```
Here is a photo of <B>my house</B>:
<P><IMG SRC = "house.gif"><BR>
See <A HREF = "morePix.html">More Pictures</A> if you
liked that one.<P>
```

into appropriate lexemes. Which lexemes should get associated lexical values, and what should those values be?

3.2 Input Buffering

Before discussing the problem of recognizing lexemes in the input, let us examine some ways that the simple but important task of reading the source program can be speeded. This task is made difficult by the fact that we often have to look one or more characters beyond the next lexeme before we can be sure we have the right lexeme. The box on "Tricky Problems When Recognizing Tokens" in Section 3.1 gave an extreme example, but there are many situations where we need to look at least one additional character ahead. For instance, we cannot be sure we've seen the end of an identifier until we see a character that is not a letter or digit, and therefore is not part of the lexeme for **id**. In C, single-character operators like `-`, `=`, or `<` could also be the beginning of a two-character operator like `->`, `==`, or `<=`. Thus, we shall introduce a two-buffer scheme that handles large lookaheads safely. We then consider an improvement involving "sentinels" that saves time checking for the ends of buffers.

3.2.1 Buffer Pairs

Because of the amount of time taken to process characters and the large number of characters that must be processed during the compilation of a large source program, specialized buffering techniques have been developed to reduce the amount of overhead required to process a single input character. An important scheme involves two buffers that are alternately reloaded, as suggested in Fig. 3.3.

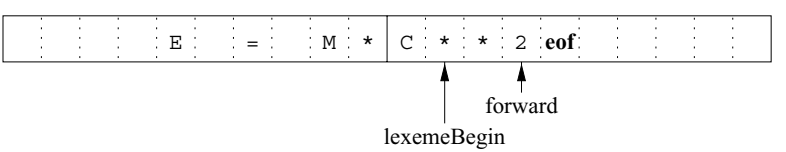

Figure 3.3: Using a pair of input buffers

Each buffer is of the same size N, and N is usually the size of a disk block, e.g., 4096 bytes. Using one system read command we can read N characters into a buffer, rather than using one system call per character. If fewer than N characters remain in the input file, then a special character, represented by **eof**,

marks the end of the source file and is different from any possible character of the source program.

Two pointers to the input are maintained:

1. Pointer `lexemeBegin`, marks the beginning of the current lexeme, whose extent we are attempting to determine.

2. Pointer `forward` scans ahead until a pattern match is found; the exact strategy whereby this determination is made will be covered in the balance of this chapter.

Once the next lexeme is determined, `forward` is set to the character at its right end. Then, after the lexeme is recorded as an attribute value of a token returned to the parser, `lexemeBegin` is set to the character immediately after the lexeme just found. In Fig. 3.3, we see `forward` has passed the end of the next lexeme, `**` (the Fortran exponentiation operator), and must be retracted one position to its left.

Advancing `forward` requires that we first test whether we have reached the end of one of the buffers, and if so, we must reload the other buffer from the input, and move `forward` to the beginning of the newly loaded buffer. As long as we never need to look so far ahead of the actual lexeme that the sum of the lexeme's length plus the distance we look ahead is greater than N, we shall never overwrite the lexeme in its buffer before determining it.

3.2.2 Sentinels

If we use the scheme of Section 3.2.1 as described, we must check, each time we advance `forward`, that we have not moved off one of the buffers; if we do, then we must also reload the other buffer. Thus, for each character read, we make two tests: one for the end of the buffer, and one to determine what character is read (the latter may be a multiway branch). We can combine the buffer-end test with the test for the current character if we extend each buffer to hold a *sentinel* character at the end. The sentinel is a special character that cannot be part of the source program, and a natural choice is the character **eof**.

Figure 3.4 shows the same arrangement as Fig. 3.3, but with the sentinels added. Note that **eof** retains its use as a marker for the end of the entire input. Any **eof** that appears other than at the end of a buffer means that the input is at an end. Figure 3.5 summarizes the algorithm for advancing `forward`. Notice how the first test, which can be part of a multiway branch based on the character pointed to by `forward`, is the only test we make, except in the case where we actually are at the end of a buffer or the end of the input.

3.3 Specification of Tokens

Regular expressions are an important notation for specifying lexeme patterns. While they cannot express all possible patterns, they are very effective in spec-

Can We Run Out of Buffer Space?

In most modern languages, lexemes are short, and one or two characters of lookahead is sufficient. Thus a buffer size N in the thousands is ample, and the double-buffer scheme of Section 3.2.1 works without problem. However, there are some risks. For example, if character strings can be very long, extending over many lines, then we could face the possibility that a lexeme is longer than N. To avoid problems with long character strings, we can treat them as a concatenation of components, one from each line over which the string is written. For instance, in Java it is conventional to represent long strings by writing a piece on each line and concatenating pieces with a `+` operator at the end of each piece.

A more difficult problem occurs when arbitrarily long lookahead may be needed. For example, some languages like PL/I do not treat keywords as *reserved*; that is, you can use identifiers with the same name as a keyword like `DECLARE`. If the lexical analyzer is presented with text of a PL/I program that begins `DECLARE ( ARG1, ARG2,...` it cannot be sure whether `DECLARE` is a keyword, and `ARG1` and so on are variables being declared, or whether `DECLARE` is a procedure name with its arguments. For this reason, modern languages tend to reserve their keywords. However, if not, one can treat a keyword like `DECLARE` as an ambiguous identifier, and let the parser resolve the issue, perhaps in conjunction with symbol-table lookup.

ifying those types of patterns that we actually need for tokens. In this section we shall study the formal notation for regular expressions, and in Section 3.5 we shall see how these expressions are used in a lexical-analyzer generator. Then, Section 3.7 shows how to build the lexical analyzer by converting regular expressions to automata that perform the recognition of the specified tokens.

3.3.1 Strings and Languages

An *alphabet* is any finite set of symbols. Typical examples of symbols are letters, digits, and punctuation. The set $\{0, 1\}$ is the *binary alphabet*. ASCII is an important example of an alphabet; it is used in many software systems. Uni-

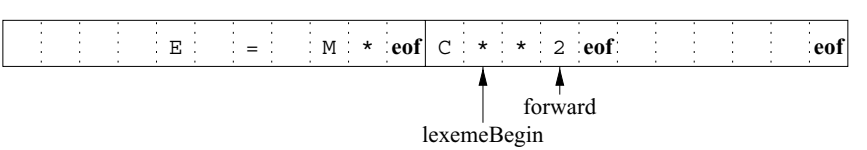

Figure 3.4: Sentinels at the end of each buffer

```
switch ( *forward++ ) {
        case eof:
                if (forward is at end of first buffer ) {
                        reload second buffer;
                        forward = beginning of second buffer;
                }
                else if (forward is at end of second buffer ) {
                        reload first buffer;
                        forward = beginning of first buffer;
                }
                else /* eof within a buffer marks the end of input */
                        terminate lexical analysis;
                break;
        Cases for the other characters
}
```

Figure 3.5: Lookahead code with sentinels

Implementing Multiway Branches

We might imagine that the switch in Fig. 3.5 requires many steps to execute, and that placing the case **eof** first is not a wise choice. Actually, it doesn't matter in what order we list the cases for each character. In practice, a multiway branch depending on the input character is made in one step by jumping to an address found in an array of addresses, indexed by characters.

code, which includes approximately 100,000 characters from alphabets around the world, is another important example of an alphabet.

A *string* over an alphabet is a finite sequence of symbols drawn from that alphabet. In language theory, the terms "sentence" and "word" are often used as synonyms for "string." The length of a string s, usually written $|s|$, is the number of occurrences of symbols in s. For example, `banana` is a string of length six. The *empty string*, denoted ϵ, is the string of length zero.

A *language* is any countable set of strings over some fixed alphabet. This definition is very broad. Abstract languages like $\emptyset$, the *empty set*, or $\{\epsilon\}$, the set containing only the empty string, are languages under this definition. So too are the set of all syntactically well-formed C programs and the set of all grammatically correct English sentences, although the latter two languages are difficult to specify exactly. Note that the definition of "language" does not require that any meaning be ascribed to the strings in the language. Methods for defining the "meaning" of strings are discussed in Chapter 5.

Terms for Parts of Strings

The following string-related terms are commonly used:

1. A *prefix* of string s is any string obtained by removing zero or more symbols from the end of s. For example, `ban`, `banana`, and ϵ are prefixes of `banana`.

2. A *suffix* of string s is any string obtained by removing zero or more symbols from the beginning of s. For example, `nana`, `banana`, and ϵ are suffixes of `banana`.

3. A *substring* of s is obtained by deleting any prefix and any suffix from s. For instance, `banana`, `nan`, and ϵ are substrings of `banana`.

4. The *proper* prefixes, suffixes, and substrings of a string s are those, prefixes, suffixes, and substrings, respectively, of s that are not ϵ or not equal to s itself.

5. A *subsequence* of s is any string formed by deleting zero or more not necessarily consecutive positions of s. For example, `baan` is a subsequence of `banana`.

If x and y are strings, then the *concatenation* of x and y, denoted xy, is the string formed by appending y to x. For example, if $x =$ `dog` and $y =$ `house`, then $xy =$ `doghouse`. The empty string is the identity under concatenation; that is, for any string s, $\epsilon s = s\epsilon = s$.

If we think of concatenation as a product, we can define the "exponentiation" of strings as follows. Define s^0 to be ϵ, and for all $i > 0$, define s^i to be $s^{i-1}s$. Since $\epsilon s = s$, it follows that $s^1 = s$. Then $s^2 = ss$, $s^3 = sss$, and so on.

3.3.2 Operations on Languages

In lexical analysis, the most important operations on languages are union, concatenation, and closure, which are defined formally in Fig. 3.6. Union is the familiar operation on sets. The concatenation of languages is all strings formed by taking a string from the first language and a string from the second language, in all possible ways, and concatenating them. The (*Kleene*) *closure* of a language L, denoted L^*, is the set of strings you get by concatenating L zero or more times. Note that L^0, the "concatenation of L zero times," is defined to be $\{\epsilon\}$, and inductively, L^i is $L^{i-1}L$. Finally, the positive closure, denoted L^+, is the same as the Kleene closure, but without the term L^0. That is, ϵ will not be in L^+ unless it is in L itself.

OPERATION	DEFINITION AND NOTATION
Union of L and M	$L \cup M = \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$
Concatenation of L and M	$LM = \{st \mid s \text{ is in } L \text{ and } t \text{ is in } M\}$
Kleene closure of L	$L^* = \cup_{i=0}^{\infty} L^i$
Positive closure of L	$L^+ = \cup_{i=1}^{\infty} L^i$

Figure 3.6: Definitions of operations on languages

Example 3.3 : Let L be the set of letters $\{\texttt{A}, \texttt{B}, \ldots, \texttt{Z}, \texttt{a}, \texttt{b}, \ldots, \texttt{z}\}$ and let D be the set of digits $\{\texttt{0}, \texttt{1}, \ldots \texttt{9}\}$. We may think of L and D in two, essentially equivalent, ways. One way is that L and D are, respectively, the alphabets of uppercase and lowercase letters and of digits. The second way is that L and D are languages, all of whose strings happen to be of length one. Here are some other languages that can be constructed from languages L and D, using the operators of Fig. 3.6:

1. $L \cup D$ is the set of letters and digits — strictly speaking the language with 62 strings of length one, each of which strings is either one letter or one digit.
2. LD is the set of 520 strings of length two, each consisting of one letter followed by one digit.
3. L^4 is the set of all 4-letter strings.
4. L^* is the set of all strings of letters, including ϵ, the empty string.
5. $L(L \cup D)^*$ is the set of all strings of letters and digits beginning with a letter.
6. D^+ is the set of all strings of one or more digits.

□

3.3.3 Regular Expressions

Suppose we wanted to describe the set of valid C identifiers. It is almost exactly the language described in item (5) above; the only difference is that the underscore is included among the letters.

In Example 3.3, we were able to describe identifiers by giving names to sets of letters and digits and using the language operators union, concatenation, and closure. This process is so useful that a notation called *regular expressions* has come into common use for describing all the languages that can be built from these operators applied to the symbols of some alphabet. In this notation, if *letter_* is established to stand for any letter or the underscore, and *digit* is

established to stand for any digit, then we could describe the language of C identifiers by:

$$letter\_ \ (\ letter\_ \mid digit \ )^*$$

The vertical bar above means union, the parentheses are used to group subexpressions, the star means "zero or more occurrences of," and the juxtaposition of *letter_* with the remainder of the expression signifies concatenation.

The regular expressions are built recursively out of smaller regular expressions, using the rules described below. Each regular expression r denotes a language $L(r)$, which is also defined recursively from the languages denoted by r's subexpressions. Here are the rules that define the regular expressions over some alphabet Σ and the languages that those expressions denote.

BASIS: There are two rules that form the basis:

1. ϵ is a regular expression, and $L(\epsilon)$ is $\{\epsilon\}$, that is, the language whose sole member is the empty string.

2. If a is a symbol in Σ, then $\mathbf{a}$ is a regular expression, and $L(\mathbf{a}) = \{a\}$, that is, the language with one string, of length one, with a in its one position. Note that by convention, we use italics for symbols, and boldface for their corresponding regular expression.[1]

INDUCTION: There are four parts to the induction whereby larger regular expressions are built from smaller ones. Suppose r and s are regular expressions denoting languages $L(r)$ and $L(s)$, respectively.

1. $(r)|(s)$ is a regular expression denoting the language $L(r) \cup L(s)$.

2. $(r)(s)$ is a regular expression denoting the language $L(r)L(s)$.

3. $(r)^*$ is a regular expression denoting $\big(L(r)\big)^*$.

4. (r) is a regular expression denoting $L(r)$. This last rule says that we can add additional pairs of parentheses around expressions without changing the language they denote.

As defined, regular expressions often contain unnecessary pairs of parentheses. We may drop certain pairs of parentheses if we adopt the conventions that:

a) The unary operator $*$ has highest precedence and is left associative.

b) Concatenation has second highest precedence and is left associative.

[1] However, when talking about specific characters from the ASCII character set, we shall generally use teletype font for both the character and its regular expression.

c) | has lowest precedence and is left associative.

Under these conventions, for example, we may replace the regular expression $(\mathbf{a})|((\mathbf{b})^*(\mathbf{c}))$ by $\mathbf{a}|\mathbf{b}^*\mathbf{c}$. Both expressions denote the set of strings that are either a single a or are zero or more b's followed by one c.

Example 3.4: Let $\Sigma = \{a, b\}$.

1. The regular expression $\mathbf{a}|\mathbf{b}$ denotes the language $\{a, b\}$.

2. $(\mathbf{a}|\mathbf{b})(\mathbf{a}|\mathbf{b})$ denotes $\{aa, ab, ba, bb\}$, the language of all strings of length two over the alphabet Σ. Another regular expression for the same language is $\mathbf{aa}|\mathbf{ab}|\mathbf{ba}|\mathbf{bb}$.

3. $\mathbf{a}^*$ denotes the language consisting of all strings of zero or more a's, that is, $\{\epsilon, a, aa, aaa, \ldots\}$.

4. $(\mathbf{a}|\mathbf{b})^*$ denotes the set of all strings consisting of zero or more instances of a or b, that is, all strings of a's and b's: $\{\epsilon, a, b, aa, ab, ba, bb, aaa, \ldots\}$. Another regular expression for the same language is $(\mathbf{a}^*\mathbf{b}^*)^*$.

5. $\mathbf{a}|\mathbf{a}^*\mathbf{b}$ denotes the language $\{a, b, ab, aab, aaab, \ldots\}$, that is, the string a and all strings consisting of zero or more a's and ending in b.

□

A language that can be defined by a regular expression is called a *regular set*. If two regular expressions r and s denote the same regular set, we say they are *equivalent* and write $r = s$. For instance, $(\mathbf{a}|\mathbf{b}) = (\mathbf{b}|\mathbf{a})$. There are a number of algebraic laws for regular expressions; each law asserts that expressions of two different forms are equivalent. Figure 3.7 shows some of the algebraic laws that hold for arbitrary regular expressions r, s, and t.

LAW	DESCRIPTION
$r\|s = s\|r$	| is commutative
$r\|(s\|t) = (r\|s)\|t$	| is associative
$r(st) = (rs)t$	Concatenation is associative
$r(s\|t) = rs\|rt;\ (s\|t)r = sr\|tr$	Concatenation distributes over |
$\epsilon r = r\epsilon = r$	ϵ is the identity for concatenation
$r^* = (r\|\epsilon)^*$	ϵ is guaranteed in a closure
$r^{**} = r^*$	* is idempotent

Figure 3.7: Algebraic laws for regular expressions

3.3.4 Regular Definitions

For notational convenience, we may wish to give names to certain regular expressions and use those names in subsequent expressions, as if the names were themselves symbols. If Σ is an alphabet of basic symbols, then a *regular definition* is a sequence of definitions of the form:

$$\begin{array}{lcl} d_1 & \rightarrow & r_1 \\ d_2 & \rightarrow & r_2 \\ & \cdots & \\ d_n & \rightarrow & r_n \end{array}$$

where:

1. Each d_i is a new symbol, not in Σ and not the same as any other of the d's, and
2. Each r_i is a regular expression over the alphabet $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$.

By restricting r_i to Σ and the previously defined d's, we avoid recursive definitions, and we can construct a regular expression over Σ alone, for each r_i. We do so by first replacing uses of d_1 in r_2 (which cannot use any of the d's except for d_1), then replacing uses of d_1 and d_2 in r_3 by r_1 and (the substituted) r_2, and so on. Finally, in r_n we replace each d_i, for $i = 1, 2, \ldots, n-1$, by the substituted version of r_i, each of which has only symbols of Σ.

Example 3.5: C identifiers are strings of letters, digits, and underscores. Here is a regular definition for the language of C identifiers. We shall conventionally use italics for the symbols defined in regular definitions.

$$\begin{array}{rcl} \mathit{letter\_} & \rightarrow & \texttt{A} \mid \texttt{B} \mid \cdots \mid \texttt{Z} \mid \texttt{a} \mid \texttt{b} \mid \cdots \mid \texttt{z} \mid \texttt{\_} \\ \mathit{digit} & \rightarrow & \texttt{0} \mid \texttt{1} \mid \cdots \mid \texttt{9} \\ \mathit{id} & \rightarrow & \mathit{letter\_} \; (\; \mathit{letter\_} \mid \mathit{digit} \;)^* \end{array}$$

□

Example 3.6: Unsigned numbers (integer or floating point) are strings such as `5280`, `0.01234`, `6.336E4`, or `1.89E-4`. The regular definition

$$\begin{array}{rcl} \mathit{digit} & \rightarrow & \texttt{0} \mid \texttt{1} \mid \cdots \mid \texttt{9} \\ \mathit{digits} & \rightarrow & \mathit{digit} \; \mathit{digit}^* \\ \mathit{optionalFraction} & \rightarrow & \texttt{.} \; \mathit{digits} \mid \epsilon \\ \mathit{optionalExponent} & \rightarrow & (\; \texttt{E} \; (\; \texttt{+} \mid \texttt{-} \mid \epsilon \;) \; \mathit{digits} \;) \mid \epsilon \\ \mathit{number} & \rightarrow & \mathit{digits} \; \mathit{optionalFraction} \; \mathit{optionalExponent} \end{array}$$

is a precise specification for this set of strings. That is, an *optionalFraction* is either a decimal point (dot) followed by one or more digits, or it is missing (the empty string). An *optionalExponent*, if not missing, is the letter E followed by an optional + or − sign, followed by one or more digits. Note that at least one digit must follow the dot, so *number* does not match `1.`, but does match `1.0`.
□

3.3.5 Extensions of Regular Expressions

Since Kleene introduced regular expressions with the basic operators for union, concatenation, and Kleene closure in the 1950s, many extensions have been added to regular expressions to enhance their ability to specify string patterns. Here we mention a few notational extensions that were first incorporated into Unix utilities such as `Lex` that are particularly useful in the specification lexical analyzers. The references to this chapter contain a discussion of some regular-expression variants in use today.

1. *One or more instances.* The unary, postfix operator $^+$ represents the positive closure of a regular expression and its language. That is, if r is a regular expression, then $(r)^+$ denotes the language $\big(L(r)\big)^+$. The operator $^+$ has the same precedence and associativity as the operator $*$. Two useful algebraic laws, $r^* = r^+|\epsilon$ and $r^+ = rr^* = r^*r$ relate the Kleene closure and positive closure.

2. *Zero or one instance.* The unary postfix operator ? means "zero or one occurrence." That is, $r?$ is equivalent to $r|\epsilon$, or put another way, $L(r?) = L(r) \cup \{\epsilon\}$. The ? operator has the same precedence and associativity as $*$ and $+$.

3. *Character classes.* A regular expression $a_1|a_2|\cdots|a_n$, where the a_i's are each symbols of the alphabet, can be replaced by the shorthand $[a_1a_2\cdots a_n]$. More importantly, when $a_1, a_2, \ldots, a_n$ form a logical sequence, e.g., consecutive uppercase letters, lowercase letters, or digits, we can replace them by a_1-a_n, that is, just the first and last separated by a hyphen. Thus, [**abc**] is shorthand for **a**|**b**|**c**, and [**a-z**] is shorthand for **a**|**b**|$\cdots$|**z**.

Example 3.7: Using these shorthands, we can rewrite the regular definition of Example 3.5 as:

$$\begin{array}{rcl} \textit{letter\_} & \rightarrow & \texttt{[A-Za-z\_]} \\ \textit{digit} & \rightarrow & \texttt{[0-9]} \\ \textit{id} & \rightarrow & \textit{letter\_}\ (\ \textit{letter\_} \mid \textit{digit}\ )^* \end{array}$$

The regular definition of Example 3.6 can also be simplified:

$$\begin{array}{rcl} \textit{digit} & \rightarrow & \texttt{[0-9]} \\ \textit{digits} & \rightarrow & \textit{digit}^+ \\ \textit{number} & \rightarrow & \textit{digits}\ (\texttt{.}\ \textit{digits})?\ (\ \texttt{E}\ \texttt{[+-]}?\ \textit{digits}\ )? \end{array}$$

□

3.3.6 Exercises for Section 3.3

Exercise 3.3.1: Consult the language reference manuals to determine (*i*) the sets of characters that form the input alphabet (excluding those that may only appear in character strings or comments), (*ii*) the lexical form of numerical constants, and (*iii*) the lexical form of identifiers, for each of the following languages: (a) C (b) C++ (c) C# (d) Fortran (e) Java (f) Lisp (g) SQL.

! **Exercise 3.3.2:** Describe the languages denoted by the following regular expressions:

a) $\mathbf{a}(\mathbf{a}|\mathbf{b})^*\mathbf{a}$.

b) $((\epsilon|\mathbf{a})\mathbf{b}^*)^*$.

c) $(\mathbf{a}|\mathbf{b})^*\mathbf{a}(\mathbf{a}|\mathbf{b})(\mathbf{a}|\mathbf{b})$.

d) $\mathbf{a}^*\mathbf{b}\mathbf{a}^*\mathbf{b}\mathbf{a}^*\mathbf{b}\mathbf{a}^*$.

!! e) $(\mathbf{aa}|\mathbf{bb})^*((\mathbf{ab}|\mathbf{ba})(\mathbf{aa}|\mathbf{bb})^*(\mathbf{ab}|\mathbf{ba})(\mathbf{aa}|\mathbf{bb})^*)^*$.

Exercise 3.3.3: In a string of length n, how many of the following are there?

a) Prefixes.

b) Suffixes.

c) Proper prefixes.

! d) Substrings.

! e) Subsequences.

Exercise 3.3.4: Most languages are *case sensitive*, so keywords can be written only one way, and the regular expressions describing their lexemes are very simple. However, some languages, like SQL, are *case insensitive*, so a keyword can be written either in lowercase or in uppercase, or in any mixture of cases. Thus, the SQL keyword `SELECT` can also be written `select`, `Select`, or `sElEcT`, for instance. Show how to write a regular expression for a keyword in a case-insensitive language. Illustrate the idea by writing the expression for "select" in SQL.

! **Exercise 3.3.5:** Write regular definitions for the following languages:

a) All strings of lowercase letters that contain the five vowels in order.

b) All strings of lowercase letters in which the letters are in ascending lexicographic order.

c) Comments, consisting of a string surrounded by `/*` and `*/`, without an intervening `*/`, unless it is inside double-quotes (").

!! d) All strings of digits with no repeated digits. *Hint*: Try this problem first with a few digits, such as $\{0, 1, 2\}$.

!! e) All strings of digits with at most one repeated digit.

!! f) All strings of a's and b's with an even number of a's and an odd number of b's.

g) The set of Chess moves, in the informal notation, such as `p-k4` or `kbp×qn`.

!! h) All strings of a's and b's that do not contain the substring abb.

i) All strings of a's and b's that do not contain the subsequence abb.

Exercise 3.3.6: Write character classes for the following sets of characters:

a) The first ten letters (up to "j") in either upper or lower case.

b) The lowercase consonants.

c) The "digits" in a hexadecimal number (choose either upper or lower case for the "digits" above 9).

d) The characters that can appear at the end of a legitimate English sentence (e.g., exclamation point).

The following exercises, up to and including Exercise 3.3.10, discuss the extended regular-expression notation from `Lex` (the lexical-analyzer generator that we shall discuss extensively in Section 3.5). The extended notation is listed in Fig. 3.8.

Exercise 3.3.7: Note that these regular expressions give all of the following symbols (*operator characters*) a special meaning:

```
\ " . ^ $ [ ] * + ? { } | /
```

Their special meaning must be turned off if they are needed to represent themselves in a character string. We can do so by quoting the character within a string of length one or more; e.g., the regular expression `"**"` matches the string `**`. We can also get the literal meaning of an operator character by preceding it by a backslash. Thus, the regular expression `\*\*` also matches the string `**`. Write a regular expression that matches the string `"\`.

Exercise 3.3.8: In `Lex`, a *complemented character class* represents any character except the ones listed in the character class. We denote a complemented class by using `^` as the first character; this symbol (caret) is not itself part of the class being complemented, unless it is listed within the class itself. Thus, `[^A-Za-z]` matches any character that is not an uppercase or lowercase letter, and `[^\^]` represents any character but the caret (or newline, since newline cannot be in any character class). Show that for every regular expression with complemented character classes, there is an equivalent regular expression without complemented character classes.

EXPRESSION	MATCHES	EXAMPLE
c	the one non-operator character c	`a`
$\backslash c$	character c literally	`\*`
`"`s`"`	string s literally	`"**"`
`.`	any character but newline	`a.*b`
`^`	beginning of a line	`^abc`
`$`	end of a line	`abc$`
$[s]$	any one of the characters in string s	`[abc]`
$[\hat{}\,s]$	any one character not in string s	`[^abc]`
$r*$	zero or more strings matching r	`a*`
$r+$	one or more strings matching r	`a+`
$r?$	zero or one r	`a?`
$r\{m,n\}$	between m and n occurrences of r	`a{1,5}`
r_1r_2	an r_1 followed by an r_2	`ab`
$r_1 \mid r_2$	an r_1 or an r_2	`a|b`
(r)	same as r	`(a|b)`
r_1/r_2	r_1 when followed by r_2	`abc/123`

Figure 3.8: `Lex` regular expressions

! **Exercise 3.3.9:** The regular expression $r\{m,n\}$ matches from m to n occurrences of the pattern r. For example, `a{1,5}` matches a string of one to five a's. Show that for every regular expression containing repetition operators of this form, there is an equivalent regular expression without repetition operators.

! **Exercise 3.3.10:** The operator `^` matches the left end of a line, and `$` matches the right end of a line. The operator `^` is also used to introduce complemented character classes, but the context always makes it clear which meaning is intended. For example, `^[^aeiou]*$` matches any complete line that does not contain a lowercase vowel.

a) How do you tell which meaning of `^` is intended?

b) Can you always replace a regular expression using the `^` and `$` operators by an equivalent expression that does not use either of these operators?

! **Exercise 3.3.11:** The UNIX shell command `sh` uses the operators in Fig. 3.9 in filename expressions to describe sets of file names. For example, the filename expression `*.o` matches all file names ending in `.o`; `sort1.?` matches all filenames of the form `sort1.`c, where c is any character. Show how `sh` filename