UFSC Livro do time (2024-2025)

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1 Data Structures

1.1 Centroid Decomposition

```
#include <algorithm>
#include <iostream>
#include <utility>
#include <vector>
using namespace std;
```

```
using pii = pair<int, int>;
// 'closest_red', query and update were used for solving xenia and the tree.
struct CentroidDecomposition {
  vector<vector<int>> tree;
  vector<int> subtrees_sz, closest_red;
  vector<vector<pii>>> parents;
  vector<bool> removed;
  CentroidDecomposition(vector<vector<int>> adj)
    :tree{adj} {
    int n = tree.size();
    subtrees sz.resize(n);
   removed.assign(n, false);
closest_red.assign(n, 1e9);
    parents.resize(n);
    centroid_decomposition(0, -1);
  void calculate_subtree_sizes(int u, int p = -1) {
    subtrees_sz[u] = 1;
    for (auto v : tree[u]) {
      if (v == p || removed[v])
        continue;
      calculate_subtree_sizes(v, u);
      subtrees_sz[u] += subtrees_sz[v];
  int find_centroid(int u, int p, int n) {
    for (auto v : tree[u]) {
      if (v == p || removed[v])
        continue;
      if (subtrees_sz[v] > n / 2)
        return find_centroid(v, u, n);
    return u;
  void calculate_distance_to_centroid(int u, int p, int centroid, int d) {
   for (auto v : tree[u]) {
  if (v == p || removed[v])
        continue;
      calculate_distance_to_centroid(v, u, centroid, d + 1);
    parents[u].push_back({centroid, d});
  void centroid_decomposition(int u, int p = -1) {
    calculate_subtree_sizes(u);
    int centroid = find_centroid(u, p, subtrees_sz[u]);
    for (auto v : tree[centroid]) {
  if (removed[v])
        continue;
      calculate_distance_to_centroid(v, centroid, centroid, 1);
    removed[centroid] = true;
    for (auto v : tree[centroid]) {
      if (removed[v])
        continue;
      centroid\_decomposition(v, u);
  int query(int u) {
  int ret = closest_red[u];
    for (auto&[p, pd] : parents[u])
  ret = min(ret, pd + closest_red[p]);
    return ret;
  void update(int u) {
    closest_red[u] = 0;
    for (auto &[p, pdist] : parents[u])
      closest_red[p] = min(closest_red[p], pdist);
```

1.2 Fenwick Tree

};

```
// 1-indexed FenwickTree
struct FenwickTree {
 FenwickTree(int n) { ft.assign(n + 1, 0); }
  FenwickTree(vector<int> &vec) {
    ft.assign(vec.size() + 1, 0);
    for (int i = 0; i < vec.size(); ++i)</pre>
      update(i + 1, vec[i]);
  inline int ls_one(int x) { return x & (-x); }
  int query(int r) {
    int sum = 0;
    while (r) {
      sum += ft[r];
      r -= ls_one(r);
    return sum;
  int query(int 1, int r) { return query(r) - query(1 - 1); }
  void update(int i, int v) {
    while (i < ft.size()) {</pre>
      ft[i] += v:
      i += ls one(i):
  // Finds smallest index i on FenwickTree such that query(1, i) >= rank.
  // I.e: smallest i for [1, i] >= k
  int select(long long k) { // O(log^2 m)
    int lo = 1, hi = ft.size() - 1;
    for (int i = 0; i < 30; ++i) {
  int mid = (lo + hi) / 2;</pre>
      (query(1, mid) < k) ? lo = mid : hi = mid;
    return hi:
  vector<int> ft;
};
// Range Update - Point query
  FenwickTree ft;
  RUPQ(int m) : ft{m} {}
  void range_update(int ui, int uj, int v) {
    ft.update(ui, v);
    ft.update(uj + 1, -v);
  int point_query(int i) { return ft.query(i); }
// Range Update - Range Query
  RUPQ rupq;
  FenwickTree ft;
  RURQ(int m) : ft{m}, rupq(m) {}
  void range_update(int ui, int uj, int v) {
    rupq.range_update(ui, uj, v);
    ft.update(ui, v * (ui - 1));
ft.update(uj + 1, -v * uj);
  int query(int j) { return rupq.point_query(j) * j - ft.query(j); }
  int query(int i, int j) { return query(j) - query(i - 1); }
```

1.3 Ordered Set

using namespace std;

```
// C++ program to demonstrate the
// ordered set in GNU C++
#include <iostream>
using namespace std;
// Header files, namespaces,
// macros as defined above
#include <ext/pb_ds/assoc_container.hpp>
```

```
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
#define ordered_set tree<int, null_type,less<int>, rb_tree_tag,tree_order_statistics_node_update>
// Driver program to test above functions
int main()
    // Ordered set declared with name o_set
    ordered_set o_set;
    // insert function to insert in 
// ordered set same as SET STL
    o_set.insert(5);
    o set insert (1):
    o_set.insert(2);
    // Finding the second smallest element
    // in the set using * because
    // find_by_order returns an iterator
    cout << *(o_set.find_by_order(1))</pre>
         << endl;
    // Finding the number of elements
    // strictly less than k=4
    cout << o_set.order_of_key(4)</pre>
         << endl:
    // Finding the count of elements less
    // than or equal to 4 i.e. strictly less
    // than 5 if integers are present
    cout << o_set.order_of_key(5)</pre>
         << endl;
    // Deleting 2 from the set if it exists
    if (o_set.find(2) != o_set.end())
        o_set.erase(o_set.find(2));
    // Now after deleting 2 from the set
    \ensuremath{//} Finding the second smallest element in the set
    cout << *(o_set.find_by_order(1))</pre>
         << endl;
    // Finding the number of
    // elements strictly less than k=4
    cout << o_set.order_of_key(4)</pre>
         << endl;
    return 0;
```

1.4 Segment Tree

```
#include <bits/stdc++.h>
#ifdef DEBUG
#define PRINT(s) std::cout << s << '\n';
#endif
using namespace std;
using 11 = long long;
#include <bits/stdc++.h>
using namespace std;
using pii = pair<int, int>;
using ll = long long;
using ull = unsigned long long;
struct SegmentTree {
  inline int left(int p) { return p * 2; }
  inline int right(int p) { return p * 2 + 1; }
  SegmentTree(vector<int> &vec) {
     n = vec.size();
    int sz = 4 * n;
    tree.assign(sz, 1e9);
    lazy.assign(sz, -1);
    build(1, 0, n - 1, vec);
  inline int merge(int a, int b) { return min(a, b); }
```

```
void build(int p, int 1, int r, vector<int> &vec) {
    if (1 == r) {
     tree[p] = vec[1];
     return;
    int m = (1 + r) / 2;
    build(left(p), 1, m, vec);
    build(right(p), m + 1, r, vec);
    tree[p] = merge(tree[left(p)], tree[right(p)]);
  void propagate(int p, int 1, int r) {
   if (lazy[p] == -1)
    tree[p] = lazy[p];
    if (1 != r)
     lazy[left(p)] = lazy[right(p)] = lazy[p];
    lazy[p] = -1;
  int query(int i, int j) { return query(1, 0, n - 1, i, j); }
  void update(int i, int j, int v) { update(1, 0, n - 1, i, j, v); }
  vector<int> lazv, tree;
  int n:
  int query(int p, int 1, int r, int i, int j) {
   propagate(p, l, r);
if (i > j) // valor impossível. merge() deve ignorá-lo
     return 1e9;
   if (1 >= i && r <= j)
     return tree[p];
   int m = (1 + r) / 2;
   void update(int p, int 1, int r, int i, int j, int v) {
   propagate(p, 1, r);
    if (i > j)
     return;
    if (1 >= i && r <= j) {
     tree[p] = v;
     lazy[p] = v;
     return;
    int m = (1 + r) / 2;
   update(left(p), 1, m, i, min(j, m), v);
update(right(p), m + 1, r, max(i, m + 1), j, v);
    tree[p] = merge(tree[left(p)], tree[right(p)]);
};
```

1.5 Sparse Table

```
#include < vector >
#include<utility>
// preprocessing: O(n log n)
// range minimum query (minimum element in [L, R] interval): O(1)
struct SparseTable {
  vector<vector<int>> st;
  int k = 25;
  int n;
  SparseTable(const vector<int>& vec) {
   n = vec.size();
    st.assign(k+1, vector<int>(n));
    st[0] = vec;
   for (int i = 1; i <= k; ++i)
for (int j = 0; j + (1 << i) <= n; ++j)</pre>
        st[i][j] = min(st[i-1][j], st[i-1][j] + (1 << (i-1))]);
  int query(int 1, int r) {
    int i = bit_width((unsigned long) (r - 1 + 1)) - 1; // change to log2 and memoization if <math>c++20 is
          not available.
```

```
return min(st[i][1], st[i][r - (1 << i) + 1]);
};
s</pre>
```

1.6 Suffix Array

```
#include <bits/stdc++.h>
#include <vector>
using namespace std;
using vi = vector<int>;
using ii = pair<int, int>;
class SuffixArray {
private:
  vi RA;
                                      // rank array
  void countingSort(int k) {
                                      // up to 255 ASCII chars
   int \max i = \max(300, n);
   vi c(maxi, 0);
for (int i = 0; i < n; ++i)</pre>
                                      // clear frequency table
                                      // count the frequency
     ++c[i+k < n ? RA[i+k] : 0]; // of each integer rank
    for (int i = 0, sum = 0; i < maxi; ++i) {
     int t = c[i];
     c[i] = sum;
     sum += t;
    vi tempSA(n);
    for (int i = 0; i < n; ++i) // sort SA
     tempSA[c[SA[i] + k < n ? RA[SA[i] + k] : 0]++] = SA[i];
    swap(SA, tempSA); // update SA
  void constructSA() { // can go up to 400K chars
    SA.resize(n);
    iota(SA.begin(), SA.end(), 0); // the initial SA
   RA.resize(n);
for (int i = 0; i < n; ++i)
     RA[i] = T[i];
                                      // initial rankings
    for (int k = 1; k < n; k <<= 1) { // repeat log_2 n times</pre>
     // this is actually radix sort
      countingSort(k); // sort by 2nd item
      countingSort(0); // stable-sort by 1st item
      vi tempRA(n);
     int r = 0;
tempRA[SA[0]] = r;
                                 // re-ranking process
      for (int i = 1; i < n; ++i) // compare adj suffixes</pre>
       (RA[SA[i] + k] == RA[SA[i - 1] + k]))
               : ++r:
     swap(RA, tempRA); // update RA
if (RA[SA[n - 1]] == n - 1)
        break; // nice optimization
public:
 const char *T; // the input string
  const int n; // the length of T
                 // Suffix Array
  SuffixArray(const char *initialT, const int _n) : T(initialT), n(_n) {
   constructSA(); // O(n log n)
  };
```

1.7 Union Find

```
#include <bits/stdc++.h>
using namespace std;

struct UnionFind {
   UnionFind(int n) {
     p.resize(n);
     rank.assign(n, 1);
     set_size.assign(n, 1);
     iota(p.begin(), p.end(), 0);
   }

int find_set(int i) {
   if (p[i] == i)
     return i;
}
```

```
return p[i] = find_set(p[i]);
}
inline bool same_set(int i, int j) { return find_set(i) == find_set(j); }

void union_set(int i, int j) {
    if (same_set(i, j))
        return;

    i = p[i];
    j = p[j];

    if (rank[i] > rank[j])
        swap(i, j);

p[i] = j;
    if (rank[i] == rank[j])
        rank[j]++;

    set_size[j] += set_size[i];
}

vector<int> p, rank, set_size;
};
```

2 Dynamic Programming

2.1 Coin Change

```
Returns minimum amount of coins from the "coins" list
such that their sum is equal to "val".
Every element on the "coins" list can be used an unlimited
amount of times.
If no sum of coins is equal to "val", returns -1.

def coin_change(coins, val):
    dp = [float("inf")] * (val + 1)
    dp[0] = 0

for amount in range(1, val + 1):
    for coin in coins:
        if amount - coin >= 0:
            temp = 1 + dp[amount - coin]
        if temp < dp[amount]:
            dp[amount] = temp
return dp[val] if dp[val] != float("inf") else -1</pre>
```

2.2 Knapsack

2.3 Longest Common Subsequence

```
'''.
Returns the length of the longest common subsequence between strings "p" and "q". The sequence doesn't need
```

3 Geometry

3.1 Circle Intersection

```
struct Circle {
   int x, y, r;
};
inline bool is_inside(Circle &a, Circle &b) {
   double d = sqrt((a.x - b.x) * (a.x - b.x) + (a.y - b.y) * (a.y - b.y));
   return d <= a.r + b.r || d <= a.r - b.r || d <= b.r - a.r;
}</pre>
```

3.2 Circuncenter

```
#include <bits/stdc++.h>
using namespace std;
struct Point {
  double x, y;
  Point (double x, double y) : x{x}, y{y} {}
inline double distance(const Point &p1, const Point &p2) {
  double x = p1.x - p2.x, y = p1.y - p2.y;
  return sqrt(x * x + y * y);
// Returns point equidistant to all vertices of the triangle
Point circuncenter (Point &A, Point &B, Point &C) {
  // LINE AB
  // ax + bv = c
  double a = B.y - A.y, b = A.x - B.x;
  double c = a * A.x + b * A.y;
  double e = C.y - B.y, f = B.x - C.x;
  double q = e * B.x + f * B.y;
  // convert AB to perpendicular bisector
  c = -b * (A.x + B.x) / 2 + a * (A.y + B.y) / 2;
 b = exchange(a, -b);
  // convert BC to perpendicular bisector
  g = -f * (B.x + C.x) / 2 + e * (B.y + C.y) / 2;
  f = exchange(e, -f);
  double determinant = a * f - e * b;
  if (determinant == 0)
   return Point (1e9, 1e9);
  return Point ((f * c - b * g) / determinant, (a * g - e * c) / determinant);
```

3.3 Tetrahedron Volume

```
#include <bits/stdc++.h>
using namespace std;
using pii = pair<int, int>;
using pdd = pair<double, double>;
using 11 = long long;
using ull = unsigned long long;
template <typename T> struct Point {
  Point(): x{0}, y{0}, z{0} {}
  Point (T x, T y, T z) : x\{x\}, y\{y\}, z\{z\} {}
  T dot(Point<T> &other) { return x * other.x + y * other.y + z * other.z; }
  T cross(Point<T> &other) {
    return x * other.x + x * other.y + x * other.z + y * other.x + y * other.y +
           y * other.z + z * other.x + z * other.y + z * other.z;
  friend istream &operator>>(istream &is, Point<T> &p) {
    is >> p.x >> p.y >> p.z;
    return is;
  Point<T> operator-(Point<T> &other) {
    return Point<T>(x - other.x, y - other.y, z - other.z);
};
double volume(Point<double> &p1, Point<double> &p2, Point<double> &p3,
              Point < double > &p4) {
  auto pa = p1 - p4, pb = p2 - p4, pc = p3 - p4;
  double determinant = pa.x * (pb.y * pc.z - pc.y * pb.z) -
                       pb.x * (pa.y * pc.z - pc.y * pa.z) +
                        pc.x * (pa.y * pb.z - pb.y * pa.z);
  return abs(determinant) / 6.0;
int main() {
  ios::sync_with_stdio(0);
  cin.tie(0);
  int t;
  cin >> t:
  while (t--) {
    Point<double> p1, p2, p3, p4;
    cin >> p1 >> p2 >> p3 >> p4;
    cout << fixed << setprecision(6) << volume(p1, p2, p3, p4) << '\n';</pre>
```

4 Graphs

4.1 Dijkstra

```
// The graph's pair<int, int> should be {distance, vertex}
#include <bits/stdc++.h>
using namespace std;
#define INF 1000000000
#define ii pair<int, int>
vector<vector<ii>>> graph;
vector<int> dijkstra(int s, int n) {
    vector<int> d(n+1, INF);
    d[s] = 0;
    priority_queue<ii, vector<ii>, greater<ii>> pq;
    pq.push({0, s});
    while (!pq.empty()) {
        auto [dist, v] = pq.top(); pq.pop();
        if (dist != d[v])
            continue:
        for (auto [len, next] : graph[v]) {
            int newDist = dist + len;
            if (newDist < d[next]) {</pre>
                d[next] = newDist;
```

```
pq.push({newDist, next});
}
}
return d;
```

4.2 Max Flow

```
#include <bits/stdc++.h>
using namespace std;
int n;
vector<vector<int>> capacity;
vector<vector<int>> adj;
int bfs(int s, int t, vector<int> &parent) {
 fill(parent.begin(), parent.end(), -1);
parent[s] = -2;
  queue<pair<int, int>> q;
  g.push({s, 1e9});
  while (!q.empty()) {
   int cur = q.front().first;
    int flow = q.front().second;
    q.pop();
    for (int next : adj[cur]) {
     if (parent[next] == -1 && capacity[cur][next]) {
        parent[next] = cur;
        int new_flow = min(flow, capacity[cur][next]);
        if (next == t)
         return new flow:
        q.push({next, new_flow});
 return 0;
int maxflow(int s, int t) {
 int flow = 0;
  vector<int> parent(n);
  int new flow;
  while (new_flow = bfs(s, t, parent)) {
    flow += new flow;
    int cur = t;
    while (cur != s) {
     int prev = parent[cur];
      capacity[prev][cur] -= new_flow;
      capacity[cur][prev] += new_flow;
      cur = prev;
  return flow;
```

4.3 Prim

```
#include <bits/stdc++.h>
#ifdef DEBUG
#define PRINT(s) std::cout << s << '\n';
#endif
using namespace std;
using pii = pair<int, int>;
using ull = unsigned long long;
using 11 = long long;
// O(E log V)
// For finding minimum spanning trees
ull prim(vector<vector<pii>>> &adj, vector<bool> &visited, int og,
         int &num_visited) {
  priority_queue<pii, vector<pii>, greater<pii>> pq;
  ull cost = 0;
  // vector<bool> visited(adj.size(), false);
  visited[og] = true;
  int n = adj.size();
```

```
for (auto &[w, v] : adj[og])
  if (!visited[v])
  pq.push({w, v});

while (!pq.empty() && num_visited != n - 1) {
  auto [w, u] = pq.top();
  pq.pop();

if (visited[u])
  continue;

visited[u] = true;
  num_visited++;
  cost += w;

for (auto &[wv, v] : adj[u])
  if (!visited[v])
    pq.push({wv, v});
}

return cost;
```

4.4 Floyd Warshall

```
for (int k = 0; k < n; ++k) {
   for (int i = 0; i < n; ++i) {
      for (int j = 0; j < n; ++j) {
            d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
      }
   }
}</pre>
```

4.5 Kahn

```
Kahn's algorithm
  O(V+E), aka linear
  vector < int > \ topological\_order \ will \ have \ the \ topological \ order \ after \ calling \ kahn ()
  if, after kahn's queue, any indegree is not 0, there is a loop (so kahn returns false)
  if there is no loop, kahn returns true
vector<vector<int>> graph;
vector<int> topological_order;
bool kahn() {
    int n = graph.size();
    vector<int> indegree(n, 0);
    topological_order.clear();
    topological_order.reserve(n);
    for (int i = 0; i < n; i++)
    for (int next : graph[i])</pre>
             indegree[next]++;
    queue<int> q;
for (int i = 0; i < n; i++)
   if (!indegree[i])</pre>
             q.push(i);
        int u = q.front(); q.pop();
        topological_order.push_back(u);
         for (int next : graph[u]) {
              indegree[next]--;
             if (!indegree[next])
                 q.push(next);
     // if any node has indeg > 0, then there is a cycle
    for (int i = 0; i < n; i++)
        if (indegree[i])
             return false; // cyclic
    return true; // acyclic
```

4.6 Kruskal

```
For maximum spanning tree, just turn the edge weights negative.
 Also works for minimum edge product.
struct UnionFind {
   vector<int> parent, size;
   UnionFind(int n) {
       parent.reserve(n);
       size.assign(n, 1);
       for (int i = 0; i < n; i++)
           parent.push_back(i);
   int find(int v) {
   if (v == parent[v])
          return v;
       return parent[v] = find(parent[v]);
    void unionSets(int a, int b) {
       a = find(a);
b = find(b);
       if (a != b)
           if (size[a] < size[b])</pre>
           parent[b] = a; // subordinate b to a (smaller to bigger) size[a] += size[b];
              swap(a, b);
};
struct Edge {
   int u, v, weight;
   bool operator<(Edge &other) const {</pre>
       return weight < other.weight;
};
// ----- MAIN -----
vector<Edge> edges;
// read edges here
sort(edges.begin(), edges.end());
int n = 0; // PUT VERTEX COUNT HERE
UnionFind uf = UnionFind(n);
vector<Edge> mst;
mst.reserve(n-1);
int mstWeight = 0;
for (Edge e : edges) {
   if (uf.find(e.u) != uf.find(e.v)) {
       mst.push_back(e);
       mstWeight += e.weight;
       uf.unionSets(e.u, e.v);
   if (mst.size() == n-1)
```

4.7 Spfa Bellman Ford

```
const int INF = 1e9;
// (vertex, distance)
vector<vector<pair<int, int>>> adj;
bool spfa(int s, vector<int>& d) {
   int n = adj.size();
   d.assign(n, INF);
   vector<int>> cnt(n, 0);
   vector<br/>dool> inqueue(n, false);
   queue<int> q;
   d[s] = 0;
```

5 Linear Sorting

5.1 Radix Sort

```
#include <bits/stdc++.h>
using namespace std;
using 11 = long long;
using ull = unsigned long long;
// If numbers are too large, MAYBE increase base.
// Once I had to use 2^15 to get AC
// Some numbers are using 11. In practice, this is only needed for very large
// bases.
// int base = 32768;
int base = 512; // IDK a good value
int get_digit(int a, int divisor) { return a / divisor % base; }
bool cmp(int a, int b, int divisor) {
  return get_digit(a, divisor) < get_digit(b, divisor);
void counting_sort(vector<int> &vec, vector<int> &output, int divisor) {
 int 1 = INT32\_MAX, r = 0;
  vector<int> aux(vec.begin(), vec.end());
  for (auto &v : aux) {
   v = get_digit(v, divisor);
l = min(l, v);
    r = max(r, v);
  vector < int > f(r - 1 + 1);
  for (auto &v : aux)
  for (int i = 1; i < f.size(); ++i)</pre>
    f[i] = f[i - 1] + f[i];
  for (l1 i = vec.size() - 1; i >= 0; --i) {
   int d = aux[i];
    \operatorname{output}[f[d-1]-1] = \operatorname{vec}[i];
    f[d - 1]--;
void radix_sort(vector<int> &vec) {
  auto [il, ir] = minmax_element(vec.begin(), vec.end());
  int 1 = *i1, r = *ir;
  int num_digits = 1;
  int tmp = r;
  while (tmp >= base) {
    num_digits++;
    tmp = tmp / base;
  vector<int> a(vec.begin(), vec.end()), b(vec.begin(), vec.end());
  auto *output = &a;
```

```
auto *aux = &b;
int divisor = 1;
for (int i = 0; i < num_digits; ++i) {
    swap(aux, output);
    counting_sort(*aux, *output, divisor);
    divisor *= base;
}
for (int i = 0; i < vec.size(); ++i)
    vec[i] = (*output)[i];
}</pre>
```

6 Math

6.1 Eratostenes

6.2 Factorize

```
Returns ascending list of prime factors of "n".
Repetitions allowed.

''

def factorize(n):
   factors = []
   while n % 2 == 0:
        factors.append(2)
        n //= 2
   i = 3
   while n > 1:
        while n % i == 0:
        factors.append(i)
        n //= i
        i += 2
   return factors
```

6.3 Is Prime

```
from math import sqrt

def is_prime(n):
    sq = int(sqrt(n))

if n % 2 == 0:
    return False
    for i in range(3, sq + 1, 2):
        if n % i == 0:
        return False
    return True
```

6.4 Mdc E Mmc

```
def mdc(a, b):
    dividendo = a
    divisor = b
    if b > a:
        dividendo = b
        divisor = a
    while 1:
        resto = dividendo % divisor
    if resto == 0:
```

```
break
dividendo, divisor = divisor, resto
return divisor

#Dont use without defining mdc().
def mmc(a, b):
    return a * b // mdc(a, b)
```

7 String Algorithms

7.1 Longest Palindromic Substring

```
,,,
Manacher's algorithm. There are two implementations of it below.
First implementation:
 - Returns the longest palindromic substring of "s" with smallest starting index.
Second implementation:
 - Returns the length of the longest palindromic substring of "s" (a little bit
  faster because doesn't need to slice the substring out of "s").
All of this in O(n) time complexity.
First time writing this algo. Do not ask me how it works.
#Returns the actual substring.
def longest_palindromic_substring(s):
   str = "#" + "#".join(s) + "#"
   c = 0
   lps = [0] * len(str)
   best_length = 0
   best_idx = 0
   for i in range(1, len(str) - 1):
       lps[i] += 1
if lps[i] > best_length:
           best_length = lps[i]
           best idx = i
       if i + lps[i] > r:
   return s[(best_idx - best_length)//2 : (best_idx + best_length)//2]
#Returns the length of the substring.
def longest_palindromic_substring(s):
   str = "#" + "#".join(s) + "#"
   c = 0
   r = 0
   lps = [0] * len(str)
   best_length = 0
   for i in range(1, len(str) - 1):
       if i < r:
           lps[i] = min(r-i, lps[2*c - i])
       while len(str) - 1 - lps[i] > i and str[i + 1 + lps[i]] == str[i - 1 - lps[i]]:
           lps[i] += 1
       if lps[i] > best_length:
           best_length = lps[i]
       if i + lps[i] > r:
           r = i + lps[i]
   return best length
```

7.2 Kmp

```
vector<int> buildLps(string &s) {
   int n = s.length();
   vector<int> lps(n);
   for (int i = 1; i < n; i++) {
      int j = lps[i-1];
      while (j > 0 && s[i] != s[j])
      j = lps[j-1];
      if (s[i] == s[j])
       j++;
      lps[i] = j;
   }
   return lps;
}
```

```
int main() {
    // every time a pattern matches, the lcs value of that position will be 'n' (the pat size)

string text, pat;
string patThenText = pat + '#' + text; // '#' should not appear on any of the strings
vector<int> lps = buildLps(patThenText);
int n = pat.length();
int m = text.length();
vector<int> indices;
for (int i = n+1; i < n+m+1; i++)
    if (lps[i] == n)
        indices.push_back(i - 2*n);
}</pre>
```

8 Utils

8.1 Binary Search

```
Returns index of target, in ascending iterables.
For descending iterables, swap "<" with ">".
If number doesn't exist, returns -1.
def binary_search(iterable, target, down_idx=0, top_idx=None):
    if top_idx is None:
        top_idx = len(iterable) - 1
    while down_idx <= top_idx:</pre>
        cur = (down_idx + top_idx) // 2
        if iterable[cur] == target:
            return cur
        elif iterable[cur] < target: #swap here</pre>
            down_idx = cur + 1
        else:
            top_idx = cur - 1
    return -1
Returns index of smallest number in iterable bigger than or equal
to target, in ascending iterables.

Swapping ">=" with "<=" returns index of biggest number in iterable
smaller than or equal to target, in descending iterables.
If number doesn't exist, returns -1.
def binary_search(iterable, target, down_idx=0, top_idx=None):
    if top_idx is None:
        top_idx = len(iterable) - 1
    while down_idx <= top_idx:</pre>
        cur = (down_idx + top_idx) // 2
        if iterable[cur] >= target: #swap here
            res = cur
            top\_idx = cur - 1
        else:
            down_idx = cur + 1
    return res
Returns index of smallest number in iterable bigger than or equal
to target, in descending iterables.
Swapping ">=" with "<=" returns index of biggest number in iterable
smaller than or equal to target, in ascending iterables. If number doesn't exist, returns -1.
def binary_search(iterable, target, down_idx=0, top_idx=None):
    if top_idx is None:
       top idx = len(iterable) - 1
    while down_idx <= top_idx:</pre>
        cur = (down_idx + top_idx) // 2
        if iterable[cur] >= target: #swap here
            down_idx = cur + 1
        else:
            top_idx = cur - 1
    return res
```

8.2 Binary Search For Smallest Possible Value

```
using namespace std;
bool valid(ull time, ull goal, vull& machines) {
  ull sum = 0;
  for (auto& m : machines)
   sum += time/m:
  return sum >= goal;
int main() {
  fast_io();
  vull machines:
  while (n--) {
   ull tmp:
   cin >> tmp;
   machines.push_back(tmp);
  ull boundary = t*(*max_element(machines.begin(), machines.end())) + 1;
  DEBUG (boundary);
  for (ull b = boundary/2; b >= 1; b /= 2) {
   DEBUG(valid(k+b, t,machines));
    while (!valid(k+b, t,machines)) k+=b;
  cout << k+1 << '\n';
```

8.3 Fast Io

```
import sys
input = lambda: sys.stdin.readline().removesuffix('\n')
print = lambda s="", end="\n": sys.stdout.write(str(s)+end)
```

8.4 Inversion Counting

```
#include <bits/stdc++.h>
using namespace std;
using pii = pair<int, int>;
using 11 = long long;
using ull = unsigned long long;
struct FenwickTree {
 FenwickTree(int n) { ft.assign(n + 1, 0); }
  FenwickTree(vector(int) &vec) {
    ft.assign(vec.size() + 1, 0);
for (int i = 0; i < vec.size(); ++i)</pre>
      update(i + 1, vec[i]);
  inline int ls_one(int x) { return x & (-x); }
  int query(int r) {
    while (r) {
      sum += ft[r];
      r -= ls_one(r);
    return sum;
  int query(int 1, int r) { return query(r) - query(1 - 1); }
  void update(int i, int v) {
    while (i < ft.size()) {</pre>
      ft[i] += v;
```

```
i += ls_one(i);
  // Finds smallest index i on FenwickTree such that query(1, i) >= rank.
  // I.e: smallest i for [1, i] >= k
  int select(long long k) { // O(log^2 m)
    int lo = 1, hi = ft.size() - 1;
    for (int i = 0; i < 30; ++i) {
     int mid = (lo + hi) / 2;
      (query(1, mid) < k) ? lo = mid : hi = mid;
   return hi;
  vector<int> ft;
};
int main() {
 ios::sync_with_stdio(0);
  cin.tie(0);
  int n:
  while (cin >> n \&\& n) {
    vector<int> seq(n);
    for (auto &v : seg)
     cin >> v:
   FenwickTree ft(n);
    int inv = 0;
    for (int i = 0; i < n; ++i) {
      inv += ft.query(seq[i] + 1, n);
      ft.update(seq[i], 1);
    cout << (inv % 2 == 0 ? "Carlos\n" : "Marcelo\n");</pre>
```

8.5 Inversion Counting

```
Conta inversões presentes em um arrav
3 e 2 representam uma inversão (2 está a frente de 3, mas é menor que ele)
from fenwick_tree import FenwickTree
def normalize(iteratable) -> dict[any, int]:
    Cria um dicionário mapeando cada elemento do array para um número no intervalo [1, len(iteratable)
    ....
    sorted_iteratable = sorted(iteratable)
    num = 1
    for val in sorted_iteratable:
       if val not in mp:
           mp[val] = num
           num += 1
    return mp
def inversion_count(iteratable) -> int:
    conta a quantidade total de inversões encontradas no array.
    A fenwick tree conta a frequência de elementos encontrados no array até o momento, permitindo
    a realização de queries para saber quantos valores já apareceram em um determinado intervalo de nú
    por exemplo:
        suponha que um loop itere sobre os valores 5 4 3 2 1.
        na terceira iteração do for loop (quando o valor for 3),
        a árvore indicará que foram encontrados dois valores no intervalo [3:5]
    isso permite encontrar inversões de forma rápida, uma vez que tudo que precisamos fazer para
          encontrar todas
```

```
as inversões de um número n é descobrir quantos números maiores que ele aparecem antes dele.
i.e: bast fazer uma query a fenwick tree no intervalo [n:len(iteratable)].

ft = FenwickTree(len(iteratable))
mp = normalize(iteratable)
inv = 0
for val in iteratable:
    inv += ft.query(mp[val], len(iteratable))
    ft.update(mp[val], 1)
return inv
```

8.6 Random Number Generation

```
#include <random>
using namespace std;
const int L = 1;
const int R = 1e9;
```

```
default_random_engine gen;
uniform_int_distribution<int> distribution(L, R);
int num() { return distribution(gen); }
```

8.7 Random

```
const long long minRand = 1;
const long long maxRand = 100;

default_random_engine generator;
uniform_int_distribution<long long> distribution(minRand, maxRand);
long long someRand = distribution(generator);
```