PYL100: Electromagnetic Waves and Quantum Mechanics (II Semester, 2016-17)

<u>Exercise Sheet No. 6</u>

Part 2: QM basics -- Wave function, Probability, Schrödinger Eq., Uncertainty principle

1. The wave function of a particle at t =0 is given by: $\psi(x) = Ce^{-\alpha^2 x^2}\ (-\infty < x < \infty)$ where C and α are constants. Calculate the probability of finding the particle in the region $(0 < x < \infty)$.

   [**Ans.** 1/2]

2. The wave function of the particle at a certain instant is given as

$$\psi(x) = A\exp\left(-\frac{x^2}{a^2} + ikx\right)$$

   If P_1 and P_2 denote the probability densities of finding the particle in the range a to a+da and 2a to 2a+da respectively. Find the ratio $P_1 : P_2$. [**Ans.** e^6]

3. Check whether the following functions represents acceptable wave function of a particle in the range $(-\infty < x < \infty)$ or not?

   (a) $\Psi(x) = 3\sin \pi x$ (b) $\Psi(x) = 4 - |x|$
   (c) $\Psi(x) = x^2$ (d) $\Psi(x) = 7\tan x$

   [**Ans.** Only (a)]

4. An one-dimensional wave function of a particle is given as

$$\psi(x) = Ae^{-x^2/a^2 + ikx}\ (A \text{ is the normalization constant})$$

   Calculate the expectation value of position and momentum of the particle.

   [**Ans.** (a) 0, (b) $\hbar k$]

5. Consider a system whose state is given in terms of an orthonormal set of three wave functions: $\Phi_1(x),\ \Phi_2(x),\ \Phi_3(x)$ as:

$$\Psi(\mathrm{x}) = \frac{\sqrt{3}}{3}\Phi_1 + \frac{2}{3}\Phi_2 + \frac{\sqrt{2}}{3}\Phi_3.$$

   (a) Verify that Ψ is normalized.
   (b) Calculate the probability of finding the system in any one of the states $\Phi_1,\ \Phi_2,\ \Phi_3$. Verify that the total probability is equal to one.
   (c) Consider now an ensemble of 810 identical systems, each one of them in the state Ψ. If measurements are done on all of them, how many systems will be found in each of the states Φ_1, $\Phi_2,\ \Phi_3$? [**Ans.** (b) $P_1 = \frac{1}{3}, P_2 = \frac{4}{9}, P_1 = \frac{2}{9}$; (c) $N_1 = 270, N_2 = 360, N_3 = 180$]

6. The wave function corresponding to a particle is given by:

$$\psi(x) = \frac{1}{\sqrt{a}} \exp\left(-\frac{|x|}{a}\right)$$

(a) Calculate the probability of finding the particle in the region $-a < x < a$.
(b) Find the value of 'b' such that probability of finding the particle between $-b < x < b$ is 0.5.

[**Ans.** (a) $1-\frac{1}{e^2}$, (b) $\frac{a\ln 2}{2}$]

7. In a region of space, a particle with mass m and with zero energy has a time-independent wave function given by

$$\psi(x) = Ae^{-x^2/L^2}$$

where A and L are constants. Determine the potential energy U(x) of the particle.

Ans: $U(x) = \frac{2\hbar^2}{mL^4}(x^2 - \frac{3L^2}{2})$

8. An electron is described by the wave function

$$\psi(x) = \begin{cases} Ce^{-x}(1-e^{-x}); x>0 \\ 0; x<0 \end{cases}$$

where x is in nm and C is a constant.

(a) Determine the value of C that normalizes ψ(x). [**Ans:** $2\sqrt{3}$ $nm^{-1/2}$]

(b) Where is the electron most likely to be found? That is, for what value of x is the probability of finding the electron the largest? [**Ans:** 0.693 nm]

(c) Calculate the average position $\langle x \rangle$ for the electron. [**Ans:** 1.083 nm]