1. Describe the row space, column space and null space, and also find their dimensions, for the following matrices.

$$A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}.$$

2. Describe the row space, column space and null space, and their dimensions, for the following matrices.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \end{pmatrix}.$$

3. Let $V$ be the subspace spanned by $(1,1,1)$ and $(2,1,0)$. Find a matrix $A$ that has $V$ as its row space. Find a matrix B that has $V$ as its null space.

4. Construct a matrix with $(1,0,1)$ and $(1,2,0)$ as a basis for its row space and its column space. Can this be a basis for the null-space of the matrix?

5. Construct a matrix whose null space contains $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, column space contains $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

6. Without multiplying matrices, find bases for the row and column spaces of matrix

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 2 & 7 \end{pmatrix} \begin{pmatrix} 3 & 0 & 3 \\ 1 & 1 & 2 \end{pmatrix}.$$

From rank argument, conclude $A$ cannot be invertible?

7. If a 7 by 9 matrix $A$ has rank 5, what are the dimensions of the row space of $A$, column space of $A$, null space of $A$, and null space of $A^T$? What is the sum of all four dimensions?

8. Compute a basis of row space and a basis of column space of the following matrices

$$A = \begin{pmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{pmatrix}$$

9. Find a basis for the row space, column space, and null space of the matrix given below:

$$\begin{pmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{pmatrix}.$$

10. What is the maximum number of linearly independent vectors that can be found in the null space of matrix

$$\begin{pmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{pmatrix}.$$