17.) $\vec{F}(x,y,z) = \left(yz[2x(y+z)+yz],\ xz[2y(x+z)+xz],\ xy[2z(x+y)+xy] \right)$

$C: \vec{r}(t) = (t\cos\pi t^2,\ t^2\sin\pi t^2,\ t^3)$

SUSPECT: $\vec{F}$ IS CONSERVATIVE.

(CHECK (BUT THIS IS NOT NECESSARY):

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy^2z+2xyz^2+y^2z^2 & 2x^2yz+2xyz^2+x^2z^2 & 2x^2yz+2xy^2z+x^2y^2 \end{vmatrix}$$

$$= (2x^2z+4xyz+2x^2y-2x^2y-4xyz-2x^2z,\ 2xy^2+4xyz+2y^2z -$$

$$- 4xyz-2y^2z-2xy^2,\ 4xyz+2yz^2+2xz^2-4xyz-2xz^2-2yz^2) =$$

$$= (0,0,0)$$)

$\Rightarrow$ $\vec{F}$ IS CONSERVATIVE

$2xy^2z+2xyz^2+y^2z^2 = \frac{\partial f}{\partial x} \Rightarrow f = x^2y^2z + x^2yz^2 + xy^2z^2 + \phi(y,z)$

$\frac{\partial f}{\partial y} = 2x^2yz + x^2z^2 + 2xyz^2 + \frac{\partial \phi}{\partial y} = 2x^2yz + 2xyz^2 + x^2z^2$

$$\Rightarrow \frac{\partial \phi}{\partial y} = 0 \Rightarrow \phi = \phi(z)$$

$$\Rightarrow \frac{\partial f}{\partial z} = x^2y^2 + 2x^2yz + 2xy^2z + \phi'(z) = 2x^2yz + 2xy^2z + x^2y^2$$

$$\Rightarrow \phi'(z) = 0 \Rightarrow \phi = C = \text{CONST.}$$

$$f = x^2y^2z + x^2yz^2 + xy^2z^2 + C$$

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = f(\vec{r}(1)) - f(\vec{r}(0))$$

$$\vec{r}(1) = (-1, 0, 1), \quad \vec{r}(0) = (0, 0, 0)$$

$$f(\vec{r}(1)) = 0, \quad f(\vec{r}(0)) = 0$$

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = 0$$

18.) $J[y] = \int_0^1 [(y')^2 + y^2]\, dx = $ EXTREMAL

$y(0)=1\ ,\ \ y(1)=0$

NO X IN INTEGRAND: $y'(2y') - [(y')^2 - y^2] = (y')^2 - y^2 = C^2$

$y' = \pm\sqrt{C^2+y^2}\ ,\ \int \frac{dy}{\sqrt{C^2+y^2}} = \pm(x+D)$

$\sinh^{-1}\frac{y}{C} = \pm(x+D) \qquad y = C \sinh(x+D)$

$y(0) = C\sinh D = 1\ ,\ \ y(1) = C\sinh(1+D) = 0 \Rightarrow D = -1$

$y(0) = C\sinh(-1) = -C\sinh 1 = 1 \Rightarrow C = -\frac{1}{\sinh 1} =$

$= -\frac{2}{e - e^{-1}} = -\frac{2e}{e^2-1}$

$y = -\frac{e}{e^2-1}\left(e^{x-1} - e^{-x+1}\right) = \frac{1}{e^2-1}\left(e^{2-x} - e^{x}\right)$

ALTERNATIVELY: EULER'S EQUATION $\frac{d}{dx}(2y') - 2y = 0$

$\Rightarrow y'' - y = 0, \quad y = e^{rt}, \quad r^2 - 1 = 0, \quad r = \pm 1$

$\Rightarrow y = Ae^{-x} + Be^{x}$

$y(0) = A + B = 1, \quad y(1) = Ae^{-1} + Be = 0$

$\Rightarrow A = -e^2 B, \quad B(1 - e^2) = 1, \quad B = -\frac{1}{e^2 - 1}, \quad A = \frac{e^2}{e^2 - 1}$

$y = \frac{1}{e^2 - 1}\left(e^{2-x} - e^{x}\right)$