PYL100: Semester II (2016-17)

Exercise Sheet # 2

Magnetostatics and Faraday's Law

1. A thick long wire of radius R and length along $\hat{z}$ carries current of uniform current density $\vec{J} \parallel \hat{z}$. Using Ampere's law deduce the magnetic field $\vec{B}$, hence, vector potential $\vec{A}$ for (i) $r < R$ and (ii) $r > R$. [Ans: (i) $r < R$: $B_\phi = \frac{\mu J r}{2}, A_z = \frac{-\mu J r^2}{4}$ (ii) $r > R$: $B_\phi = \frac{\mu J R^2}{2r}, A_z = \frac{-\mu J R^2}{4}(1 + 2ln\,(r/R))$]
2. An ion beam of charge q, density n_o, velocity $v_o\hat{z}$ and radius R propagates in free space. Obtain the electric and magnetic fields of the beam for $r < R$. [Ans: $E_r = \frac{n_o q r}{2\epsilon_o}, B_\phi = \frac{\mu_o n_o q v_o r}{2}$]
3. A mirror machine comprises two coaxial current carrying coils separated by a long distance. On the axis, magnetic force is $\vec{B} = \hat{z}\, B_o\, (1 + z^2/L^2)$. Using $\nabla . \vec{B} = 0$, deduce B_r at points slightly off axis $(r \neq 0)$. [Ans: $B_r = \frac{-r}{2}\frac{\partial B_z}{\partial z}$]
4. A magnetically insulated diode has parallel plates at $x = 0,\; x = d$ maintained at a potential difference of V_o. It also has a static magnetic field $B_s\hat{z}$. An electron originates at *z=0* with zero velocity. Estimate the minimum value of B_s that stops the electron from reaching the other plate. [Ans: $B_s = (\frac{mV_0}{ed^2})^{1/2}$]
5. The vector potential due to a dipole antenna of length $dl\hat{z}$ and current $I = I_0 e^{-i\omega t}$ is $\vec{A}(r) = \hat{z}\frac{\mu_0}{4\pi}\frac{I_0 dl}{r}e^{-i(\omega t - \frac{\omega}{c}r)}$. Obtain the magnetic field. Here *r* refers to spherical polar coordinate system. At what value of *r* does the induction field (that goes as $1/r^2$) equal the radiation field (that goes as 1/r)? [Ans: $\hat{\varphi}\left(i\frac{\omega}{c} - \frac{1}{r}\right)A(\vec{r})$]
6. In a region of space electric field is

   i) $\vec{E} = A(\hat{x} + \hat{y})e^{-i(\omega t - \frac{\omega}{2c}(x+y))}$

   ii) $\vec{E} = A(\hat{x} - \hat{y})e^{-i(\omega t - \frac{\omega}{2c}(x+y))}$

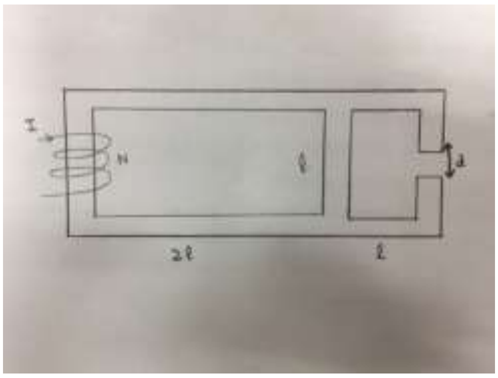
   iii) $\vec{E} = A(\hat{x} + i\hat{y})e^{-i(\omega t - \frac{\omega}{c}z)}$

   Deduce the corresponding magnetic field.

   [Ans: i) 0, ii)$-\frac{A}{c}(\hat{z})e^{-i(\omega t - \frac{\omega}{2c}(x+y))}$, iii) *(-i/c)* $\vec{E}$

7. A long cylinder of radius R is uniformly magnetized with magnetization $M\,\hat{z}$. Obtain the magnetic field for r<R and r>R. [Ans: $\mu_0 M\hat{z}\ for\ r > r\ and\ 0\ for\ r < R$

8. A magnetic circuit made of rods of cross section S and permeability μ is shown in the figure. In one arm it has a coil of N turns carrying current I. extimate the magnetic field B in the air gap of length d<<l. [Ans: $\frac{\mu NI}{(23l+6\mu d/\mu_0)}$]



9. A em wave propagates in a dielectric with $\vec{E} = A\hat{y}e^{-i(\omega t - \frac{\omega}{c}(2x+z))}$. Obtain i) $\vec{k}$ ii) $\vec{B}$ iii) ϵ_r iv) $\vec{D}$ v) $\vec{S}_{avg}$. [Ans: i) $\frac{\omega}{c}(2\hat{x}+\hat{z})$ ii) $\vec{k}X\vec{E}/\omega$ iii) 5 iv) $5\epsilon_0\vec{E}$ v) $\vec{S}_{avg} = \frac{A^2}{2\mu_0\omega}\vec{k}$

10. An em wave propagating through a dielectric of relative permittivity $\epsilon_r$has $\vec{E} = A\hat{y}e^{-i(\omega t - \vec{k}.\vec{r})}$. However, $\vec{k}$ can be complex, $\vec{k} = \vec{k}_r + i\vec{k}_i$. Show that the plane of constant amplitude is perpendicular to the plane of constant phase. Such a wave is called inhomogeneous wave.

11. A wave has $\vec{E} = A(\hat{z} + \frac{ik_z}{\alpha}\hat{x})e^{-\alpha x}e^{-i(\omega t - k_z z))}$. Obtain $i)\,\vec{B}\quad ii)\,\vec{S}_{avg}$. [Ans: i) $-iA(\alpha + \frac{{k_z}^2}{\alpha})e^{-\alpha x}e^{-i(\omega t - k_z z))}$

12. A em wave in a plasma has $\vec{E} = A\hat{x}e^{-i(\omega t - \frac{\omega}{2c}z)}$. Obtain: i) ω_p ii) v_{ph} iii) E/H

[Ans: $i)\ \frac{\sqrt{3}\omega}{2}$ *ii) 2c iii)* $2c/\mu_0$