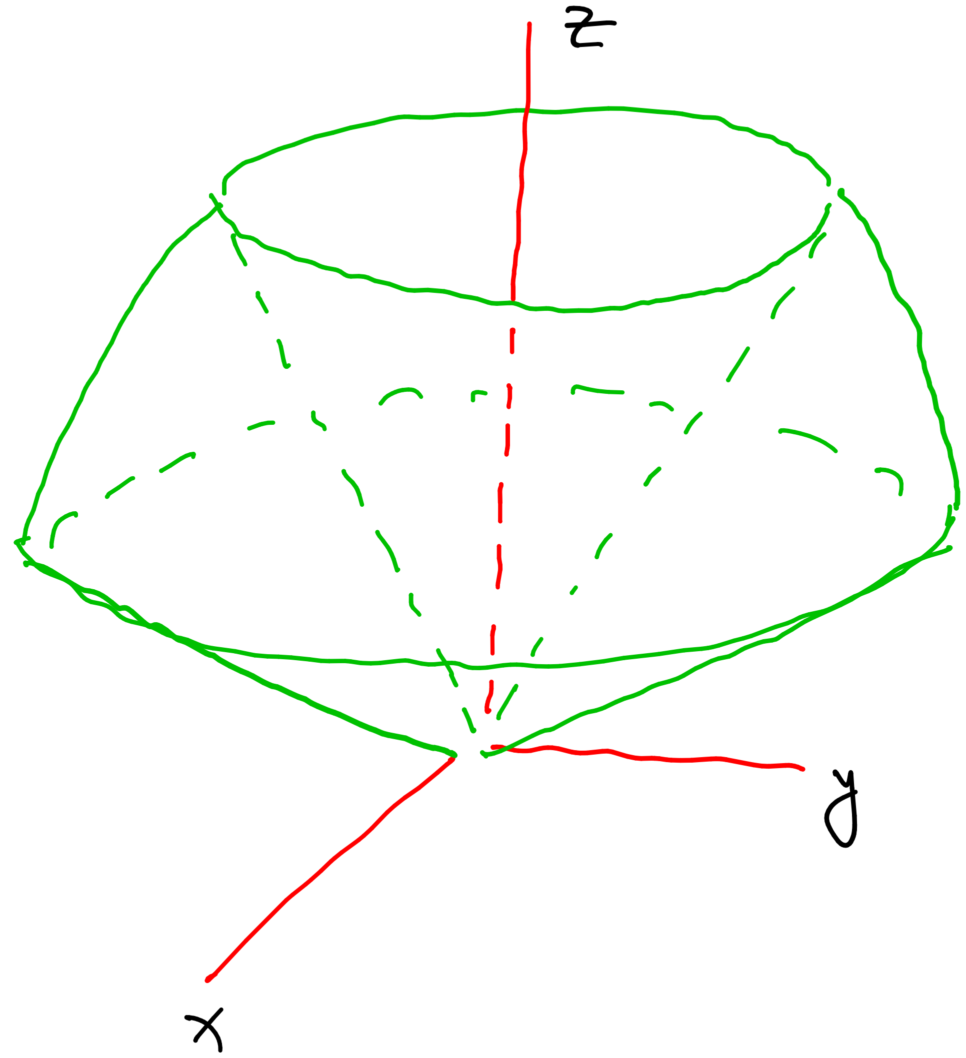
$z \geq 0$.

SPHERICAL COORDINATES:

$x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$

$z^2 = \alpha^2(x^2+y^2)$: $\cos^2\phi = \alpha^2 \sin^2\phi \Rightarrow \tan\phi = \frac{1}{\alpha}$

$\alpha = \frac{1}{\sqrt{3}} \Rightarrow \phi = \frac{\pi}{3}$; $\alpha = \sqrt{3} \Rightarrow \phi = \frac{\pi}{6}$



$$I = \iiint_D z\, dV = \int_0^{2\pi} d\theta \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d\phi \int_0^a d\rho\, \rho\cos\phi\, \rho^2 \sin\phi =$$

$$= 2\pi \frac{a^4}{4} \frac{\cos^2\phi}{2}\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi a^4}{4}\left(\frac{3}{4} - \frac{1}{4}\right) = \frac{\pi a^4}{8}$$