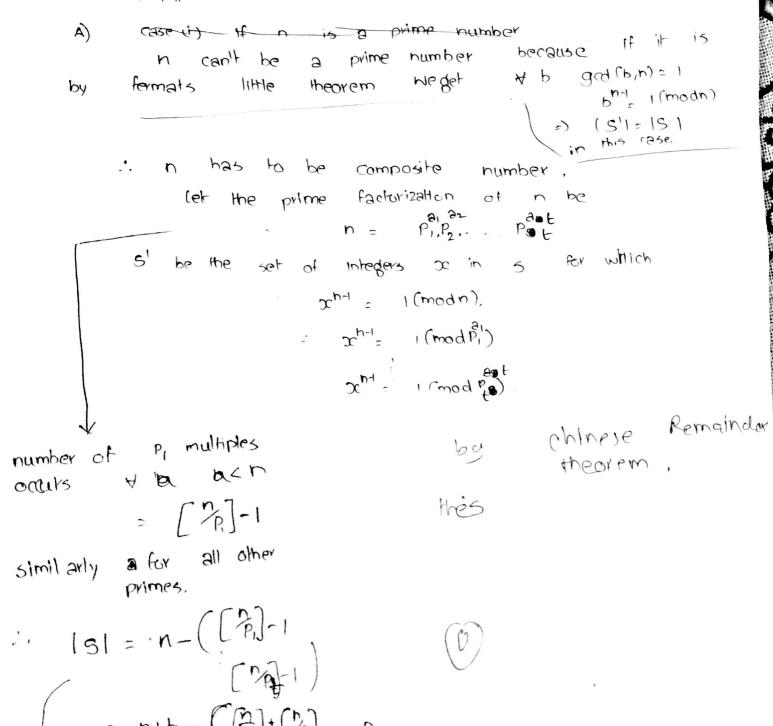


- 2. You are NOT allowed to leave the exam hall during the exam period unless on medical emergency.
- 3. Calculators and phones are NOT allowed.
- 4. You are NOT allowed to ask any questions during the exam. If in doubt, make (and state) your assumptions.
- Q1. (6 marks) Let n be a positive integer. Show that given any set of n+2 distinct integers, either there are two integers in the set whose sum is divisible by 2n, or there are two integers in the set whose difference is divisible by 2n.
- divisible by an the A) when number Go {0,1,2..., 2n-1}.= S remainders ave If 2 any two of the given set of n+2 Case 1) same remainder with distinct integers give take difference of can then which is divisible by 2n. a= r (mod 2n) = a-b = r (mod 2n) = o (mod (2n)) give distinct remainders organise the set of possible caseli) all n+2 Allows. , left remainders ( (a), (1,2n-1), (2,2n-2).... (n-1,n+1), n n+1 entityles

numbers Es are divided by an the terms distinct uhen (n+2' in number) left by them are pigeons and nainders entityles (n+1) entitles are holes. As all remainders above the (n+2)th number's remainder must be distinct

are Scanned by CamScanner 2n/a+b

Q2. (8 marks) Recall that a positive integer m is a Carmichael number if  $b^{m-1} = 1 \pmod{m}$  for all integers b, such that 1 < b < m and gcd(b, m) = 1. Let n be a positive integer which is NOT a Carmichael number. Let S denote the set of integers b such that gcd(b, n) = 1 and 1 < b < n. Let S' be the set of integers x in S for which  $x^{n-1} = 1 \pmod{n}$ . Prove that  $|S'| \le |S|/2$ .



Q3. (6 marks) Let S be a set of positive real numbers which have the following property: countable (you can use the fact that the union of a countable number of countable sets is countable).

i) If s is influ Anthe A) HIVI at set then 2) Let s be infinite Anite every With set A of s, the numbers Sum ct S, take two finete subsets of Ap, Aq cot 921, -- -. , 2p3 = Ap {b<sub>1</sub>, ..., b<sub>q</sub>} = Aq 5 b1 5 1 2 ai <1 map all by to 249 ai to 6 /2 ki map such that Kp and only if aliebi let all elements All Anlly subsels take We 51  $\frac{2}{1}$   $\frac{2}{2}$   $\frac{2}{1}$   $\frac{2}{2}$   $\frac{2}$ SHIL S= AiUA, ... shows