

Note:

- a) Attempt any 8 questions. All question carry 5 marks
- b) Only 10% Marks are allocated towards the correctness of the process and technique used to solve the problem

Q1. $x[n]$ is a real valued causal sequence with discrete time Fourier Transform $X(e^{j\omega})$. Determine $x[n]$ if the imaginary part of $X(e^{j\omega})$ is given by : $\text{Im} \{ X(e^{j\omega}) \} = 3 \sin(2\omega) - 2 \sin(3\omega)$.

Q2. The following information is known about a discrete time LTI system with input $x[n]$ and output $y[n]$.

- a) If $x[n] = (-2)^n$ for all n , then $y[n] = 0$ for all n
- b) If $x[n] = (1/2)^n u[n]$ for all n , then $y[n]$ for all n is of the form $y[n] = \delta[n] + a(1/4)^n u[n]$, where "a" is a constant
- c) Determine the value of the constant "a"
- d) Using z transform, determine the response $y[n]$ if the input $x[n] = 1$ for all n .

Q3. For an LTI system the input $x[n]$ and output $y[n]$ are related by the difference equation: $y[n-1] - (5/2)y[n] + y[n+1] = x[n]$. This system may or may not be stable or causal. Considering the pole-zero pattern associated with this difference equation, determine three possible choices of the unit impulse response of the system.

Q4. Using Laplace Transform technique solve the initial value problem given below:

$$\frac{d^2 y(t)}{dt^2} + 4 \frac{dy(t)}{dt} = \cos(t-3) + 4t \quad y(3)=0 \quad y'(3)=7$$

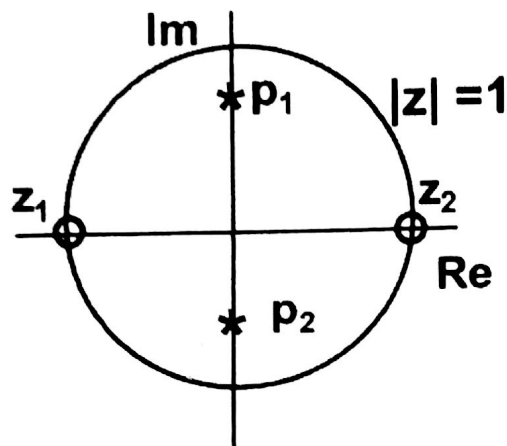
Q5. Using the Laplace transform solve the third order initial value problem given below:

$$\frac{d^3 y(t)}{dt^3} - \frac{d^2 y(t)}{dt^2} + \frac{dy(t)}{dt} - y = 0$$

Where $y(0) = 1$, $dy(t)/dt = 0$ and $d^2 y(t)/dt^2 = -1$

Q6. The pole zero configuration of a filter are shown in fig.1. The zeros's are located on the unit circle $z = \pm 1$. The poles are complex conjugate located at $\pm 0.80365j$. Find
a) predict heuristically the typical frequency response expected from the pole zero configuration, b) find the transfer function $H(z)$ c) Find the difference equation describing the system behavior as a causal and stable system, d) If the sampling frequency $\omega_s = 1000\pi$, predict the centre frequency of the filter and bandwidth

defined between $\pm\pi/8$, c) draw the estimated frequency response from 0 to 2π (ω_s). f) what is magnitude of frequency response at $\pi/2$.



Q7. We have the following information about a signal $x[n]$:

- $x[n]$ is a real and even signal
- $x[n]$ has a period $N=10$ and Fourier coefficients a_k .
- $a_{11}=5$
- $\frac{1}{10} \sum_{n=0}^9 |x[n]|^2 = 50$

Show that $x[n] = A \cos(Bn + C)$. Find the values of A, B and C

Q8. We are given the following information about the real signal $x(t)$ with Laplace transform $X(s)$.

- $X(s)$ has exactly two poles
- $X(s)$ has no zeros in the finite s plane
- $X(s)$ has a pole at $s = -1 + j$
- $e^{2t} x(t)$ is not absolutely integrable
- $X(0) = 8$

Determine $X(s)$ and find its region of convergence.

Q9. Consider an input $x[n]$ and the unit impulse response $h[n]$ as given below:

$$x[n] = (1/2)^{n-2} u[n-2] \text{ and } h[n] = u[n+2]$$

what is the convolved output $y[n] = x[n] * h[n]$?