DEPARTMENT OF MATHEMATICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

MTL 100: CALCULUS

TOTAL MARKS: 20   MINOR-II   TIME: 1 H

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(1) (a) Find all values of $x \in \mathbb{R}$ for which the following power series converges:

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{2n+1}}.$$

(b) Using Gamma function, evaluate the following integral:

$$\int_0^1 x(\log x)^4 dx.$$

(2) (a) Let $a_n = \dfrac{1}{n^{\frac{5}{2}}} \sum_{i=1}^{n} i^{\frac{3}{2}}$. Determine $\lim\limits_{n \to \infty} a_n$ by expressing a_n as a Riemann sum o suitable function.

(b) Let f be such that $x \cos(\pi x) = \int_0^{x^2} f(t)dt$ for any $x > 0$. Find $f(4)$.

(3) Find all values of $p \in \mathbb{R}$ for which the following improper integral converges.

$$\int_0^{\infty} \frac{x^p e^{-x}}{\log(1+x)} dx.$$

(4) Test the convergence of the following improper integral: [2+

$$\int_0^{\infty} \cos(tx) e^{-\frac{x^2}{2}} dx \text{ for } t > 0.$$

Also, prove that

$$\int_0^{\infty} \cos(tx) e^{-\frac{x^2}{2}} dx = \sqrt{\frac{\pi}{2}} e^{-\frac{t^2}{2}}.$$

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