Tutorial Sheet-1 PYL100 Course

Q 1. Write the equation of a plane that is : (i) perpendicular to z-axis and passes through the point (2,1,1), (ii) perpendicular to vector $4\hat{x} + 2\hat{y}$ and passes through the point (1,1,1), (iii) perpendicular to vector $\vec{k} = \hat{x} + \hat{y} + 2\hat{z}$ and passes through the origin. [Ans. $(i) z = 1, (ii) 4x + 2y = 6, (iii) x + y + 2z = 0.$]

Q 2A. Find the unit vector normal to the plane (i) $x + y = 4$,(ii) $x - y = 4$,(iii) $x + y + 2z = 4 [Ans. (i) \frac{\hat{x}+\hat{y}}{\sqrt{2}}, (ii) \frac{\hat{x}-\hat{y}}{\sqrt{2}}, (iii) \frac{\hat{x}+\hat{y}+2\hat{z}}{\sqrt{6}}]$

Q 2B. Obtain $\bigtriangledown.\vec{r}, \bigtriangledown(\vec{r}), \bigtriangledown(\vec{k}.\vec{r}), \bigtriangledown \times \vec{r}, \bigtriangledown.\frac{\vec{r}}{r^2}$ $[Ans. (i) 3, (ii) \frac{\vec{r}}{r}, (iii) \vec{k}, (iv) 0, (v) - \frac{1}{r^2}]$

Q 3. An electrostatic wave in a plasma has potential $\phi = A\cos(\omega t - kz)$. Obtain $\vec{E} = - \bigtriangledown \phi, \bigtriangledown.\vec{E}, \bigtriangledown \times \vec{E}$.[Ans.$(i) \hat{z} Ak \sin(\omega t - kz), (ii) k^2\phi, (iii) 0$]

Q 4. An electromagnetic wave has $\vec{E} = A\cos(\omega t - 2\frac{\omega}{c}x - \frac{\omega}{c}z)\hat{y}$. Obtain $\bigtriangledown.\vec{E}, \bigtriangledown \times \vec{E}$. Can you construct a potential ϕ such that $\vec{E} = - \bigtriangledown \phi$? [Ans. $(i) 0, (ii) (2\hat{z} - \hat{x})\frac{\omega}{c} A\sin(\omega t - 2\frac{\omega}{c}x - \frac{\omega}{c}z)$, (iii) No ϕ is possible.]

Q 5. Current density in a medium is $\vec{J} = Ae^{-\frac{r^2}{r_0^2}}\hat{z}$ where r refers to cylindrical polar coordinates. Obtain $\bigtriangledown.\vec{J}$ and $\bigtriangledown \times \vec{J}$.

Q 6. $\vec{J} = A(\hat{x} + 2\hat{z})\cos(\omega t - 4\omega(\frac{x+2z}{c}))$. Obtain $\bigtriangledown.\vec{J}$ and $\bigtriangledown \times \vec{J}$.

Q 7. A sphere of radius R and center at $\vec{r_0}$ has charge density ρ. Using Gauss's theorem write the electric field at an interior point .[Ans. $\vec{E} = \frac{\rho}{3\epsilon_0}(\vec{r} - \vec{r_0})$]

Q 8. A conducting sphere of radius R can be viewed as a superposition of two overlapping spheres, an ion sphere of charge density ρ and a free electrron sphere of charge density $-\rho$. The free electron sphere is given a displacement $\Delta\hat{x}$ with respect to the ion sphere, obtain the electric field in the overlap region. Take permittivity of the sphere as ϵ_0.[Ans. $\vec{E} = \frac{\rho}{3\epsilon_0}\vec{\Delta}$]

Q 9. In problem 8, Δ <<R and electron sphere is released after giving a displacement, it executes simple harmonic motion. Take electron mass as m, electron charge -e, electron density n_0. Obtain the frequency of oscillation.[Ans. $\omega = \frac{\omega_p}{\sqrt{3}}, \omega_p = (\frac{n_0 e^2}{m\epsilon_0})^{\frac{1}{2}}$]

Q 10. A charged particle of mass m and charge q is attached to a spring of spring constant k and lenght along $\hat{x}$. It is subjected to an electric field $\vec{E} = A\cos\omega t\hat{x}$. Obtain (i) the displacement x of the charge, (ii)restoration force $\vec{F}$ on the charge due to the spring.[Ans.$x = \frac{-qA\cos\omega t}{m(\omega^2-\omega_0^2)}, \vec{F} = \frac{q\vec{E}\omega_0^2}{\omega^2-\omega_0^2}, \omega_0 = \sqrt{\frac{k}{m}}$]

Q 11. A sphere of radius R has charge density ρ. Obtain the force on the upper half of the sphere due to the lower half.[Ans. $\frac{\pi\rho^2 R^4}{12\epsilon_0}$]

Q 12. A conductor of conductivity σ and permittivity ϵ is given a charge of charge density $\rho_0(\vec{r})$. Estimate the time scale on which the charge density moves to the surface. [Ans. The charge density goes as $\rho = \rho_0 e^-(\frac{\sigma}{\epsilon})t$.]

Q 13. A dipole (comprising -q and q charges separated by a distance $\vec{d}$) is placed in a non-uniform electric field $\vec{E}(\vec{r})$. Show that the net force on the dipole is $\vec{p}. \bigtriangledown \vec{E}$, where $\vec{p} = q\vec{d}$.

Q 14. Consider a plasma sheet of thickness d$\hat{x}$, electron density n_0, electron charge -e, electron mass m, ion density n_0 and ion charge +e. The electron sheet is given a small displacement $\Delta\hat{x}$ while ions are immobile. Obtain the natural frequency of oscillations.[Ans.$(\frac{n_0 e^2}{m\epsilon_0})^{\frac{1}{2}}$]

Q 15. A long wire of radius R has current density $\vec{J} = Ae^-(\frac{r^2}{r_0^2})\hat{z}$(where r refers to cylindrical polar coordinate system). Obtain (i) $\vec{B}(\vec{r})$ and (ii)vector potential $\vec{A}(\vec{r})$ for $r < R$, $r > R$.[Ans. $B_\phi = \frac{\mu_0 A r_0^2}{2r}(1 - e^-(\frac{r^2}{r_0^2}))$]