2½

COL 202 | Minor 1 | Aug 28

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Q1. Let S be the set of students in a class. Each pair of them play a game exactly once (assume that one of the player always wins, there is no possibility of draw in the game). Write the following logical statements using quantifiers. All quantifiers should appear at the beginning of the sentence. You can use the proposition $C(x,y)$ which denotes x defeated y in the game, and $G(x)$ which denotes x is a girl. You can use the usual logical operators $(\wedge, \vee, =, \neq, \rightarrow, \neg)$.

(i) No girl lost to a boy. **(1 mark)**

$(\forall x \forall y) \in S \quad (G(x) \wedge (\neg G(y))) \longrightarrow (C(x,y))$

(ii) There is exactly one girl who defeated all the boys. **(2 marks)**

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$(\exists x_1 \forall y \forall z) \in S \ (G(x) \wedge (\neg G(y)) \wedge C(x,y)) \wedge ((G(z) \wedge (\neg G(z)) \wedge C(x,y)) \longrightarrow (y = x_1))$

(iii) There are two students such that everyone else lost to at least one of these two students. **(2 marks)**

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$(\exists x_1, x_2 \ \forall y \ \exists z) \in S \quad (C(x_1,y) \vee C(x_2,y)) \wedge (C(z, \ldots)) \rightarrow ((z = x_1) \vee (z = x_2))$

z_1

0

(iv) No two boys lost to the same girl. **(2 marks)**

$(\forall x \forall y \forall z) \in S \quad (G(x) \wedge (\neg G(y)) \wedge (\neg G(z))) \wedge (C(x,y) \wedge \ldots \longrightarrow (y = z)$

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Q2 **(6 marks)** You play a game where you start with the number 18. At each step, you perform one of the following 2 operations:

- Multiply the current number by 2.
- Multiply the current number by 3.
- Divide the current number by 2 (if it is even)
- Divide the current number by 3 (if it is divisible by 3)

For example, you can divide 18 by 2 in Step 1 to get 9. In Step 2, you could multiply it by 3 to get 27, and so on. Prove that you will never get the number 96 after 60 steps.

A)

$18 = 2 \times 3^2$

$96 = 2^5 \times 3$

every n^{th} step can be denoted by multiply $\times (2^{x_n} \cdot 3^{y_n})$

where (x_n, y_n) can be $(1,0)$, $(0,1)$, $(-1,0)$, $(0,-1)$

$\therefore$ after 60 steps we have,

$96 = 18 \times (2^{x_1} \cdot 3^{y_1}) \times (2^{x_2} \cdot 3^{y_2}) \dots (2^{x_{60}} \cdot 3^{y_{60}})$

$2^5 \times 3 = 2 \times 3^2 \times (2^{x_1} \cdot 3^{y_1}) \times (2^{x_2} \cdot 3^{y_2}) \dots (2^{x_{60}} \cdot 3^{y_{60}})$

$\therefore$ 2's power should increase by 4
3's power should decrease by 1

formal proof

there should be atleast 4 $\times (2^1 \cdot 3^0)$ steps and 1 $\times (2^0 \cdot 3^{-1})$ step

remaining steps are 55 to get the final answer as 96 from 18

for every extra ~~(1x...)~~ $(2^1 \cdot 3^0)$ step we must have one extra $(2^{-1} \cdot 3^0)$ step and vice versa

similarly for $(2^0 \cdot 3^1)$ and $(2^0 \cdot 3^{-1})$

... last

n^{+}

n
$n-1$
$\le 2n-2$

Q3. (6 marks) Let n be a positive integer. You are given a set of $n+1$ distinct positive integers, each of which is less than or equal to $2n$. Use mathematical induction to show that there is at least one integer in this set that divides another integer in this set.

A) for $(n=1)$ we have $n+1 = 2$ $2n = 2$

$\therefore$ possible +ve integer sets are. (with distinct numbers)

is (1, 2)

in each of these cases atleast one divides other.

let us assume this to be true for $1 \le r \le n$

now we have to prove for $n+1$ case.

we have $n+2$ distinct +ve integer each

$\le 2(n+1)$.

NOW ~~we~~ ~~remove~~

let us consider two numbers $(2n+1, 2n+2)$

se(i):- If the maxima of the $n+2$ distinct +ve inte

is either of $2n+1$ or $2n+2$ but not both

are present ~~then~~, we remove that number from the set

$n+2$. Now we have $n+1$ distinct numbe

with maximum $\le 2n$. By above assum

(~~Wali~~ we can find at least one number n

this set that divides other.

e(ii):- If the $(n+2)$ distinct numbers contain both

$(2n+1$ and $2n+2)$ we remove both numb

but we are left with n numbers ≤ 2

If we continue reducing n by removing

the maximum number or numbers

numbers