## custom-e-book-5

Written by consumer consumer

## **Table of Contents**

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## Type Systems for Programming Languages

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This is preliminary draft of a book in progress. Comments, suggestions, and corrections are welcome.

## **Contents**

Pr	Preface					
1	Intr	oduction	13			
	1.1	What is a Type System?	13			
	1.2	A Brief History of Type	14			
	1.3	Applications of Type Systems	16			
	1.4	Related Reading	16			
2	Mathematical Preliminaries					
	2.1	Sets and Relations	18			
	2.2	Induction	18			
	2.3	Term Rewriting	19			
3	Unt	yped Arithmetic Expressions	20			
	3.1	Basics	20			
		Syntax	21			
		Evaluation	21			
	3.2	Formalities	21			
		Syntax	21			
		Evaluation	25			
	3.3	Properties	27			
	3.4	Implementation	27			
		Syntax	27			
		Evaluation	28			
	3.5	Summary	28			
	3.6	Further Reading	30			
4	The Untyped Lambda-Calculus					
	4.1	Basics	32			
		Syntax	33			
		Operational Semantics	34			
	4.2	Programming in the Lambda-Calculus	34			

	4.3	Is the Lambda-Calculus a Programming Language?	39
	4.4	Formalities	39
		Syntax	39
		Substitution	40
		Operational Semantics	43
		Summary	43
	4.5		44
	4.3	Further Reading	44
5	Imp	lementing the Lambda-Calculus	45
•	5.1	Nameless Representation of Terms	45
	0.1	Syntax	46
		Shifting and Substitution	48
		· · · · · · · · · · · · · · · · · · ·	49
	r 0	Evaluation	49
	5.2	A Concrete Realization	
		Syntax	50
		Shifting and Substitution	50
		Evaluation	51
	5.3	Ordinary vs. Nameless Representations	51
6	Т	ad Auithmetic Europeasions	52
U	- I	ed Arithmetic Expressions	
	6.1	Syntax	52
	6.2	The Typing Relation	52
	6.3	Properties of Typing and Reduction	53
		Typing Derivations	53
		Typechecking	54
		Safety = Preservation + Progress	54
	6.4	Implementation	55
	6.5	Summary	56
		·	
7	Sim	ply Typed Lambda-Calculus	<b>58</b>
	7.1	Syntax	58
	7.2	The Typing Relation	59
	7.3	Summary	61
	7.4	Properties of Typing and Reduction	62
		Typechecking	62
		Typing and Substitution	64
		Type Soundness	64
	7.5	Implementation	65
	7.6	Further Reading	66
	7.0	rururer iveauring	υU

8	Exte	nsions	67			
	8.1	Base Types	67			
	8.2	Unit type	67			
	8.3	Let bindings	68			
	8.4	Records and Tuples	68			
	8.5	Variants	72			
	8.6	General recursion	72			
	8.7	Lists	73			
	8.8	Lazy records and let-bindings	75			
9	References 76					
	9.1	Further Reading	79			
10	Exce	eptions	80			
	10.1	Errors	80			
	10.2	Exceptions	80			
11	Тур	e Equivalence	81			
12	Defi	nitions	83			
	12.1	Type Definitions	83			
		Term Definitions	85			
13	Subtyping 86					
		The Subtype Relation	87			
		Variance	88			
		Summary	89			
	13.2	Metatheory of Subtyping	91			
		Algorithmic Subtyping	91			
		Minimal Typing	92			
	13.3	Implementation	96			
		Meets and Joins	97			
	13.5	Primitive Subtyping	99			
	13.6	The Bottom Type	99			
	13.7	Other stuff	99			
14			100			
	14.1	Objects	100			
		Object Generators				
		Subtyping				
		Basic classes				
	14.5	Extending the Internal State	105			
		Classes with "Self"				

15	Recu	ırsive Types	109
	15.1	Examples	109
		Lists	109
		Hungry Functions	109
		Recursive Values from Recursive Types	110
		Untyped Lambda-Calculus, Redux	110
		Recursive Objects	112
	15.2	Equi-recursive Types	
		ML Implementation	
	15.3	Iso-recursive Types	
		Subtyping and Recursive Types	
16	Case	e Study: Featherweight Java	117
17	Tyne	Reconstruction	118
		Substitution	
		Universal vs. Existential Type Variables	
		Constraint-Based Typing	
		Unification	
		Principal Typings	
		Further Reading	
	17.0	rutulei keading	123
18		versal Types	130
		Motivation	
		Varieties of Polymorphism	
	18.3	Definitions	132
	18.4	Examples	135
		Warm-ups	
		Polymorphic Lists	136
		Impredicative Encodings	136
	18.5	Metatheory	
		Soundness	
		Strong Normalization	
		Erasure and Typeability	
		Type Reconstruction	
	18.6	Implementation	
	_0.0	Nameless Representation of Types	
		ML Code	
	18 7	Further Reading	
	18.7	Further Reading	143