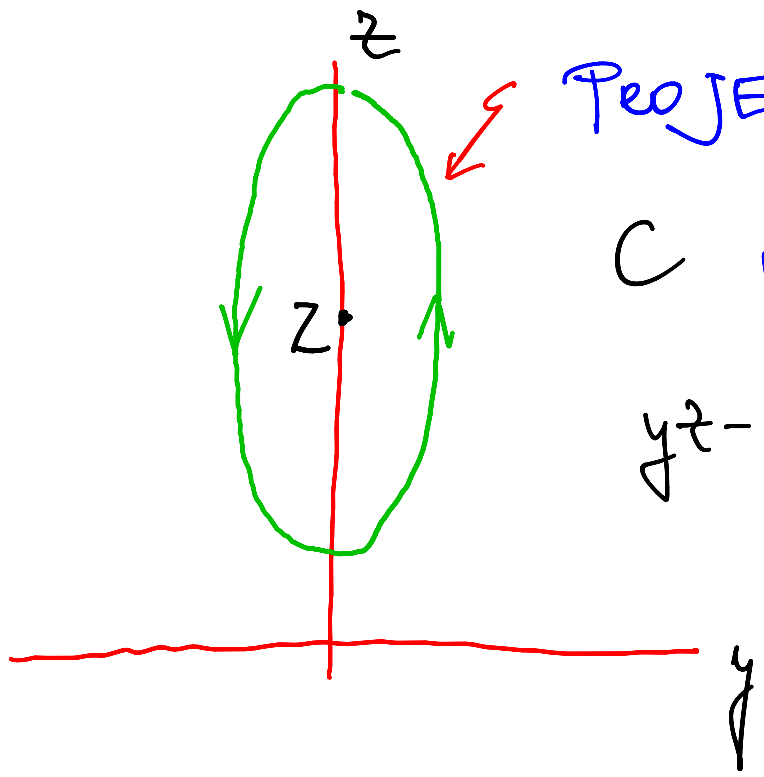
13.) $C: \{(x,y,z) \mid 9(x-1)^2 - 27y^2 + (z-2)^2 = 1, \ x-2y=1\}$

$$I = \int_C (2-z)\,dx + (2-z)\,dy + (x+y-1)\,dz$$

$\Rightarrow$ $x = 2y+1$, $\quad 36y^2 - 27y^2 + (z-2)^2 = 1$, $\quad 9y^2 + (z-2)^2 = 1$



PROJECTION OF C ONTO yz-PLANE

LET $3y = \cos\theta$, $\quad z = 2 + \sin\theta$, $\quad 0 \le \theta \le 2\pi$

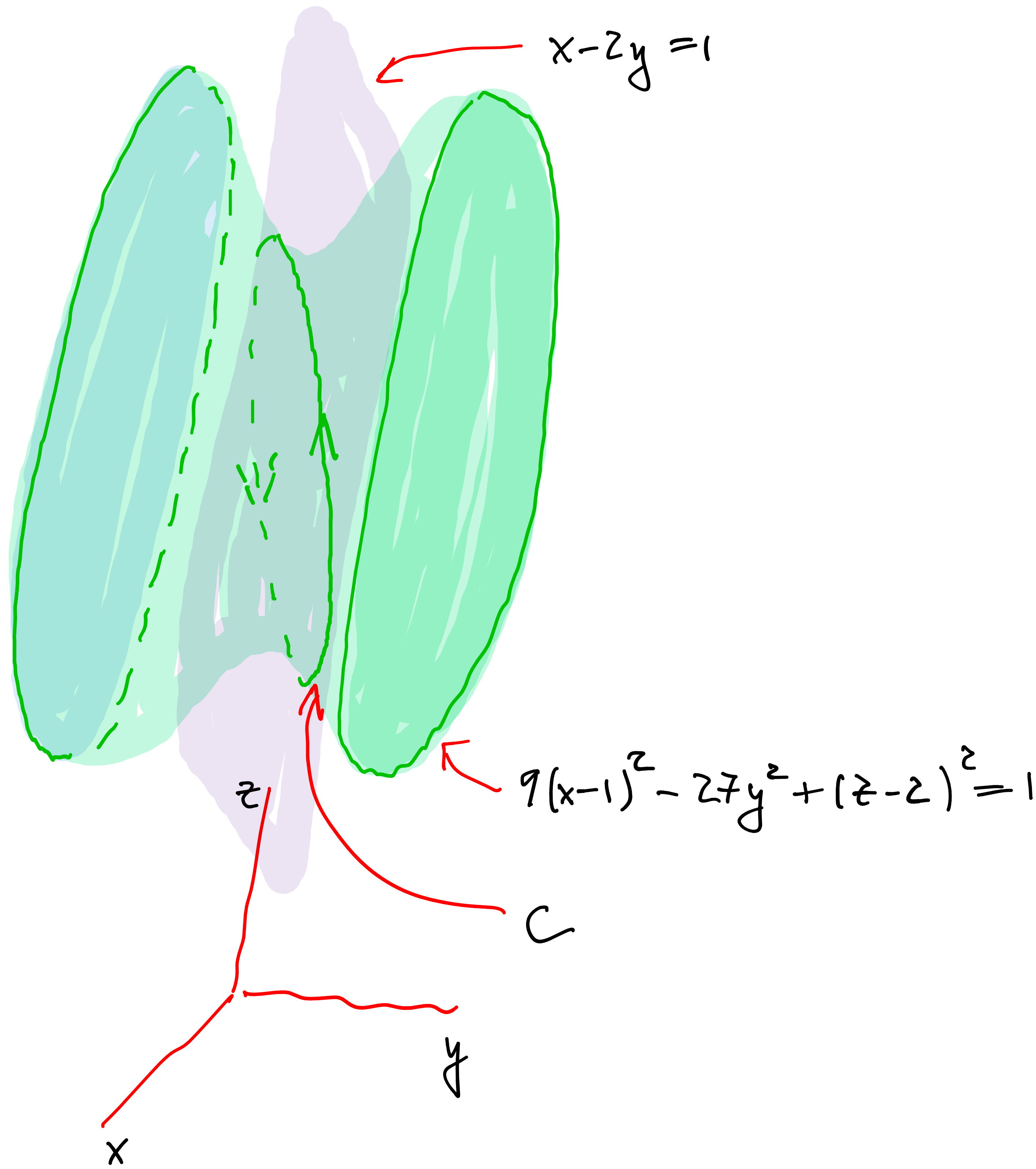
$\Rightarrow$ $x = \frac{2}{3}\cos\theta + 1$

$dx = -\frac{2}{3}\sin\theta\,d\theta$, $\quad dy = -\frac{1}{3}\sin\theta\,d\theta$, $\quad dz = \cos\theta\,d\theta$

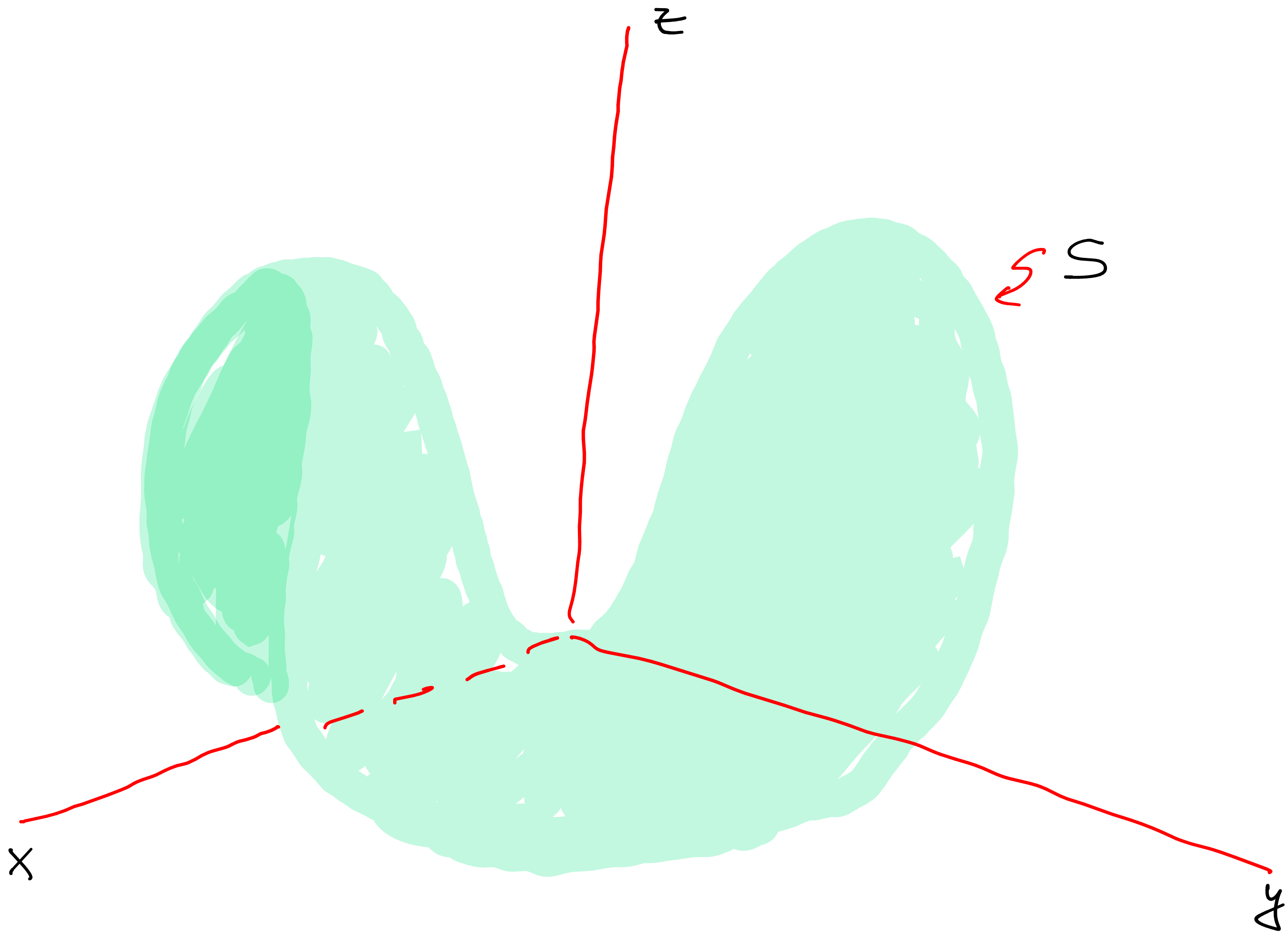
$$I = \int_0^{2\pi} \left[(-\sin\theta)\left(-\tfrac{2}{3}\sin\theta - \tfrac{1}{3}\sin\theta\right) + \cos\theta\cos\theta \right] d\theta =$$

$$= \int_0^{2\pi} \left[\sin^2\theta + \cos^2\theta \right] d\theta = 2\pi$$

$$9(x-1)^2 - 27y^2 + (z-2)^2 = 1 \Rightarrow 27y^2 = 9(x-1)^2 + (z-2)^2 - 1$$



4.) $z = f(x,y) = \frac{b}{2}(x^2 - y^2)$, $\quad x^2+y^2 \le a^2$



$$A(S) = \iint_{x^2+y^2\le a^2} \sqrt{f_x^2 + f_y^2 + 1}\, dx\,dy$$

$f_x = bx$, $\quad f_y = -by$, $\quad f_x^2 + f_y^2 + 1 = b^2(x^2+y^2) + 1$

POLAR COORDINATES:

$$x = r\cos\theta, \quad y = r\sin\theta$$

$$A(S) = \int_0^{2\pi} d\theta \int_0^a \sqrt{b^2r^2+1}\; r\,dr = \frac{\pi}{b^2}\int_0^{b^2a^2} \sqrt{s+1}\, ds =$$

$$= \frac{2\pi}{3b^2}\sqrt{s+1}^{\,3}\Big|_0^{b^2a^2} = \frac{2\pi}{3}\left(\sqrt{b^2a^2+1}^{\,3} - 1\right)$$

$$a \text{ OR } b \to 0: \sqrt{1+b^2a^2}^{\,3} = 1 + \binom{3/2}{1} b^2a^2 + \mathcal{O}(b^4a^4) =$$

$$= 1 + \frac{3}{2} b^2a^2 + \mathcal{O}(b^4a^4)$$

$$A(S) = \frac{2\pi}{3b^2}\left(1 + \frac{3}{2}b^2a^2 + \mathcal{O}(b^4a^4) - 1\right) =$$

$$= \pi a^2 + \mathcal{O}(b^2a^4)$$

THIS IS THE AREA OF THE PARAMETER-DOMAIN CIRCLE $r \le a$.