DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MTL 100: CALCULUS

TOTAL MARKS: 20   MINOR-I   TIME: 1 HOUR

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(1) Let $f(x) = \sqrt{1+x}$.   [3+2=5]

     (a) Write the Taylor's polynomial $P_2(x)$ of order 2 about 0 of the function $f(x)$.

     (b) Find the maximum error while approximating $f(x)$ by $P_2(x)$ in the interval $(-0.5, 0.5)$.

(2) Test the convergence/divergence of the following infinite series.   [3+3=6]

     (a) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n \ln(\ln n)}$.

     (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{20}}{20^n}$.

(3) Let $\{x_n\}$ be a sequence of real numbers such that $\displaystyle\lim_{n\to\infty} (x_{n+1} - x_n) = 5$. Then find the following limits (if exist):   [2+2=4]

     (a) $\displaystyle\lim_{n\to\infty} \frac{x_{n+1} - x_1}{n}$.

     (b) $\displaystyle\lim_{n\to\infty} \frac{x_n}{n \log n}$.

(4) Prove or disprove the uniform continuity of the following functions.   [3+2=5]

     (a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$.

     (b) $g : \mathbb{R} \to \mathbb{R}$, $g(x) = x \sin x$

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