

Part 1:

3. For the selection sort:

```
public int selectionSort( )  
{  
  
    int counter = 0; (1)  
    int temp, indexOfMax; (2)  
  
    for ( int i = 0; i < arr.length - 1; i++ ) (3)  
    {  
        indexOfMax = 0; (4)  
  
        for ( int j = 1; j < arr.length - i; j++ ) (5)  
        {  
            if ( arr[j] > arr[indexOfMax] ) (6)  
                indexOfMax = j; (7)  
            counter++; (8)  
        }  
  
        temp = arr[indexOfMax]; (9)  
        arr[indexOfMax] = arr[arr.length - i - 1]; (10)  
        arr[arr.length - i - 1] = temp; (11)  
    }  
    return counter; (12)  
}
```

| Statement | #Time Executed | Operations |
|-----------|----------------------------|---|
| (1) | 1 | Assignment |
| (2) | 0 | |
| (3) | $1+(n+1)+1+n = 2n+3$ | Assignment, comparison and array length, Math "-", variable increment |
| (4) | 1 | Assignment |
| (5) | $n(1+n+1+1+n) = 2n^2 + 3n$ | Assignment, comparison and array length, Math "-", variable increment |
| (6) | $3n*n$ | 2 x Access array, comparison |
| (7) | 0 to $n*n$ | Assignment |
| (8) | $n*n$ | Increment |
| (9) | $2n$ | Assignment, access array |
| (10) | $6n$ | Assignment, 2 x access array, array length, 2 x math "-" |
| (11) | $5n$ | 2 x Math "-", Array access, assignment, array length |
| (12) | 1 | Return |

Worst case: $7n^2 + 18n + 6$

Best case: $6n^2 + 18n + 6$

Average case: $6.5n^2 + 18n + 6$

Its running time is $O(n^2)$

Part 2:

2.1 Algorithm A executes $10n \log n$ operations, while algorithm B executes n^2 operations. Determine the minimum integer value n_0 such that A executes fewer operations than B for $n \geq n_0$.

Firstly, $10n \log n < n^2$, which is equal to $10 \log n < n$, if we divide both sides by n . So we can just compare $10 \log n$ and n .

Secondly, because log is related to the power of 2 ($\log 8 = 3$, $2^3 = 8$ for example), it will be easier to compare the runtimes using numbers which are the result of powers of 2. The table below shows where n finally becomes larger than $10 \log n$.

| n | n | $10 \log n$ |
|-------|-----|-------------|
| 2^0 | 1 | 0 |
| 2^1 | 2 | 10 |
| 2^2 | 4 | 20 |
| 2^3 | 8 | 30 |
| 2^4 | 16 | 40 |
| 2^5 | 32 | 50 |
| 2^6 | 64 | 60 |

From this table, we can see that when $n \leq 32$, $10 \log n > n$, and when $n = 64$, $10 \log n < n$, so I can see that the minimum number " n " where $10 \log n$ executes fewer operations than n is between 50 and 60.

Then I used log base 2 calculator:

$n = 55$:

$$10 \log 55 = 57.81 > 55$$

$n = 57$:

$$10 \log 57 = 58.32 > 58$$

$n = 58$:

$$10 \log 58 = 58.58 > 58$$

$n = 59$:

$$10 \log 59 = 58.83 < 59$$

In the range of 50 to 60, I assigned different integer value to n . Finally, I got the minimum integer value is **59** that A executes fewer operations than B.

2.2 Comment and justify the running time of the following algorithm:

Algorithm Foo (a, n):

Input: two integers, a and n

Output: ?

$k \leftarrow 0$ (1)

$b \leftarrow 1$ (2)

while $k < n$ **do** (3)

$k \leftarrow k + 1$ (4)

$b \leftarrow b * a$ (5)

return b (6)

The output is a^n

| Statement | Time executed | Operations |
|-----------|---------------|------------------------|
| (1) | 1 | Assignment |
| (2) | 1 | Assignment |
| (3) | n | Iterate the while loop |
| (4) | 0 to 2n | Math "+", assignment |
| (5) | 0 to 2n | Math "-", assignment |
| (6) | 1 | return |

Best case: $3+5n$. (when $k = 0$, it didn't have any operations after the while loop)

Worst case: $3+3n$

Average case: $3+4n$

Its running time is **$O(n)$**

2.3 Comment and justify the running time of the following algorithm:

Algorithm Bar (a, n):

Input: two integers, a and n

Output: ?

$k \leftarrow n$

$b \leftarrow 1$

$c \leftarrow a$

while $k > 0$ **do**

if $k \bmod 2 = 0$ **then**

$k \leftarrow k/2$

$c \leftarrow c * c$

else

$k \leftarrow k - 1$

$b \leftarrow b * c$

return b

The output is a^n

Situation 1:

If n is even, it get halved, else it minus 1 and get halved in the next iteration.

A number can only divided in half $\log n$ times (e.g. if $n = 8$, $\log 8 = 3$, it can be halved 3 times, so the running time is $\log n$), when n is even, the situation of “else” part’s operation is constant, it is always 1, because the decrement only operates when $k = 1$.

Situation 2:

If a number n is odd, the divide part’s running time is still $\log n$, the decrement part’s running time is constant, it is always 2, because the decrement only operates when k equals the odd number and 1 (e.g. if $n = 9$, $9-1=8$, is an operation and then iterate the division process when $n = 8$). Then decrement when $k=1$.

Situation 3:

When a number needs to be decremented in between halving it, (e.g. $n = 6$. $6/2$ is 3, so it will still do decrement once, and do another time when $k = 1$), the longest running time is still $\log n + 2$.

Its running time is **$O(\log n)$**