19.) $J = \int_\alpha^\beta \sqrt{\dot{x}^2+\dot{y}^2}\, dt = \text{MIN}$

$$K = \frac{1}{2}\int_\alpha^\beta (y\dot{x} - x\dot{y})\, dt = A$$

$$F = \sqrt{\dot{x}^2+\dot{y}^2} + \lambda(y\dot{x} - x\dot{y})$$

$$H = \dot{x}F_{\dot{x}} + \dot{y}F_{\dot{y}} - F = 0$$

B/C F IS HOMOGENEOUS OF DEGREE 1.

$$\frac{d}{dt}\left[\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} + \lambda y\right] + \lambda\dot{y} = 0$$

$$\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} + 2\lambda y = C_1 \Rightarrow \frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} = C_1 - 2\lambda y \;\Big|\; (-\dot{y})$$

$$\frac{d}{dt}\left[\frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} - \lambda x\right] - \lambda\dot{x} = 0$$

$$\frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} - 2\lambda x = C_2 \Rightarrow \frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}} = C_2 + 2\lambda x \;\Big|\; \dot{x}$$

$$(2\lambda y - C_1)\dot{y} + (2\lambda x + C_2)\dot{x} = 0$$

$$\frac{1}{4\lambda}(2\lambda y - C_1)^2 + \frac{1}{4\lambda}(2\lambda x + C_2)^2 = D^2$$

$$(x-x_0)^2 + (y-y_0)^2 = \frac{D^2}{\lambda} = R^2 = \frac{A}{\pi}$$

CIRCLE

20.) $\ddot{x} + x^2 - x = 0$

(i) $\ddot{x} = x - x^2 = x(1-x)$

KINETIC ENERGY: $T = \frac{1}{2}\dot{x}^2$

POTENTIAL ENERGY: $f(x) = x - x^2 \Rightarrow U(x) = -\int (x - x^2)\, dx$

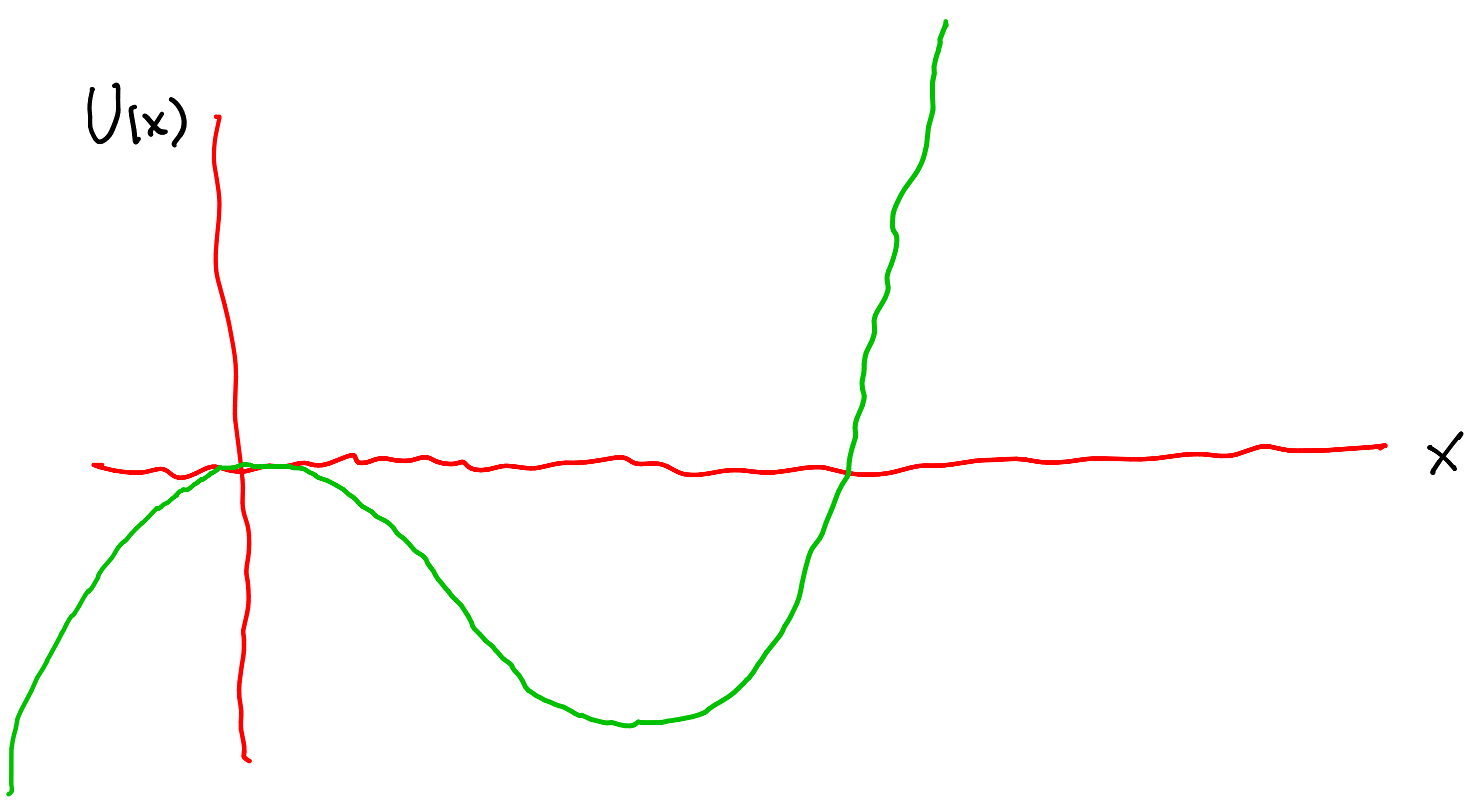
$$= -\frac{x^2}{2} + \frac{x^3}{3}$$

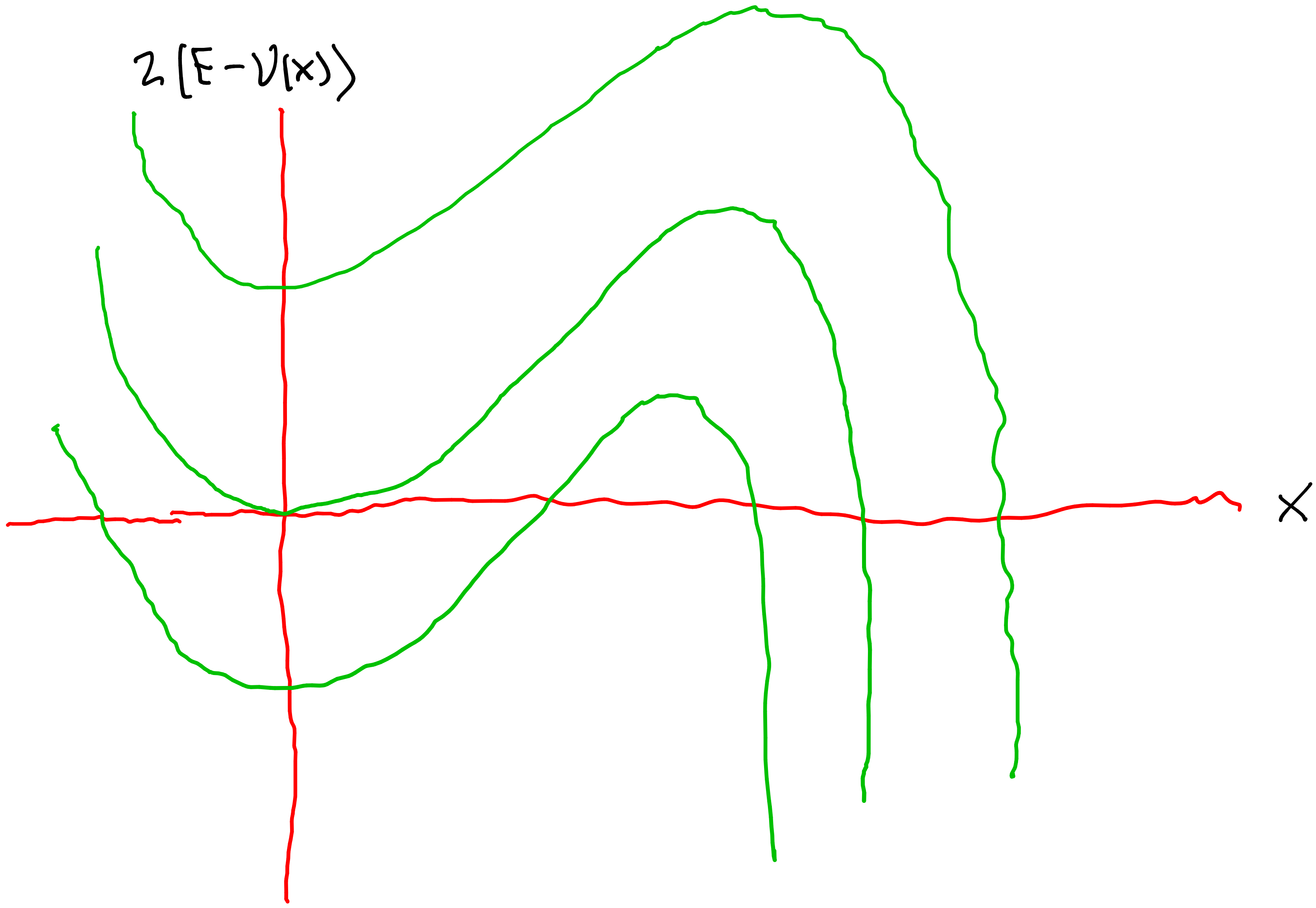
EQUILIBRIA: $f(x) = 0 \Rightarrow x = 0, \quad x = 1$

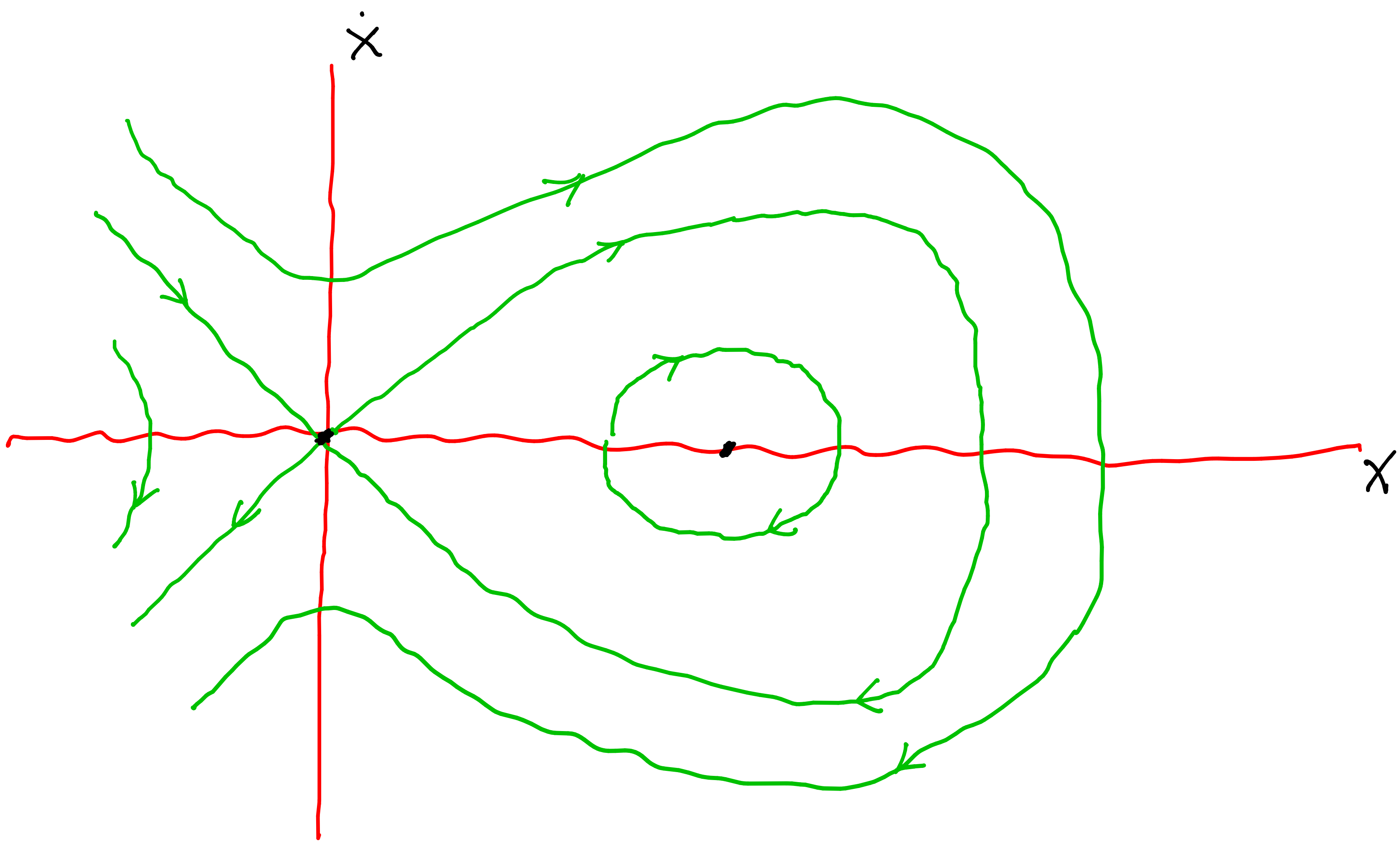
(ii) $\ddot{x} + x^2 - x = 0 \quad | \, \dot{x}$

$$\dot{x}\ddot{x} + x^2\dot{x} - x\dot{x} = 0$$

$$\frac{1}{2}\dot{x}^2 + \frac{1}{3}x^3 - \frac{1}{2}x^2 = E$$ TOTAL ENERGY




$2(E - V(x))$
$x$


$\dot{x}$
$x$

(iii) AT THE EQUILIBRIUM POINT AT $x = 0$, $\dot{x} = 0$,

$E = 0$

$\Rightarrow$ BY CONTINUITY, ALONG THE SEPARATRIX LOOP $E = 0$.