Lab 9, Fall 2023, Simulation of Boundary Dominated Flow

Simulate the following diffusivity equation problem using the simulator provided in the Ch_30_Diffusivity_Implicit_Thomas_Doolittle_LU.xlsm/.py files.

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t};\ \ k = 1;\ \ 0 \le x \le 1;\ \ \Delta x = 1/20;\ \ \ \Delta t = 1/200$$

$$I.C.:\ T(x,0) = 0$$

$$B.C.:\ \left\{ \begin{array}{c} T(0,t) = 1;\ \text{Dirichlet} \\ \frac{\partial T}{\partial x}(1,t) = 0;\ \ \text{Neumann} \end{array} \right\}$$

The analytic solution to this problem (*k*=1) yields $\partial T/\partial x$, at the left boundary (at *x*=0); and is $-1/\sqrt{\pi t}$ when $t < 0.2$, and$-2 * \mathrm{Exp}(-t\pi^2/4)$ when $t \geq 0.2$.

After running the simulation to time of *t*=1, use the 2nd order forward difference approximation (see Fig. 23.1) to approximate $\partial T/\partial x$ at the left boundary using simulation results, and compare to the analytic solution using a log-log plot of -$\partial T/\partial x|_{x=0}$ vs. *t*. For the analytic solutions use the following description of the *t*-axis, let the value go from 0.01 to 1, with at least 20 evenly spaced points per log-cycle (time factor = 10^(1/20)).

Experiment: How do the simulation results change in comparison to the analytic solution when you use the "consistent refinement" approach (Ch_30_Lecture_1) to reduce the truncation error by a factor of 4, then by another factor of 4. Copy the "Results" worksheet for each set of results and rename appropriately.

Reporting: Discuss changes you made to the "Data" worksheet to define this problem for both cases run. Discuss how you approximated the partial derivative at the left edge. Discuss the comparison of your simulation results to the analytic solution for all three cases.