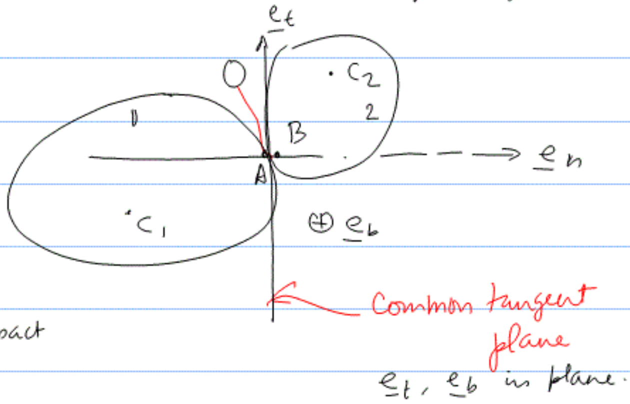
Note Title 15-10-2016

Impact of Rigid bodies

1 General smooth impact of two unconstrained rigid objects.



Contact at one point.

Common points A, (on 1)
B (on 2)

O fixed to space

Coincident at the time of impact

$\underline{e_n}$ → normal to the tangent plane (at O)

The line along $\underline{e_n}$ through O is called the line of impact.

$\underline{e_t}$, $\underline{e_b}$ in the tangent plane;

Central impact: C_1, C_2 on the line of impact

non-central impact -"- not on -"- -"-

Impact - assumed to happen in zero time

$\underline{\omega_i}$ → angular velocity of body i

3+3 $\underline{V_{ci}}$ → velocity of centre of mass of body i

$\underline{\omega_i'}$ → ang. vel. (after impact) of body i

3+3 $\underline{V_{ci}'}$ - vel. of C.O.M. (-"-) -"- -"-

12 unknowns.

→ Momenta for each body in direction $\underline{e_t}$, $\underline{e_b}$ are conserved (Since the external impulse to each body is along $\underline{e_n}$ only (smooth impact)).

$$V_{1t}' = V_{1t} \; ; \; V_{1b}' = V_{1b} \; ; \quad V_{2t}' = V_{2t} \; ; \; V_{2b}' = V_{2b}. \qquad \text{— ⓐ}$$

$\underline{V}_c = \underline{V}_{1n}\, \underline{e}_n + \underline{V}_{1t}\, \underline{e}_t + \underline{V}_{1b}\, \underline{e}_b$; and so on.

→ No no net external impulse in $\underline{e}_n$ direction also for the system of two bodies. ⇒ momentum along $\underline{e}_n$ for the system is conserved.

$$m_1 V_{1n}' + m_2 V_{2n}' = m_1 V_{1n} + m_2 V_{2n} \qquad \text{—— ⓑ}$$

$\underline{H}_0$ of each object is conserved since the external angular impulse about the fixed point O is zero. (impulse is through O)

$$\underline{H}_0' = \underline{H}_{C_1}' + \underline{r}_{C_1 0} \times m \underline{V}_{C_1}' = \underline{H}_{C_1} + \underline{r}_{C_1 0} \times m \underline{V}_{C_1}$$

⇒ from ⓐ

$$\underline{H}_{C_1}' + \underline{r}_{C_1 0} \times m V_{1n}' \underline{e}_n = \underline{H}_{C_1} + \underline{r}_{C_1 0} \times m V_{1n} \underline{e}_n$$

$$\underline{H}_{C_2}' + \underline{r}_{C_2 0} \times m \underline{V}_{C_2}' = \underline{H}_{C_2} + \underline{r}_{C_2 0} \times m \underline{V}_{C_2}$$

$$\underline{H}_{C_2}' + \underline{r}_{C_2 0} \times m V_{2n}' \underline{e}_n = \underline{H}_{C_2} + \underline{r}_{C_2 0} \times m V_{2n} \underline{e}_n \qquad ⓒ$$

ⓒ → 6 eqns; ⓐ — 4 eqns. ⓑ — 1 eqn ⇒ 11 eqns

12 unknowns! ⇒ One equation short

Define an empirical parameter called the co-efficient of restitution.

$$e = \frac{V_s}{V_a} = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

just after impact

from A towards B along $\underline{e}_n$.

just before impact

$$e = -\frac{(V_{Bn}' - V_{An}')}{(V_{Bn} - V_{An})}$$

Note: These are defined at A & B; not at C_1, C_2.

$$v'_{An} = \underline{v}'_A \cdot \underline{e}_n = (\underline{v}'_{C_1} + \underline{\omega}'_1 \times \underline{r}_{AC_1}) \cdot \underline{e}_n$$

$$v'_{Bn} = \underline{v}'_B \cdot \underline{e}_n = (\underline{v}'_{C_2} + \underline{\omega}'_2 \times r_{BC_2}) \cdot \underline{e}_n \quad \text{— (e)}$$

$$v_{An} = \underline{v}_A \cdot \underline{e}_n = (\underline{v}_{C_1} + \underline{\omega}_1 \times r_{AC_1}) \cdot \underline{e}_n$$

$$v_{Bn} = \underline{v}_B \cdot \underline{e}_n = (\underline{v}_{C_2} + \underline{\omega}_2 \times \underline{r}_{BC_2}) \cdot \underline{e}_n$$

If body 2 is massive; $m_2 \cong \infty \Rightarrow \underline{\omega}'_2 = \underline{\omega}_2$,
$\underline{v}'_{C_2} = v_{C_2}$; $v'_{Bn} = v_{Bn}$

(b) and (c) do not yield non-trivial equations.

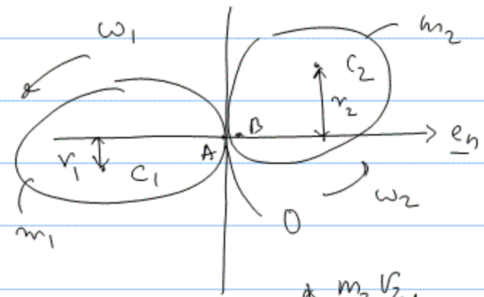
Six scalar unknowns of body 1 ($\underline{v}'_{C_1}$ & $\underline{\omega}'_1$)
are obtained from (a), (c) (for body (1)) and (d)

$v'_{1t} = v_{1t}$, $v'_{1b} = v_{1b}$; $\underline{H}'_{C_1} + r_{C_1O} \times m v'_{1n} \underline{e}_n = \underline{H}_{C_1} + r_{C_1O} \times m v_{1n} \underline{e}_n$

$(v'_{Bn} - v'_{An}) = e\,(v_{Bn} - v_{An})$ (1+1+3+1 eqn).

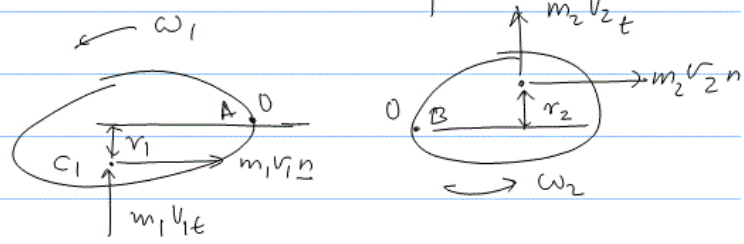
If the massive body (2) is at rest; then:
$v'_{An} = -e\, v_{An}$ (A is the point of impact on 1).

Smooth impact of two unconstrained rigid bodies in plane motion at a point.



6 post impact unknowns. (scalars)
$v'_{1n}, v'_{1t}, v'_{2n}, v'_{2t}, \omega_1, \omega_2$

$\Rightarrow$ 6 eqns needed.



ⓐ, ⓑ ⇒

$v_{1t}' = v_1,\quad v_{2t}' = v_2,\quad m_1 v_{1n}' + m_2 v_{2n}' = m_1 v_{1n} + m_2 v_{2n}$ — ⓐ'

(FOR THE CONFIGURATION SHOWN!)

ⓒ, ⓓ.

$$\left.\begin{aligned} I_{33}^{C_1}\,\omega_1' + m v_{1n}' r_1 &= I_{33}^{C_1}\,\omega_1 + m v_{1n} r_1 \\ I_{33}^{C_2}\,\omega_2' - m v_{2n}' r_2 &= I_{33}^{C_2}\,\omega_2 - m v_{2n} r_2 \end{aligned}\right\} \text{ⓑ}'$$

$(v_{Bn}' - v_{An}') = -e\,(v_{Bn} - v_{An}) \Rightarrow$

$$\left[(v_{2n}' + \omega_2' r_2) - (v_{1n}' - \omega_1' r_1)\right] = -e\left[(v_{2n} + \omega_2 r_2) - (v_{1n} - \omega_1 r_1)\right].$$

ⓒ'

If body 2 is a massive body. $m_2 = \infty$, $\underline{\omega_2'} = \underline{\omega_2}$; $\underline{v_{C_2}'} = v_{C_2}$.

$v_{Bn}' = v_{Bn}$.

Then ⓐ' for the momentum of the system of bodies;

and ⓑ' for body ② are not non-trivial equations.

3 scalar unknowns $\underline{v_{A_1}'}\ \to 2$; $\underline{\omega_1'}$ (1) ⇒ (3 unknowns)

are determined from ⓐ first part (for body 1).

ⓑ' and ⓒ'

↑ for body ①.

$$v_{1t}' = v_{1t};\quad I_{33}^{C_1}\,\omega_1' + m_1 v_{1n}' r_1 = I_{33}^{C_1}\,\omega_1 + m_1 v_{1n} r_1$$

$$\left[(v_{2n}' + \omega_2' r_2) - (v_{1n}' - \omega_1' r_1)\right] = -e\left[(v_{2n} + \omega_2 r_2) - (v_{1n} - \omega_1 r_1)\right]$$

body ② — massive. & at rest → last eqn ⇒

$$(v_{1n}' - \omega_1' r_1) = -e\,(v_{1n} - \omega_1 r_1).$$