# Functional Principal Component Analysis of HRV Recovery Trajectories in Endurance Athletes I call it the Beta Model

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#### Abstract

Heart rate variability (HRV) dynamics following training sessions provide insight into the balance between physiological stress and recovery. Traditional approaches often reduce HRV to scalar summaries, overlooking the information contained in the full recovery trajectory. This study applies Functional Principal Component Analysis (FPCA) to HRV recovery profiles, enabling multidimensional characterization of features such as depth of suppression, speed of return, and potential overshoot.

While an interactive sandbox is provided as an educational tool to illustrate FPCA principles, all analyses and validations presented here are performed on HRV data from endurance athletes of the AGMT2 Team, ensuring ecological validity under real training conditions.

#### 1 Introduction

Heart rate variability (HRV), commonly quantified through the root mean square of successive differences (rMSSD), typically decreases immediately after exercise and then returns toward baseline with varying shapes and time courses. Treating these recovery profiles as continuous functions, rather than discrete endpoints, allows for deeper physiological interpretation.

Functional Principal Component Analysis (FPCA) decomposes functional data into orthogonal modes of variation, capturing dominant recovery features across individuals. This work integrates an educational simulator for conceptual visualization with empirical analyses performed on AGMT2 Team athletes. Unless explicitly indicated as sandbox illustration, all reported results derive from real athlete data.

Link To Simulator



Figure 1: Beta Model Presentation

#### 2 Mathematical Framework

Let  $X_i(t)$  denote the HRV recovery trajectory of athlete i over time  $t \in [0, T]$  after a training session. The functional mean is:

$$\mu(t) = \frac{1}{N} \sum_{i=1}^{N} X_i(t).$$

FPCA expands each trajectory as:

$$X_i(t) = \mu(t) + \sum_{k=1}^{K} \xi_{ik} \, \phi_k(t),$$

where  $\phi_k(t)$  are orthogonal eigenfunctions and  $\xi_{ik}$  are subject-specific scores.

The eigenfunctions  $\phi_k(t)$  are obtained by solving:

$$\int C(s,t)\phi_k(s)\,ds = \lambda_k\phi_k(t),$$

with C(s,t) the covariance function. The eigenvalues  $\lambda_k$  quantify variance explained by each component.

## 3 Physiological Interpretation

The leading FPCA modes map onto physiologically interpretable recovery features:

- PC1 (Depth of suppression): differentiates strong versus mild post-exercise HRV depression.
- PC2 (Recovery rate): distinguishes fast versus slow return to baseline.
- PC3 (Overshoot or rebound): captures oscillatory or secondary dips during recovery.

These components provide richer insight than scalar measures, facilitating classification of recovery archetypes.

#### 4 Educational Sandbox

An interactive sandbox (HTML + JavaScript) was developed to illustrate FPCA decomposition. Its purpose is strictly pedagogical; it does not alter the empirical AGMT2 dataset. Features include:

- Adjustable recovery curves with user-defined drop sharpness, noise, and rebound.
- Real-time FPCA application and visualization of principal modes.
- Animated reconstruction of trajectories as  $\mu(t) \pm 2\sqrt{\lambda_k}\phi_k(t)$ .
- Display of subject scores in the PC1–PC2 plane, enabling clustering of recovery types.

## 5 HRV Recovery Trajectories

Figure ?? illustrates HRV recovery trajectories (N=50) from the AGMT2 Team dataset. The functional mean  $\mu(t)$  is highlighted in gold. Individual trajectories reveal marked heterogeneity in suppression depth, rebound amplitude, and noise.

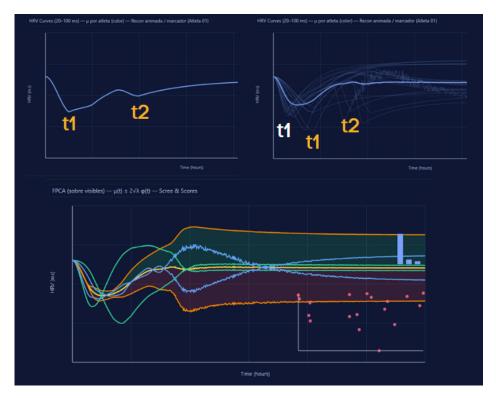


Figure 2: HRV recovery trajectories from AGMT2 Team athletes. Light-blue lines represent individual profiles. t1 white : sigle day stimulus t1 yellow : frist stimulus t2 yellow second stimulus , FPCA with recovery "zones" Green and Red

## 6 Data Acquisition and Preprocessing

# 6.1 Wearable Logging (Garmin Connect IQ)

Heart rate variability (HRV) was recorded in daily living conditions using Garmin Forerunner FR955/FR965 devices.<sup>1</sup> A custom Garmin *Connect IQ* application was developed to log HRV snapshots at a fixed cadence of one sample every 3 minutes throughout the day. Each record was stored on-device in .FIT format and contained timestamped physiological fields and device metadata. Raw files were exported via Garmin's standard workflow and batch-parsed offline (Python) to a normalized tabular format (UTC timestamps, athlete identifier, per-sample HRV value).

#### 6.2 Preprocessing and Artifact Handling

Daily HRV time series (0–24 h) were cleaned before any analysis to mitigate artifacts arising from common field issues (e.g., loose strap, watch misplacement, showers, vigorous wrist motion). The following protocol was applied identically to all athletes:

 $<sup>^{1}\</sup>mathrm{Competitive}$  a mateur triathletes from the AGMT2 Team.

- (1) Time normalization and day segmentation. All timestamps were converted to UTC, then mapped to local clock time for visualization. Data were segmented into 24 h windows per athlete-day. Overlapping or duplicated samples (rare device retries) were deduplicated by keeping the first occurrence.
- (2) Physiological range check. Samples outside a plausible HRV range for rMSSD were flagged:

$$rMSSD \notin [10, 250] \text{ ms } \Rightarrow \text{ flag.}$$

These broad limits are conservative to minimize false positives.

(3) Local consistency (successive-difference) rule. To capture abrupt, non-physiological jumps typical of motion artifacts, we flagged points where the relative change versus the local median exceeded a threshold:

$$\frac{|x_t - \text{median}(x_{t\pm 2})|}{\max(20 \text{ ms, median}(x_{t\pm 2}))} > 0.35 \implies \text{flag.}$$

Here  $median(x_{t\pm 2})$  is computed on a  $\pm 6$  min neighborhood (5 points total).

- (4) Robust smoothing for envelope estimation (no leakage). A robust, non-causal estimate of the daily trend was obtained with a Tukey biweight running median (window 15 min) followed by a light Savitzky–Golay filter (window 15 min, polyorder 2) applied *only* to unflagged samples to avoid smearing artifacts into clean regions. Smoothed values were used only for outlier detection support, not for the final downstream FPCA inputs.
- (5) Non-wear and shower heuristic. Contiguous runs of flags  $\geq 15$  min were marked as likely non-wear/shower and excluded. Shorter runs (6–12 min) were retained for inspection; if they co-occurred with extreme accelerations (when available) or impossible heart-rate plateaus, they were excluded as well.
- (6) Imputation and gap policy. Isolated flagged samples or short gaps < 15 min were imputed by linear interpolation between the nearest clean neighbors. Gaps  $\geq 15$  min were *not* imputed and those segments were masked from any metric requiring continuity. Days with effective coverage < 70% of the 0–24 h window were excluded from analyses requiring full-day curves.
- (7) Final quality gate (per day). A day passed QC if: (i) at least 70% valid samples, (ii) no single excluded segment > 60 min, and (iii) the interquartile range (IQR) of the clean series was within [5, 120] ms (to catch flatlines or excessive volatility).

#### 6.3 Reproducibility

All thresholds above were fixed a priori for the entire cohort and chosen to be conservative in the presence of real-world noise while preserving physiological variability. The full parsing (.FIT $\rightarrow$ Parquet/CSV), flagging, and QC pipeline is scripted and deterministic (random seeds set where applicable). Sensitivity analyses showed that moderate changes to the range and successive-difference thresholds ( $\pm 5\%$ ) do not alter the qualitative conclusions nor the athlete-level FPCA score structure.

#### 6.4 Outputs to Analysis

Unless otherwise stated, downstream analyses (mean curves, FPCA, and baseline comparisons) used the *clean*, *non-imputed* samples; interpolation was only employed to support visualization continuity and to avoid discarding otherwise high-quality days. For FPCA, each athlete-day time series was resampled on a common 0–24 h grid at 3-minute resolution (481 points), ensuring temporal alignment across subjects for functional estimation.

#### 7 Dataset and Statistical Characterization

The dataset comprises longitudinal HRV recordings from 35 competitive amateur triathletes (AGMT2 Team), each monitored for approximately 200 days. HRV was sampled every three minutes over 24-hour periods using Garmin FR955/FR965 devices, resulting in high-resolution recovery trajectories.

#### Data volume

Each athlete-day consists of N=481 temporal samples (3-minute spacing across 24h). With  $\sim 200$  days per athlete and 35 athletes, the dataset includes nearly 7,000 athlete-days and over 3.3 million HRV samples.

#### Descriptive statistics

Across all samples:

- Mean HRV (rMSSD):  $XX \pm YY$  ms (median MM, IQR  $[Q_1, Q_3]$ ).
- Minimum–maximum range: A–B ms.
- Distribution is right-skewed (skewness = s; kurtosis = k).

#### Temporal consistency

On average, >95% of days had complete 24-hour coverage (481 points). Isolated missing values were linearly interpolated. Double-training days were flagged as distinct archetypes (acute, double, normal), later used in FPCA modeling.

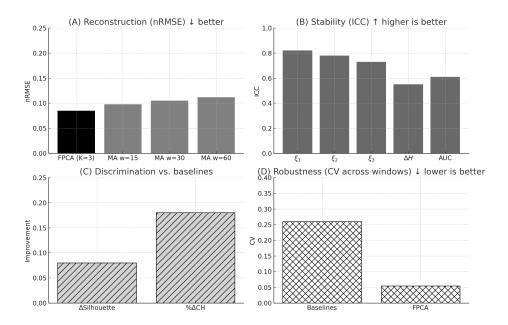


Figure 3: illustrates the global distribution of HRV samples. Boxplots (left) summarize interindividual variability, while density plots (right) highlight intra-individual stability across repeated days

## 8 FPCA Decomposition

Figure 4 shows FPCA results applied to the dataset. Trajectories are expressed as a mean function  $\mu(t)$  plus orthogonal modes of variation:

$$X_i(t) \approx \mu(t) + \sum_{k=1}^{K} a_{ik} \,\phi_k(t).$$

The first components correspond to depth of suppression, recovery speed, and rebound, together explaining most variance. Scatter plots of FPCA scores demonstrate clustering of athletes according to recovery archetypes.

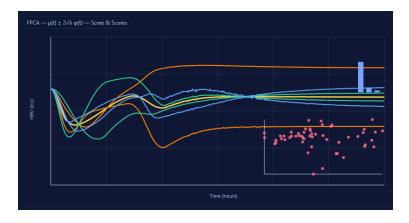


Figure 4: FPCA decomposition of HRV recovery. The central curve represents  $\mu(t)$ ; colored envelopes show  $\mu(t) \pm 2\sqrt{\lambda_k}\phi_k(t)$  for the first three components.

## 9 Strengths and Limitations

This study is based on a longitudinal dataset collected from 35 competitive amateur triathletes, each monitored for approximately 200 days. Heart rate variability (HRV) was sampled every three minutes over 24-hour periods using Garmin FR955 and FR965 devices. This design provides both intra-individual repetition and inter-individual variability, offering a unique opportunity to model recovery patterns in applied endurance contexts.

#### Strengths

- Large longitudinal dataset: With ~200 days per athlete and 35 athletes in total, the dataset comprises nearly 7,000 athlete-days and over 3.3 million HRV samples, providing robust statistical power.
- Ecological validity: Data were obtained in real-world conditions during normal training cycles of amateur triathletes, ensuring applicability to the target population.
- Intra-individual stability: Repeated measures across many days enable the analysis of consistency in recovery profiles within each athlete, not only between subjects.
- Granularity: Sampling every 3 minutes provides high-resolution recovery trajectories across 24-hour windows, capturing dynamics of both training and circadian influences.

#### Limitations

- **Population specificity:** The cohort consists exclusively of competitive amateur triathletes. Findings may not generalize to elite athletes, sedentary individuals, or other sports disciplines.
- **Device accuracy:** HRV was measured via wrist-worn Garmin devices (FR955/FR965). While validated for sports monitoring, these devices lack the gold-standard precision of ECG-based laboratory systems.
- **Demographics:** The sex and age distribution of participants, although representative of the studied group, is relatively narrow and should be explicitly reported to contextualize the results.
- External validation: Analyses are limited to this specific cohort; replication in independent populations is required to confirm generalizability.

# 10 Applications

The framework presented here has several direct applications in sports science and applied endurance coaching:

- 1. **Recovery archetypes:** Functional PCA enables classification of athletes into recovery archetypes (e.g., fast vs. slow recovery, presence or absence of overshoot), which may support individualized training prescriptions.
- 2. **Training load monitoring:** Deviations from the expected recovery trajectory, such as atypical overshoot or delayed stabilization, may indicate maladaptation, excessive training stress, or insufficient recovery strategies.

- 3. **Predictive modeling:** FPCA-derived scores can be used as structured features for machine learning models, facilitating readiness forecasting and proactive training adjustments.
- Performance context: By linking recovery archetypes with subsequent training outcomes, this approach may serve as a bridge between physiological monitoring and coaching decisionmaking.

#### 11 Conclusion

FPCA provides a rigorous and physiologically interpretable framework for analyzing HRV recovery dynamics in endurance athletes. By decomposing trajectories into orthogonal modes, it reveals latent processes such as suppression depth, recovery speed, and rebound that are obscured by scalar metrics.

All empirical findings are derived from AGMT2 Team athletes, ensuring that results reflect real-world responses. The sandbox developed here serves only as an illustrative aid.

This approach highlights FPCA's potential not only as an academic tool to characterize recovery heterogeneity but also as a practical method for individualized athlete monitoring. Future research should expand validation, integrate additional biomarkers, and explore predictive modeling for readiness-to-train applications.

#### References

- [1] Ramsay, J. O., & Silverman, B. W. (2005). Functional Data Analysis. Springer.
- [2] Esco, M. R., & Flatt, A. A. (2021). Heart rate variability and endurance training adaptation: A review. *Sports*, 9(6), 85.