PYL 100 : Tut. Sheet # 4

Reflection / Refraction, Applications

1. A 100 GHz em wave normally impinged on a collisionless plasma suffers 100% reflection with phase change of $\pi/3$. Estimate ω_p.
[Ans. $\omega_p = 2\omega/\sqrt{3} = 7.3\times10^{11}$ rad/s.]

2. An em wave of frequency ω propagates through a semiconductor with electron-phonon collision frequency $\nu < \omega$. Taking the average energy loss of an electron in a collision as $(3\delta/2)(T_e - T_o)$, where T_e is the electron temperature and T_o is the lattice temperature, estimate T_e.
[Ans. $T_e = T_o\,(1 + e^2|E|^2/3\delta m\omega^2 T_o)$.]

3. In an overdense plasma laser field is
$\vec{E} = \hat{x}A_o e^{-\alpha z} e^{-i\omega t}$, $\alpha = (\omega/c)(\omega_p^2/\omega^2 - 1)^{1/2}$.
Obtain the time average ponderomotive force $\vec{F}_p = \frac{1}{2}\mathrm{Re}[-e\vec{v}\times\vec{B}^*]$ on an electron.
[Ans. $\vec{F}_p = -(e^2/4m\omega^2)\nabla|E|^2$.]

4. A laser of intensity I is normally incident on a collisionless overdense plasma. Show that the total ponderomotive force on electrons per unit area equals $2I/c$.

5. A coaxial cable comprises a hollow cylinder of inner and outer radii r_1, r_2 filled with a dielectric of relative permittivity ϵ_r. A TEM mode propagates through the cable along its axis $\hat{z}$ with $E_r, H_\phi \sim e^{-i(\omega t - kz)}$. Deduce the r dependence of E_r, H_ϕ assuming them to be independent of ϕ and $k = (\omega/c)\,\epsilon_r^{1/2}$.

Hint: The wave eq. for E_x is $\nabla^2 E_x + (\omega^2/c^2)\,\epsilon_r E_x = 0$. Take $E_x = E_r(r)\cos\phi$, $\nabla^2 = \partial^2/\partial r^2 + (1/r)\,\partial/\partial r + (1/r^2)\,\partial^2/\partial\phi^2 + \partial^2/\partial z^2$ and deduce an eq. for E_r. Try $E_r \sim r^l$ and deduce l. [$E_r, H_\phi \sim 1/r$.]

6. An em wave of low freq. is normally incident on the earth. Plot E/H as a function of ω.

7. A s-polarized em wave is obliquely incident on a dielectric - vacuum interface ($z=0$) from dielectric of refractive index $\eta = 3$. The transmitted field in vacuum is $\vec{E}_T = \hat{y}\, A\, e^{-2\omega z/c}\, e^{-i(\omega t - \sqrt{5}\,\omega x/c)}$.

Deduce the incident field. [$\vec{E}_i = \hat{y}\,\frac{1+i}{2}\, A\, e^{-i(\omega t - \sqrt{5}\frac{\omega x}{c} - \frac{2\omega z}{c})}$.]

8. A surface wave propagates over a metal - vacuum interface ($x=0$) with effective relative permittivity of metal $\epsilon_m = 9 - \omega_p^2/\omega^2$. Plot field amplitude as a function of x in vacuum and metal for $\omega = \omega_p/\sqrt{11}$ and $\omega = \omega_p/\sqrt{14}$.

9. Estimate the intensity of laser that would impart electrons an oscillatory velocity $|v| \sim 0.1\,c$ when $\omega = \sqrt{2}\,\omega_p$.