Exploring Recovery Patterns in HRV using Functional PCA (FPCA)

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Abstract

Heart rate variability (HRV) dynamics after training sessions reflect the balance between stress and recovery. Traditional analysis methods often reduce HRV to a scalar summary (e.g., baseline vs. post-training differences), ignoring the full recovery curve. Functional Principal Component Analysis (FPCA) provides a multidimensional representation of HRV trajectories, enabling the identification of patterns such as magnitude of initial drop, recovery rate, and overshoot. This document describes the mathematical foundations, visual outputs, and simulation sandbox for exploring FPCA on HRV data.

For a practical illustration, a sandbox is provided in the following link, allowing users to interact with simulated data and better understand these concepts. HRV + FPCA Sandbox.

1 Introduction

HRV, typically quantified via rMSSD, drops after training and returns toward baseline with variable speed and shape. Analyzing these curves as functions rather than isolated points enables deeper physiological insights. FPCA is a method that decomposes functional data into orthogonal components, capturing major modes of variation in recovery dynamics.

2 Mathematical Framework

Let $X_i(t)$ denote the HRV trajectory of athlete i over time $t \in [0, T]$ after a training session. The functional mean is:

$$\mu(t) = \frac{1}{N} \sum_{i=1}^{N} X_i(t).$$

FPCA expands each trajectory as:

$$X_i(t) = \mu(t) + \sum_{k=1}^{K} \xi_{ik} \, \phi_k(t),$$

where $\phi_k(t)$ are orthogonal eigenfunctions and ξ_{ik} are subject-specific scores.

The eigenfunctions $\phi_k(t)$ are obtained by solving:

$$\int C(s,t)\phi_k(s)ds = \lambda_k \phi_k(t),$$

with $C(s,t)$ the covariance function. The eigenvalues λ_k quantify variance explained by each component.

3 Physiological Interpretation

FPCA modes correspond to physiologically meaningful recovery features:

- **PC1 (Depth of drop):** distinguishes athletes with strong vs. mild post-exercise HRV depression.
- **PC2 (Recovery speed):** differentiates fast vs. slow return to baseline.
- **PC3 (Overshoot / rebound):** captures oscillations or secondary dips during recovery.

These modes provide a richer interpretation than scalar summaries, allowing classification of recovery phenotypes.

4 Simulation Sandbox

An interactive sandbox was built (`HTML + JavaScript`) to visualize FPCA decomposition:

- Generate synthetic HRV curves with adjustable *sharpness of drop*, noise, and rebound.
- Apply FPCA in real-time to extract principal modes.
- Animate reconstruction of trajectories as $\mu(t) \pm 2\sqrt{\lambda_k}\phi_k(t)$.
- Visualize subject scores on PC1–PC2 plane, enabling clustering of recovery patterns.

5 Simulation Controls and Mathematical Meaning

The interactive sandbox allows the user to manipulate the shape of synthetic HRV recovery curves and observe the effect on Functional PCA (FPCA) decomposition. Each parameter corresponds to a specific mathematical modification in the generative model:

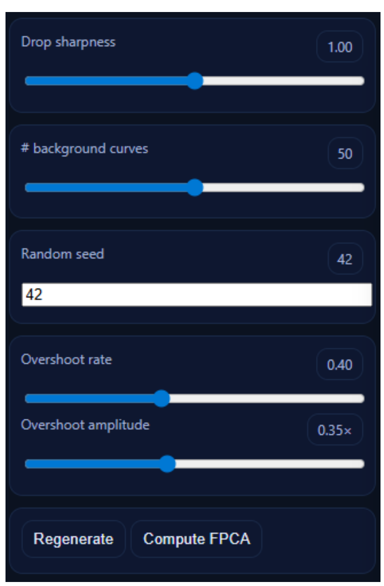

Figure 1: Parameters Panel

- **Drop sharpness:** Controls how steeply HRV decreases immediately after exercise. Mathematically, this modifies the exponential decay constant α in

$$H(t) = H_0 - A\,e^{-\alpha t},$$

where larger α produces sharper drops.

- **Number of background curves:** Sets the number N of simulated HRV trajectories used to compute FPCA. Increasing N improves stability of the covariance function

$$C(s,t) = \frac{1}{N}\sum_{i=1}^{N}(X_i(s) - \mu(s))(X_i(t) - \mu(t)).$$

- **Random seed:** Fixes the random generator to ensure reproducibility of noise perturbations added to synthetic curves. Each HRV trajectory includes stochastic variability

$$\epsilon(t) \sim \mathcal{N}(0, \sigma^2),$$

whose realization depends on the seed.

- **Overshoot rate:** Determines the temporal frequency β of rebound oscillations. This adds a damped sinusoidal term:

$$H(t) = H_0 - Ae^{-\alpha t} + B\,e^{-\gamma t}\sin(\beta t),$$

where β is increased with higher overshoot rate.

- **Overshoot amplitude:** Scales the magnitude B of the rebound component. Larger amplitude produces stronger HRV rebounds above baseline.

- **Animation mode:** Defines how FPCA reconstructions are visualized:
  - *Sweep over $\mu(t)$:* Animates $\mu(t) \pm 2\sqrt{\lambda_k}\phi_k(t)$, showing each principal mode.
  - *Morph FPCA:* Animates reconstructions

  $$X(t) \approx \mu(t) + a_1\phi_1(t) + a_2\phi_2(t) + a_3\phi_3(t),$$

  where (a_1, a_2, a_3) are interactively varied.

5.1 Practical Use

1. Adjust **Drop sharpness** to simulate athletes with different acute stress responses.
2. Modify **Overshoot rate/amplitude** to represent recovery rebound (overshoot vs. monotonic return).
3. Use **Regenerate** to create a new synthetic cohort and **Compute FPCA** to extract dominant recovery patterns.
4. Visualize the contribution of each FPCA mode via animation, linking mathematical decomposition to physiological interpretation.

This interactive approach links physiological hypotheses (sharp drop, rebound, recovery speed) to mathematical modes extracted by FPCA, providing both pedagogical and analytical insight.

6 HRV Curves and Animated Reconstruction

Figure **??** shows the simulated Heart Rate Variability (HRV) recovery curves after an exercise bout, with $N = 50$ background trajectories (light blue) and the functional mean $\mu(t)$ highlighted in gold. Each individual curve represents a possible recovery profile with varying drop magnitude, rebound intensity, and noise perturbations.

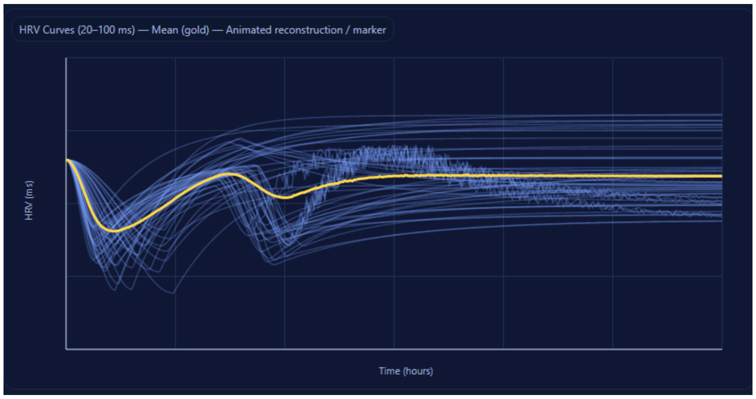

Figure 2: Simulated HRV trajectories with functional mean $\mu(t)$ (gold). The light-blue lines correspond to individual realizations of recovery patterns.

The gold curve $\mu(t)$ represents the expected recovery pattern across the cohort, obtained by averaging all individual HRV curves:

$$\mu(t) = \frac{1}{N}\sum_{i=1}^{N} X_i(t).$$

The background curves illustrate inter-individual variability, generated by modifying parameters of the underlying model:

$$X_i(t) = H_0 - A_i e^{-\alpha_i t} + B_i e^{-\gamma_i t}\sin(\beta_i t) + \epsilon_i(t),$$

where A_i controls drop depth, α_i the sharpness of the fall, B_i and β_i the overshoot amplitude and frequency, and $\epsilon_i(t)$ is Gaussian noise.

The animation overlays reconstructions obtained via Functional Principal Component Analysis (FPCA). This decomposition expresses each curve as

$$X_i(t) \approx \mu(t) + \sum_{k=1}^{K} a_{ik}\phi_k(t),$$

where $\phi_k(t)$ are the principal modes of variation and a_{ik} their individual scores.

Thus, the figure serves two purposes:

1. It provides an intuitive view of variability in recovery dynamics (sharp vs. smooth drop, monotonic vs. oscillatory return).

2. It allows comparison between the empirical mean trajectory and FPCA-based reconstructions, linking physiological interpretation (recovery depth, speed, rebound) with mathematical components.

7 Functional Principal Component Analysis (FPCA) of HRV Recovery

Figure **??** displays the results of Functional Principal Component Analysis (FPCA) applied to the simulated HRV recovery curves. The analysis decomposes the ensemble of trajectories into a mean function $\mu(t)$ and orthogonal modes of variation $\phi_k(t)$, each weighted by a subject-specific score a_{ik}:

$$X_i(t) \approx \mu(t) + \sum_{k=1}^{K} a_{ik}\, \phi_k(t).$$

Each component has a physiological interpretation:

- **First component** (ϕ_1): captures the overall *drop magnitude* of HRV after exercise. Large positive scores indicate athletes with a shallow decline and fast stabilization, while large negative scores reflect deep drops and prolonged suppression.
- **Second component** (ϕ_2): represents the *speed of recovery*. Positive values correspond to faster rebounds toward baseline, while negative values reflect a slower, more gradual normalization.
- **Third component** (ϕ_3): encodes *oscillatory behavior* such as overshoot or rebound. Positive scores indicate strong post-exercise overshoot (HRV rising above baseline before settling), while negative scores correspond to monotonic recovery without overshoot.

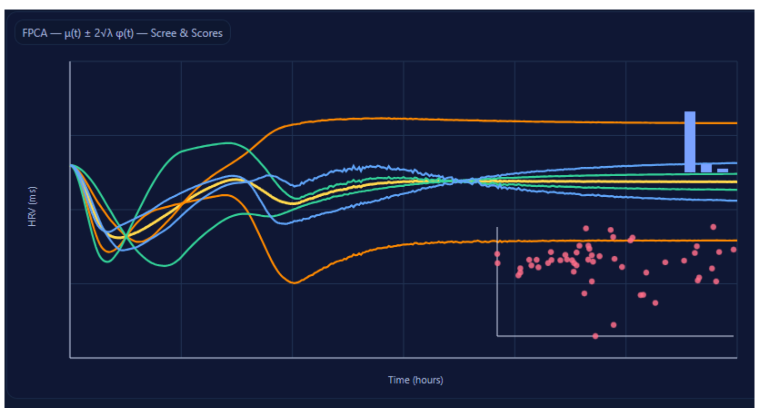

Figure 3: FPCA decomposition of HRV recovery. The central curve corresponds to $\mu(t)$, while the colored envelopes show $\mu(t) \pm 2\sqrt{\lambda_k}\, \phi_k(t)$ for the first three components. The bar chart (top right) indicates the proportion of variance explained by each component, and the scatter plot (bottom right) shows subject-specific FPCA scores.

The bar plot shows the variance explained by each mode $\lambda_k / \sum_j \lambda_j$, typically with the first two components accounting for the majority of inter-individual differences. The scatter plot illustrates how individual athletes distribute in the FPCA score space, clustering according to recovery strategies (e.g., "fast but shallow" vs. "slow but deep" recovery profiles).

This decomposition enables a compact yet physiologically meaningful representation of recovery, reducing the complexity of high-dimensional time-series data to a few interpretable axes.

The scatter plot (pink points) represents the subject-specific FPCA scores $\{a_{ik}\}$ for a given component. Each point is the numerical signature of one athlete: horizontal displacement reflects differences in drop magnitude, while vertical spread reflects recovery speed or overshoot tendencies. This provides a compact coordinate system where athletes can be directly compared in terms of their recovery dynamics.

8 Applications

1. **Identify recovery archetypes:** athletes can be grouped by their FPCA scores.
2. **Monitor training load:** abnormal overshoot or delayed recovery curves indicate maladaptation.
3. **Personalized modeling:** FPCA-derived features can feed into machine learning models for readiness prediction.

9 Conclusion

FPCA provides a powerful, mathematically rigorous framework to analyze HRV recovery curves. By decomposing trajectories into orthogonal modes, it captures latent physiological processes beyond scalar metrics. The simulation sandbox enables intuitive exploration of these dynamics, offering both pedagogical and applied value for sports science.

References

[1] Ramsay, J. O., & Silverman, B. W. (2005). *Functional Data Analysis*. Springer.

[2] Esco, M. R., Flatt, A. A. (2021). Heart rate variability and endurance training adaptation: A review. *Sports*, 9(6), 85.