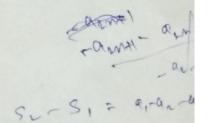
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Department of Mathematics MTL 100: Calculus 2016-17: Semester I Minor 1

30 August 2016



Please begin the answer to each question on a new page, and give adequate explanation for full credit.

- Let A and B be two nonempty sets of real numbers such that $A \cup B = (0,1)$. Prove that this implies $(\inf A) \cdot (\inf B) = 0$, or give an example of sets A and B for which $(\inf A) \cdot (\inf B) > 0$.
- 2. If $|a_n| < 2$ and $|a_{n+2} a_{n+1}| \le \frac{1}{8} |a_{n+1}^2 a_n^2|$ for $n \ge 1$, prove that the sequence $\{a_n\}$ converges.
 - 3. Discuss the convergence for the following two series:
 - (a) $\sum_{n=1}^{\infty} n^{-1-1/n}$.
 - (b) $\sum_{n=2}^{\infty} (\ln n)^p$, where p < 0.

[2+2]

4 Prove that an alternating series $\sum (-1)^{n-1}a_n$ converges if $\{a_n\}$ is a nonincreasing sequence of positive real numbers that tends to 0. Further prove that if the series converges to S, then

$$\left| S - S_n \right| < a_{n+1}$$

for each
$$n \ge 1$$
 and $S_n = \sum_{i=1}^n (-1)^{i-1} a_i$.

[4]

S. Let $f:[0,1] \to [0,1]$ be given by

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational;} \\ 1 - x & \text{if } x \text{ is irrational.} \end{cases}$$

Discuss the continuity of f in [0,1] giving an $\epsilon - \delta$ argument

[4]